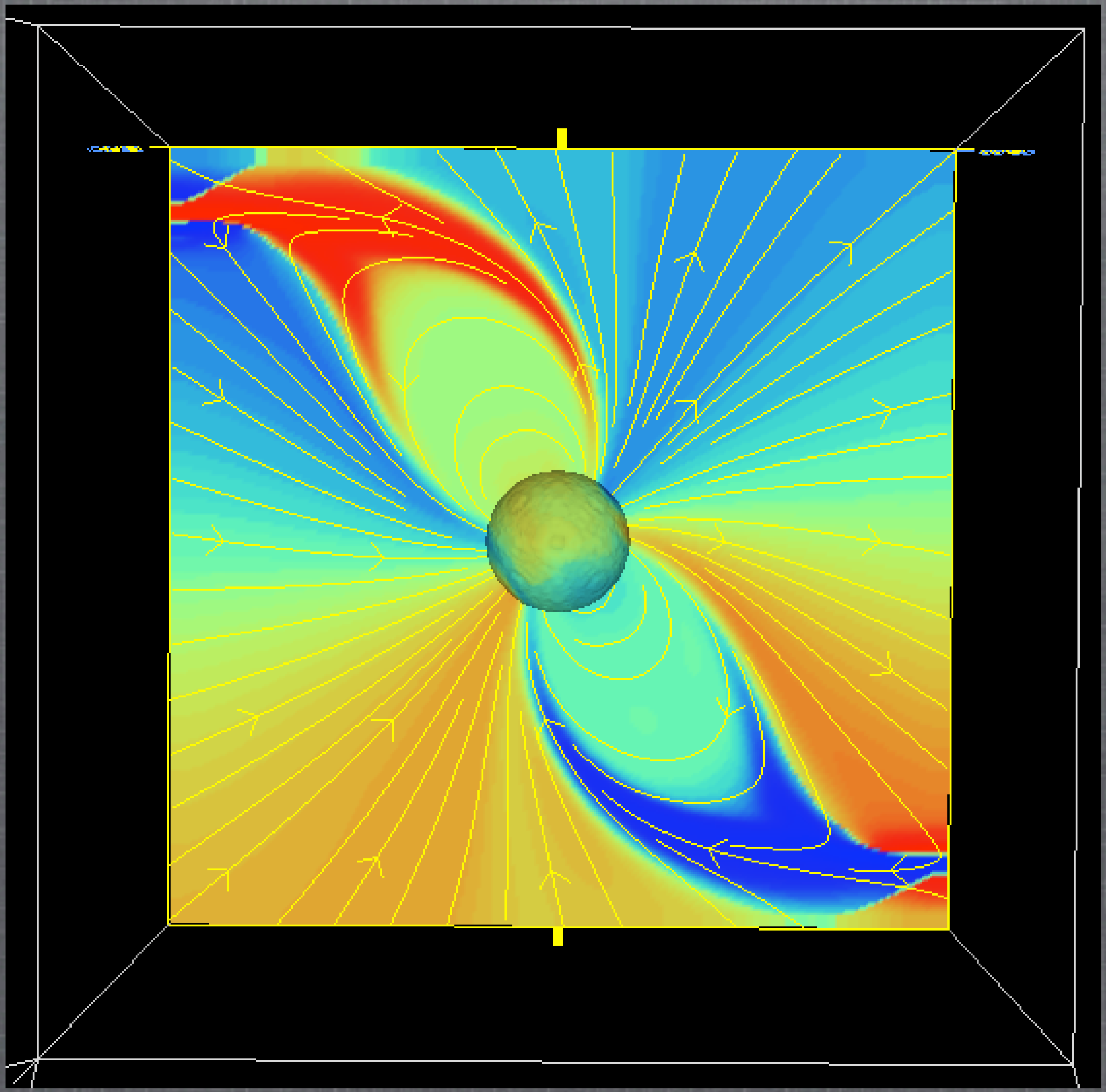
FORCE-FREE MAGNETOSPHERE

Anatoly Spitkovsky



Collaborators: Xuening Bai, Jon Arons

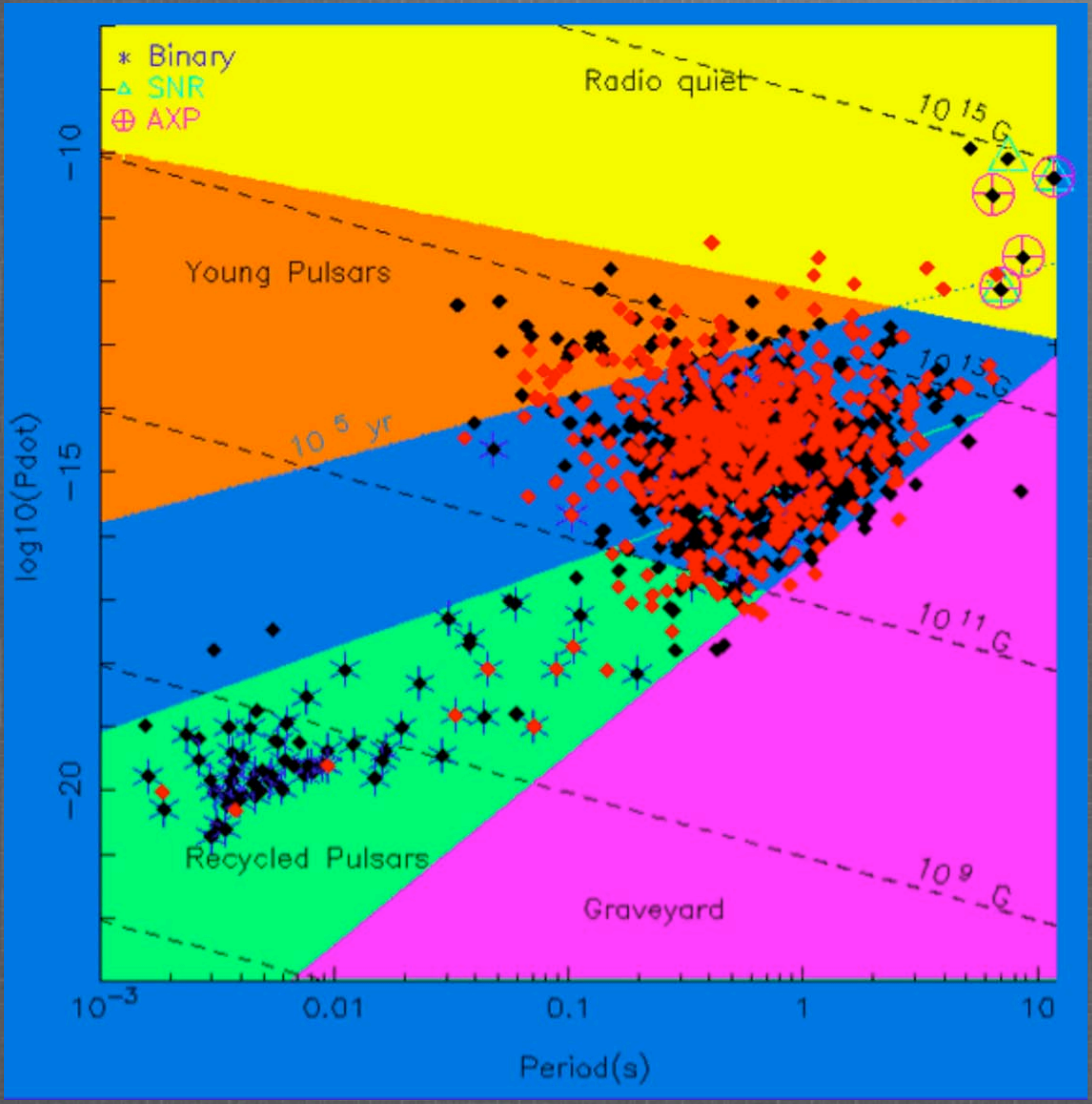
FORCE-FREE MAGNETOSPHERE

Anatoly Spitkovsky

1. The problem --> Magnetospheric setup
2. The solution --> Structure of the magnetosphere
3. More problems --> Getting the light curves

THE PROBLEM

How do pulsars lose energy?



Spin-down age:

$$\tau = \frac{P}{\dot{P}}$$

$$I\Omega\dot{\Omega} = \dot{E}_{loss}$$

Vacuum spin-down:

$$\dot{E}_{vac} = \frac{2}{3}\frac{\mu^2\Omega^4}{c^3}\sin^2\theta$$

Surface magnetic field:

$$B = 3.2\times10^{19}\sqrt{P\dot{P}}\ \mathrm{G}$$

Typical 10^{12} G

THE PROBLEM

How do pulsars lose energy?

Really simple question:

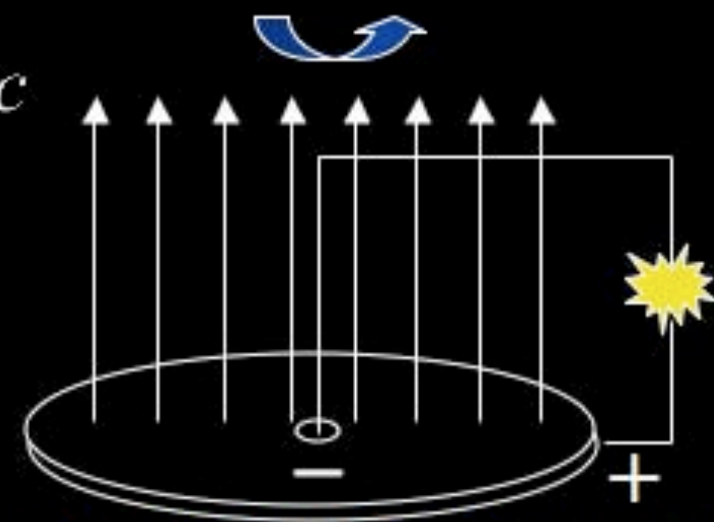
You have a spherical conductor with dipolar magnetic field. Conductor rotates. What happens in the limit of strong field and very small work function on the surface?

Plasma happens

Plasma source

Where does the plasma come from?

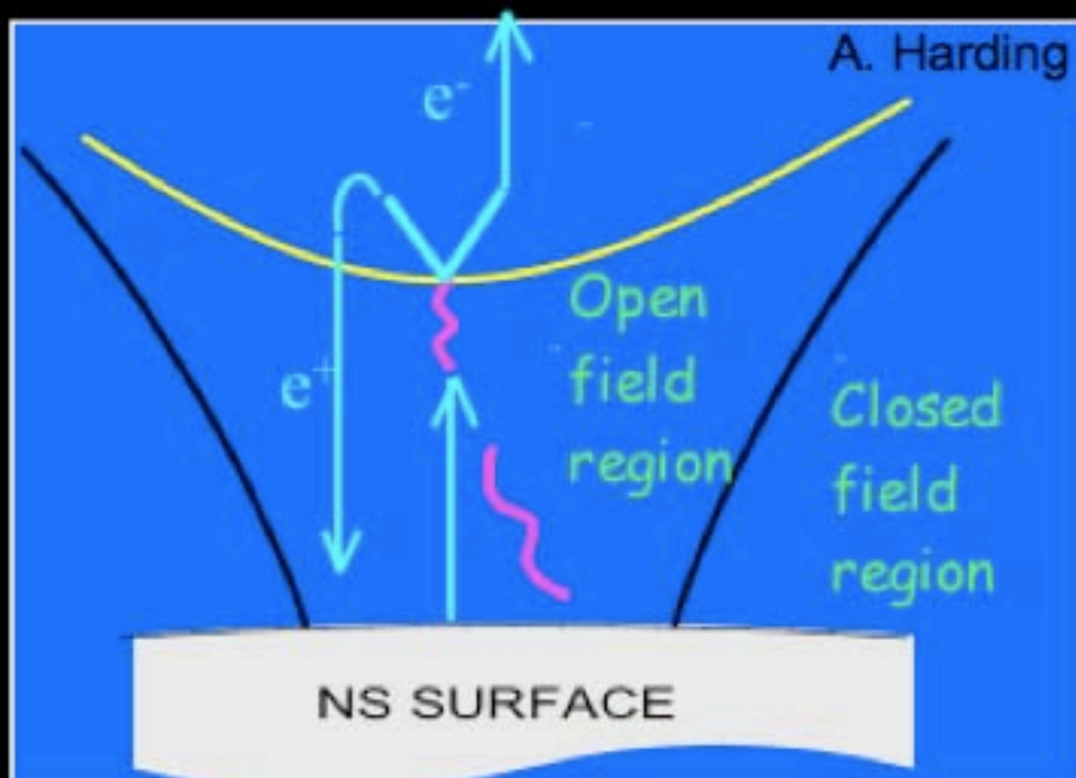
$\phi_0 = \Omega B a^2 / c$



Polar cap is a space-charge limited accelerator.
Accelerated primary particles radiate curvature radiation, and pair produce in the strong field.
Pair cascade shorts out E*B.

$\gamma_{\rm primary} \sim 10^7 \quad \gamma_{\rm secondary} \sim 10^{2-3} \quad \sigma_{\rm LC} \sim 10^4$

Faraday disk: unipolar induction



Arons & Scharleman 79, Muslimov & Harding 03

Electrostatic accelerator, non-MHD region

$$\vec{E} = -\frac{\vec{v}}{c} \times \vec{B} = -\frac{\vec{\Omega}}{c} \times \vec{R} \times \vec{B}$$

$$\frac{1}{4\pi} \nabla \cdot \vec{E} = \rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c}$$

$$j_{GJ} = \rho_{GJ} c = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi}$$

After pair formation front -- enough plasma to use MHD.

TWO PATHS

Is there dense ($n \gg n_{GJ}$) plasma in the magnetosphere?

No!

Charge separated magnetosphere

as in Golderich & Julian '69

Michel et al 1980s+

Yes, but not everywhere, and not always

Reality

Your Name (2009)

Yes!

MHD / force-free

Contopoulos et al 1999 +

many others

TWO PATHS

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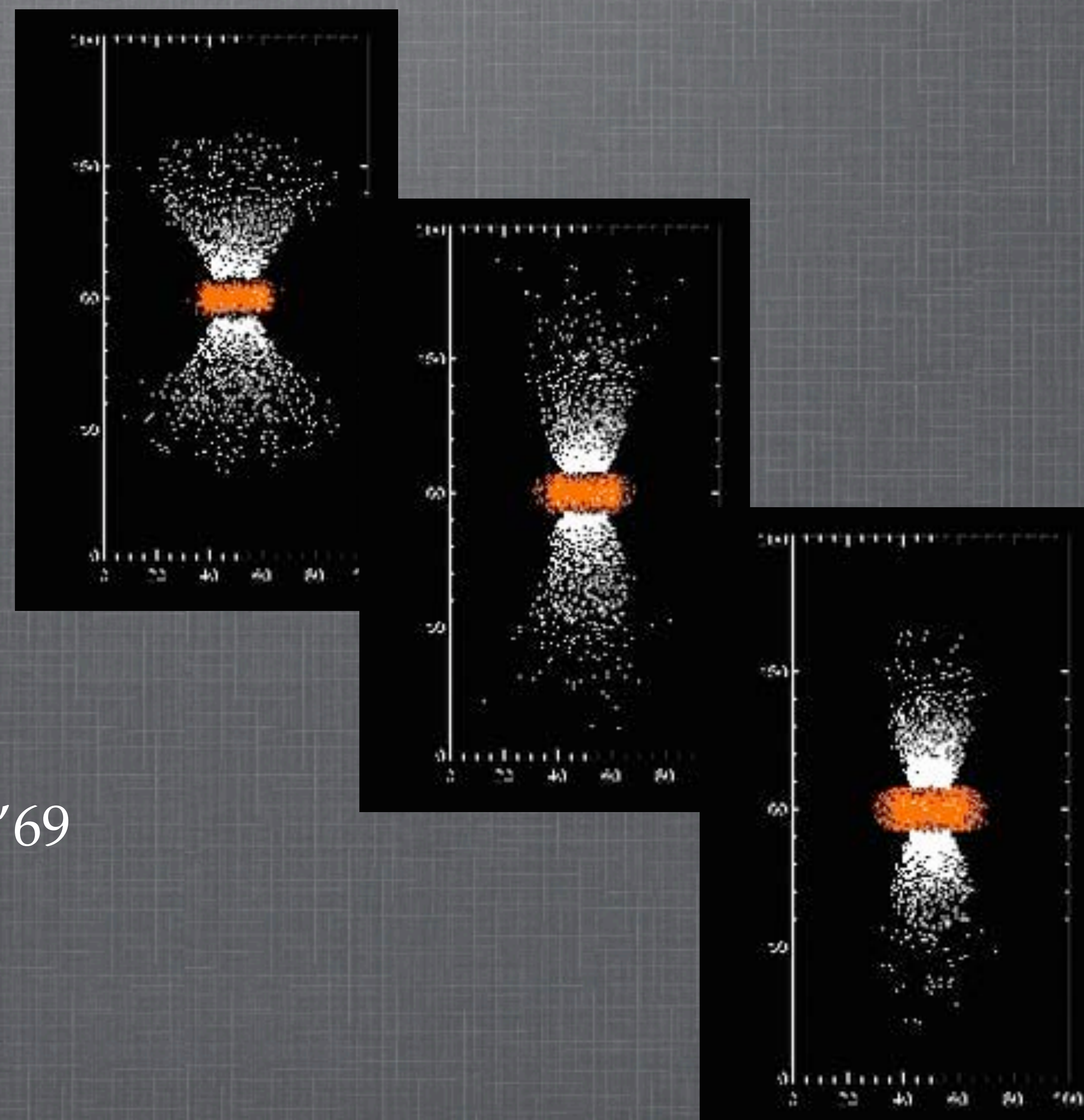

No!

