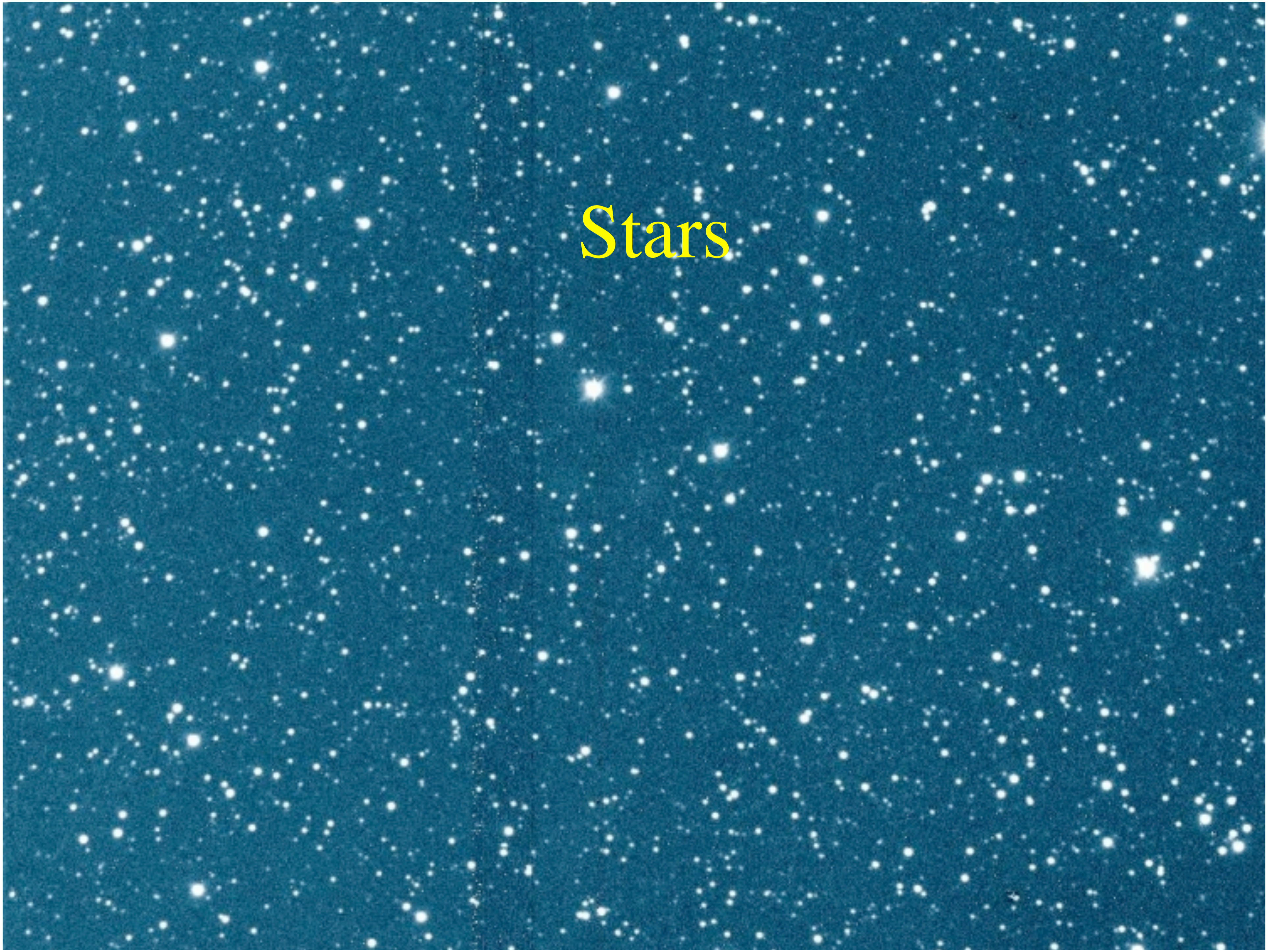
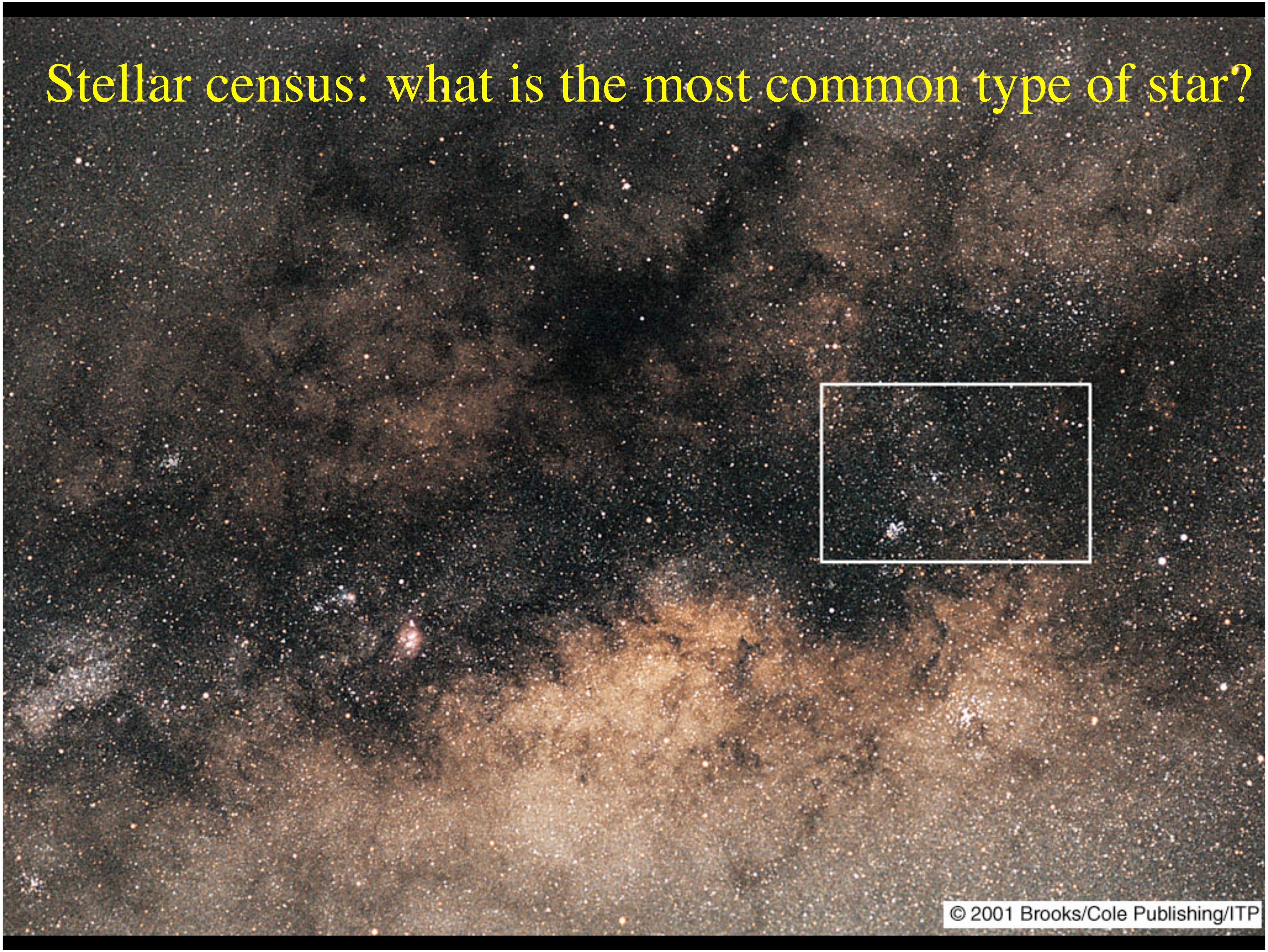
Stars



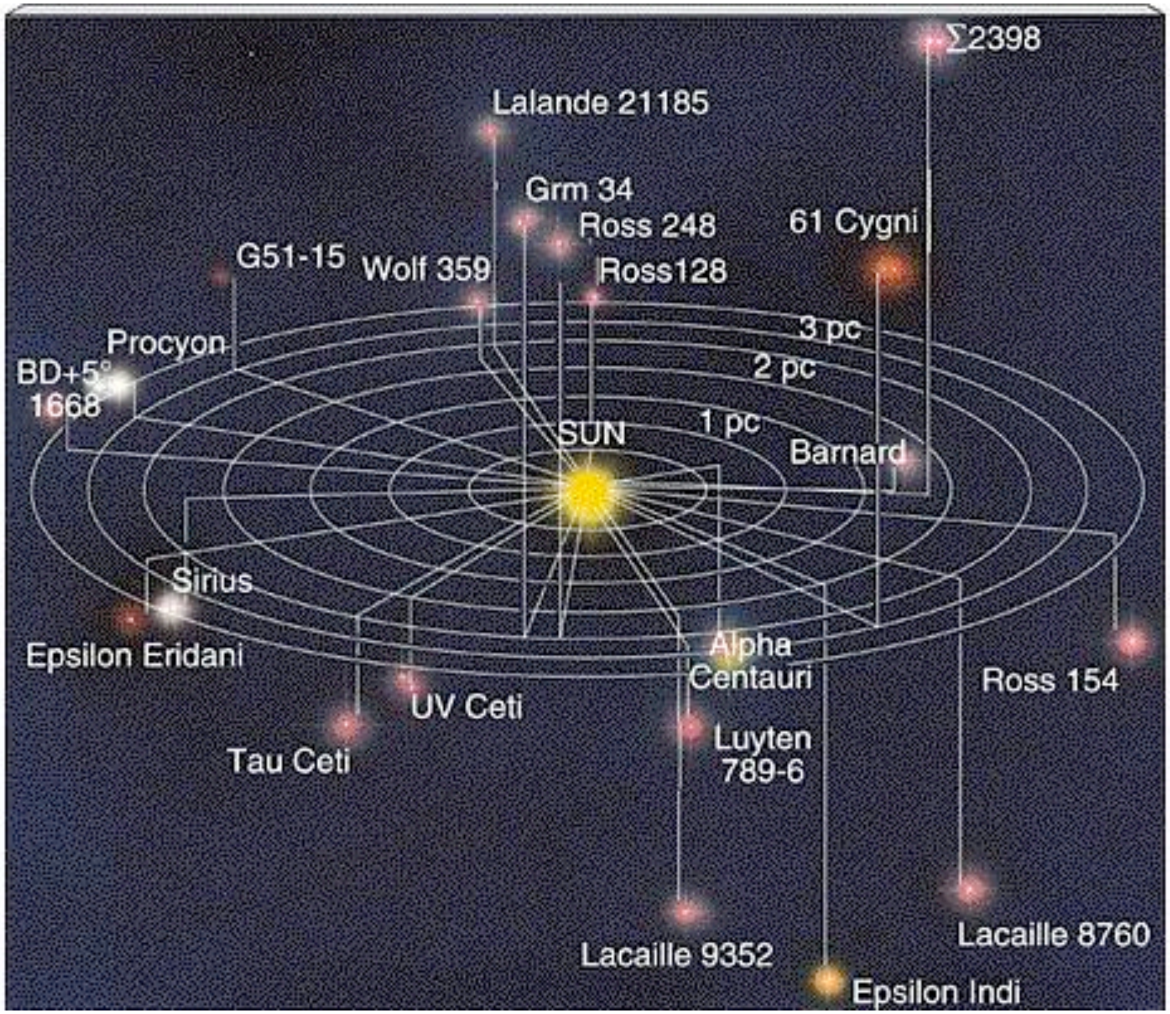
Stellar census: what is the most common type of star?



Stellar Census

- Most common stars:
  - ✶ The *lowest mass* main-sequence stars
  - ✶ *White dwarfs*: hot, but small radius (very faint)
- Nearly all stars are low-mass main-sequence stars or white dwarfs, but are so faint a large telescope is needed to see them.

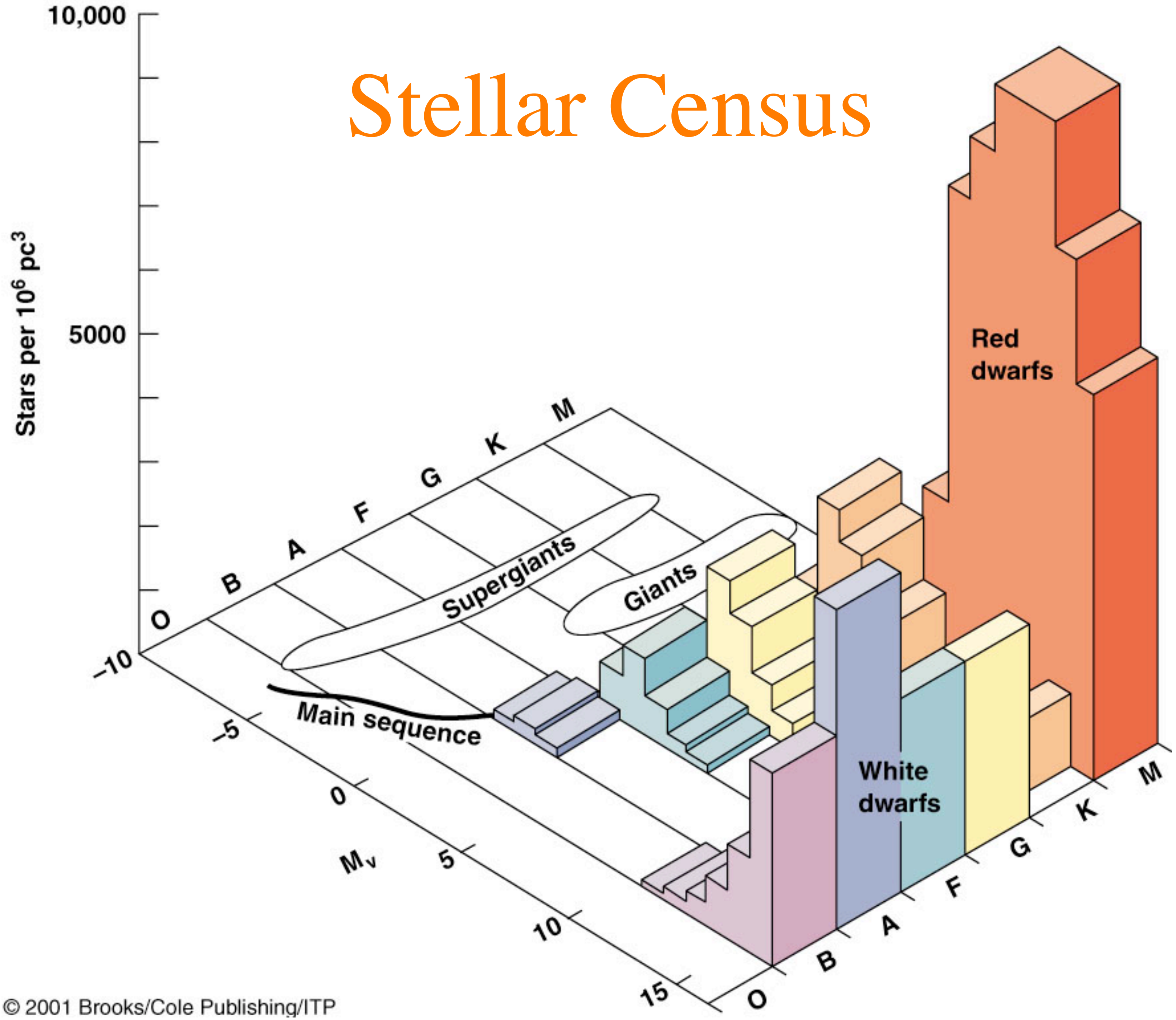
Nearest stars are almost all low mass stars



