Relativistic Pulsar Winds and Magnetospheres
Anatoly Spitkovsky (Princeton)

Outline:

1. Pulsar and their winds: observations, properties and problems

2. Emerging unified picture:
   a) 3D pulsar magnetosphere
   b) Wind-nebula interaction
   c) Acceleration of particles at shocks

3. Conclusions and future

The life of pulsars

All pulsars lose rotational energy and slow down

Spindown age:

\[ \tau = \frac{P}{2\dot{P}} \]

Surface magnetic field

\[ B = 3.2 \times 10^{19} \sqrt{P\dot{P}} \text{ G} \]

Typical value \( 10^{12} \text{G} \)
(depend on spin-down formula!!).

Energy loss in radiation tiny .01-10% of spin-down.
Most energy is in the wind.
Our main source of information about the wind is Pulsar Wind Nebulae in young supernova remnants. Box calorimeter for the wind. Most of spindown energy ends up in the wind.

Properties of pulsar winds:
- Highly relativistic ($\gamma \sim 10^6$) upstream, $\sim c/2$ downstream
- Kinetic energy dominated at the nebula (hard!)
- $\sigma_B = \frac{B^2}{(4\pi n\gamma mc^2)} \sim 10^{-3}$
- Pole-equator asymmetry and collimation (hard!)
- Produce nonthermal particles (how?)

Center-filled morphology, nonthermal spectrum, linear polarization
Our main source of information about the wind is Pulsar Wind Nebulae in young supernova remnants. Box calorimeter for the wind. Most of spindown energy ends up in the wind.

Crab (Weisskopf et al. 00)
B1509 (Gaensler et al. 02)
Vela (Pavlov et al. 01)

Properties of pulsar winds:
- Highly relativistic ($\gamma \sim 10^6$) upstream, $\sim c/2$ downstream
- Kinetic energy dominated at the nebula (hard!)
- $\sigma = B^2 / (4\pi n \gamma mc^2) \sim 10^{-3}$
- Pole-equator asymmetry and collimation (hard!)
- Produce nonthermal particles (how?)

What are pulsar wind properties at the source?

Bow shock nebulae

(a) Chandra (0.5–8.0 keV)
Gaensler et al. 03
The mouse

(b) VLA (4.8 GHz)

Chatterjee et al. 03
The guitar

Van der Swaluw, 03

Fruchter (1995)
Our main source of information about the wind is Pulsar Wind Nebulae in young supernova remnants. Box calorimeter for the wind. Most of spin down energy ends up in the wind.

Properties and puzzles of pulsar winds:

- Highly relativistic ($\gamma \sim 10^6$) upstream, $\sim c/2$ downstream
- Kinetic energy dominated at the nebula (“$\sigma$-problem”)
  \[ \sigma = \frac{B^2}{(4\pi n \gamma mc^2)} \sim 10^{-3} - 10^{-1} \]
- Pole-equator asymmetry and collimation
- Produce nonthermal particles (how?)
**CRAB NEBULA SN1054**

Synchrotron emission:

- **Lifetime:** X-rays -- few years, γ-rays -- months. Need energy input!
- **Crab pulsar:** $E_R = 5 \times 10^{38}$ erg/s, 10-20% efficiency of conversion to radiation.
- **Max particle energy** $> 3 \times 10^{15}$ eV, comparable to pulsar voltage.
- Nebular shrinkage indicates one accelerating stage:
  - require $10^{38.5} - 10^{39}$ e$^\pm$/s, radio mystery
  - PSR also injects B field into nebula ($\sim 10^{-4}$ G)

\[ S_{\nu} \propto \nu^{-0.3} \text{(radio)}; \nu^{-1.0} \text{(X-ray)}; \text{break} \]
How is the wind produced at the source?

Where does acceleration/collimation happen?

How is flow energy converted into radiation?

Goal:

Use modelling of PWN data and ab-initio simulations of magnetospheres to construct a self-consistent picture of wind injection, transport and deposition, and infer wind properties (speed, magnetization, composition).

Ultimately, use the wind to get a handle on physics at the source.
Going for the source: pulsar magnetosphere

• Pulsars are not in vacuum!