Charge separated magnetosphere

as in Golderich & Julian '69

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Dead end

TWO PATHS

Is there dense ($n \gg n_{GJ}$) plasma in the magnetosphere?

Assume abundant plasma with small inertia (force-free), but with current & charge:

$$mn\frac{\partial \gamma \vec{v}}{\partial t} = \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} \approx 0$$

“Pulsar equation” (Michel ‘73; Scharleman &Wagoner ‘73):

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{1+x^2}{x(1-x^2)}\frac{\partial \Psi}{\partial x} = -\frac{I(\Psi)I'(\Psi)}{R_L^2(1-x^2)}$$

Yes!

MHD / force-free

Contopoulos et al 1999 + many others

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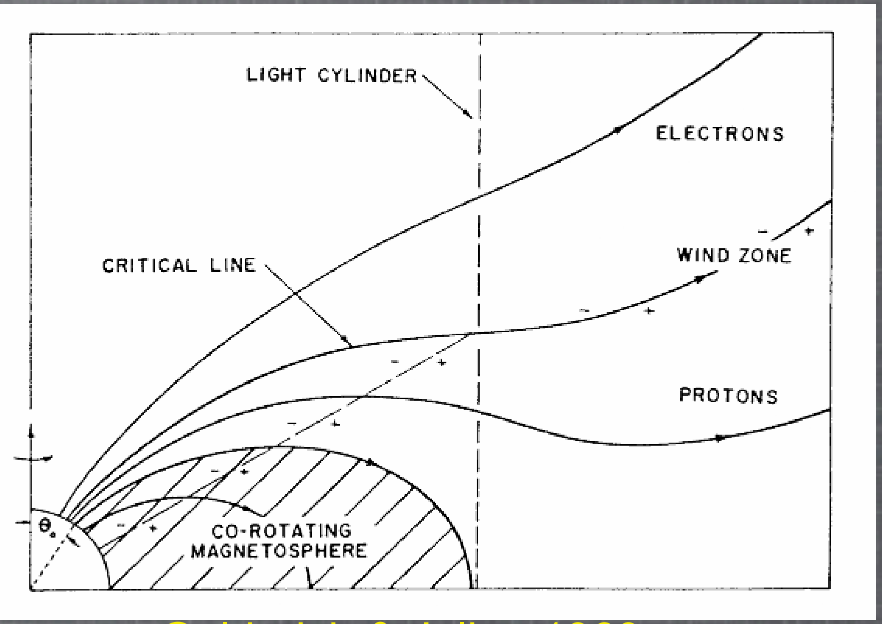

Goldreich & Julian 1969

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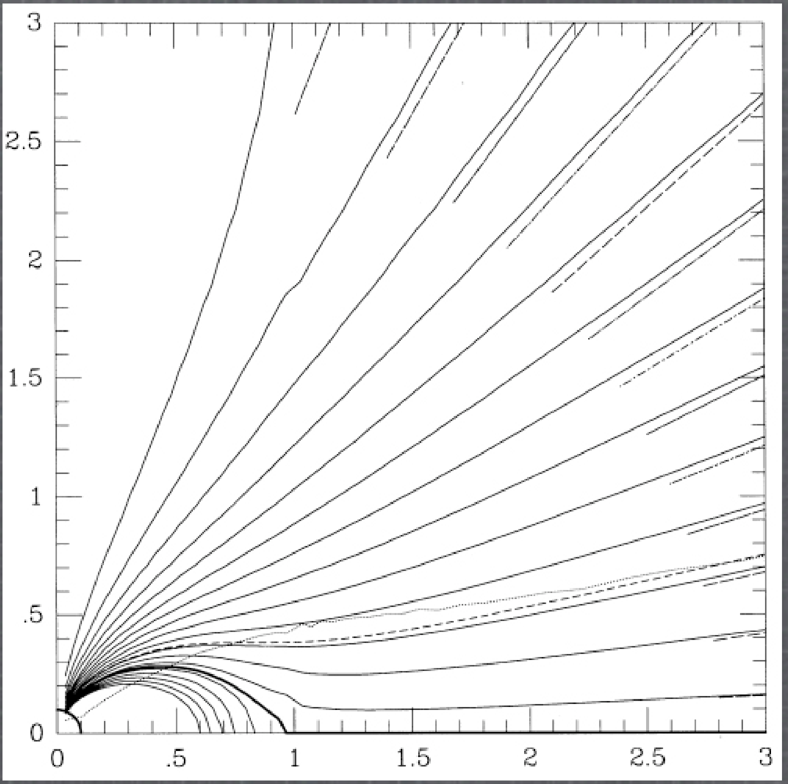
Properties:
Closed-open configuration;
Corotation of the closed zone, extending upto 1Rlc
(other solutions also possible [Timokhin 05])



Contopoulos et al 99, 05, Gruzinov 05, Timokhin 05

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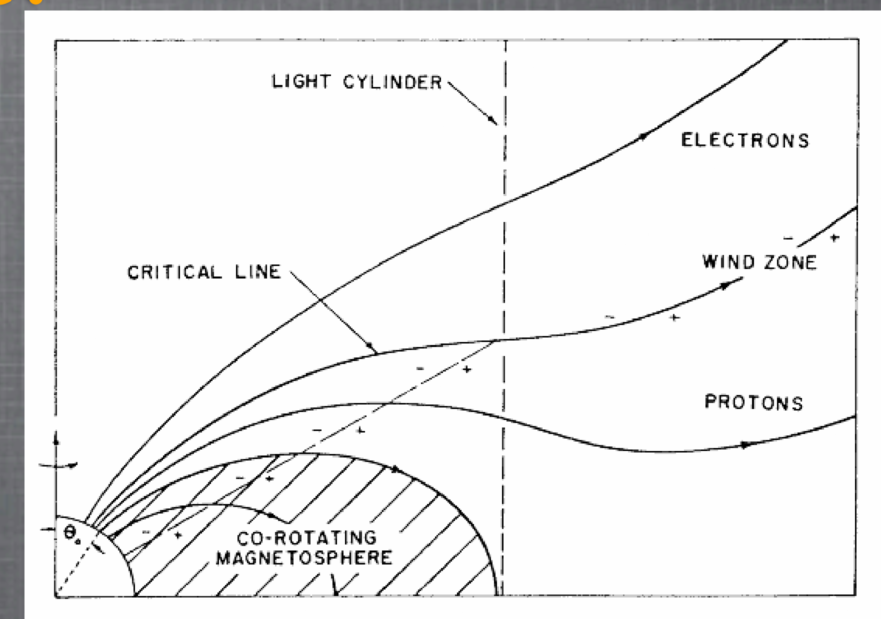

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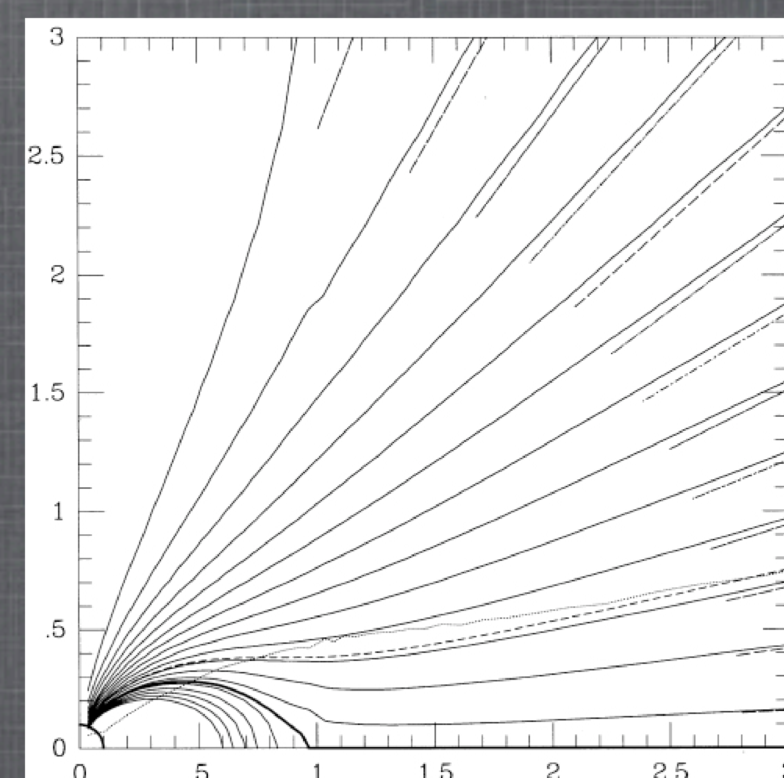
Properties:

Despite no time variability -- loses energy! Poynting flux in the wound up toroidal field



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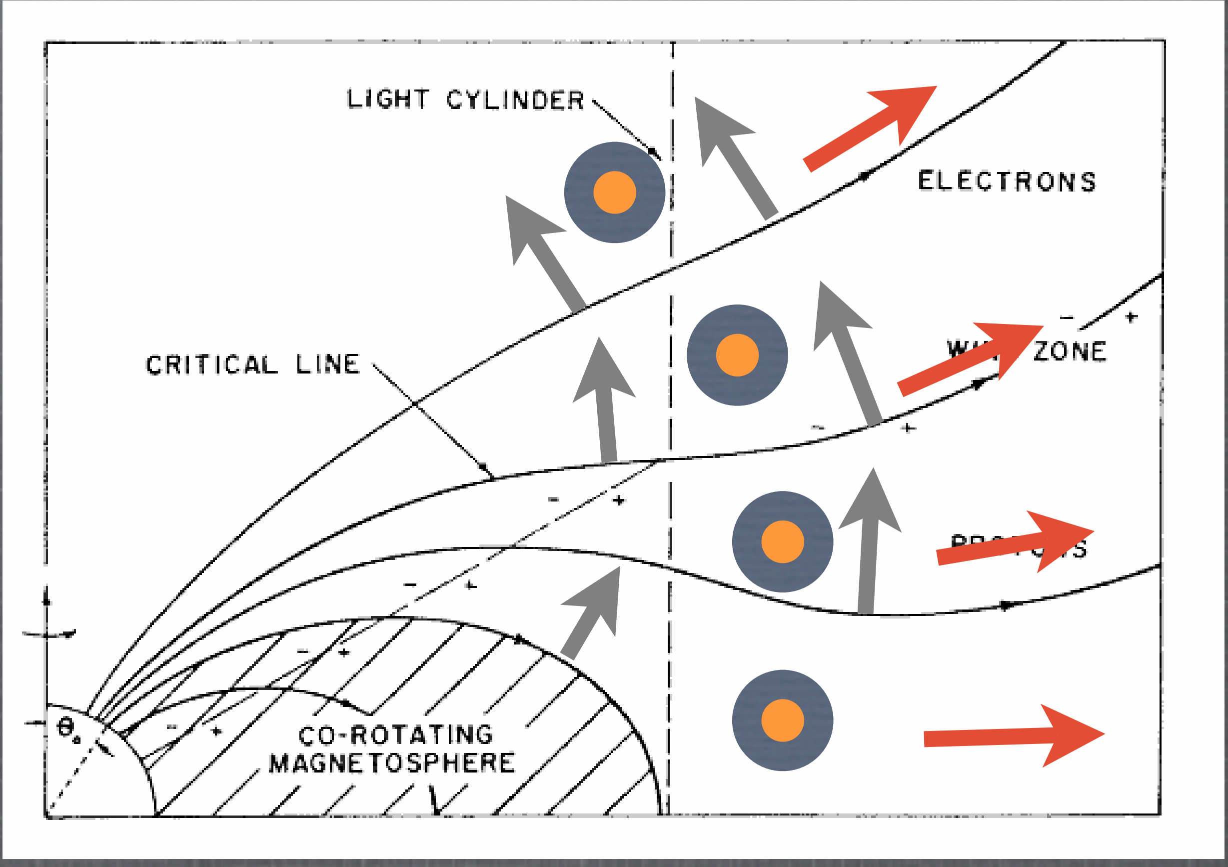


Properties:

Despite no time variability -- loses energy! Poynting flux in the wound up toroidal field

Toroidal field means there is poloidal current!

TIME-DEPENDENT EVOLUTION

Full RMHD equations become stiff for high magnetization

$$mn\frac{\partial\gamma\vec{v}}{\partial t} = \rho\vec{E} + \frac{\vec{j}}{c}\times\vec{B} \approx 0$$

Derive dynamical set of equations by ignoring particle inertia but retaining plasma charges and currents.

$$\left.\begin{aligned}
\frac{1}{c}\frac{\partial E}{\partial t} &= \nabla\times\vec{B} - \frac{4\pi}{c}\vec{j} \\
\frac{1}{c}\frac{\partial B}{\partial t} &= -\nabla\times\vec{E} \\
\rho\vec{E} + \frac{\vec{j}}{c}\times\vec{B} &= 0 \\
\frac{\partial}{\partial t}\vec{E}\cdot\vec{B} &= 0
\end{aligned}\right\} \quad \vec{j} = \frac{c}{4\pi}(\nabla\cdot\vec{E})\frac{\vec{E}\times\vec{B}}{B^2} + \frac{c\vec{B}(\vec{B}\cdot\nabla\times\vec{B} - \vec{E}\cdot\nabla\times\vec{E})}{4\pi B^2}$$