Stellar Census

- Luminous stars are very rare:
  - ✶High-mass main-sequence stars (O stars) are fewer than 1 in a million
  - ✶Supergiants are similarly rare
- Since they are rare, most are very far away.
- Most stars seen with naked eye are intrinsically luminous = *Malmquist bias*

Stellar Census

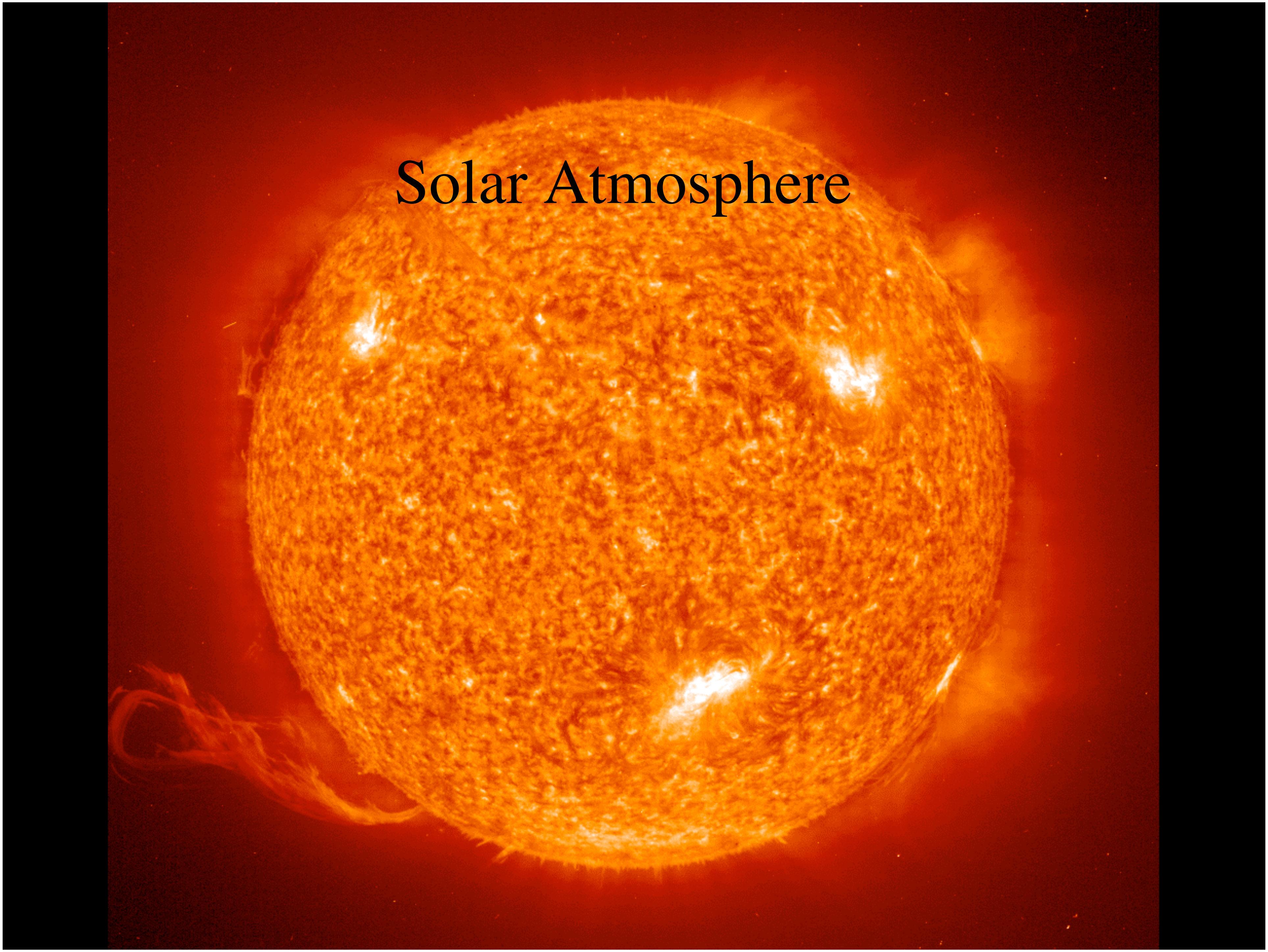


Stellar Physics and Interiors

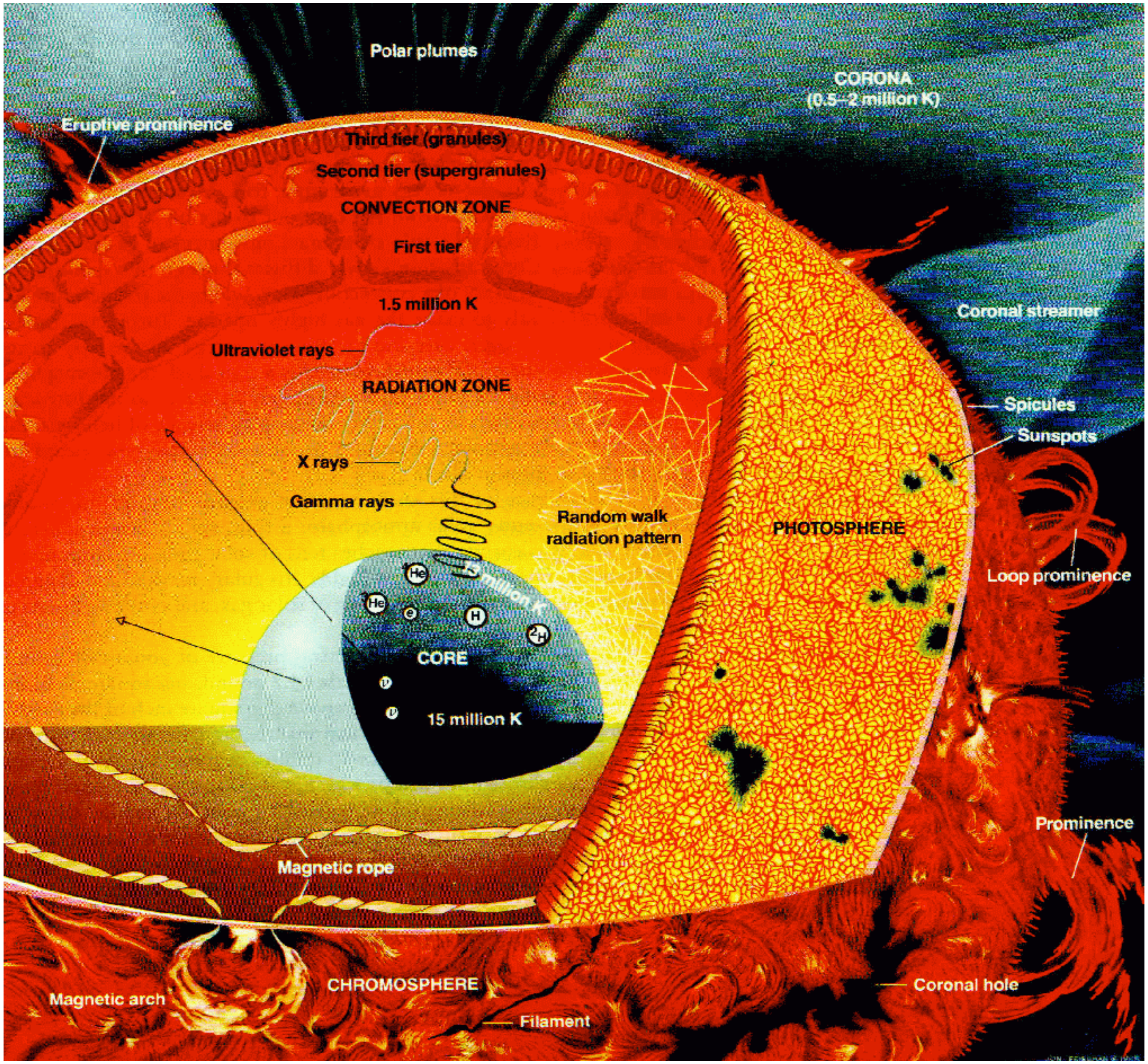
Solar Interior

- Sources of energy for Sun
- Nuclear fusion
- Solar neutrino problem
- Helioseismology

Solar Atmosphere



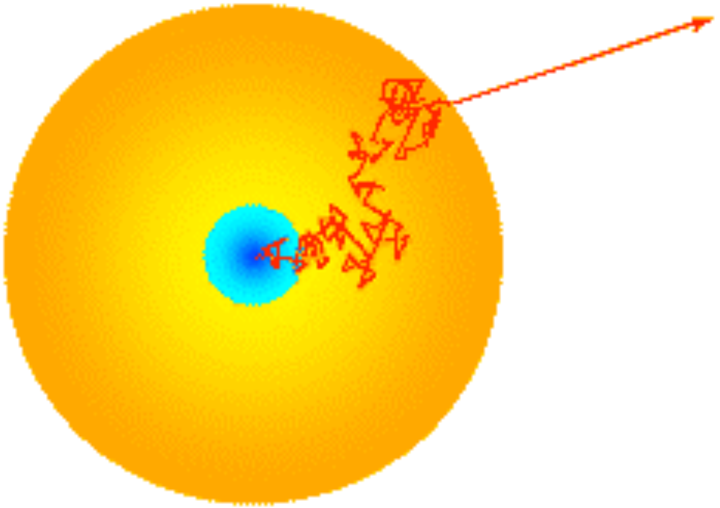
Solar interior



Solar facts

- **Luminosity**: 3.8×10^{26} J/s
- **Mass**: 2.0×10^{30} kg
- **Composition**: 73% Hydrogen, 25% Helium, 2% "heavy elements" (by mass)
- **Radius**: 7.0×10^{8}m
- **Avg Density**: 1400 kg/m^3
- $\mathbf{T_{eff}} = (L/(\sigma 4\pi R^2))^{1/4} = 5800^{o}K$ (How does this compare with the average and central temperatures?)

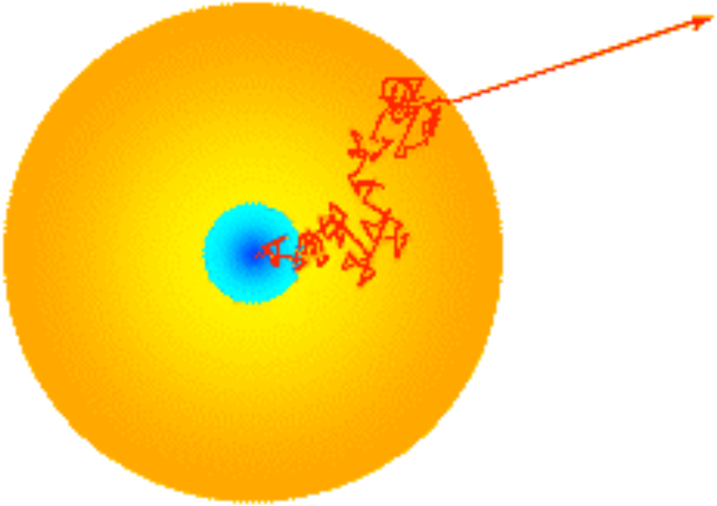
Solar interior



- At avg density of 1400 kg/m^3 and avg temp of 4.5x10^{6}K "mean free path" of photon before interacting with matter is < 1 cm. Optical depth is very high, effective path length is much longer than $R_\odot$; time to escape is much longer than $R_\odot/c \sim 2$ sec

Solar interior

- Timescale for radiation to diffuse out is > few 10^4 years
- Slow leakage of photons regulates $L_\odot$



Solar Energy

- Sun is radiating copious amounts of energy
- What is the source of this radiation?
- What if we just consider the fact that the sun is a ball of hot gas, no additional heat source?
- According the ideal gas law, the thermal energy of a gas at temperature T is:

$$E=3/2(NkT)$$

(N=# of particles, k=Boltzmann's constant)

Solar Energy

- (see derivation on board about K-H timescale)
- Heating from gravitational contraction can only sustain sun for ~10^7 years (K-H timescale)
- Yet we know that the age of the solar system is ~4.5 billion years