• Equator-pole potential difference ($10^{15}$V for Crab)

• Charge extraction from the surface (E field $>>$ gravity)

• Currents, strong magnetization

• Corotating zone; Light cylinder

• Throwing away toroidal field -- energy loss (Poynting flux)

• Plasma currents modify field. How can we model this?

$\phi_0 = \Omega B a^2 / c$
Pulsars are not in vacuum!

Equator-pole potential difference (10^{15} V for Crab)

Charge extraction from the surface (E field $\gg$ gravity)

Currents, strong magnetization

Corotating zone; Light cylinder

Throwing away toroidal field -- energy loss (Poynting flux)

Plasma currents modify field. How can we model this?
**Relativistic MHD:**
Evolves electromagnetic fields and plasma self-consistently.

**Equations:**
\[
\nabla_\beta \left( T^{\alpha\beta}_{(m)} + T^{\alpha\beta}_{(f)} \right) = 0
\]
\[
\nabla_\beta * F^{\alpha\beta} = 0
\]
\[
\nabla_\alpha (nu^\alpha) = 0
\]
\[
F_{\nu\mu}u^\mu = 0 \quad \text{- perfect conductivity}
\]
\[
mn \frac{\partial \gamma^\nu}{\partial t} = \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} \approx 0
\]

**Stiff for high magnetization**

**Force-free:**
Simplification for strong magnetization
Evolves only EM fields, plasma implicit

**Equations:**
\[
\nabla_\mu T^{\mu\mu}_{(f)} = 0
\]
\[
\nabla_\beta * F^{\alpha\beta} = 0
\]

**Breaks down in low magnetization**

\[
T^{\alpha\beta}_{(m)} \ll T^{\alpha\beta}_{(ef)}
\]

\[
F_{\mu\nu} * F^{\mu\nu} = 0
\]
\[
F_{\mu\nu} F^{\mu\nu} > 0
\]
\[
E \cdot B = 0
\]
\[
B^2 - E^2 > 0
\]

(Komissarov 2002, 2005)
Force-free equations

Full RMHD equations become stiff for high magnetization

\[ mn \frac{\partial \gamma \vec{v}}{\partial t} = \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} \approx 0 \]

Derive dynamical set of equations by ignoring particle inertia but retaining plasma charges and currents.

\[ \begin{aligned}
\frac{1}{c} \frac{\partial E}{\partial t} &= \nabla \times \vec{B} - \frac{4\pi}{c} \frac{\vec{j}}{c} \\
\frac{1}{c} \frac{\partial B}{\partial t} &= -\nabla \times \vec{E} \\
\rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} &= 0 \\
\frac{\partial}{\partial t} \frac{\vec{E} \cdot \vec{B}}{B} &= 0
\end{aligned} \]

\[ \vec{j} = \frac{c}{4\pi} \left( \nabla \cdot \vec{E} \right) \frac{\vec{E} \times \vec{B}}{B^2} + \frac{c \vec{B} \cdot (\vec{B} \cdot \nabla \times \vec{B} - \vec{E} \cdot \nabla \times \vec{E})}{4\pi B^2} \]

Where is plasma? Assumed to flow with ExB velocity, but velocity along the field is undefined. Plasma provides only charges and currents, no inertia.

Hyperbolic eqs. Use electromagnetic solvers to advance the system in time.

Gruzinov 99, Blandford 01
Evolve force-free equations in time: spin up a sphere with dipole field

With plasma, even aligned rotator loses energy.

**Properties of the solution (A.S. 06):**
- Spontaneous formation of equatorial current sheet ($10^{14}$ Amps).
- Reconnection necessary to reach LC
- Y-point near LC (cf McKinney 06)
- Field is divergent at Y-point
- Field is zero in the equatorial plane
- Asymptotically -- split monopole
- Closed zone expands to LC over 10 period timescale.

**Spin-down:** (cf Contopoulos et al ‘99)

\[
\dot{E} = \frac{\mu^2 \Omega^4}{c^3} = c B_{LC}^2 R_{LC}^2
\]

**Vacuum formula:**

\[
\dot{E}_{\text{vac}} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta
\]
Dynamic magnetosphere

Time-dependent magnetosphere solution allows qualitatively different approach to understanding pulsar physics.