Perpendicular current

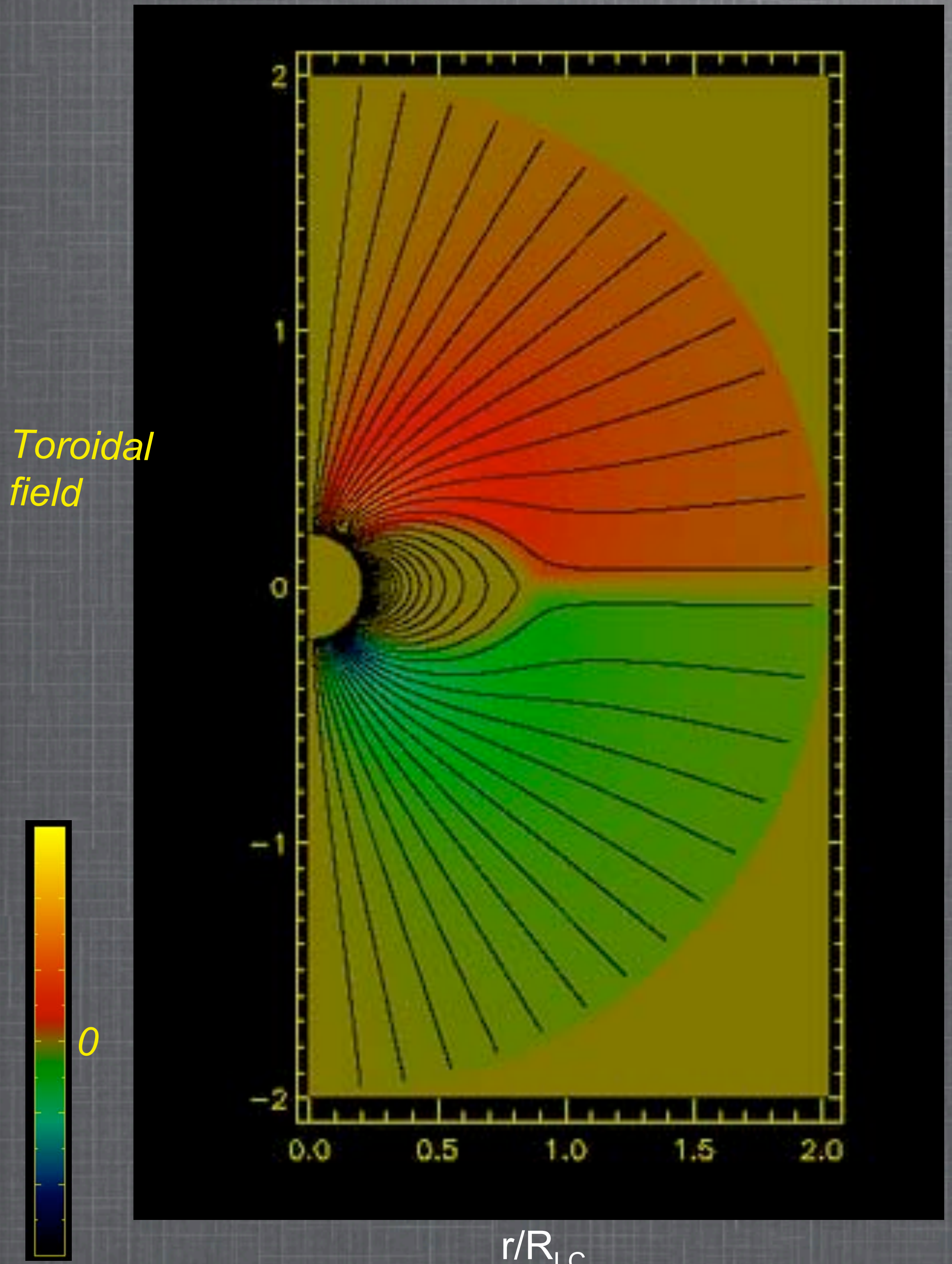
Parallel current

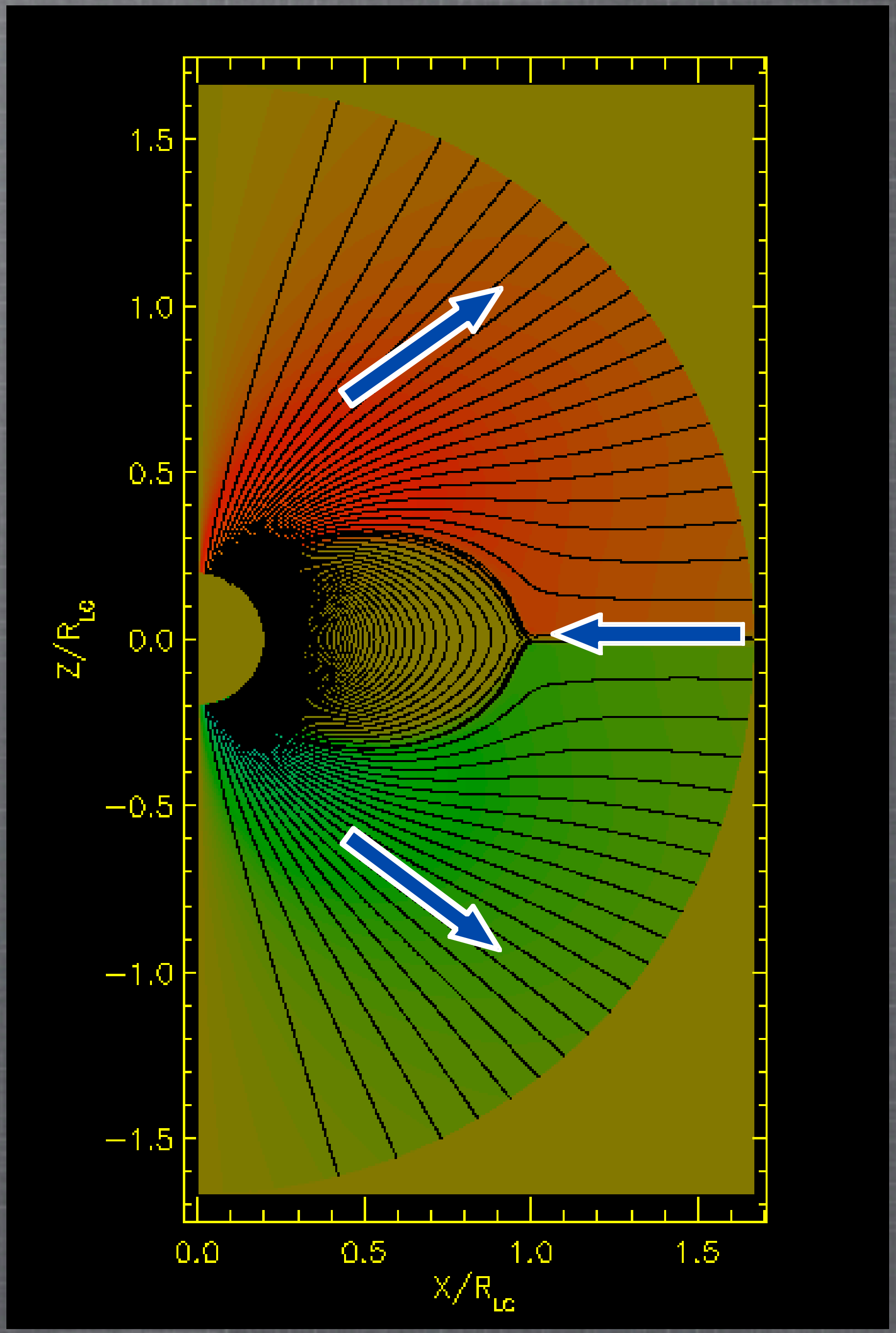
Gruzinov 99, Blandford 01

Where is plasma? Assumed to flow with ExB velocity, but velocity along the field is undefined. Plasma provides only charges and currents, no inertia.

Hyperbolic eqs. Use electromagnetic solvers to advance the system in time.

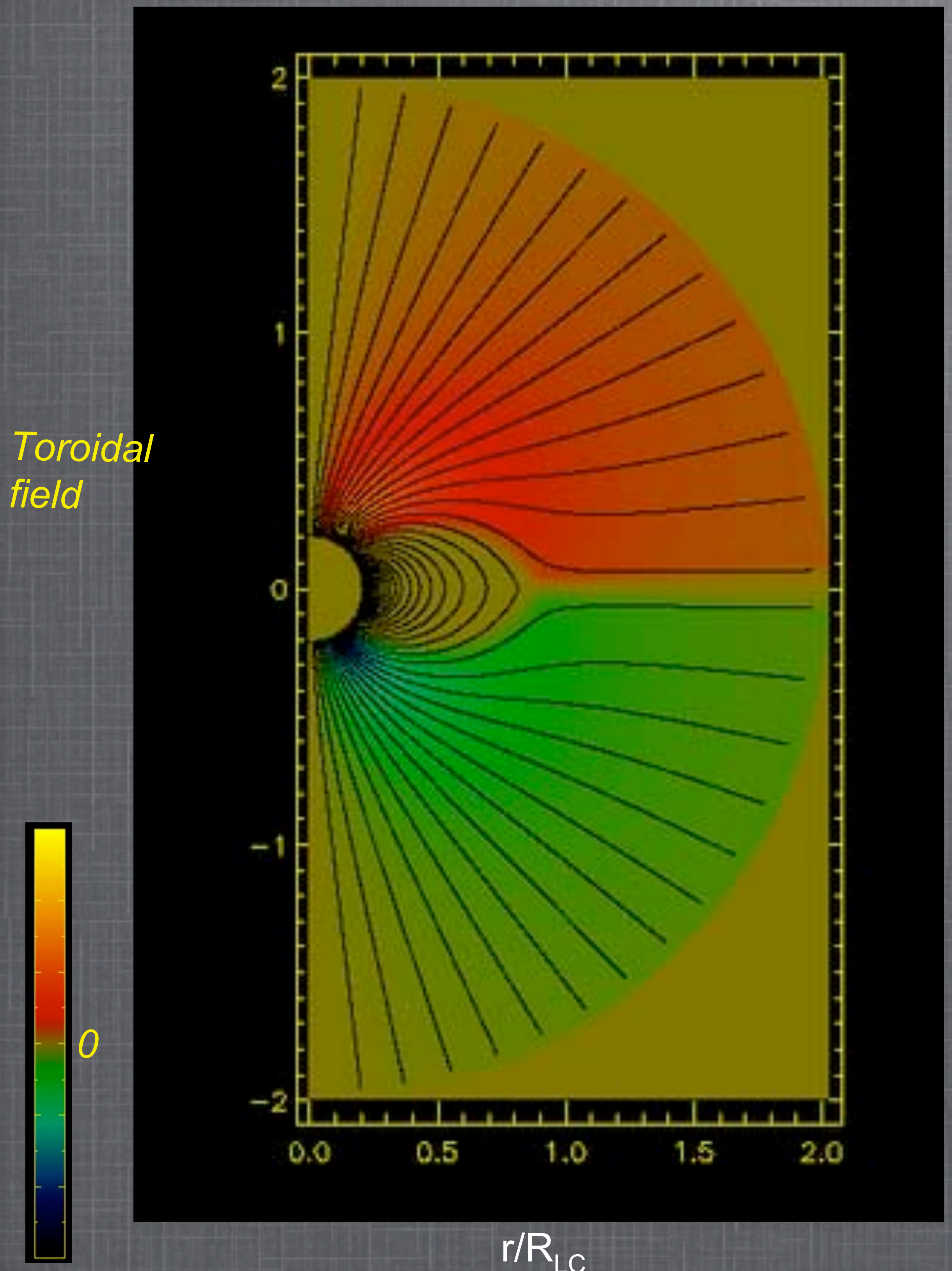
Aligned rotator: plasma magnetosphere

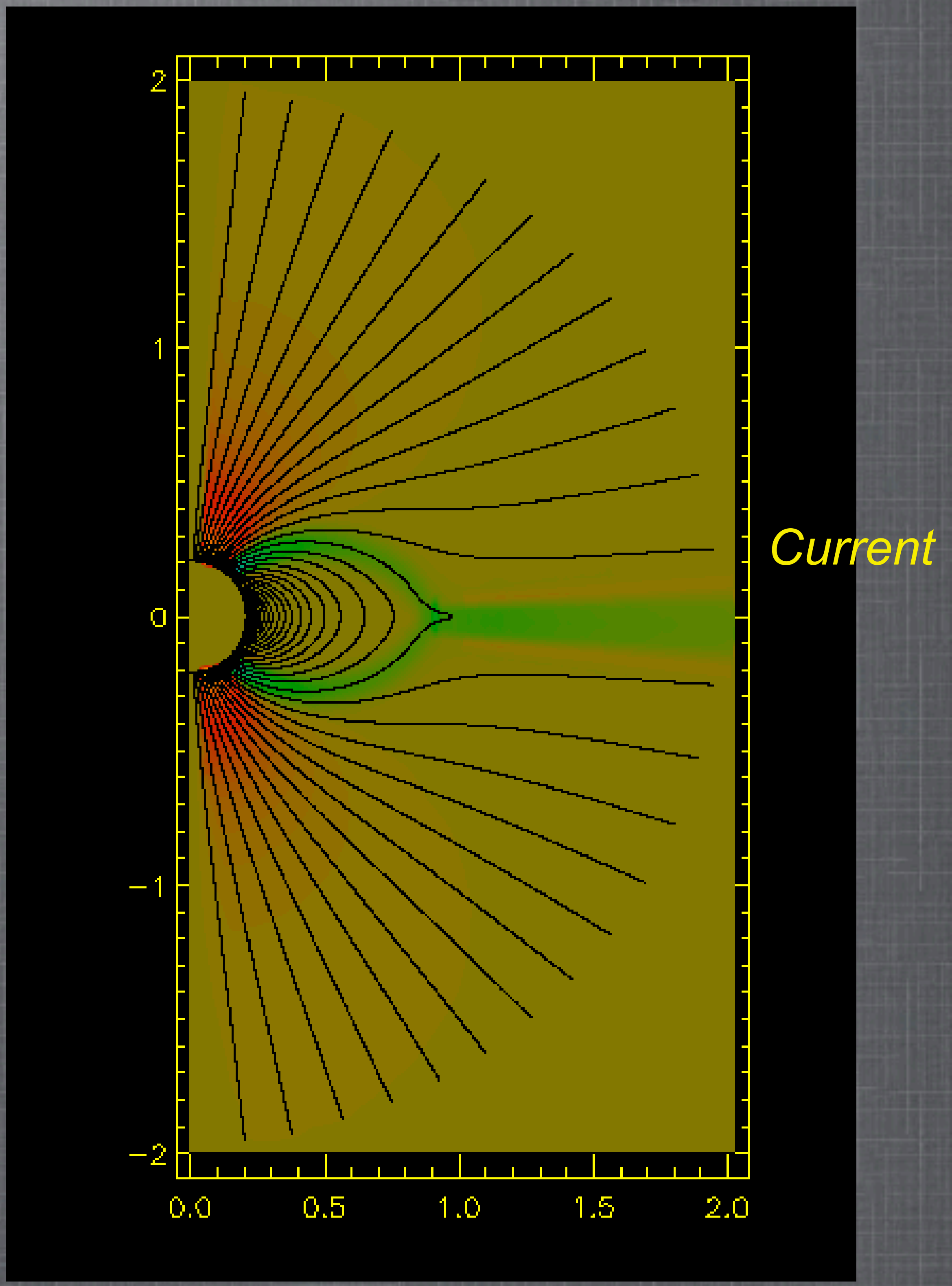




$$\dot{E} = \frac{\mu^2\Omega^4}{c^3} = cB_{LC}^2 R_{LC}^2 \quad \text{VS} \quad \dot{E}_{vac} = \frac{2}{3}\frac{\mu^2\Omega^4}{c^3}\sin^2\theta$$

Aligned rotator: plasma magnetosphere





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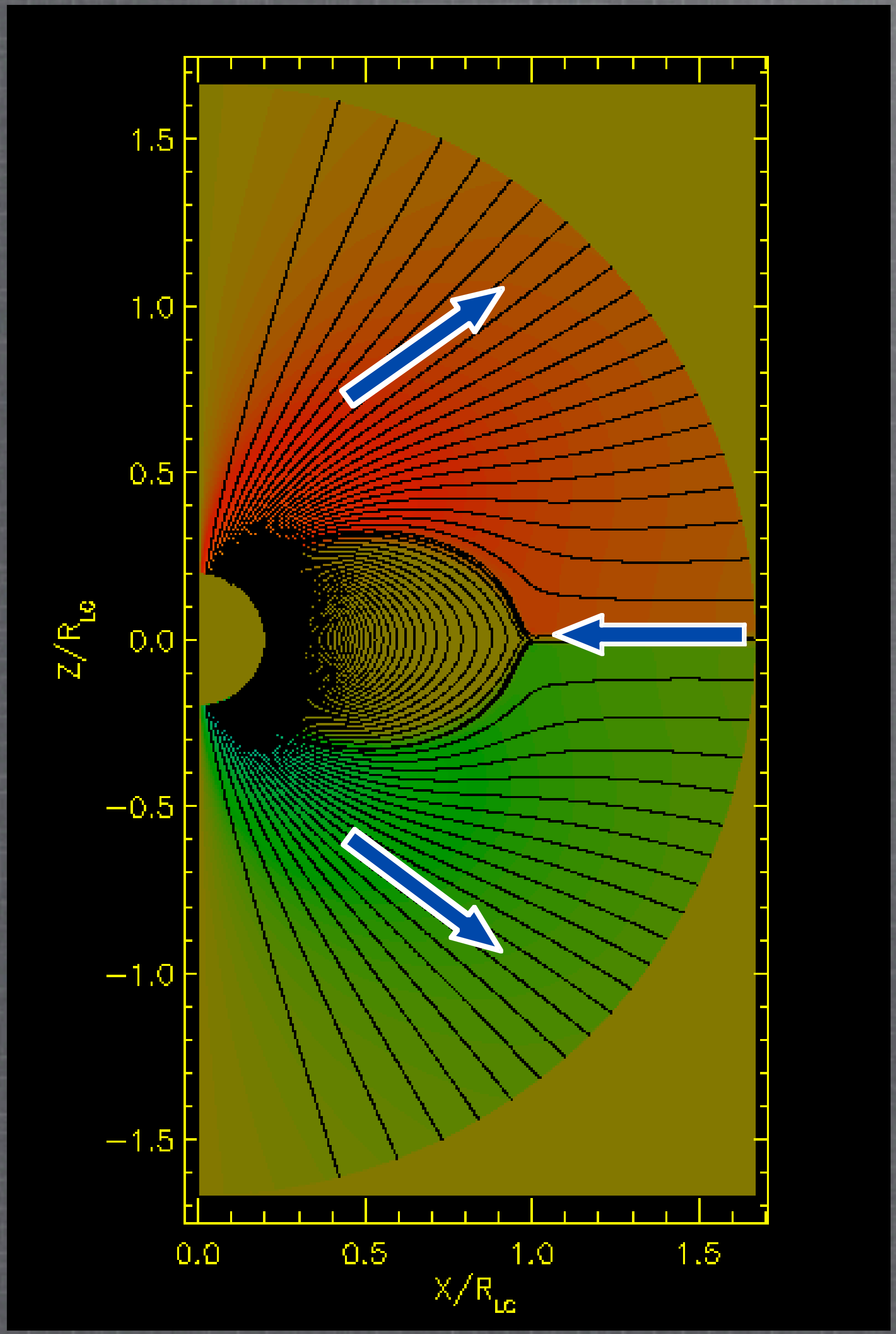
ALIGNED ROTATOR

Solution properties



Spontaneous formation of current sheet with field reversal

Asymptotically split monopole

Y-point field divergence; current splits around the closed zone

Closed zone extends to LC

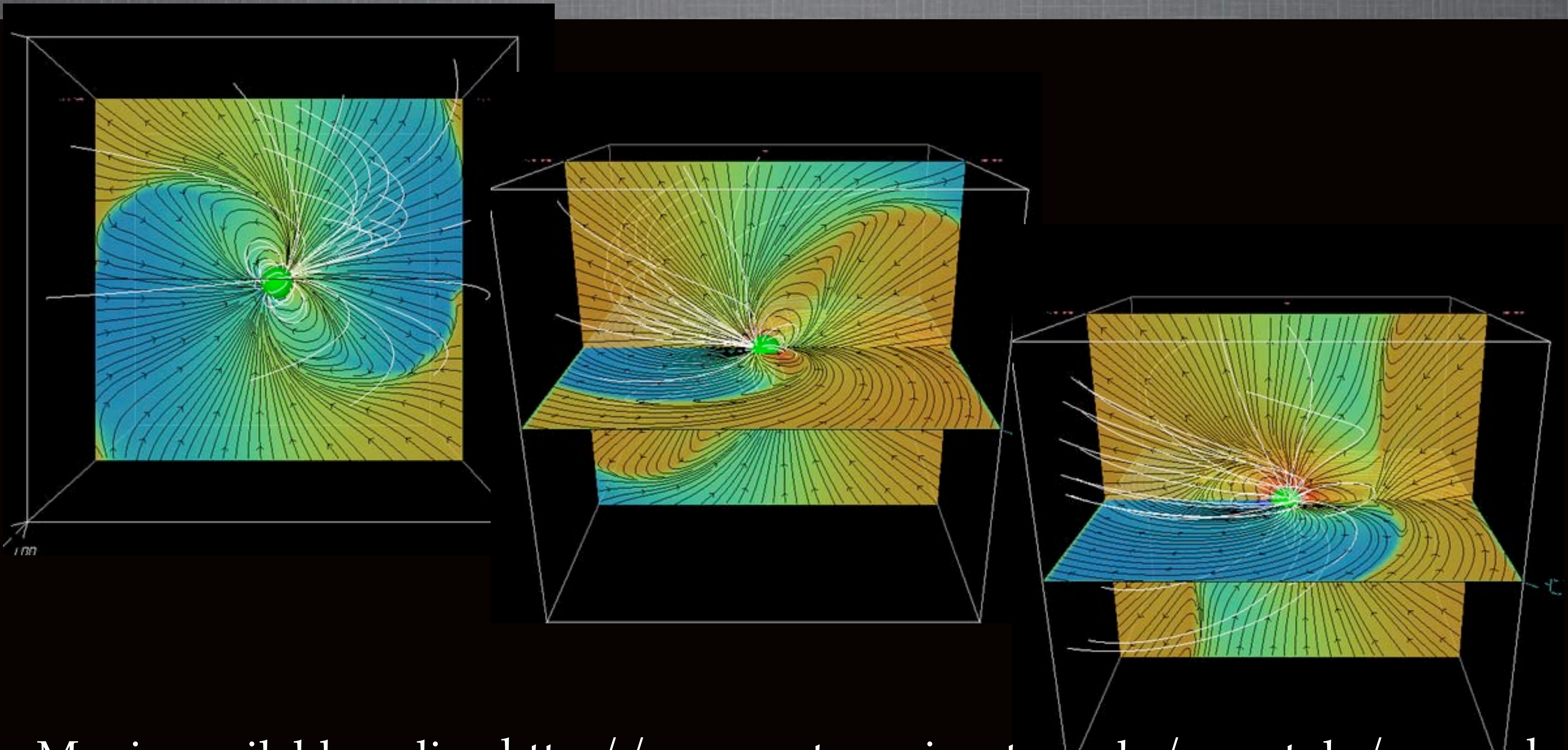
Spin-down is nonzero!