Radioactive dating

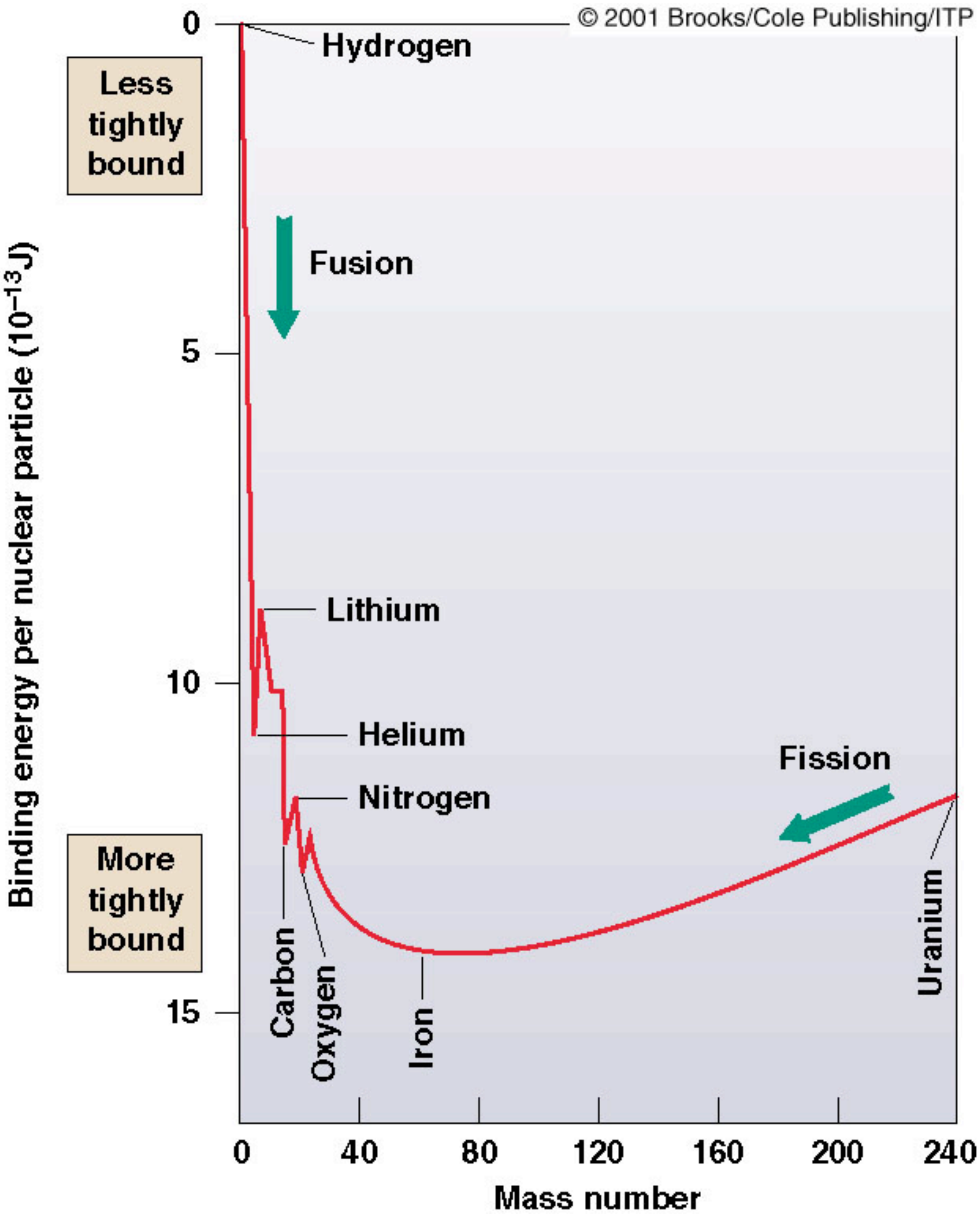
- Oldest rocks found so far:
  - ✶Earth: 3.9 billion years
  - ✶Moon: 4.5 billion years
  - ✶Mars: 4.5 billion years
  - ✶Meteorites: 4.6 billion years
- The *smaller* an object, the *faster* it cools and therefore solidifies, the *older* it is
  - ➢Planets, moon, meteorites (entire solar system) formed 4.6 billion years ago (Sun too)

Solar Energy

- (see derivation on board about K-H timescale)
- Heating from gravitational contraction can only sustain sun for ~10^7 years (K-H timescale)
- Yet we know that the age of the solar system is ~4.5 billion years
- We need another source of energy: nuclear fusion!

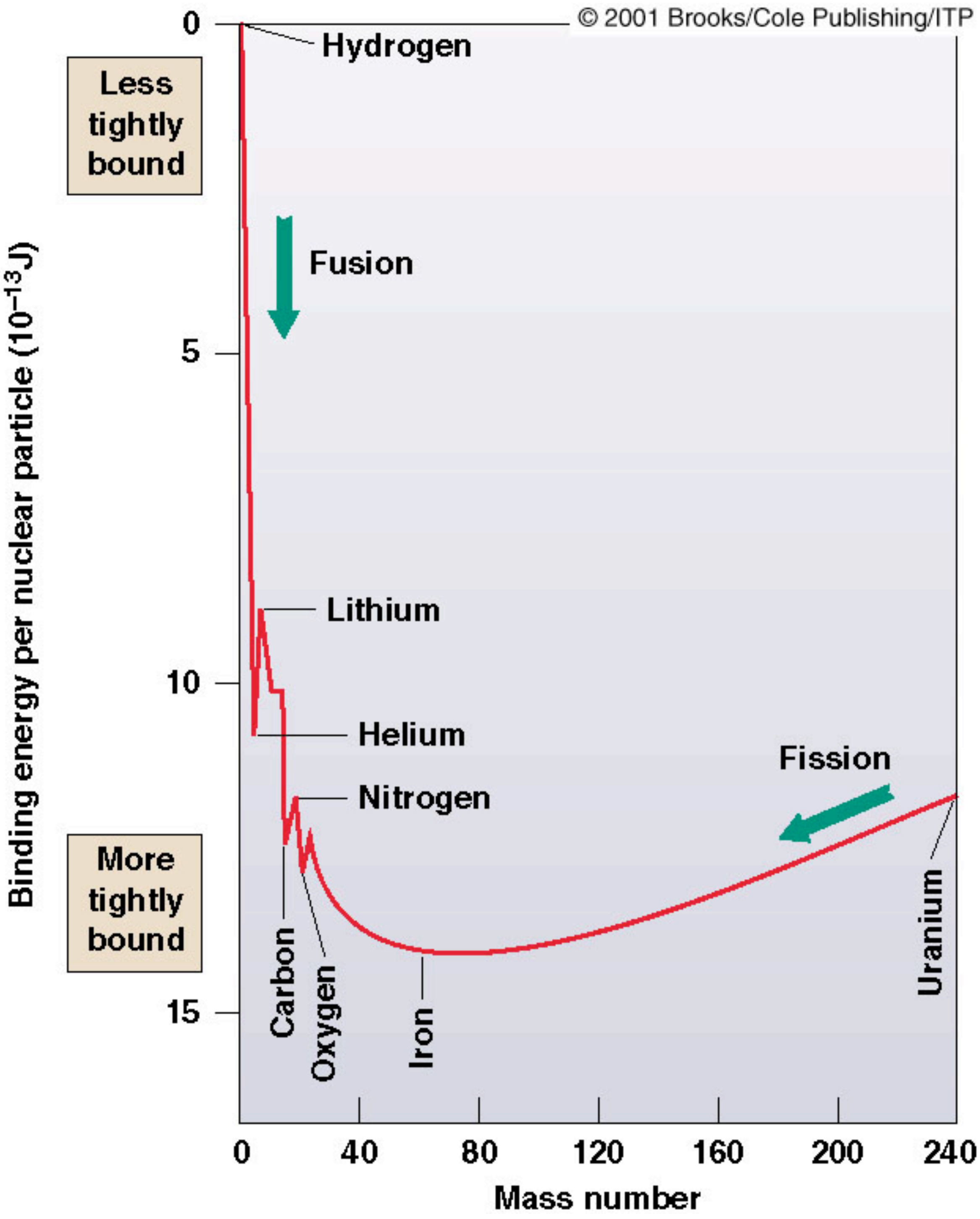
Fusion & Fission

- ***Fusion*** (joining) of ***light*** elements results in more tightly bound elements
    - ✶Releases energy up to Fe



Fusion & Fission

- ***Fission*** (splitting) of ***heavy*** elements results in more tightly bound element
  - ✶ Releases energy above Fe

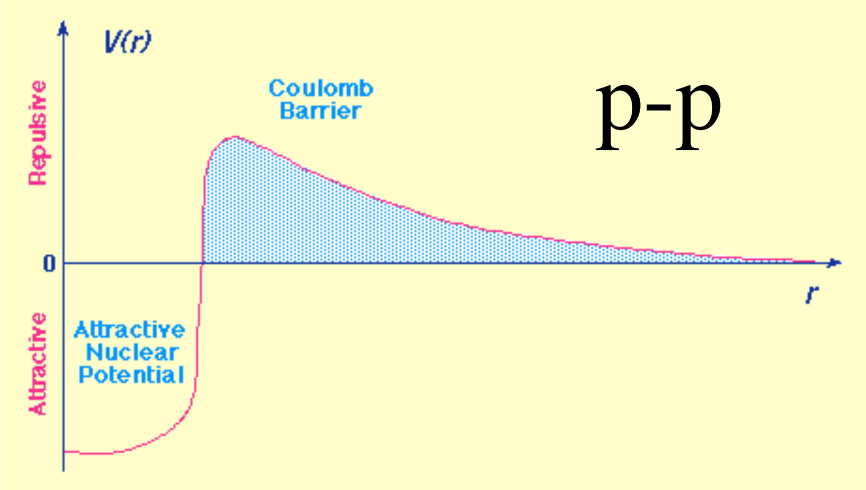


How does fusion release energy?

- So 0.7% of mass of H in Sun is converted into energy.
- Total energy available $E_{nuc} = 0.007 M_{Sun} c^2$ (compare with $E_{therm} = (3/2) NkT$ or $E_{grav} = (3/10) GM_{sun}^2 / R_{sun}$)
- Nuclear lifetime $t = E_{nuc}/L_{sun} = 10^{11}$ yrs
- Actually, we'll see drastic things happen to Sun once H in core exhausted, which is only 10% of total
- So Sun "lives" for about 10^{10} yrs.

How does fusion happen?

- In order for reaction to occur, colliding nuclei must have enough KE to overcome Coulomb repulsion of like charges

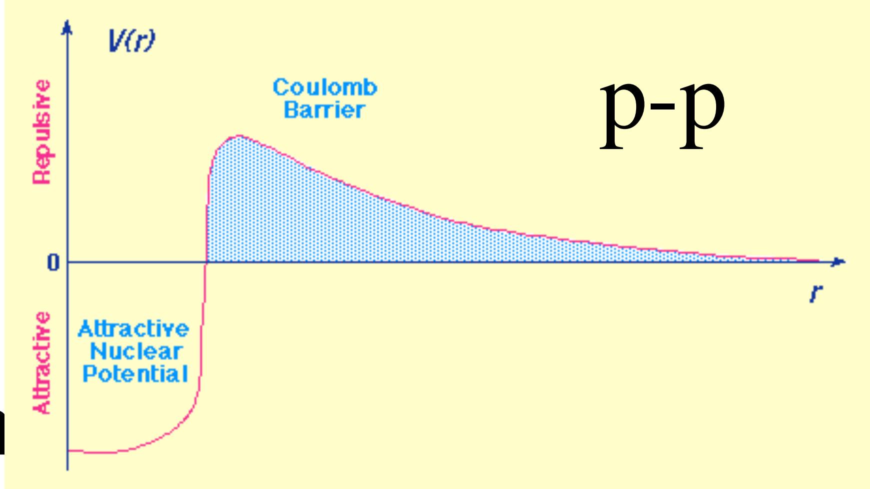


- First, think classically: energy required to overcome barrier is provided by gas thermal energy…

How does fusion happen?

- Including effects of QM tunneling indicates that fusion can happen in the center of the sun, with T~10^7 K

- Next, calculate reaction and energy generation rates, which are strong functions of temperature.