Adjustment of magnetosphere happens on scales longer than rotation period (drifting subpulses?)

How is force-free current provided by space-charge gaps? Time-dependence? (Levenson et al)

Closed-open boundary is a candidate site for high-energy emission based on geometry. Are return current instabilities responsible for high energy emission in “outer gaps”? Important for GLAST.

What happens in the current sheet and at the Y-point? More physics needed!

Return current potentially carried by ions

Structure of currents
(cf Contopoulos, Kazanas, Fendt 99, Timokhin 06, McKinney 06, Komissarov 06)
3D force-free magnetosphere: 30 degrees inclination

Real pulsars have to be oblique: otherwise they would not pulse! Go to 3D!

30 degrees force-free solution
(with plasma!)
Superposition of aligned component
+ oscillating wave

30 degrees vacuum solution
(no plasma!)
3D force-free magnetosphere: 30 degrees inclination

30 degrees force-free current density  
30 degrees force-free inner magnetosphere
3D force-free magnetosphere: 60 degrees inclination

60 degrees force-free solution (with plasma)
60 degrees vacuum solution (no plasma!)
3D force-free magnetosphere: 60 degrees inclination

60 degrees force-free current density

60 degrees inner magnetosphere
3D force-free magnetosphere: 60 degrees inclination
3D force-free magnetosphere: 60 degrees inclination
Meanwhile in the rotating frame: 60 degrees inclination

Magnetic field, plane of \( \mu - \Omega \)
Steady pattern in corot. frame

Current density, plane of \( \mu - \Omega \)
Spin-down of oblique rotator

\[ \dot{E} \approx \frac{\mu^2 \Omega^4}{c^3} \left(1 + \sin^2 \theta \right) \]

Vacuum formula

\[ \dot{E}_{\text{vac}} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta \]

Braking index is still \( n=3 \ldots \)
Pulsar spindown

What happens as pulsar ages?
Gap potential takes larger fraction of the available voltage: suppression of (aligned) spin down power.

This makes pulsars curve in their P-Pdot tracks. Can explain P-Pdot diagram without B field decay! (Contopoulos & Spitkovsky 06)

\[
\dot{E} \approx \frac{\mu^2 \Omega^4}{c^3} \left( 1 + \sin^2 \theta \right)
\]

\[
\dot{E} \approx \frac{\mu^2 \Omega^4}{c^3} \left( 1 - \frac{\Omega_d}{\Omega} + f \left( \frac{\Omega_d}{\Omega} \right) \sin^2 \theta \right)
\]

\[
\dot{E}_{\text{vac}} = \frac{2 \mu^2 \Omega^4}{3 c^3} \sin^2 \theta
\]

\@ \quad \Omega = \Omega_{\text{death}} : \quad \dot{E}_{\text{vac}} = \frac{2 \mu^2 \Omega_{\text{death}}^4}{3 c^3} \sin^2 \theta
**3D magnetosphere implications**

- Correct spin-down formula can yield surface B field up to 1.7 times smaller than vacuum formula.
  
  \[ B_{\text{PSR}} = 2.6 \times 10^{19} \sqrt{P\dot{P}} / (1 + \sin^2 \theta) \ G \]
  
  \[ B_{\text{vac}} = 3.2 \times 10^{19} \sqrt{P\dot{P}} \ G \]

- Oblique pulsars evolve faster in P-Pdot diagram than aligned pulsars, therefore, expect excess of aligned pulsars near death line (Contopoulos & Spitkovsky 06). Don’t need decay of B field!

- Correct spin-down formula should be used in population synthesis models
Implications of 3D magnetosphere

3D solution asymptotically split-monopole

\[ B_\varphi \propto \sin \theta \]

Energy \( \propto \sin^2 \theta \)

Equator wedge special. Reconnection in the equator leads to annihilation of B field, e.g.:

\[ B_\varphi \propto \sin \theta \left( 1 - \frac{2\theta}{\pi} \right) \]

This is needed to explain torus-jet morphology form postshock (next talk)

Requires annihilation of field -- possible transfer of energy to particles
The Role of Ions

Ions are natural candidates for return current layer in the magnetosphere.