Several codes (time-dep and time-indep) agree. Even RMHD solution exists (Komissarov 2006)

Nothing interesting at null charge point

OBLIQUE ROTATOR

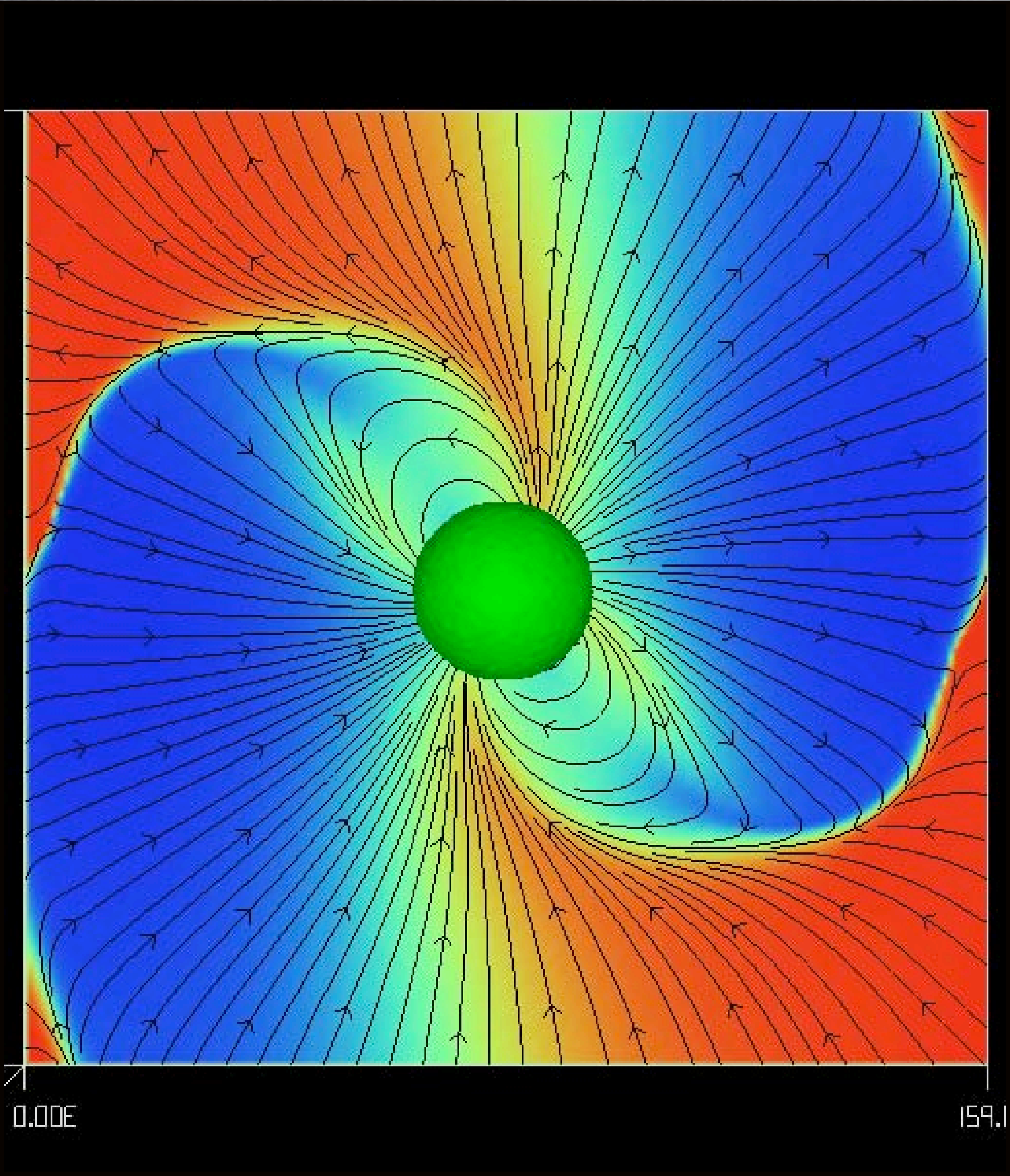
Force-free solution

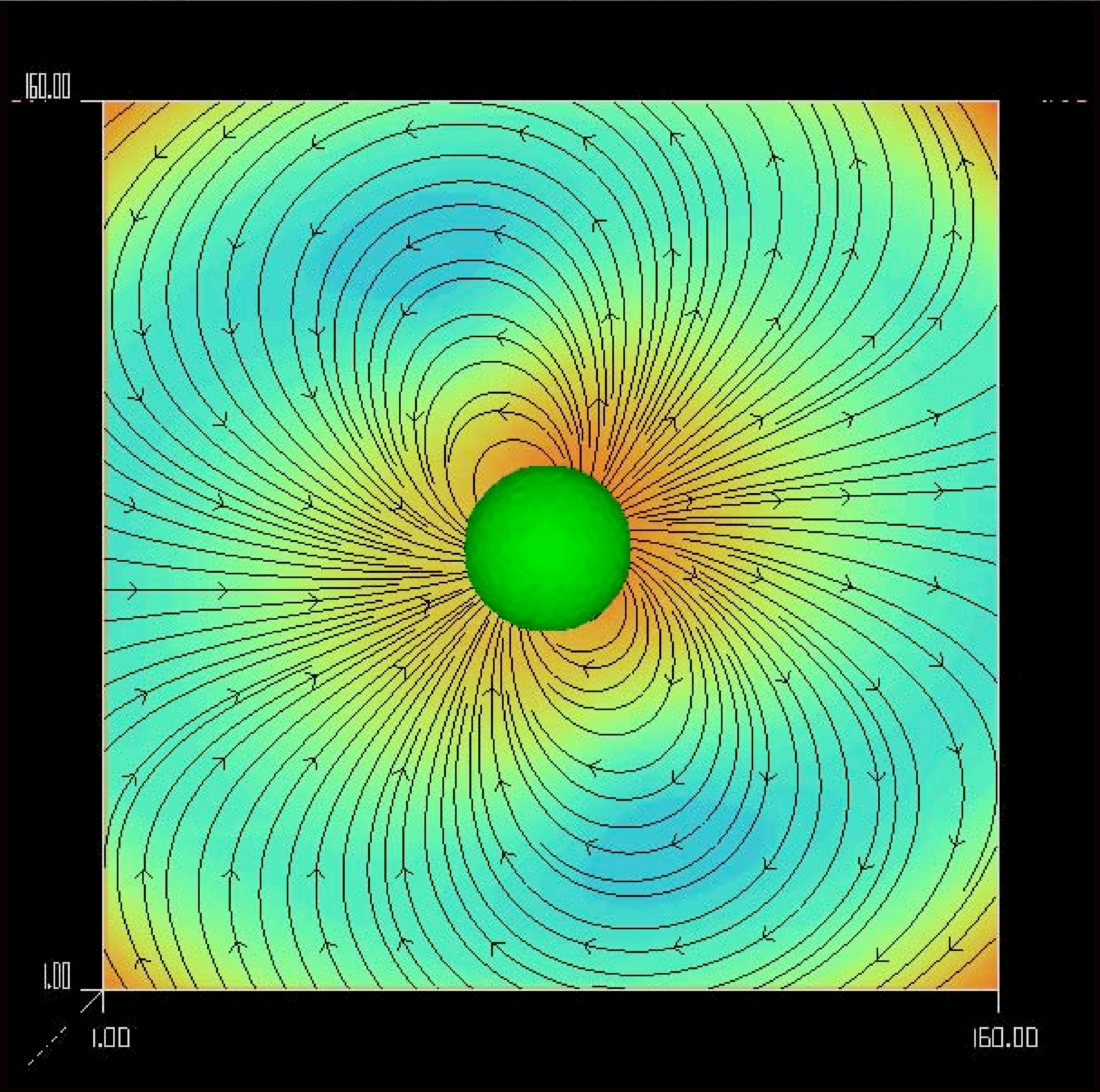


Movie available online http://www.astro.princeton.edu/~anatoly/research

IN COROTATING FRAME

60 degree inclination



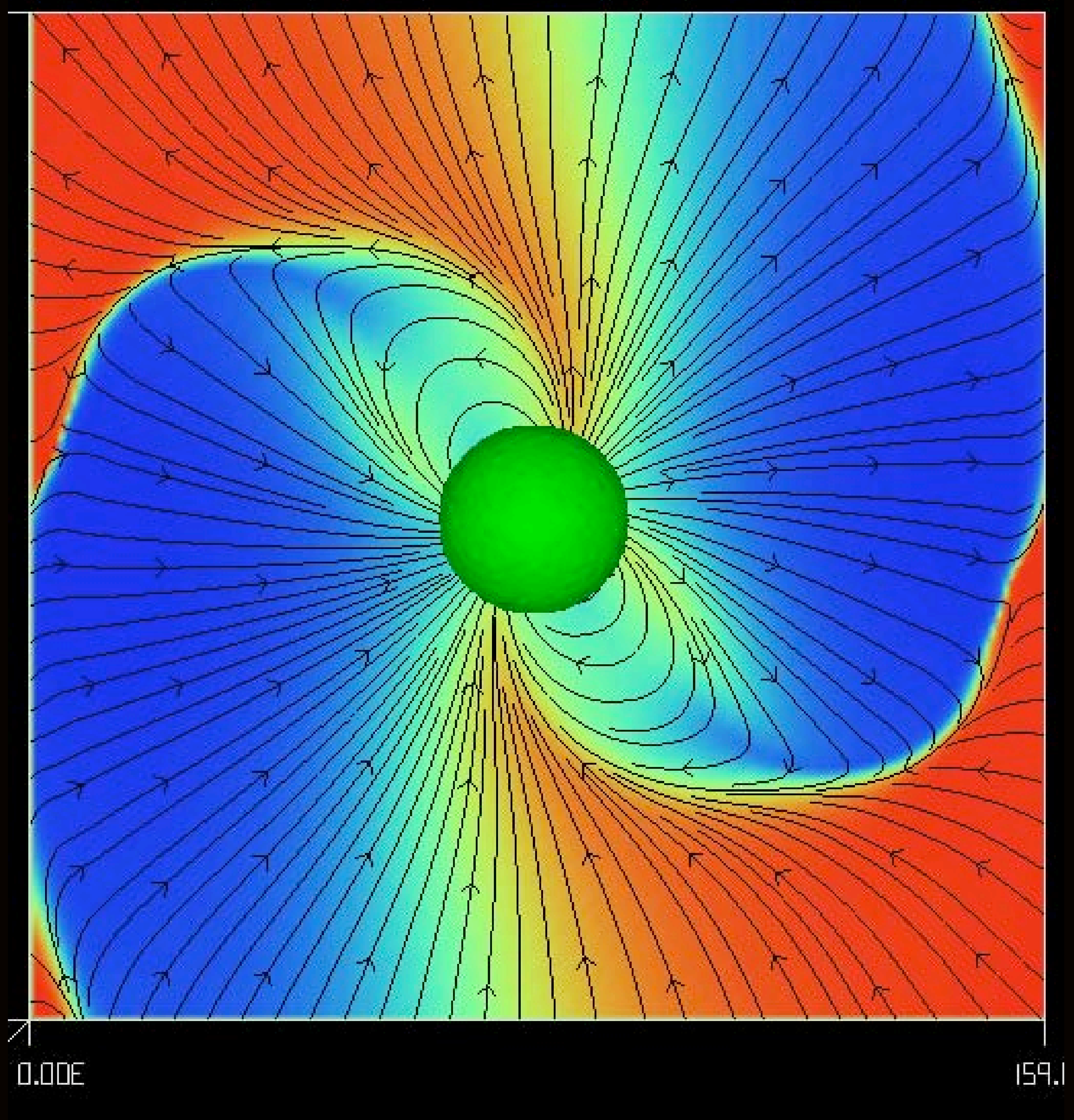


Force-free

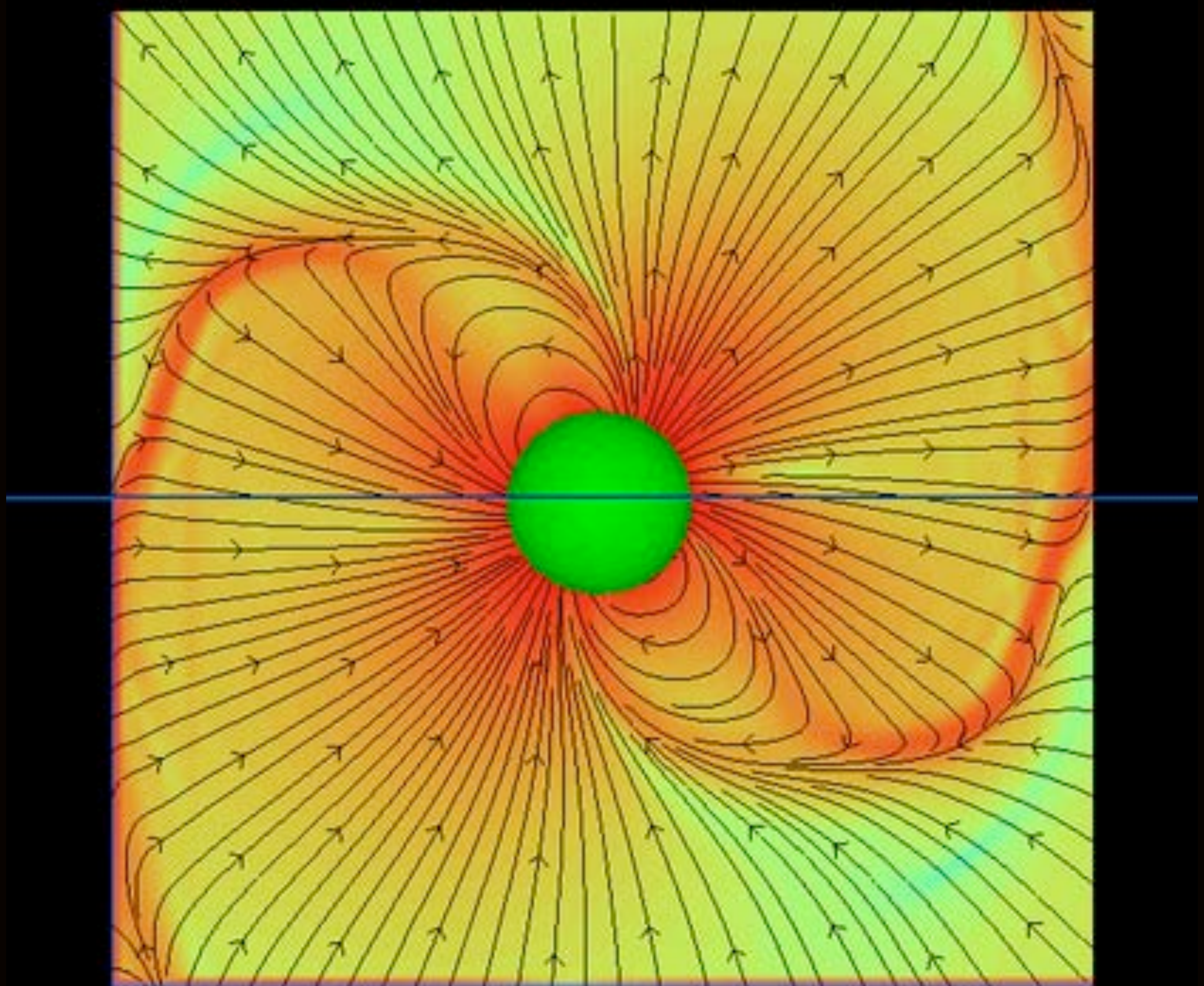
Vacuum in mu-Omega plane

IN COROTATING FRAME

60 degree inclination



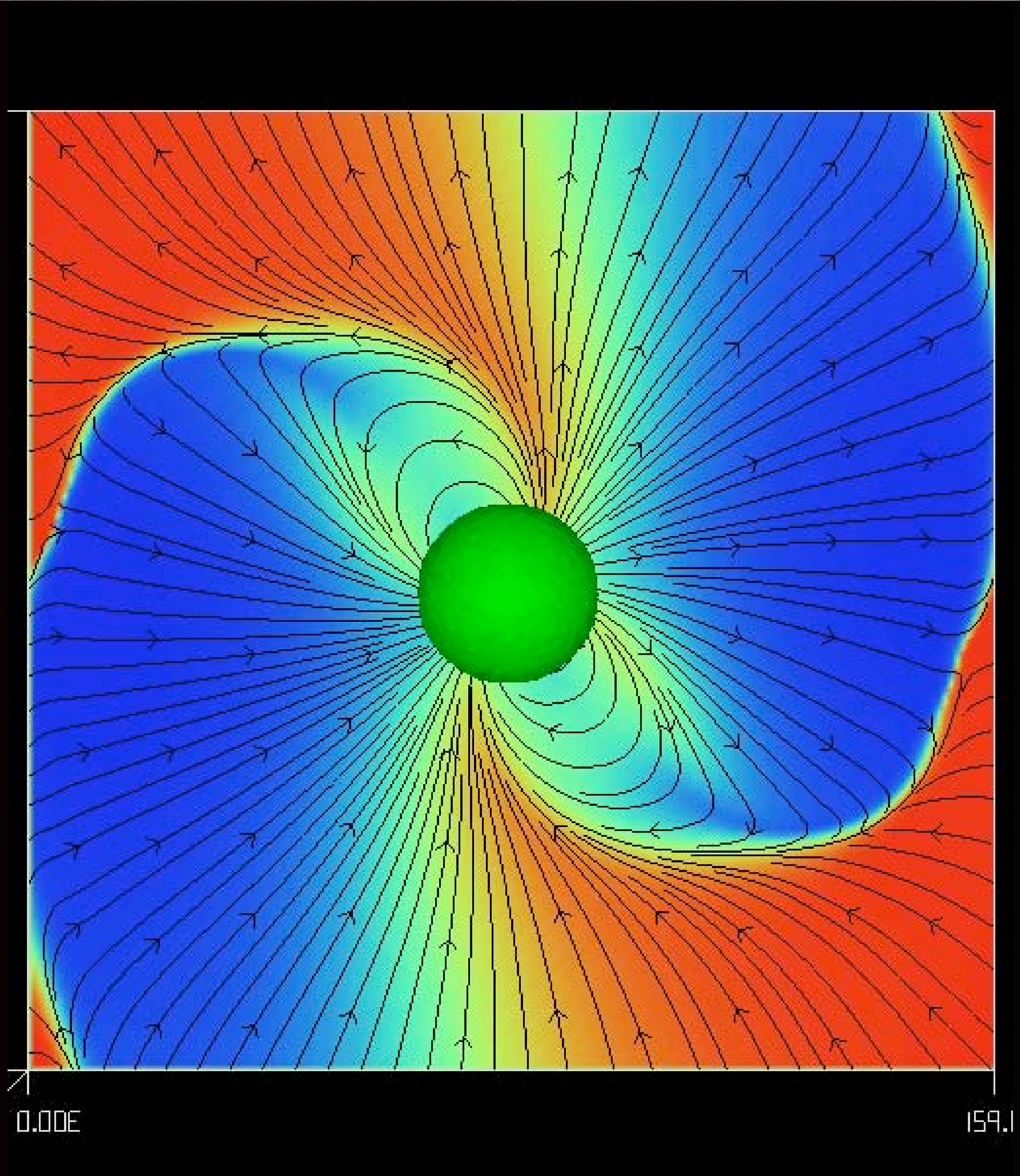
Force-free

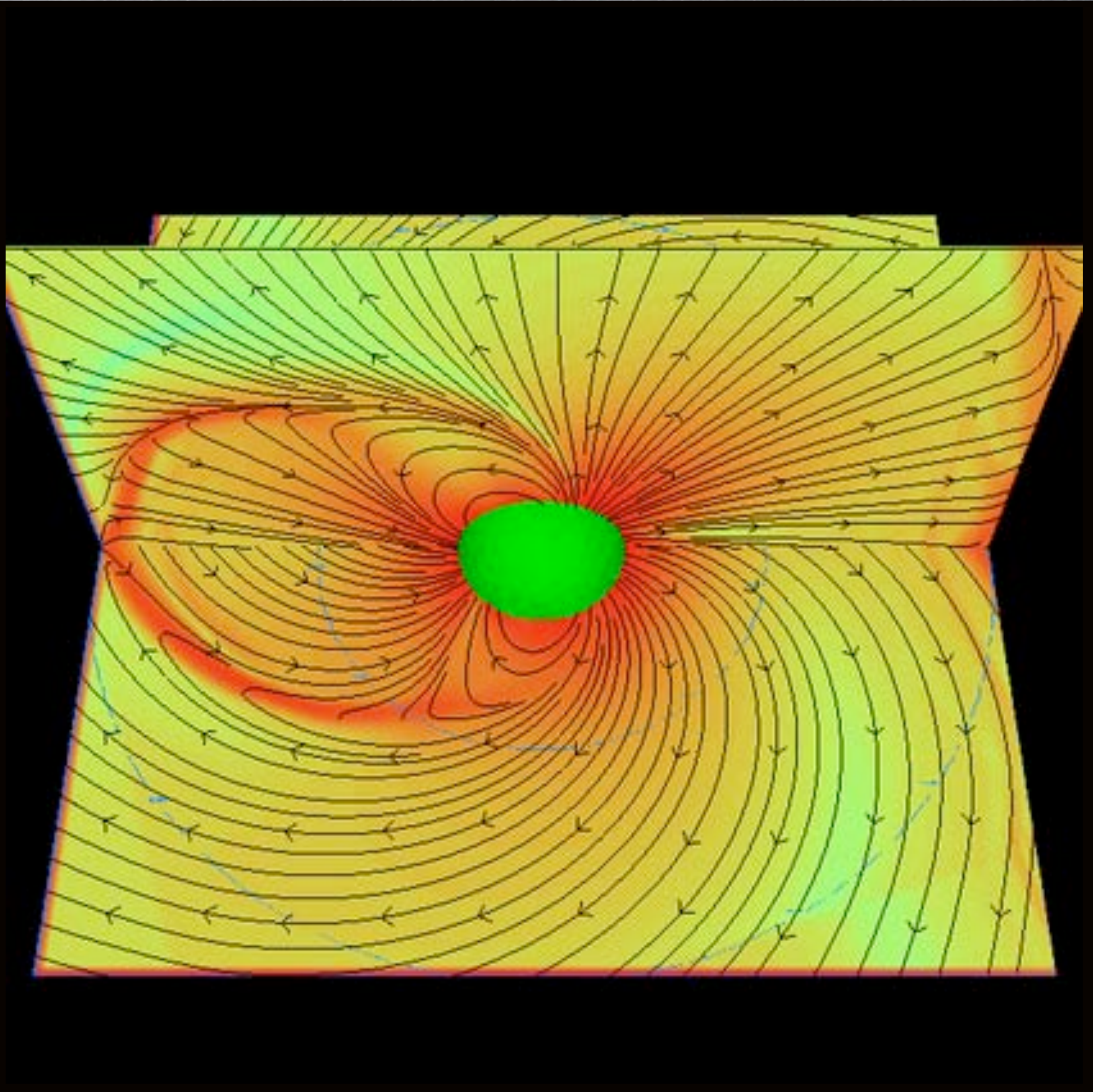


Force-free current density

IN COROTATING FRAME

60 degree inclination