Proton-proton chain

Most important reaction in Sun is PPI chain

$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu$

e^{+} = positron

ν = neutrino

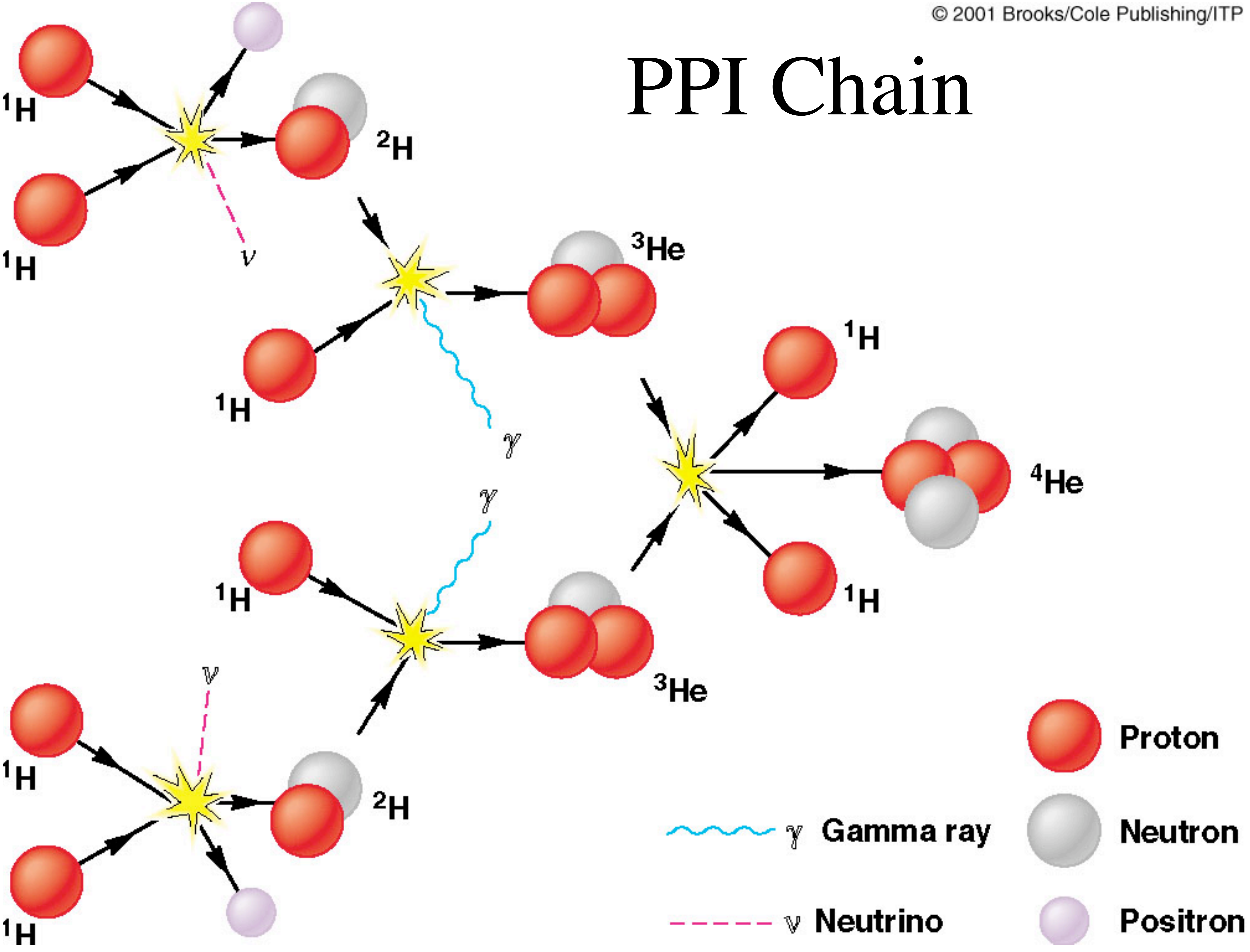
γ = photon

$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma$

$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H$

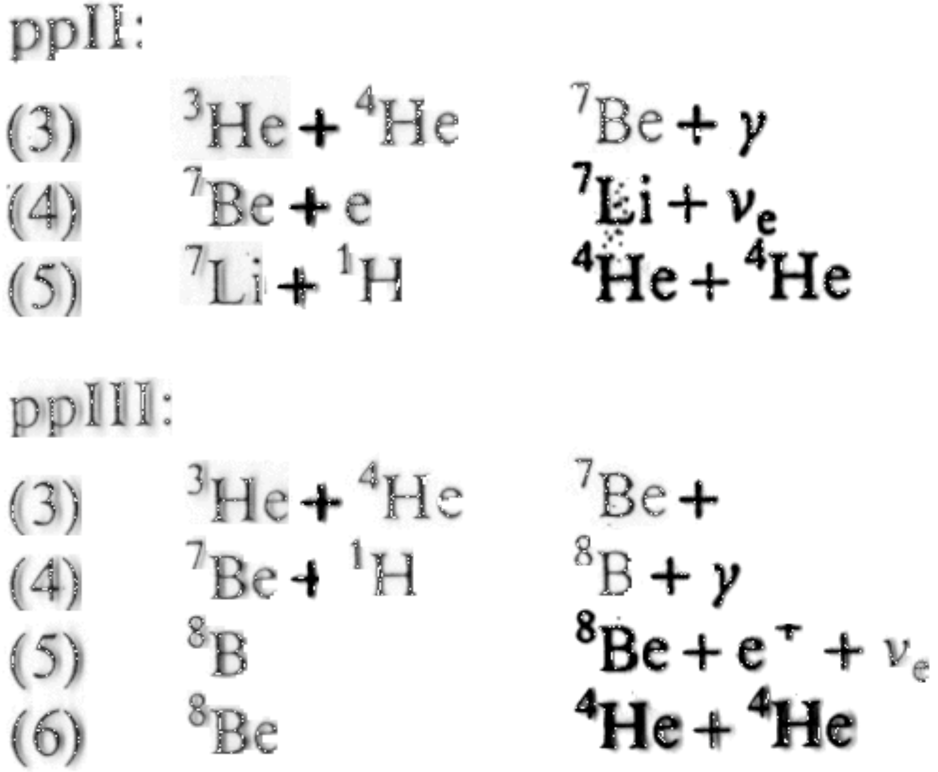
Net result is 4H fused into ^{4}He

PPI Chain


^{1}H
^{1}H
ν
^{2}H
^{1}H
γ
^{3}He
^{1}H
^{4}He
^{1}H
γ
^{1}H
^{3}He
ν
^{1}H
^{2}H
^{1}H
γ Gamma ray
ν Neutrino
Proton
Neutron
Positron

PPII & PPIII

- After first 2 steps, 31% of reactions proceed with $^3He + ^4He \rightarrow ^7Be + \gamma$, and further branch between:



ppII:

(3) $^3He + ^4He$ $^7Be + \gamma$
(4) $^7Be + e$ $^7Li + \nu_e$
(5) $^7Li + ^1H$ $^4He + ^4He$

ppIII:

(3) $^3He + ^4He$ $^7Be +$
(4) $^7Be + ^1H$ $^8B + \gamma$
(5) ^8B $^8Be + e^+ + \nu_e$
(6) ^8Be $^4He + ^4He$

- 69% of the time, H fusion occurs via PPI chain in Sun

- 31% of the time, PPII chain occurs

$^{1}H + {}^{1}H \;\; -> \;\; {}^{2}H + e^{+} + \nu$

$^{2}H + {}^{1}H \; –> \; {}^{3}He + \gamma$

$^{3}He + {}^{4}He \; -> \; {}^{7}Be + \gamma$

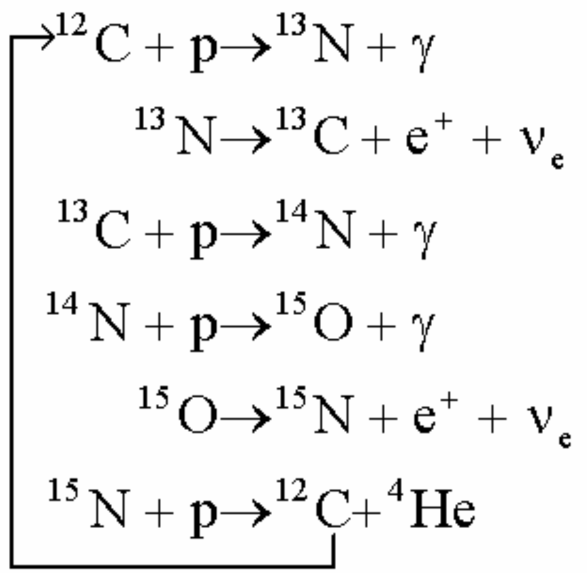
$^{7}Be + e^{-} \; -> {}^{7}Li + \nu$

$^{7}Li + {}^{1}H \; -> 2\; {}^{4}He$

- Other chains (CNO cycle) are negligible, but important in other stars

CNO Cycle

- H can also be converted to ^{4}He through the CNO cycle
- Carbon, Nitrogen, Oxygen used as catalysts
- Much more T-dependent than P-P chain, CNO cycle occurs in stars slightly more massive than the Sun



$$^{12}C + p \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_{e}$$

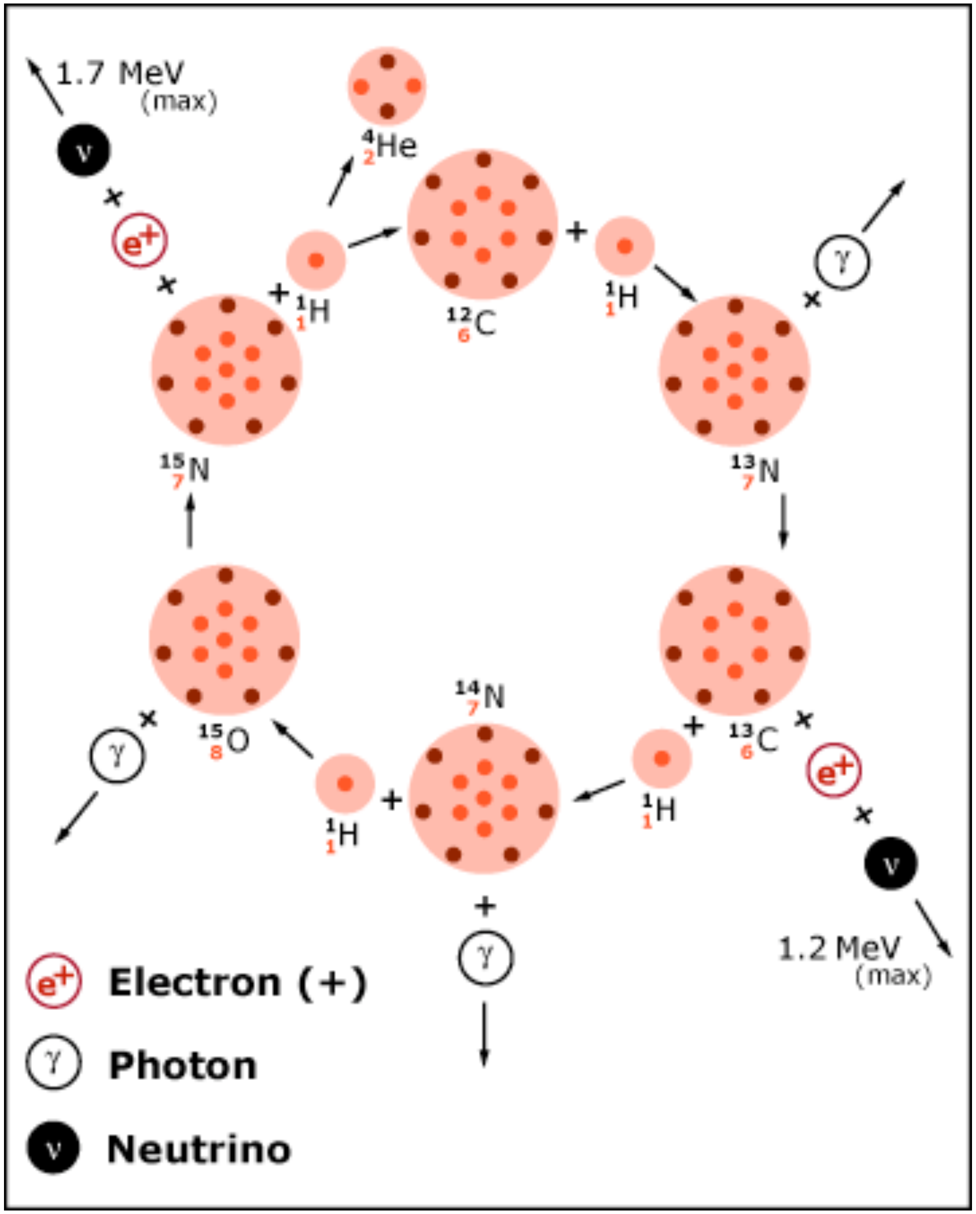
$$^{13}C + p \rightarrow {}^{14}N + \gamma$$

$$^{14}N + p \rightarrow {}^{15}O + \gamma$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu_{e}$$

$$^{15}N + p \rightarrow {}^{12}C + {}^{4}He$$

CNO cycle: stars with $M > 1.2M_{Sun}$



Uses C, N, and O nuclei to catalyze fusion of H into He

Solar neutrino problem

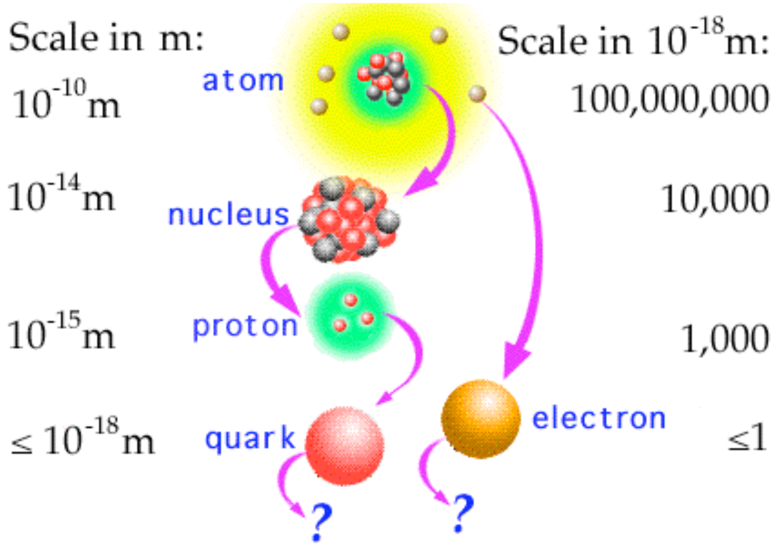
- 4 fundamental forces: gravitational, electromagnetic, strong, weak

- Elementary particles

Leptons (electrons, muons, taus, 3 neutrinos) participate in weak and EM interactions

Quarks, 3 at a time make up protons and neutrons (baryons), participate in strong, weak, EM interactions

Both leptons and quarks are acted on by gravity



Solar neutrino problem

- Neutrinos: zero charge, non-zero mass, each type ν_e, ν_μ, ν_τ, is associated with a charged particle (electron, muon, tau)

- Rarely interact with matter: 10^{11} neutrinos pass through your thumb every second. For every 10^{11} neutrinos from the sun that pass through the earth, only one interacts with terrestrial material

Solar neutrino problem

- Spectacular confrontation of solar and theoretical particle physics

- Background: the P-P chain predicts the production of *neutrinos* along the way to forming ^{4}He, e.g.:

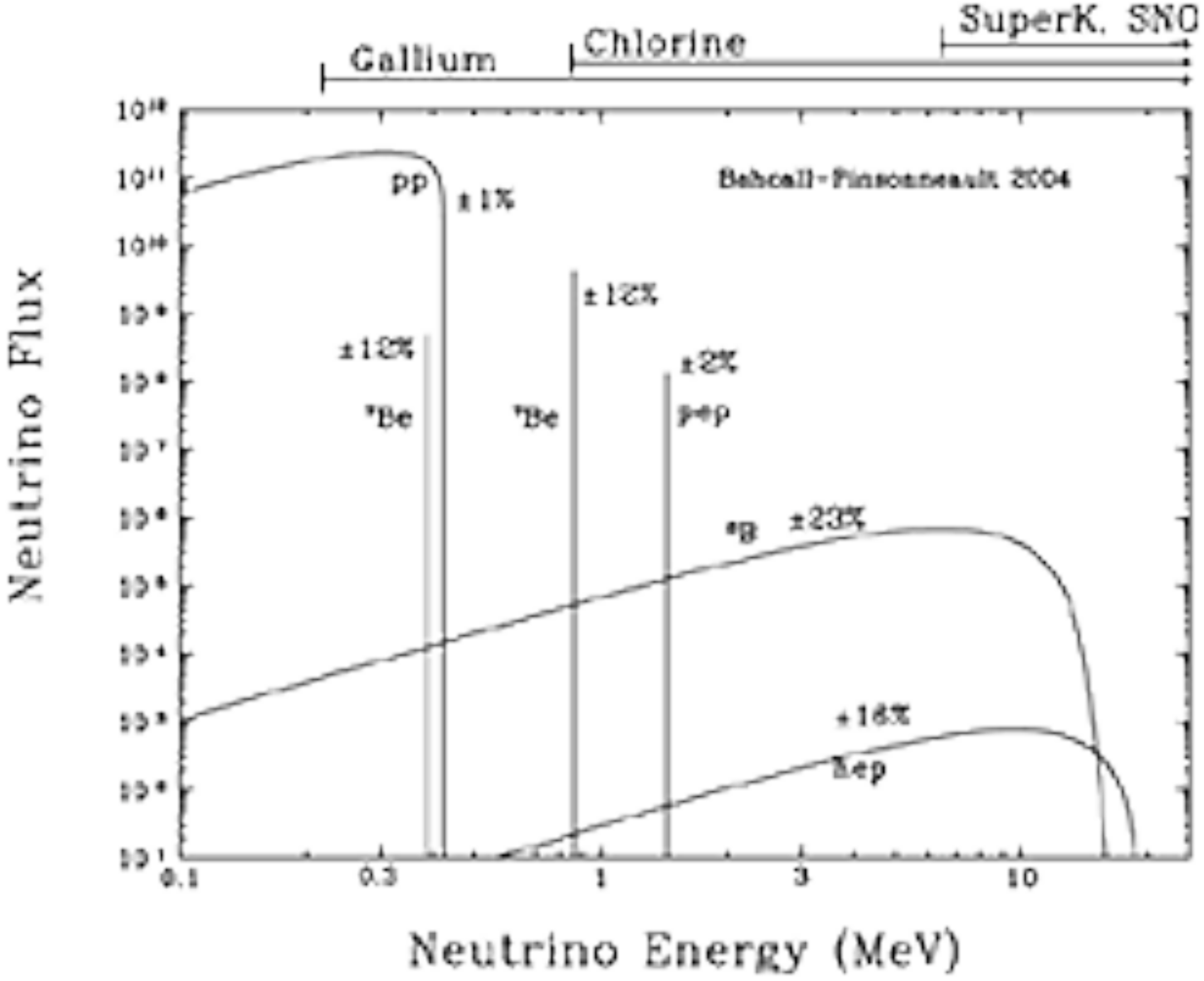
$$^8B \rightarrow {}^8Be + e^+ + \nu_e$$

Ultimately results in:

$4\ {}^1H \rightarrow {}^4He + 2e^+ + 2\ \nu_e + 2\gamma$

(e^+ positrons, γ photons, ν_e neutrinos)

Energy spectrum of neutrinos produced in different reactions



In 1970, in the underground Homestake Gold Mine in SD, physicists started measuring solar neutrino flux produced by p-p chain

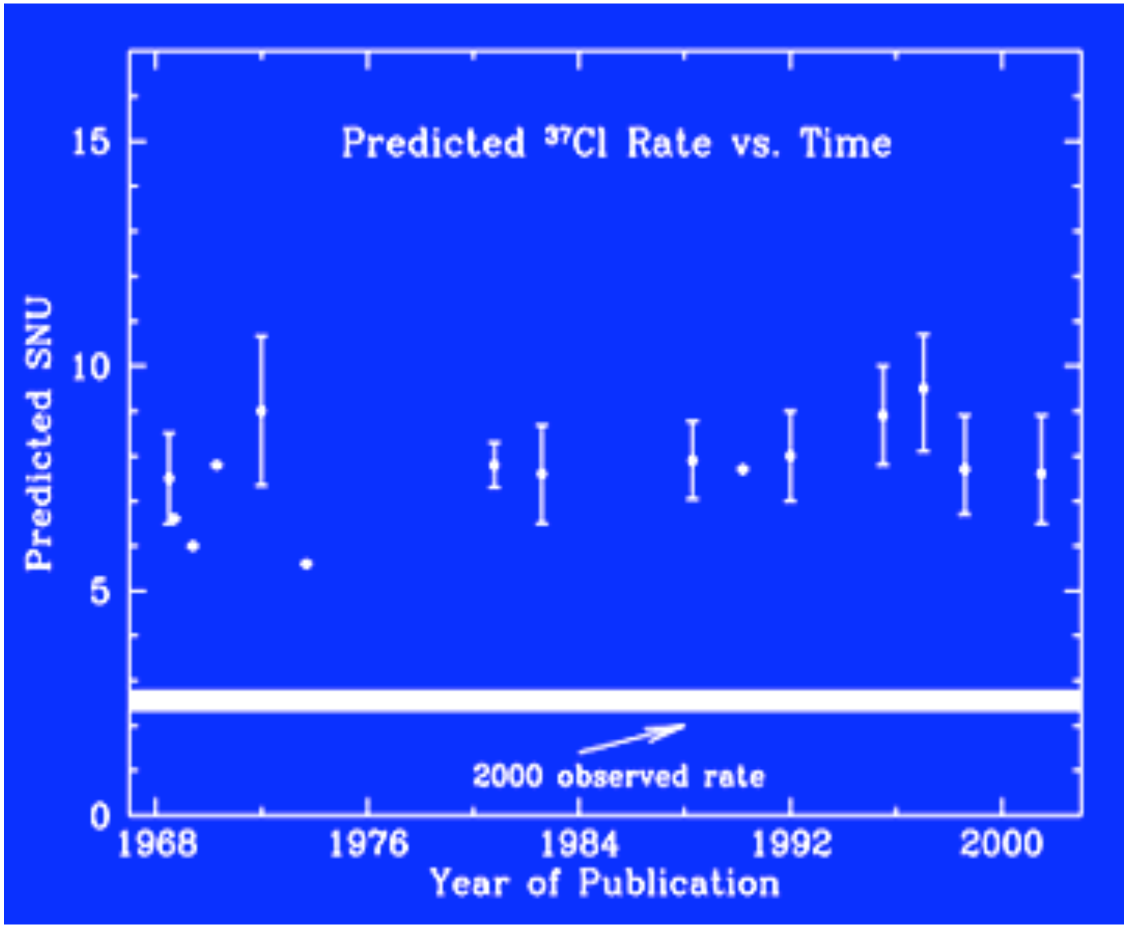
- Underground tank of 615,000 kg of C_2Cl_4
- Detects ^{37}Ar produced by reaction between ^{37}Cl and ν
- Sensitive to production of few Ar atoms per month

Solar neutrino results

- Measured rate was ~1/3 predicted rate from standard solar model = "**solar neutrino problem**"

- Physicists blamed this on Solar models

- But experiments measured only electron neutrinos …



Solar neutrino solutions?

3 possibilities:

1. Predicted # of ν from the solar interaction, or predicted # of argon incorrect
2. Experimental data wrong
3. Theory of neutrinos incorrect: in terms of behavior while traversing large distances

Solar neutrino results

- For 30 years, measured rate was ~1/3 predicted rate from standard solar model = “**solar neutrino problem**”

- 3 possibilities:

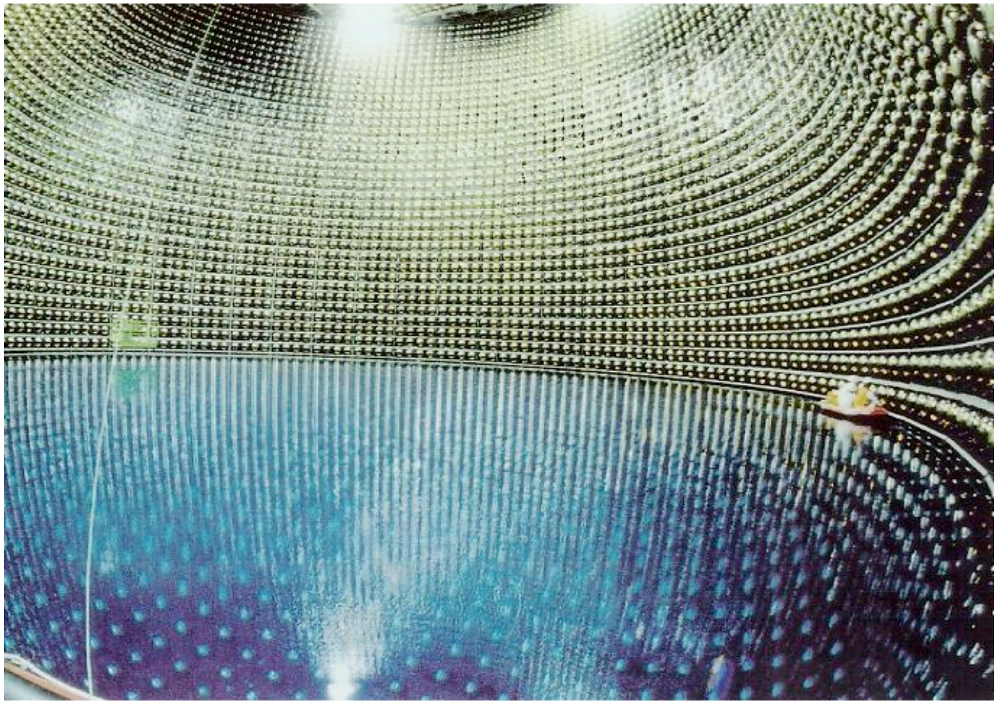
1. ~~Predicted # of ν from the solar interaction, or predicted # of argon incorrect~~
2. ~~Experimental data wrong~~
3. Theory of neutrinos incorrect: in terms of behavior while traversing large distances

Solar neutrino results

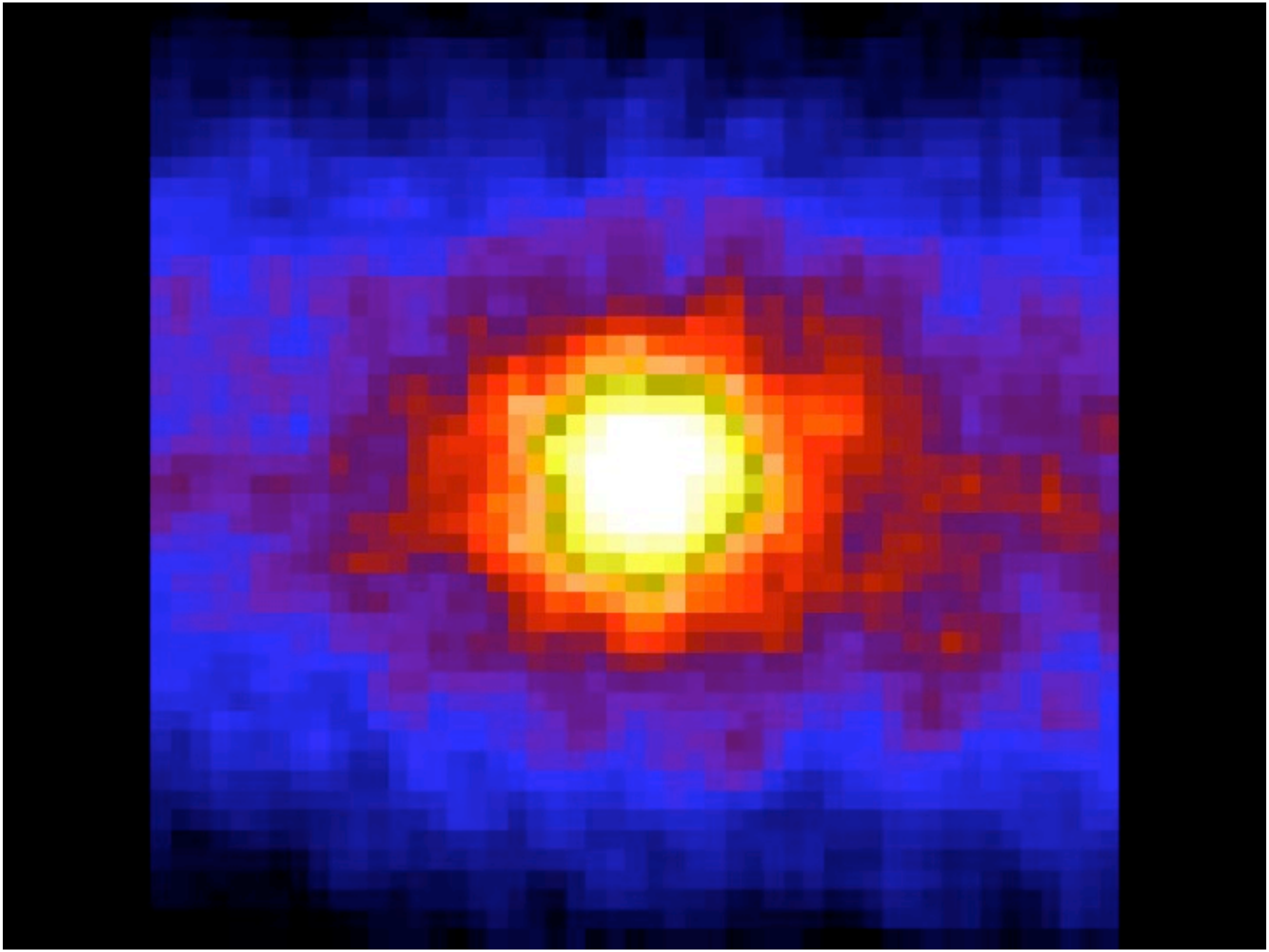
- The problem was not with the solar model, but our understanding of particle physics!
- Solution: while traveling from the sun at close to the speed of light, electron neutrinos change "flavor" to muon or tau neutrinos. Detector is only sensitive to electron neutrinos, hence the lower detection rate than expected
- Verified this model with recent observations of (Superkamiokande, Sudbury Neutrino observatory)

Solar neutrino observatories

- Sudbury Neutrino Obs.