They are mapped into the equatorial region.

Evidence for presence of ion current on the order of $I_{GJ}$ comes from three sources:

1) Wisp dynamics in the Crab nebula and B1509 (Spitkovsky & Arons 04, Gaensler et al 02)

2) Particle acceleration in magnetized collisionless shocks in pair+ions plasma (Hoshino & Arons 92, Amato & Arons 06).

3) Hadronic component useded to explain the HESS TeV observations of Crab and Vela (Horns et al 06, Bednarek et al 03, 06, Amato et al 03)

Composition of the wind has yet to be determined unambiguously!

Can shock physics put more constraints?
Shock Acceleration: internal structure of relativistic collisionless shocks

3D Particle-in-Cell simulations of collision of relativistic plasma shells.

Shock structure is sensitive to ambient magnetization, \( \sigma = \frac{\omega_c^2}{\omega_p^2} = \frac{B^2}{4\pi n \gamma mc^2} \).

For low magnetization (\( \sigma < 5 \times 10^{-3} \)) shock is mediated by streaming Weibel instability, which generates magnetic field on plasma scale.

Larger magnetization shocks (\( \sigma > 5 \times 10^{-3} \)) are mediated by magnetic reflection of particles.

Shock structure affects particle acceleration.
Nonthermal shock acceleration

- Shocks are dominated by magnetic reflections -- particles don’t cross field lines.
- Particles thermalize by emission and absorption of cyclotron waves.
- No upstream-downstream bouncing as in Fermi acceleration. Not enough magnetic turbulence!

\[ \sigma = 0.1, \text{ initial flow } \gamma = 15 \]
**The case for ions -- nonthermal shock acceleration**

- Ions in the flow provide free energy to accelerate nonthermal electrons; \( m_i/m_e = 16 \) (1D simulations Hoshino & Arons 92, Amato & Arons 2005). Same true in 3D.
- \( N(\gamma) = \gamma^{2-3} \) (sensitive to ratio of densities). Max. energy \( \gamma_e = m_p/m_e \gamma_{sh} \)

Even with ions acceleration is non-Fermi: combination of cyclotron harmonics absorption and electrostatics at the head of the shock

\( \sigma = 0.1, \text{ initial flow } \gamma = 15, \text{ } m/m_e = 16 \)
Alternative -- unmagnetized shocks?

- For low $\sigma$, shocks are mediated not by reflections, but by Weibel instability
- Turbulent B field is generated in the shock up to 10% equip., decaying to <1%
- Critical magnetization $\sigma < 5 \times 10^{-3}$ -- just the range for PWNs near equator.

Unmagnetized shocks may exist in the equatorial plane, and energize particles there.
Tentative detection of self-consistent Fermi acceleration

Self-consistent PIC simulations of unmagnetized pair shocks produce Fermi acceleration
Conclusions

• All stages of pulsar winds can now be modeled:
• 3D shape of pulsar magnetosphere with plasma is now known.
• Spin-down power scales as \((1 + \sin^2 \theta)\). It was not clear whether oblique rotators in plasma would spin down at all! (e.g. Beskin et al)
• Expect older pulsars to be preferentially aligned.
• Split-monopolar spiral magnetic field with low-magnetization equator seems to work best to model the nebula -- consistent with force-free magnetospheric models.

Ion component: evidence for large energy fraction carried in the ions. Evidence from shock acceleration, nebular dynamics (wisps), pulsar electrodynamics (current sheet) and TeV hadronic emission.

• Shock acceleration still a puzzle: magnetized \((\sigma > 0.01)\) perpendicular shocks are very unlikely to accelerate particles in pair plasma. Possible with ions, but hard to get universal powerlaws. “Unmagnetized” (Weibel-mediated) shocks have better chance at Fermi acceleration. Can fit into equator of PWNs.
• \(\sigma\)-problem is still there. Likely solution -- non-ideal acceleration in dissipative equator (e.g., Kirk et al; Melatos). Pure MHD acceleration is unlikely (Vlahakis et al).