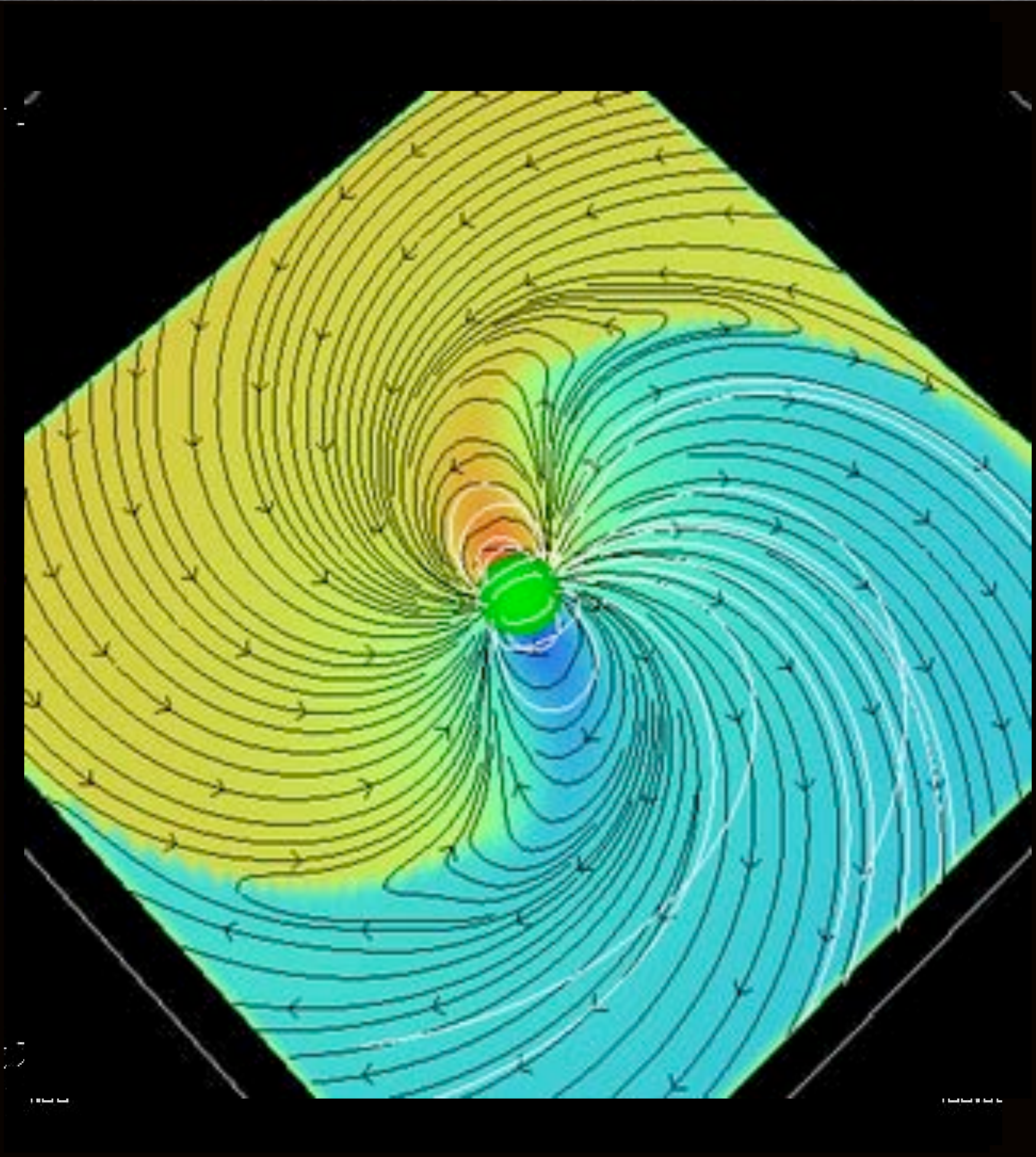


Force-free

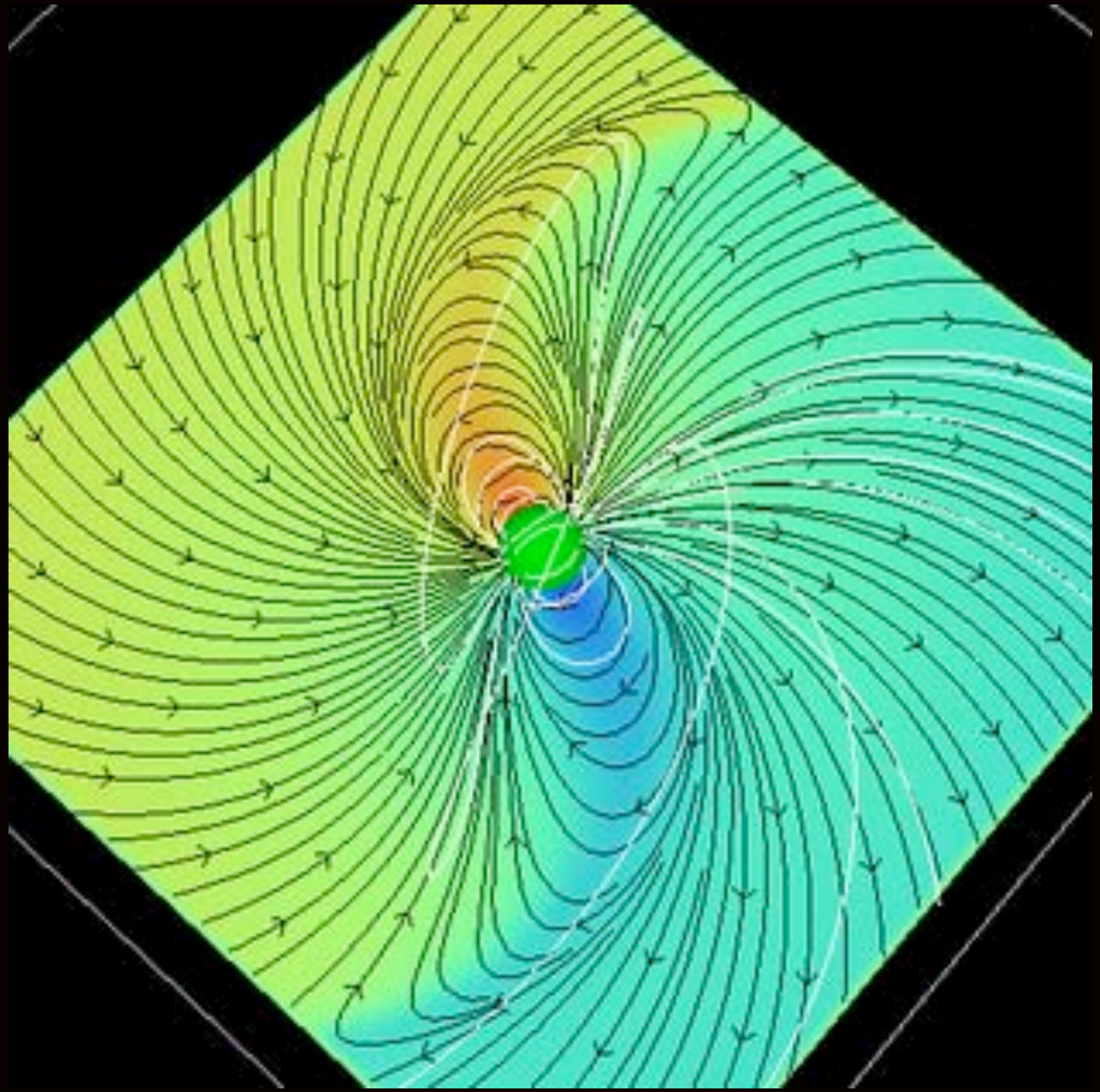
Force-free current density

Comparison with vacuum

90 degree inclination



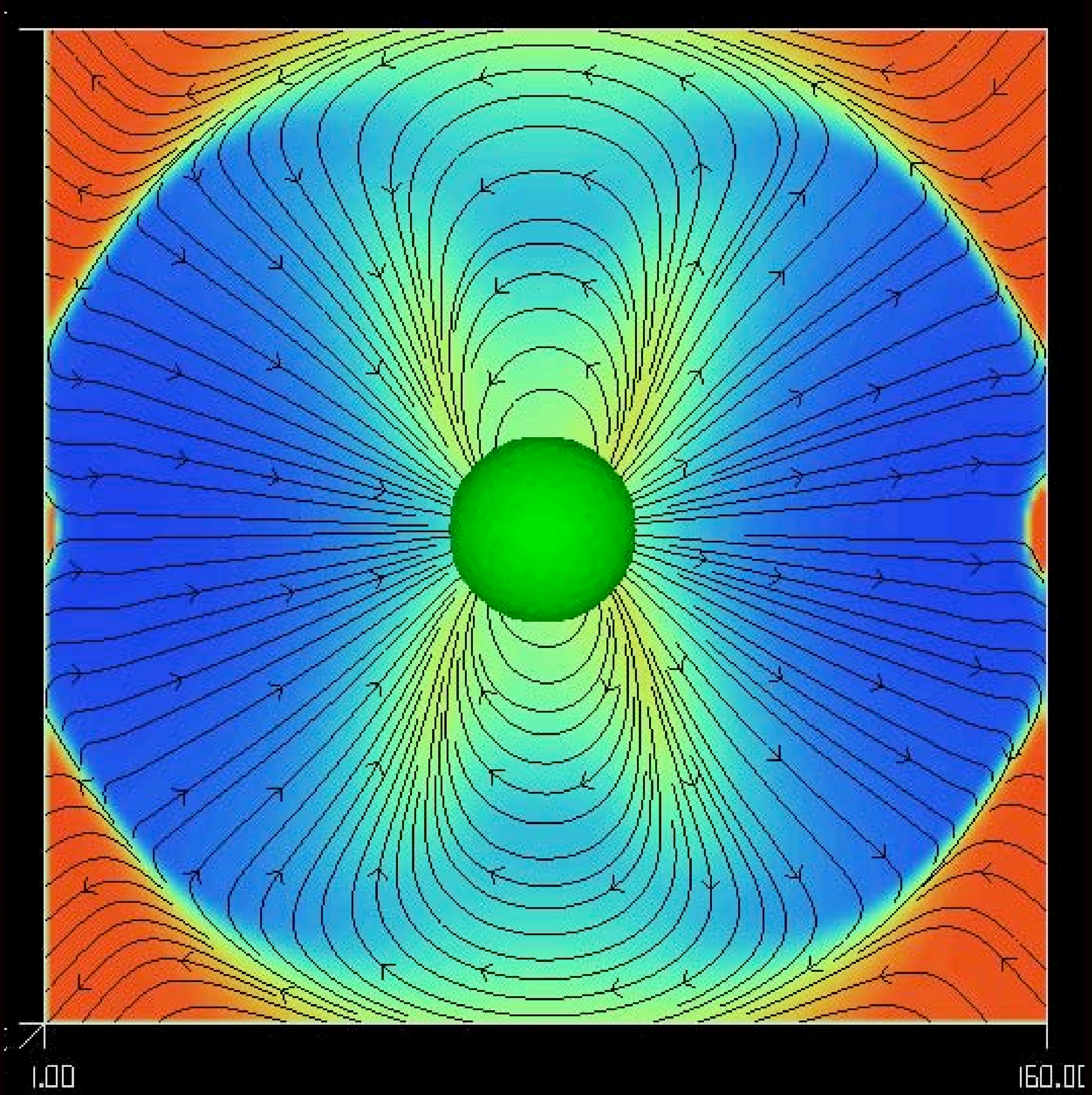
Force-free



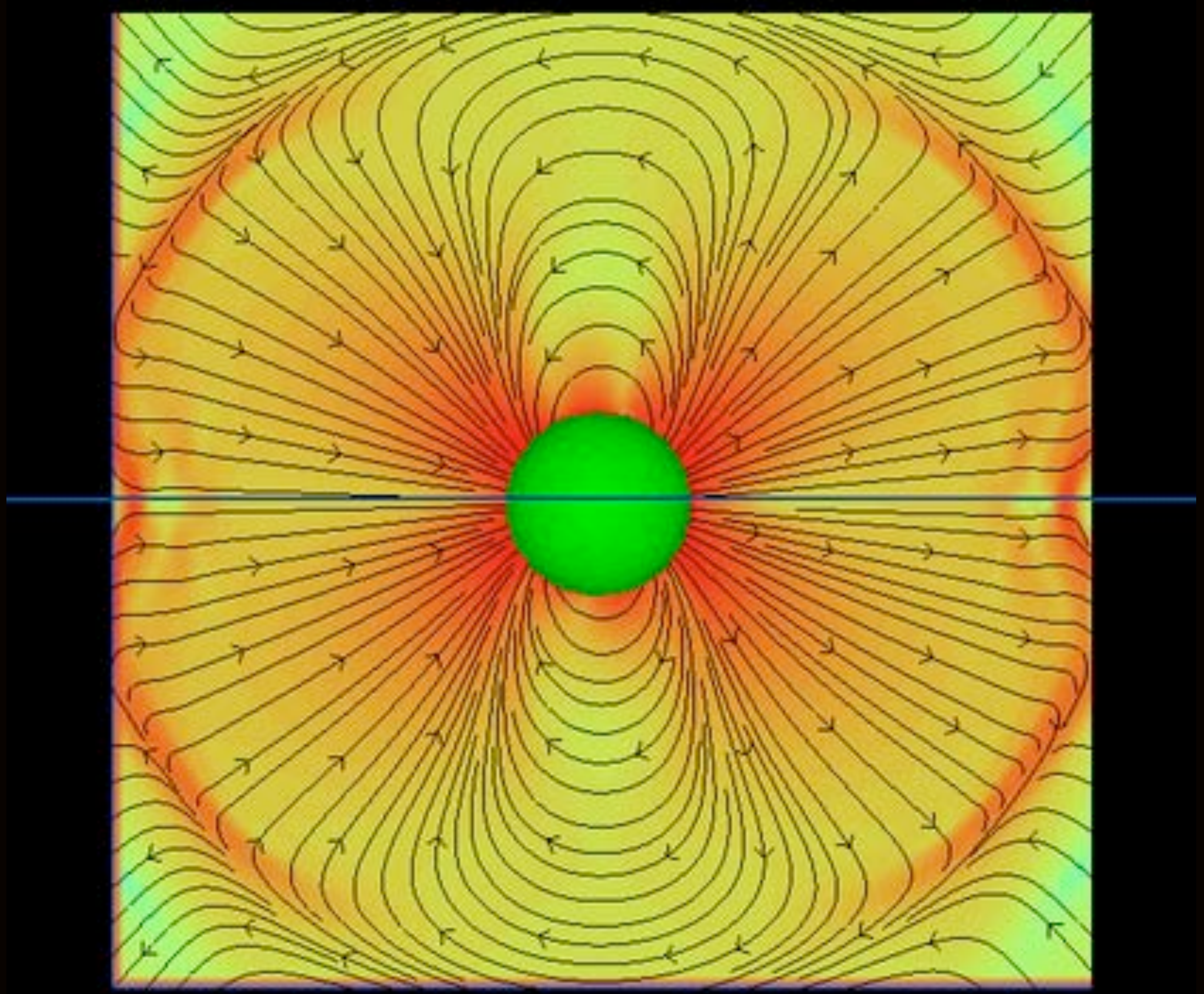
Vacuum

IN COROTATING FRAME

90 degree inclination



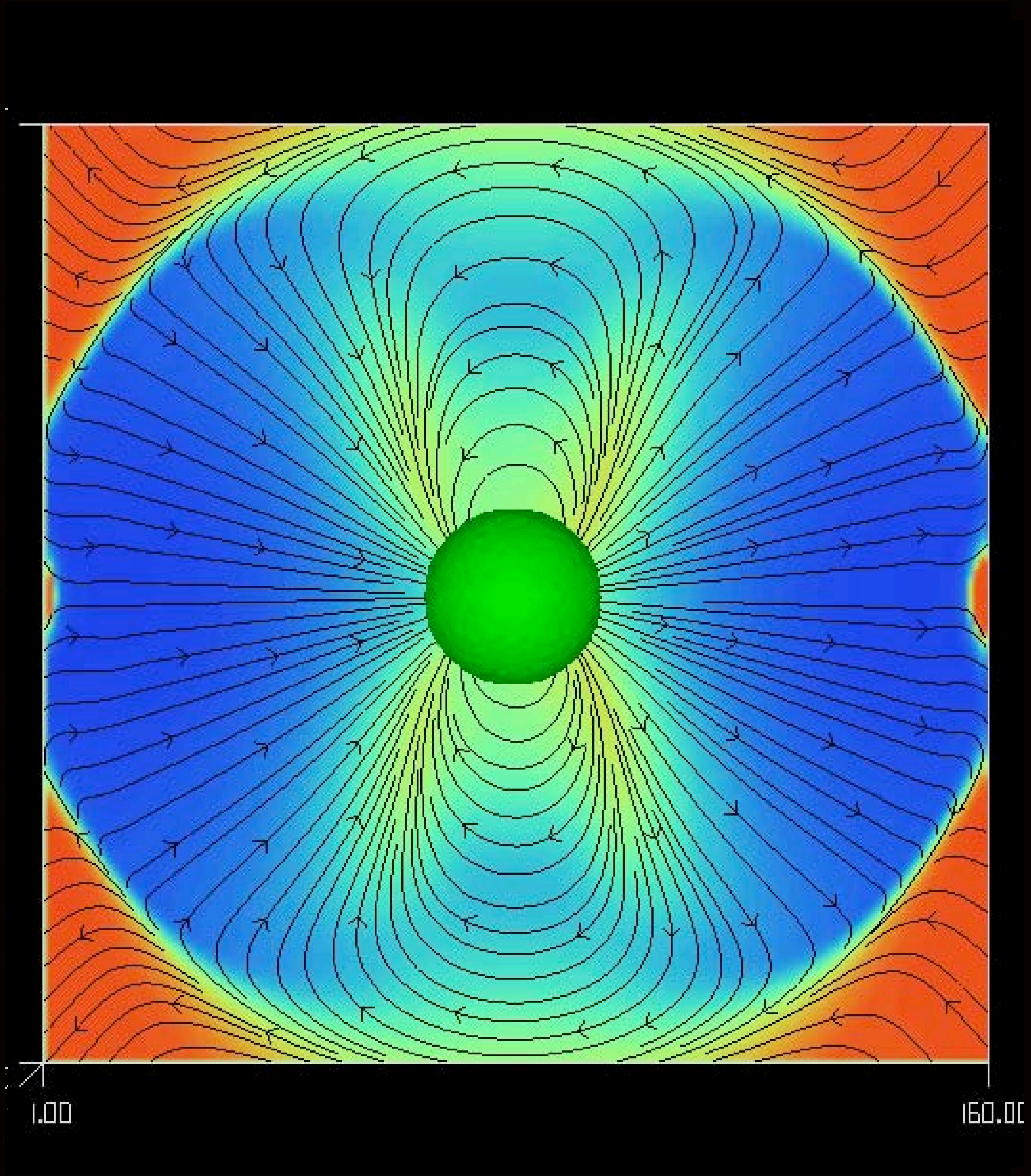
Force-free

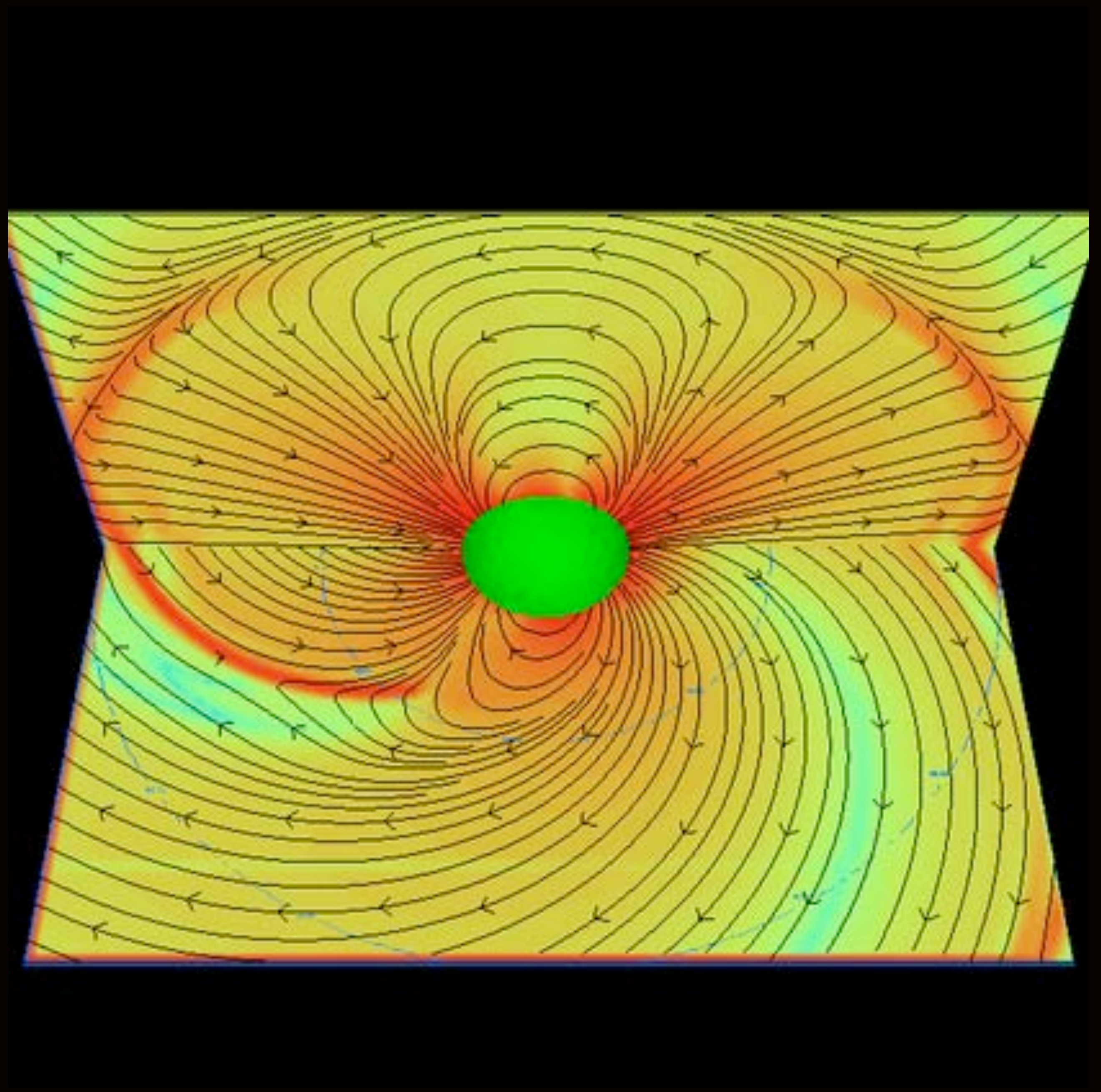


Force-free current density

IN COROTATING FRAME

90 degree inclination

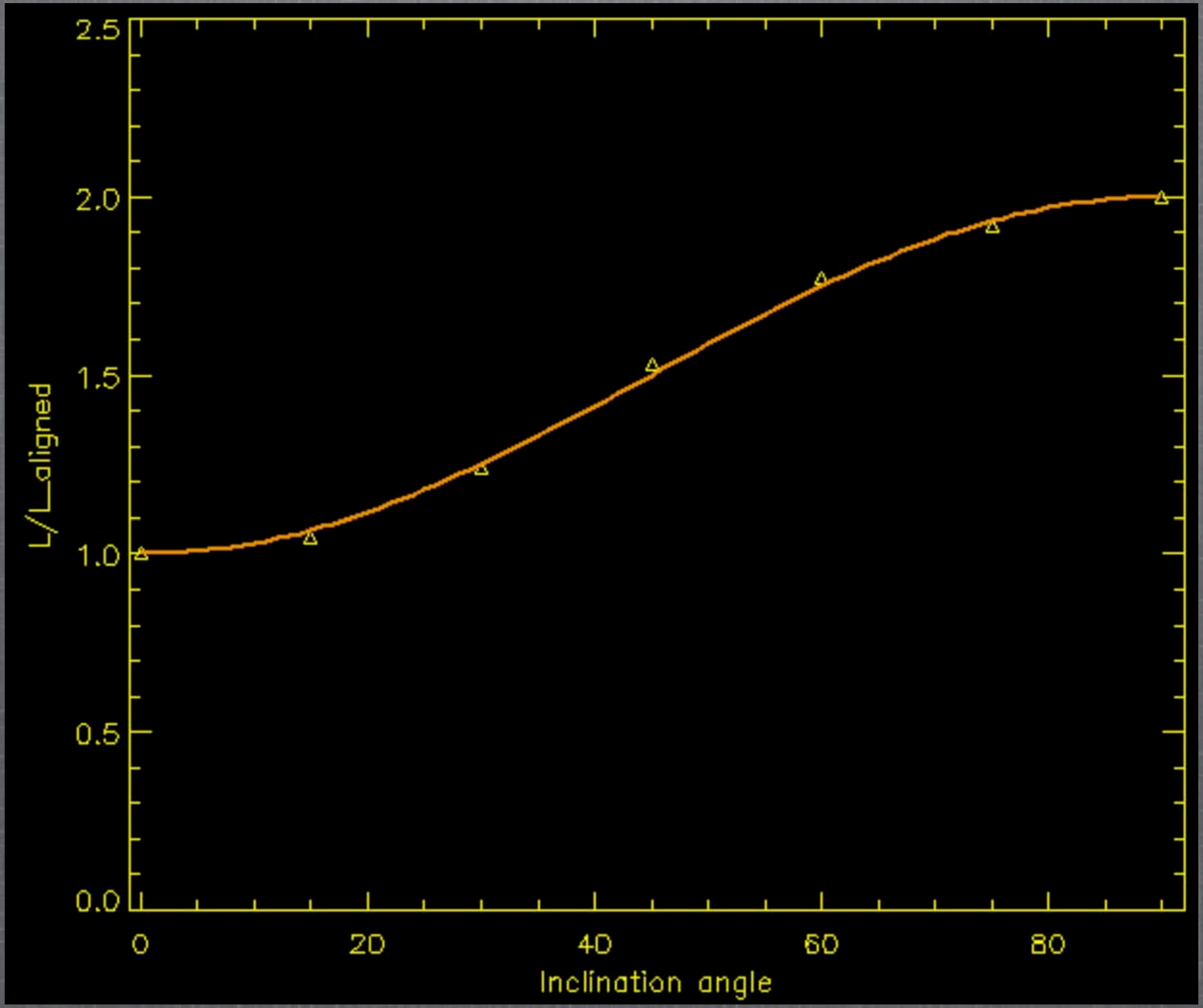




Force-free

Force-free current density

SPIN-DOWN POWER



Spin-down of oblique rotator

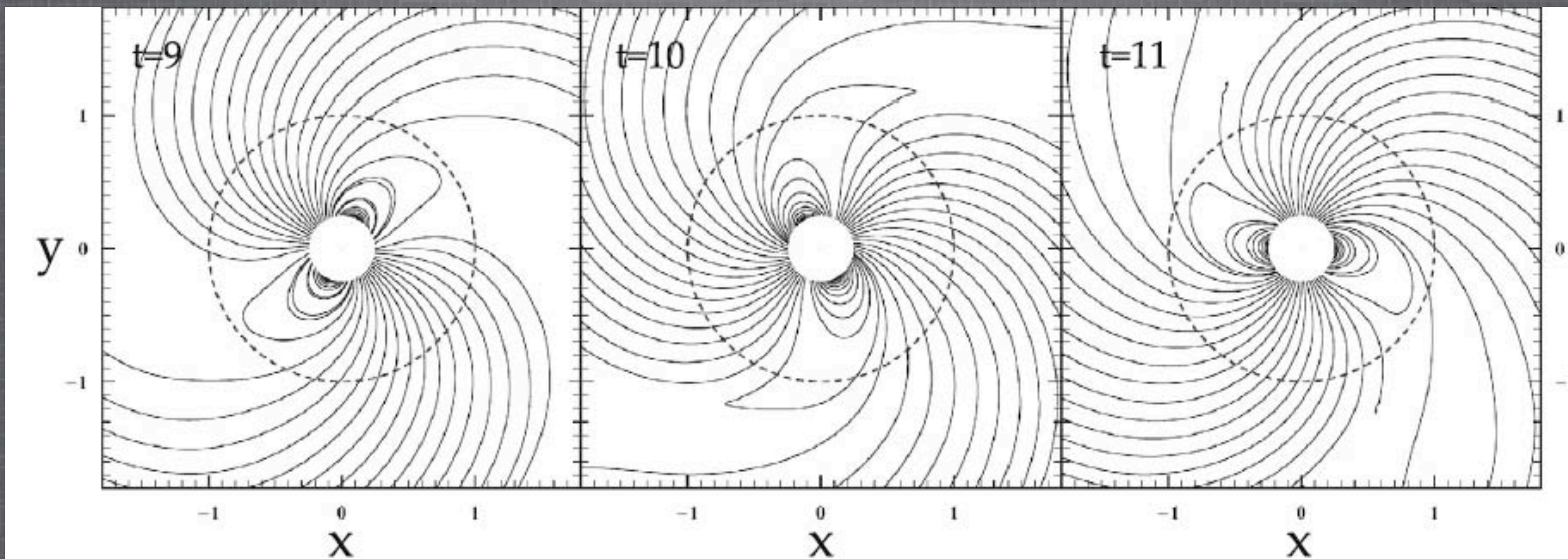