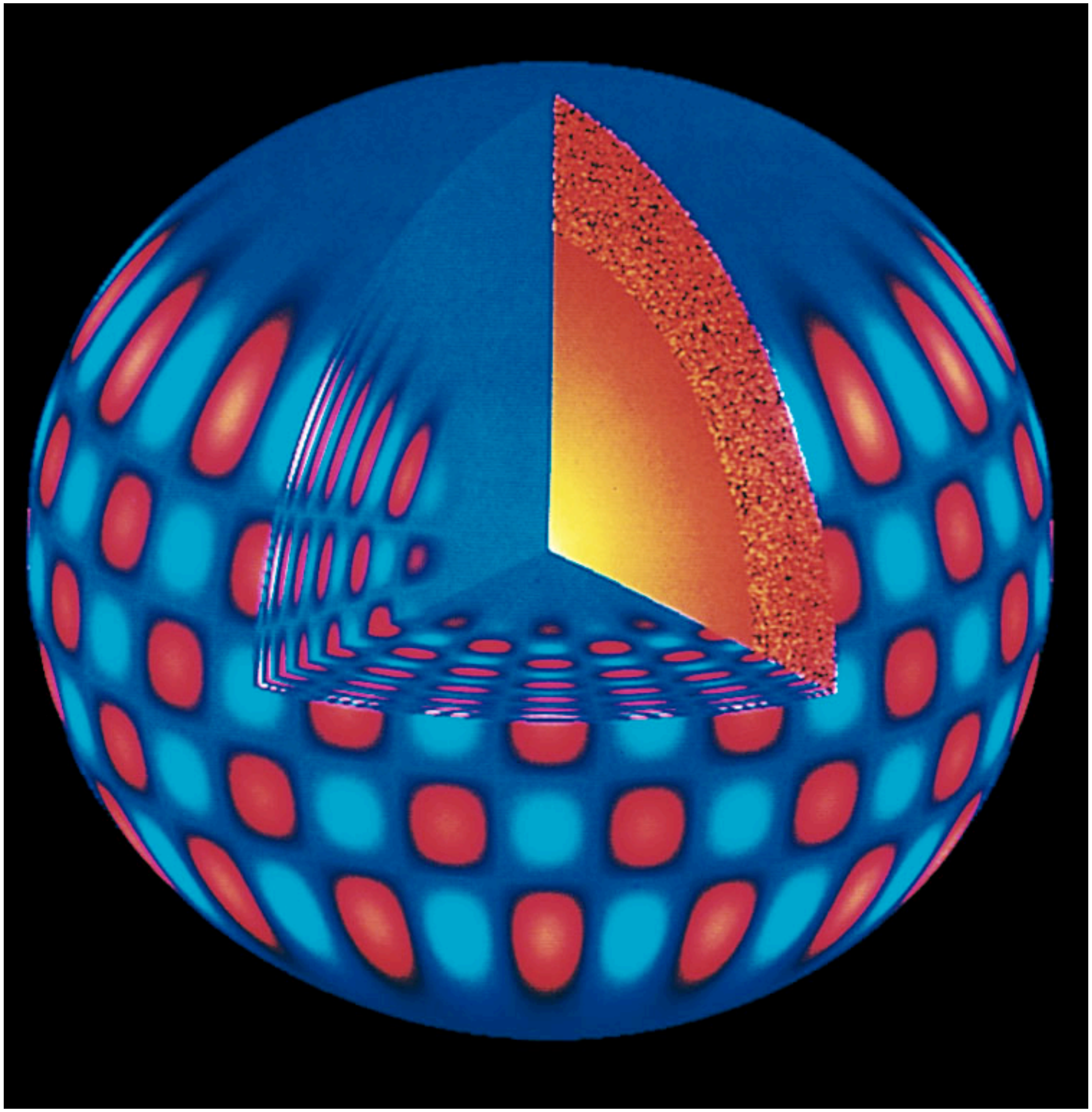


- Super-Kamiokande

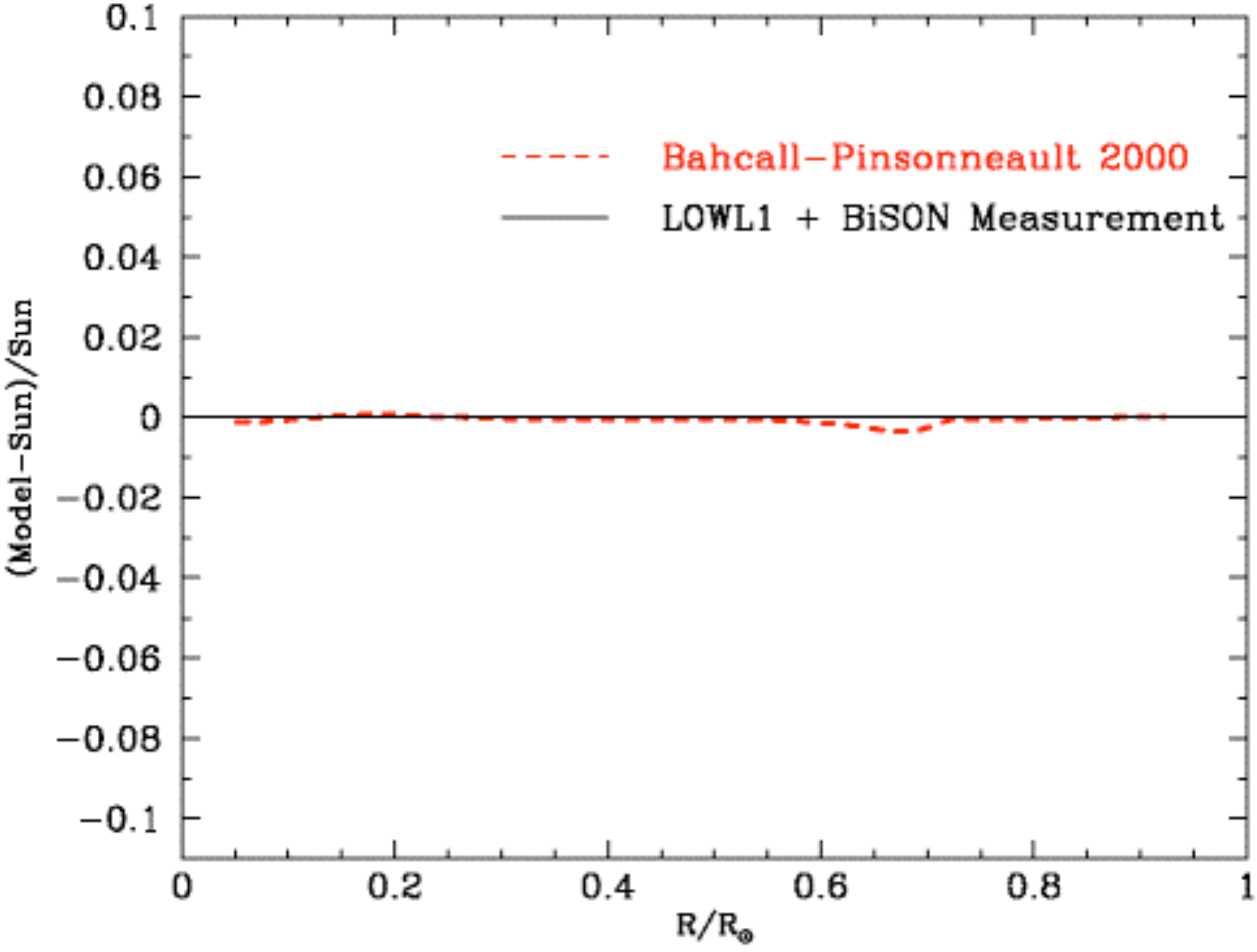


Helioseismology

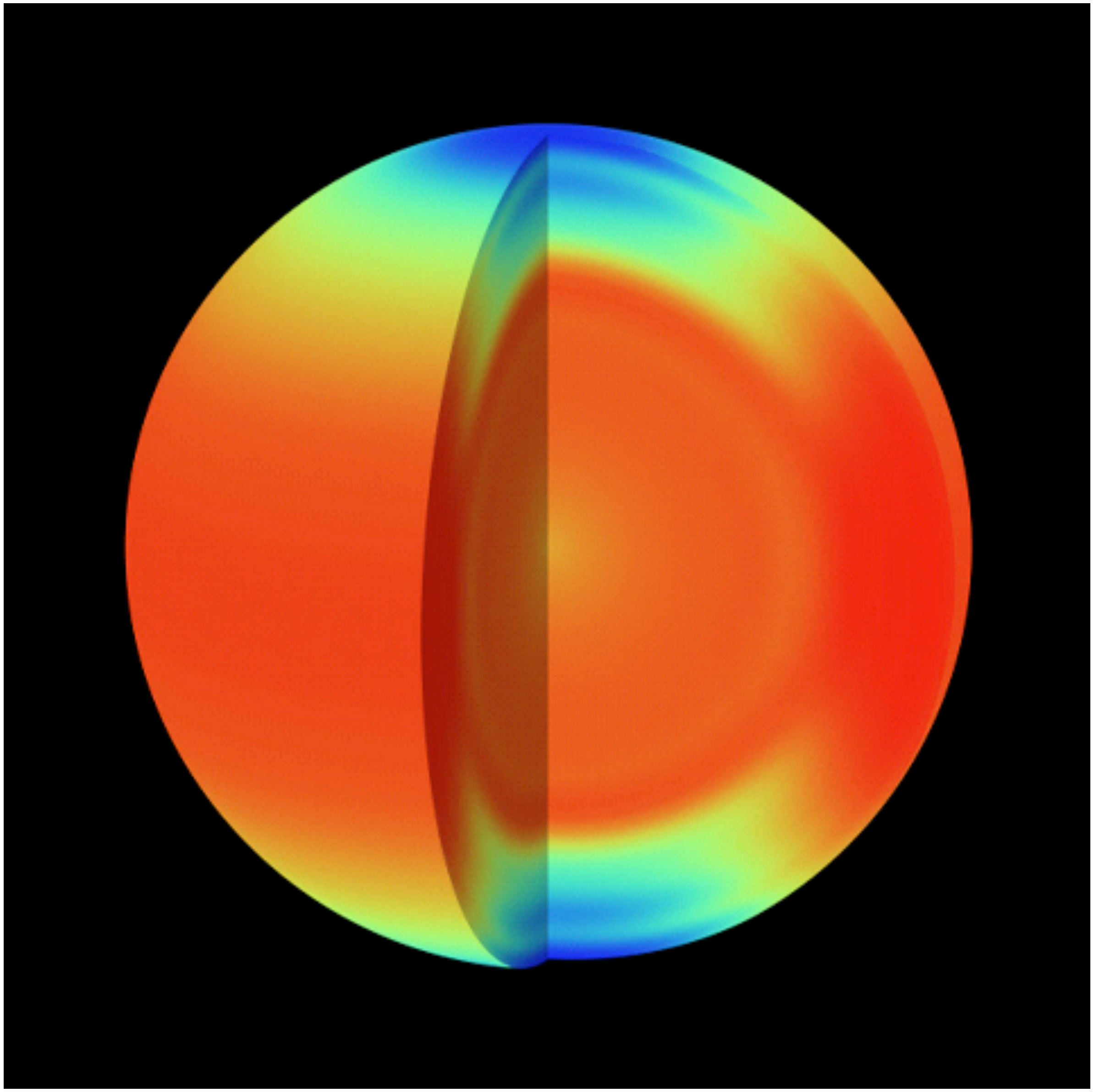
- Vibrations of solar atmosphere measured by Doppler shifts
- Pattern of observed frequencies tells us about sun’s interior (e.g. sound speed)



Measured vs. predicted sound speed in Sun



Internal rotation rate of Sun measured through helioseismology



Convection and radiative zones rotate at different rates

Perhaps leads to generation of magnetic field

Red = fast rot.

Blue = slow rot.

Stars come in all different masses and sizes…

But fundamentally, they all have the same structure – they are just giant balls of gas, mostly H and He.