$$\dot{E} = \frac{\mu^2\Omega^4}{c^3}(1+\sin^2\theta)$$

$$\dot{E}_{vac} = \frac{2}{3}\frac{\mu^2\Omega^4}{c^3}\sin^2\theta$$

NB: this is a fit!

Independent Confirmation

Kalapotharakos & Contopoulos 2009



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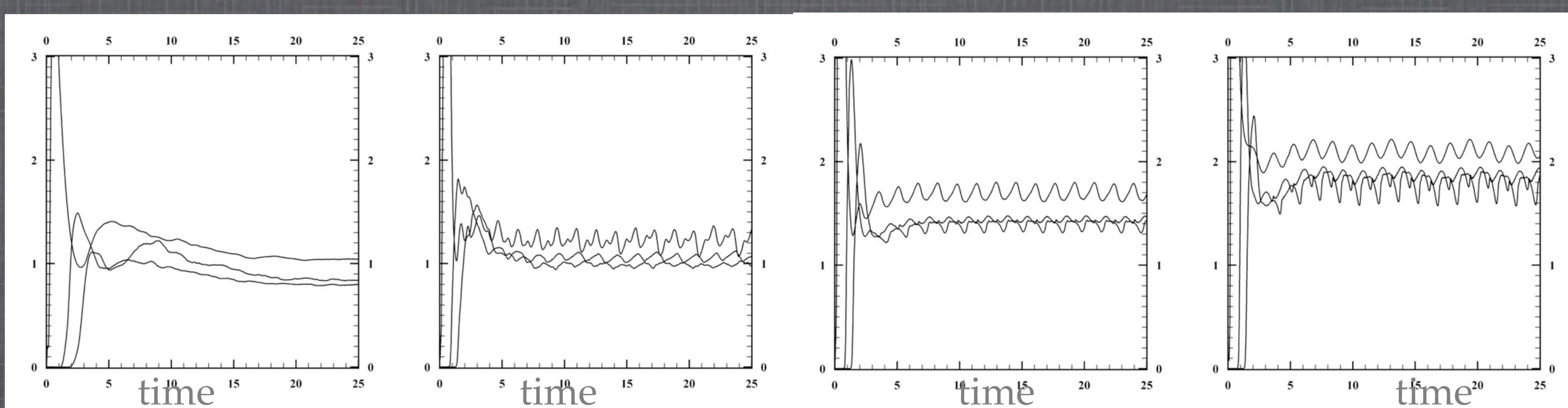


IMPLICATIONS

Force-free pulsar solution

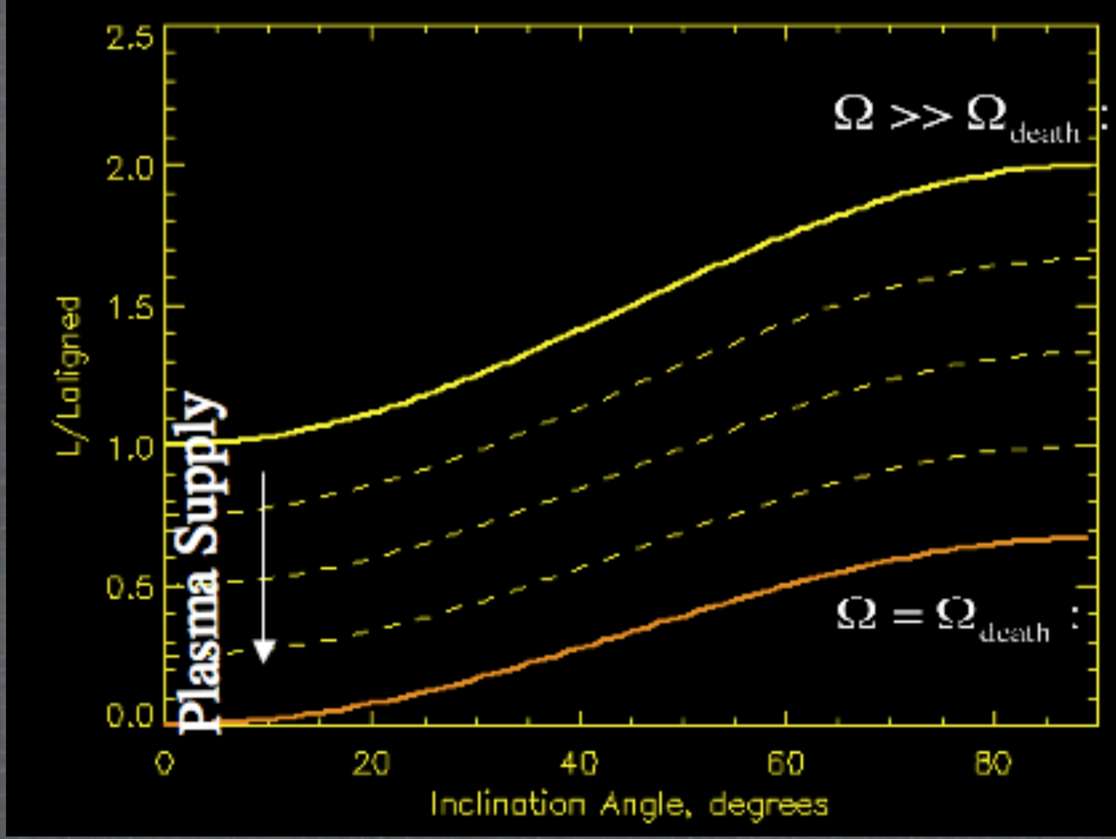
B field estimates with vacuum field are close to truth (if do not use sin^2 term)

$$B_{PSR} = 2.6 \times 10^{19} \sqrt{P\dot{P}/(1+\sin^2\theta)}\ \mathrm{G}$$

$$B_{vac} = 3.2 \times 10^{19} \sqrt{P\dot{P}}\ \mathrm{G}$$

Oblique rotators spin down faster -- expect excess of aligned rotators near the death line, as the oblique ones move out.

Starvation of plasma supply will lead to modification of spindown: applications to pulsars that turn on and off?



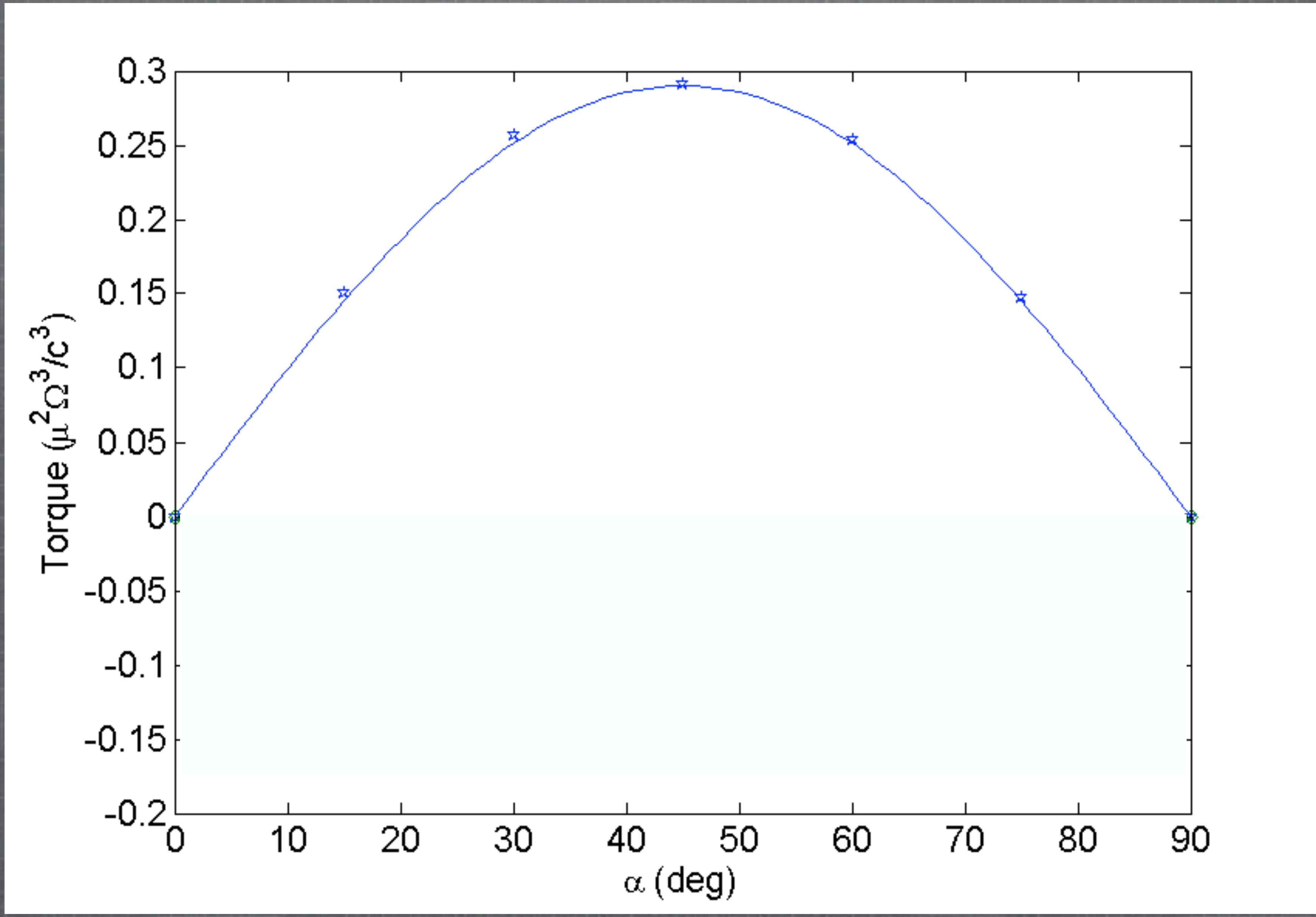
$$\dot{E} \approx \frac{\mu^2\Omega^4}{c^3}\left(1+\sin^2\theta\right)$$

$$\dot{E} \approx \frac{\mu^2\Omega^4}{c^3}\left(1 - \frac{\Omega_d}{\Omega} + f\left(\frac{\Omega_d}{\Omega}\right)\sin^2\theta\right)$$

$$@\ \Omega = \Omega_{\rm death} : \quad \dot{E}_{vac} = \frac{2}{3}\frac{\mu^2\Omega^4}{c^3}\sin^2\theta$$

Torque

Force-free pulsar solution

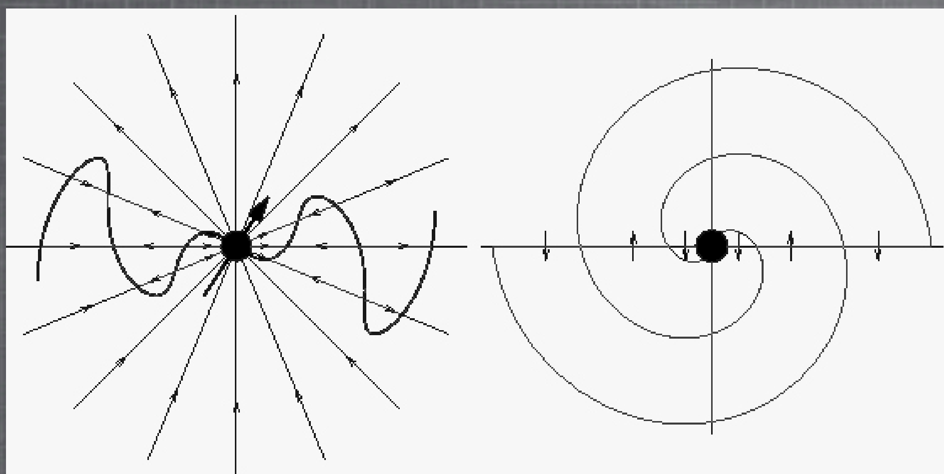


Torque is always aligning, but varies with angle (max at 45 degrees)
Alignment time (neglecting dissipation in NS) is 10x spin-down time.