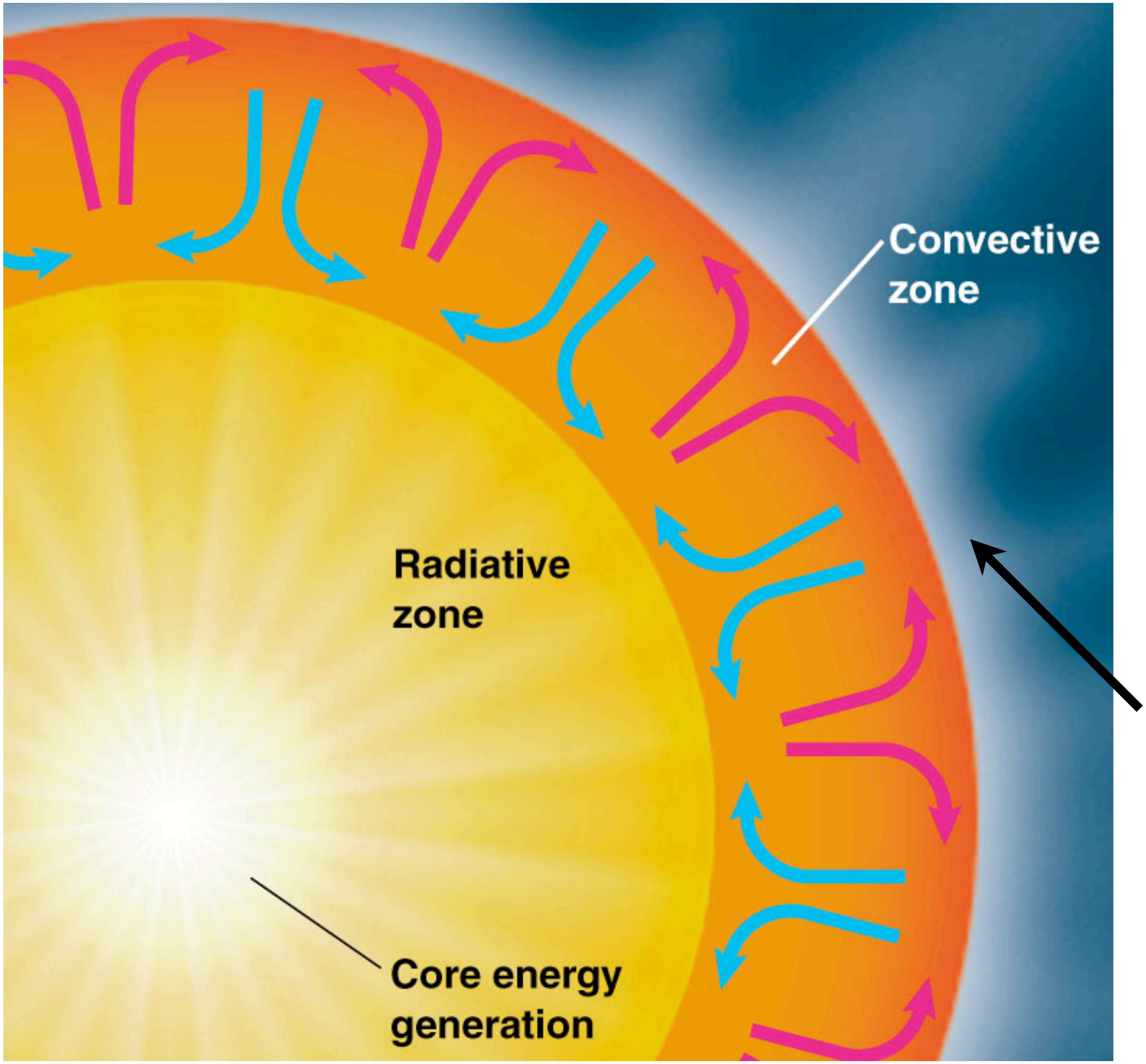
Stars of different masses differ primarily in

- Internal energy transport mechanism
- Primary H fusion network
  - PP chain in low mass stars
  - CNO cycle in high mass stars

Energy Transport in stars

- **Radiation**: energy is carried towards surface via photons
- **Convection**: energy transported via hot buoyant mass elements rising outwards while cool elements fall inwards
- **Conduction**: energy transported via collisions through particles

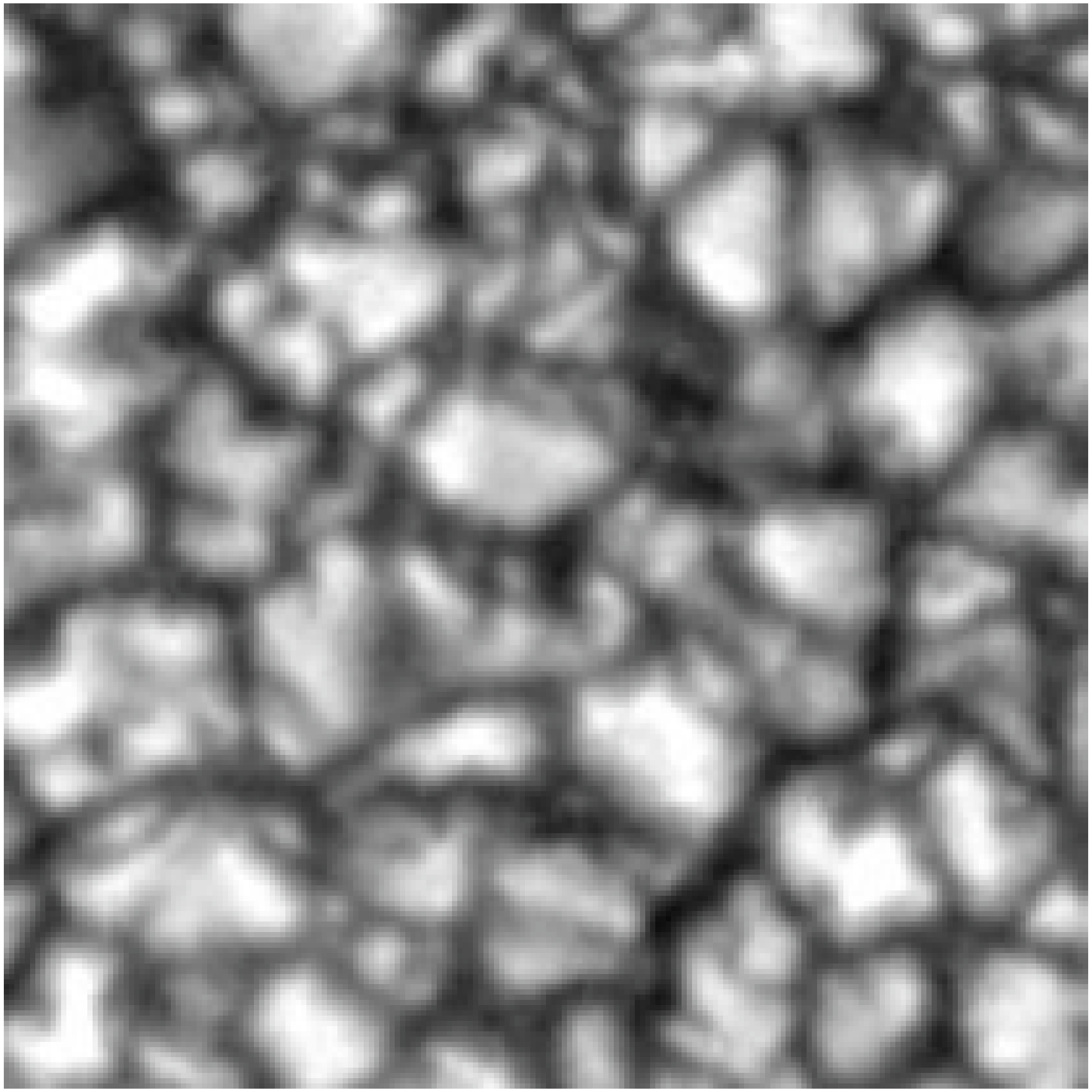
Energy Transport in Sun



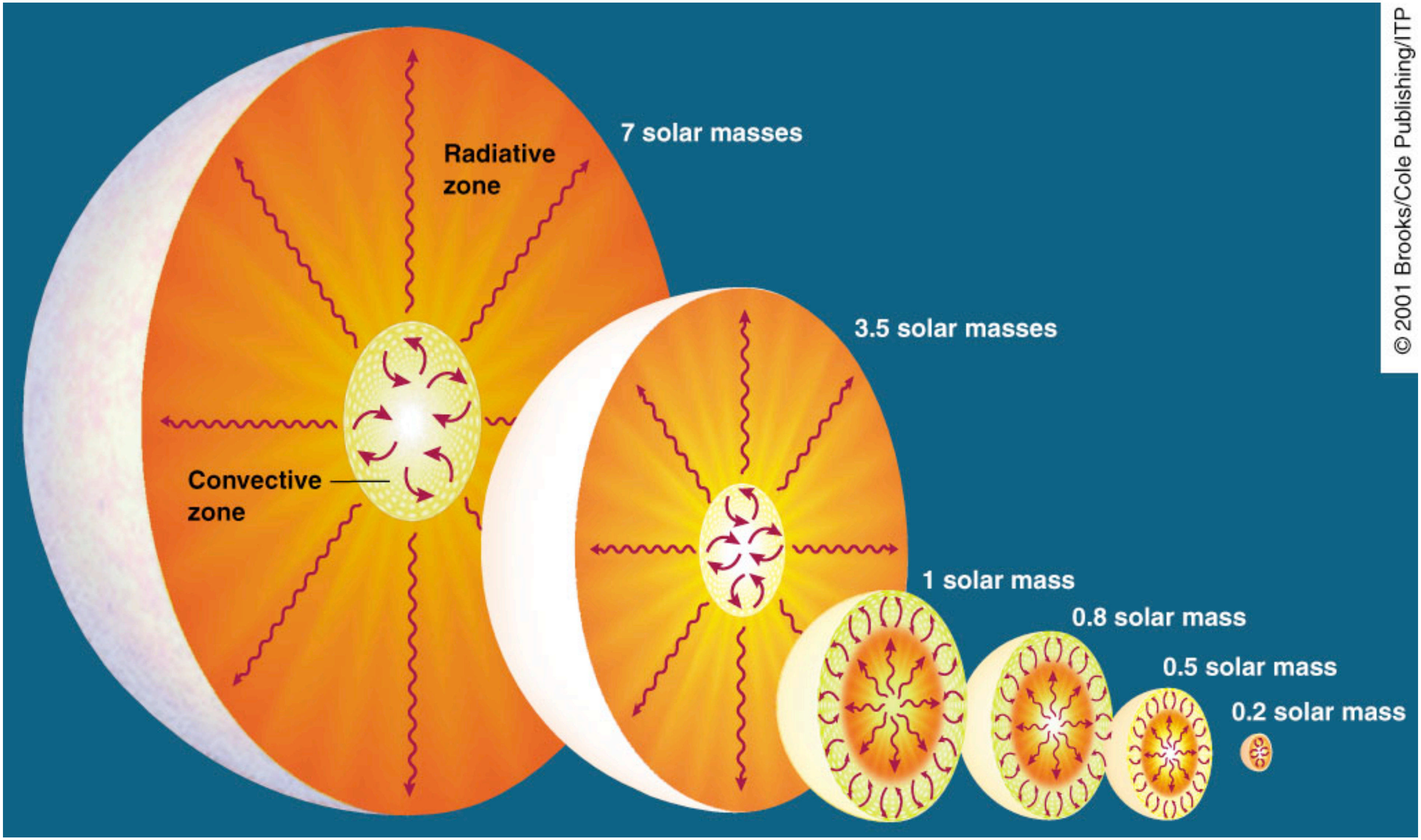
Photosphere:

Radiates to space

Solar Convection:



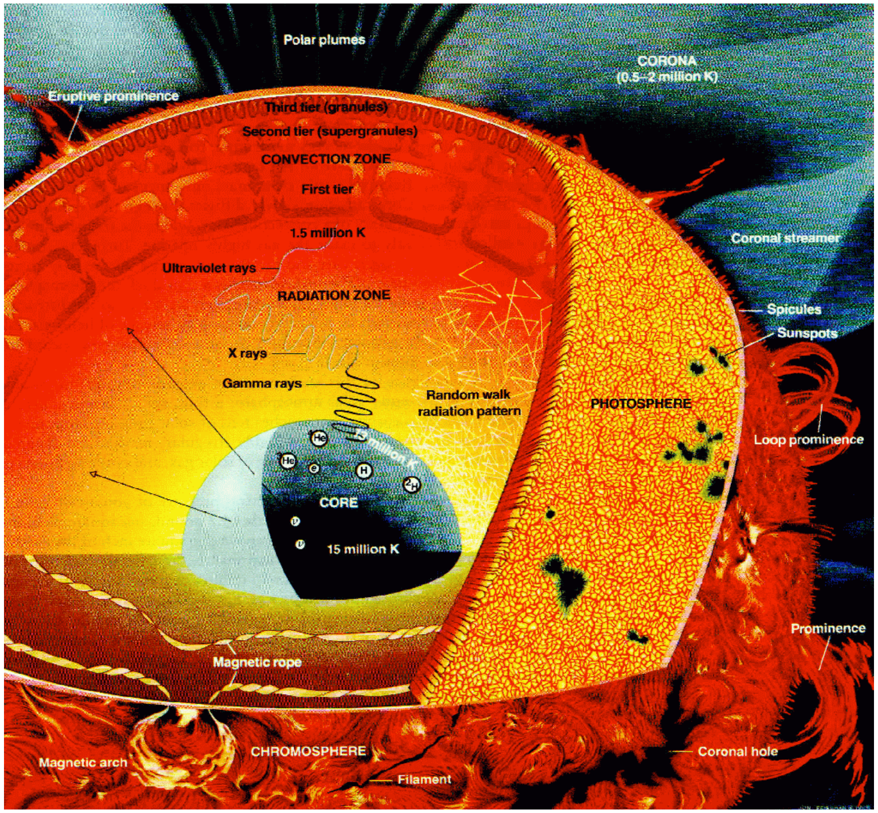
Energy Transport in other stars



Energy Transport in other stars

- High-mass stars (M>1.2-1.3 M_{sun}): convective core, radiative envelope
- Low-mass stars (M<1.2-1.3 M_{sun}): radiative core, convective envelope
- Really low-mass stars (M<0.3 M_{sun}): entirely convective
- Relative importance of convection and radiation, depends on opacity and temperature gradient

How can we calculate conditions inside Sun and Stars?…(M, L, T, P, ρ)



…using equations of stellar structure.

Luminosity increases with *Mass*

- Bigger mass means higher gravity
- Higher gravity means higher pressure in core
- Higher pressure means higher T and density
- This means MUCH more nuclear fusion
- This means star is MUCH more luminous
- This means mass determines luminosity

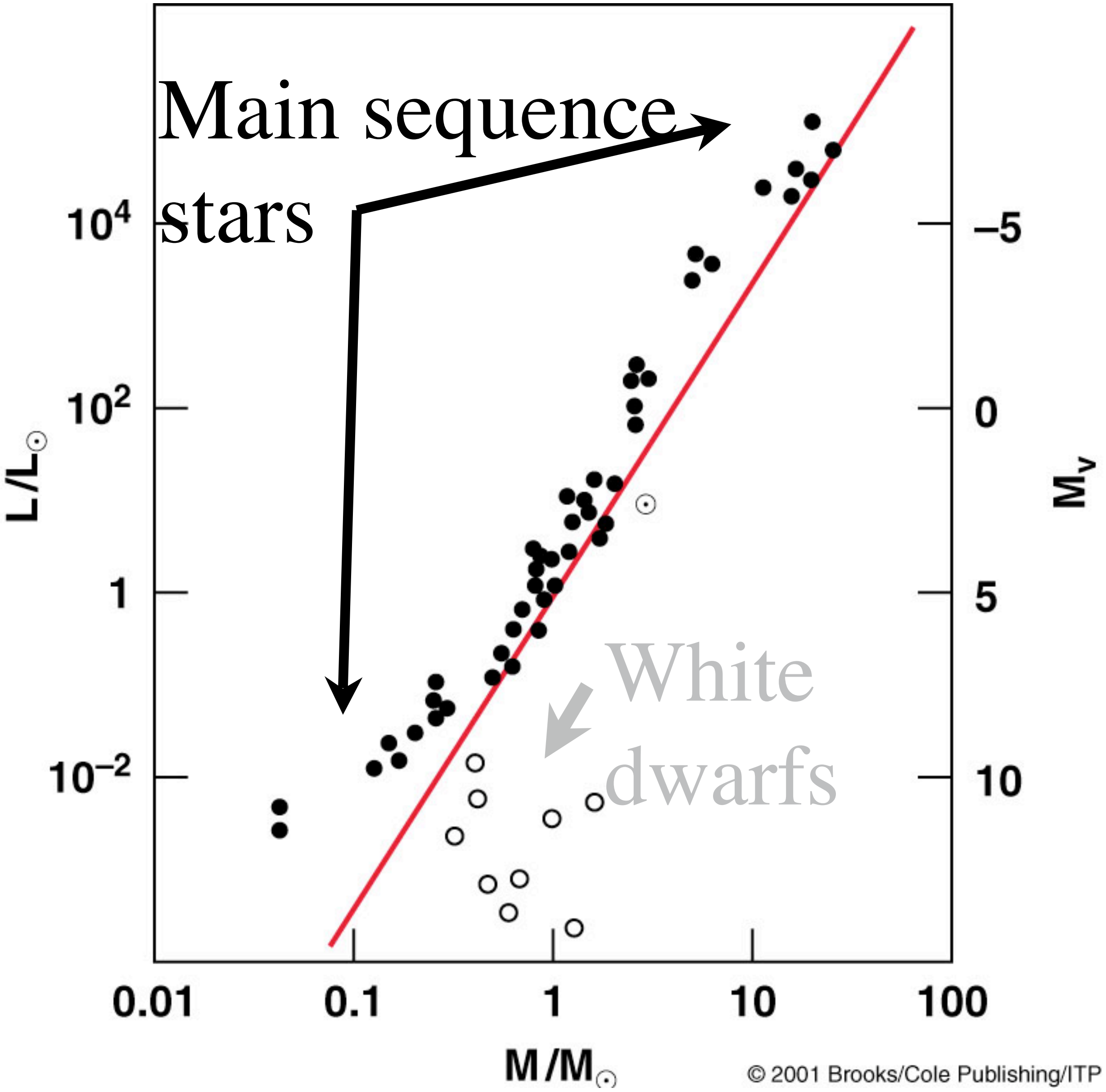
Mass – Luminosity Relation

On the main sequence
high-mass stars are MUCH more luminous than *low-mass* stars:

$0.1\ M_{sun} \rightarrow 0.0005\ L_{sun}$

$40\ M_{sun} \rightarrow 400{,}000\ L_{sun}$

$L \propto M^{3.5}$



Main sequence stars

White dwarfs

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Equations of Stellar Structure

Equation of state (EOS)

- EOS – relation between pressure, density and temperature of the gas.

- At moderate T and P ideal gas EOS works well:

$$P_i = \frac{\rho k_B T}{\mu}$$

- At very high T radiation pressure is important:

$$P_{rad} = \frac{aT^4}{3}$$

- Two combined:

$$P = P_i + P_{rad} = \frac{\rho k_B T}{\mu} + \frac{aT^4}{3}$$

Degeneracy pressure

- When material is highly compressed Pauli exclusion principle is important, states $\Delta p \times \Delta x \sim \hbar$
- In dense gas uncertainty in position is $\Delta x \sim n^{-1/3}$
- Thus, particles have minimum impulse $p_d = \Delta p = \hbar n^{1/3}$
- When $p_d > m v_{th}$ gas becomes degenerate:

$$\hbar n^{1/3} > m\sqrt{k_B T / \mu}, \quad n > \left(\frac{m}{\hbar} \sqrt{\frac{k_B T}{\mu}} \right)^3, \quad \Rightarrow p = p_d$$

- Leads to degeneracy pressure:

$$P_d \sim n p_d v$$

- In non-relativistic gas

$$v = p / m, \;\Rightarrow\; P_d \sim n p_d^2 / m \sim \frac{\hbar^2}{m} n^{5/3}$$

Governing equations

- Hydrostatic equilibrium

$$\boxed{\frac{dP}{dr} = -G\frac{M(r)\rho}{r^2}}$$

- Mass equation

$$\boxed{\frac{dM(r)}{dr} = 4\pi r^2 \rho}$$

- Energy transfer

$$\boxed{\frac{d\ln T}{d\ln P} = \nabla_{rad} \quad or \quad \nabla_{ad}}$$

- Energy production

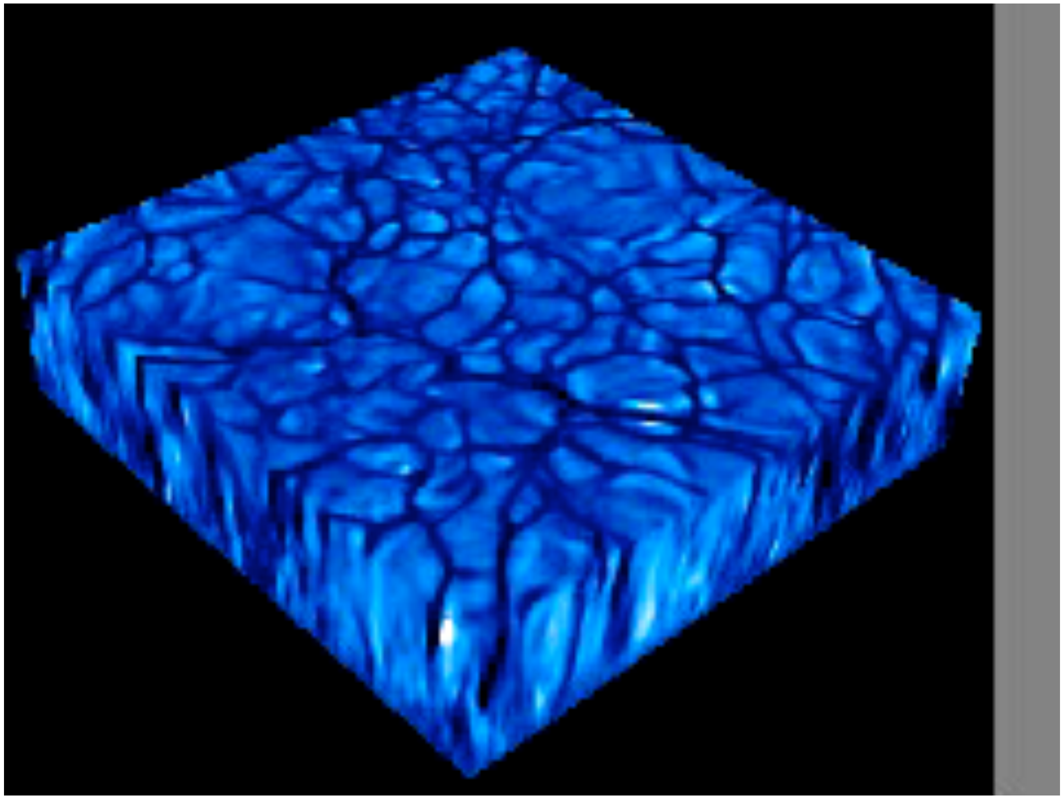
$$\boxed{\frac{dl(r)}{dM} = \varepsilon(\rho, T)}$$

Convection

$$\nabla = \frac{\partial \ln T}{\partial \ln P} > \frac{\gamma - 1}{\gamma} = \nabla_{ad}$$

Convective instability – vertically perturbed fluid elements are buoyantly unstable

- Fluid motions carry energy with them
- Both upward and downward motions



Additional relations

- Boundary conditions

$$\boxed{L(0)=0,\ \ M(0)=0,\ \ M(R)=M_*,\ \ P(R)=0}$$

- Opacity behavior

$$\boxed{\kappa=\kappa(\rho,T)}$$

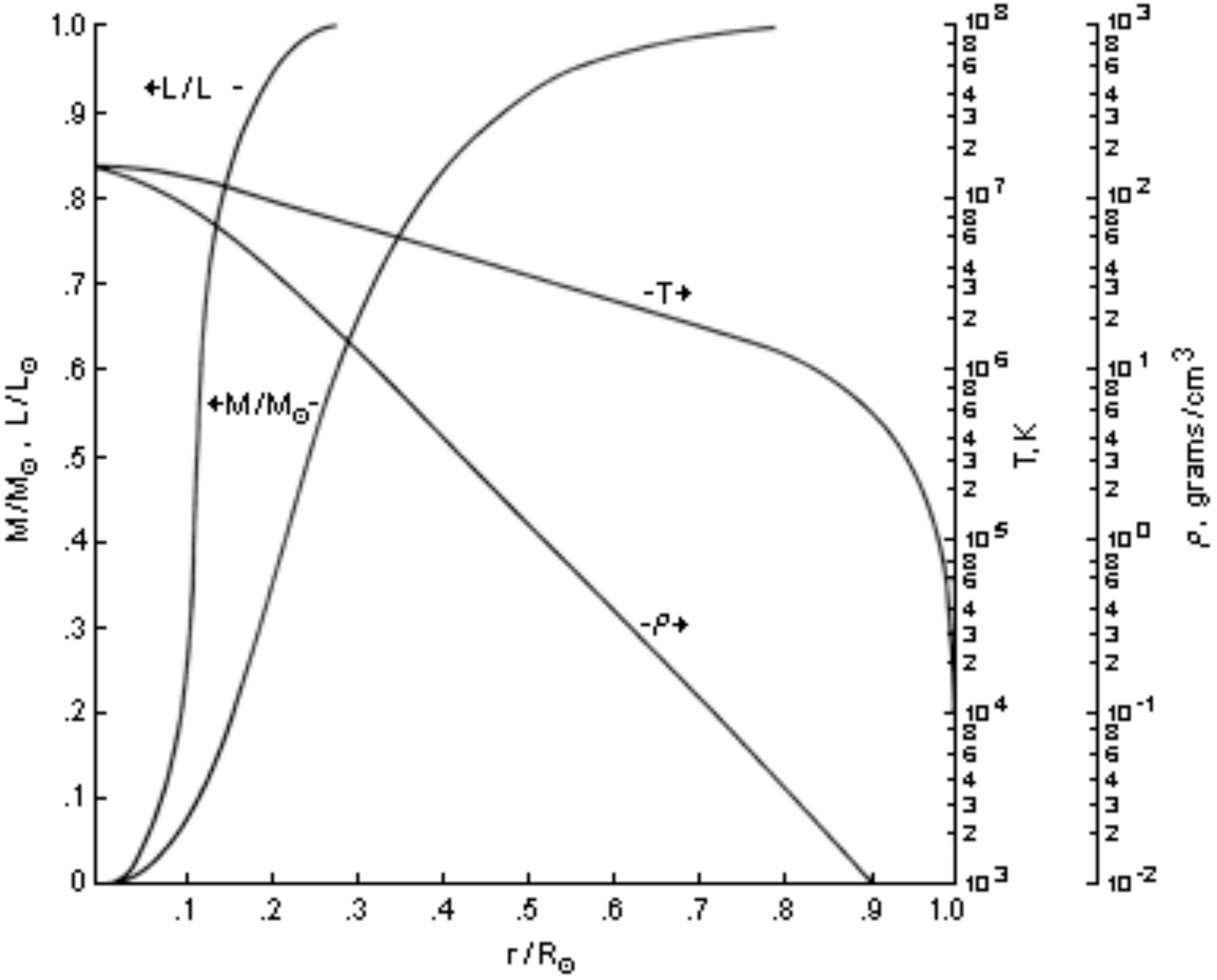
- Equation of state

$$\boxed{P=P(\rho,T)}$$

- Energy generation rate

$$\boxed{\varepsilon=\varepsilon(\rho,T)}$$

Result: standard solar model



T_c = 15x10^6 K

Fusion occurs out to 0.3R_{Sun}

Convection zone starts at 0.7R_{Sun}

The thermal properties of self-gravitating objects

Cooling reduces pressure

- so interior shrinks (compresses) → gets hotter

Conversely, heating increases pressure

- so interior expands → gets cooler

So why doesn’t the Sun explode?

- Stars like the sun have a thermostat:
  - **If fusion rate increases:**
    - gas gets hotter, expands, cools down, fusion slows
  - **If fusion rate decreases:**
    - gas cools, contracts, compresses, heats up, fusion increases
- Stars are *stable*

How Long Can a Star Shine?

- ✶ Time = Amount of Fuel / Fuel-consumption-rate
  - Amount of Fuel: Star’s mass
  - Fuel consumption rate: Luminosity $L \propto M^{3.5}$
- ✶ Time $\propto$ Mass / Luminosity $\propto M/M^{3.5} \propto M^{-2.5}$
- ✶ massive stars are ***VERY*** luminous and short-lived

Main Sequence Lifetimes

Spectral Type	Mass (Sun=1)	Luminosity (Sun=1)	Years on Main Sequence
O5	40	400,000	1×10^6
G0	1.1	1.4	8×10^9
M0	0.5	0.08	56×10^9