IMPLICATIONS

Force-free pulsar solution

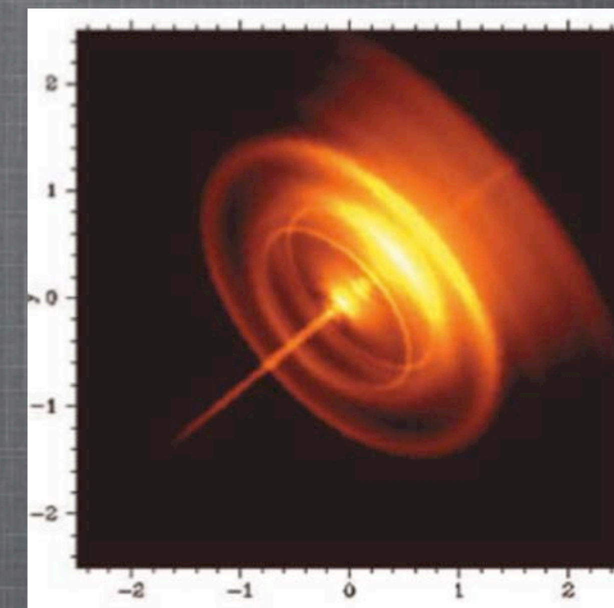
Bogovalov 1999

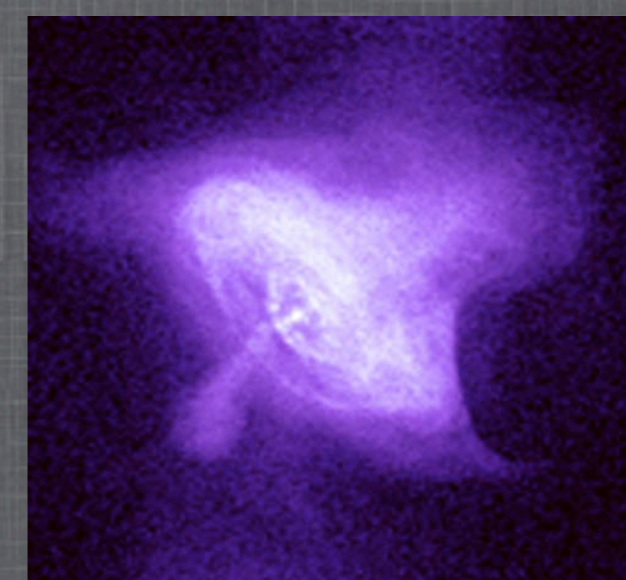


Simulations of PWN require latitude-dependent energy injection, which is naturally provided by the model with reconnecting reversing fields in the equatorial zone.

$$B_{\varphi} \propto \sin\theta\left(1-\frac{2\theta}{\pi}\right)$$

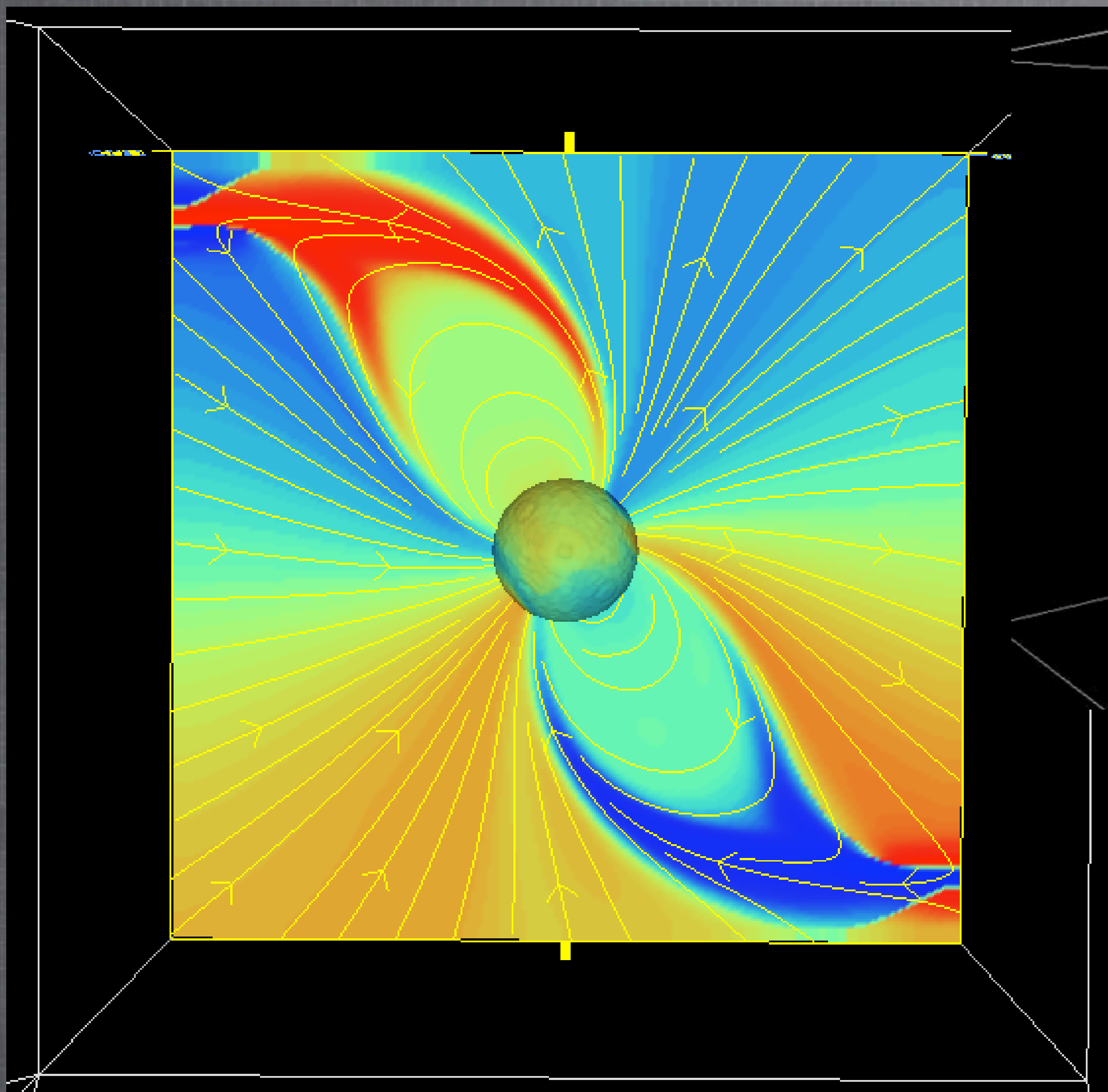
Komissarov & Lyubarsky

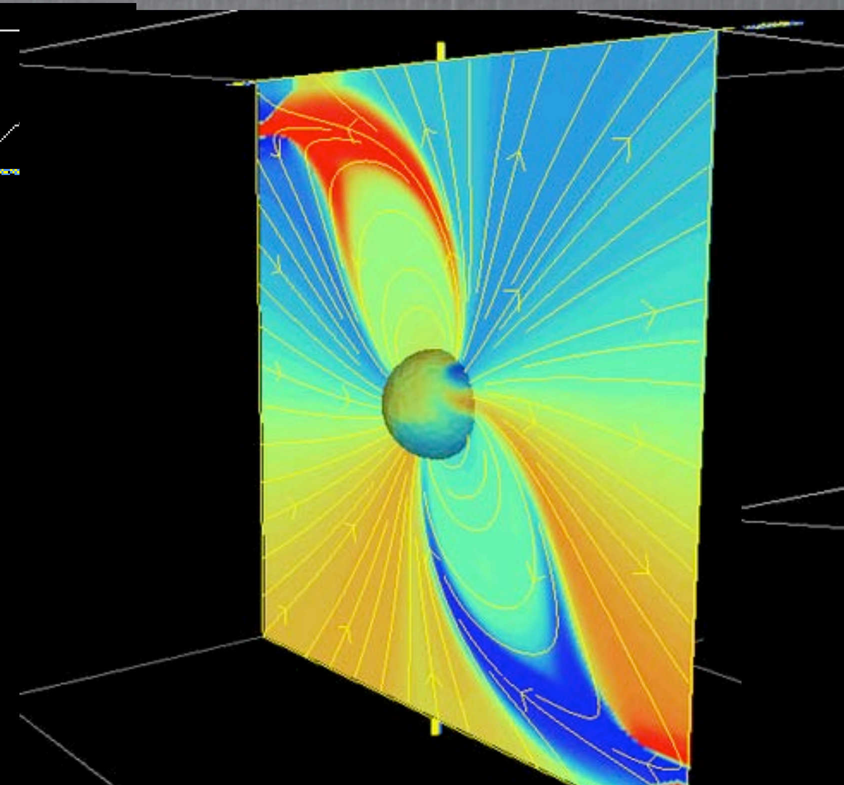




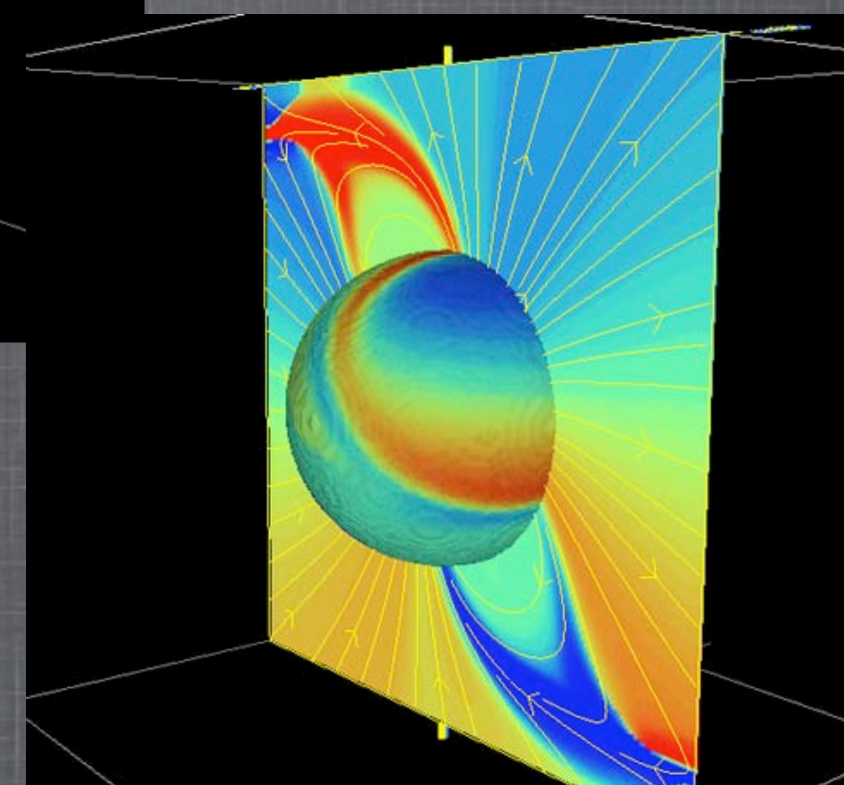
Magnetospheric Currents

Force-free MHD





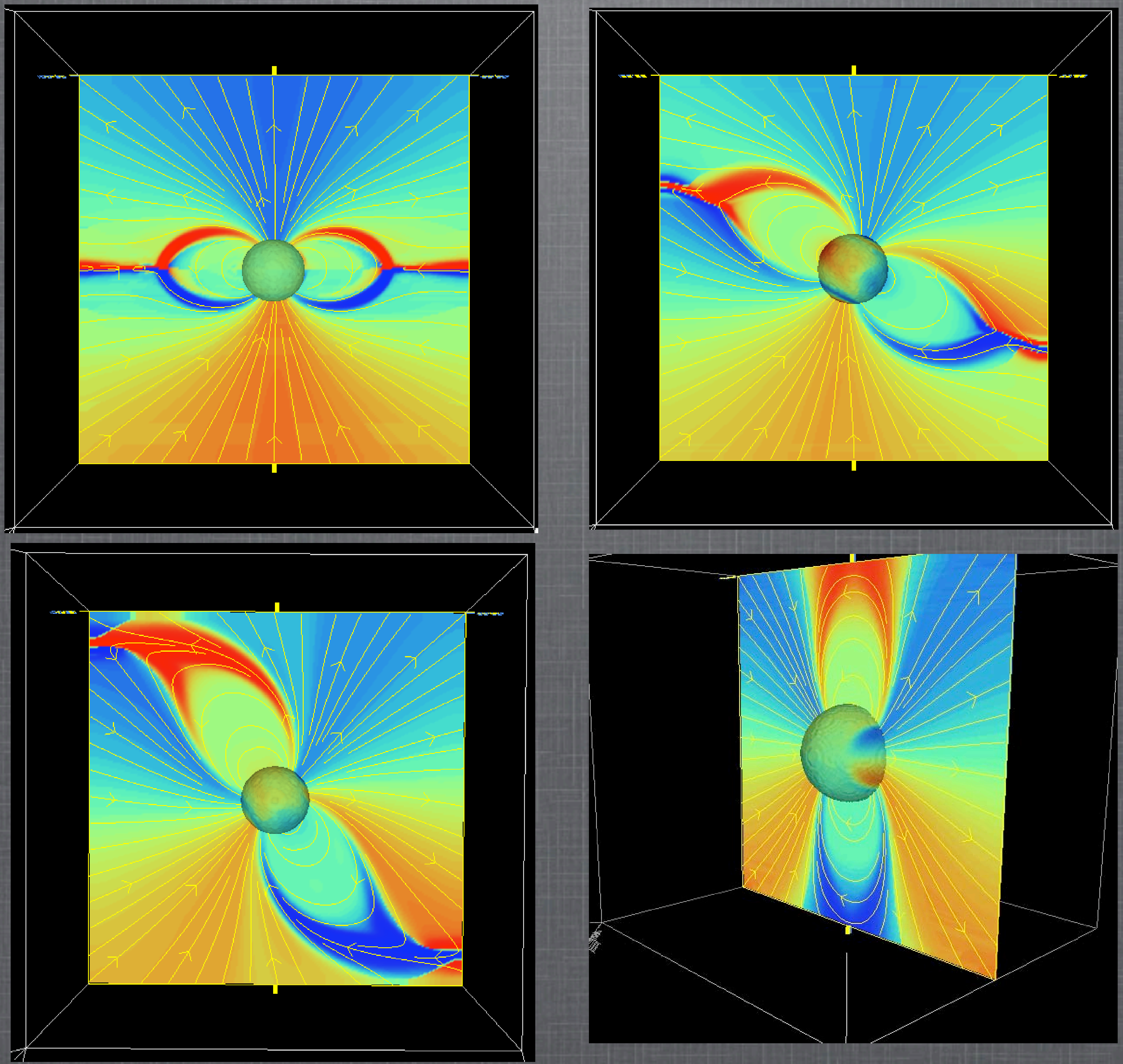
Current at different radii



$$\lambda = \nabla \times (\mathbf{B} + \mathbf{V} \times (\mathbf{V} \times \mathbf{B})) \cdot \mathbf{B} / \mathbf{B}^2; \quad \mathbf{V} = \mathbf{\Omega} \times \mathbf{R}$$

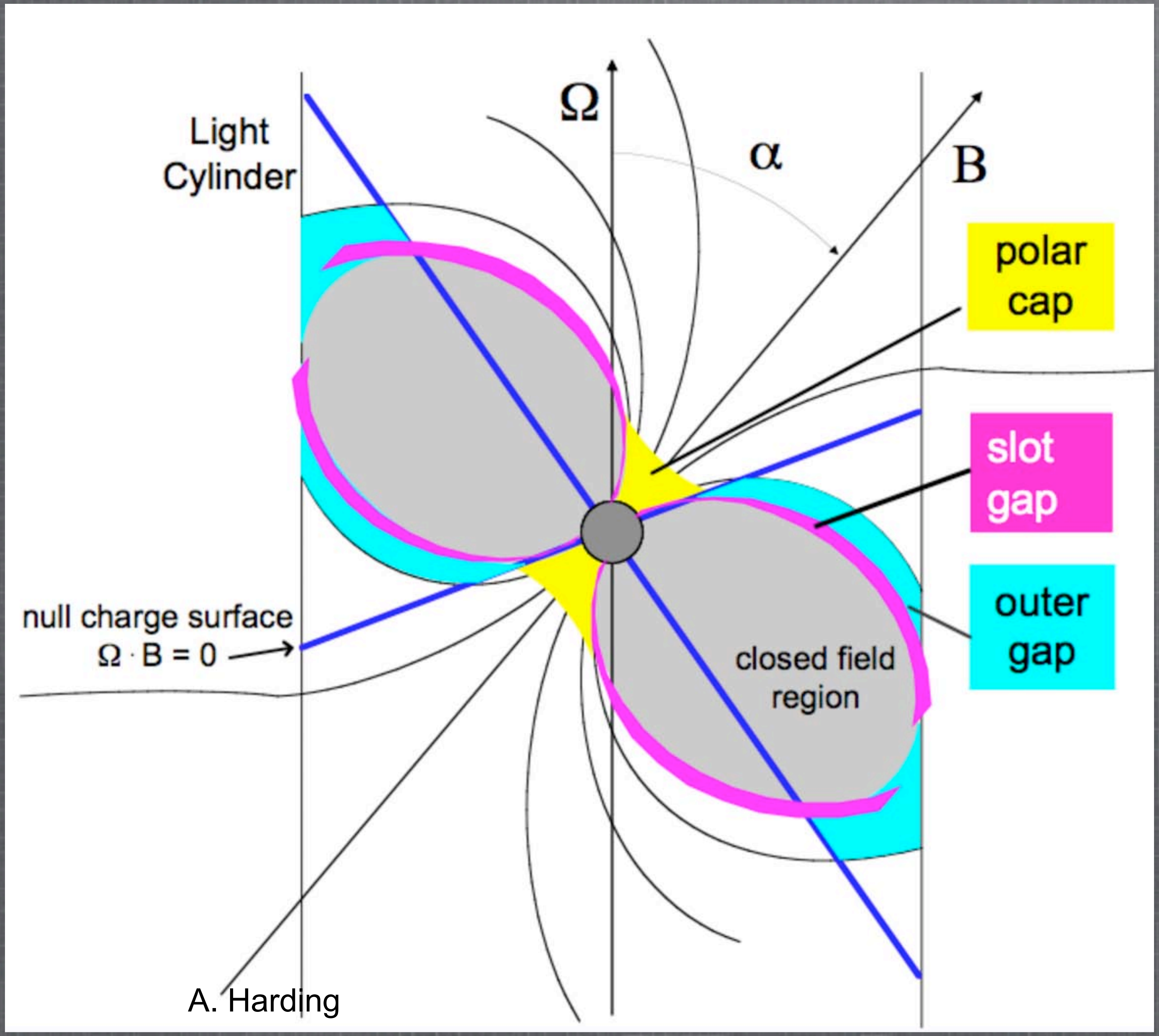
Gruzinov (2005) found an invariant on field lines, which has relation to current carried.

Magnetospheric Currents



Inclinations
0,30,60,90
degrees

It is tempting to associate the current sheets in the magnetosphere with the emission regions of high energy photons, normally thought to be generated in gaps.



Geometrically, it seems that the current flows in the inferred region for the gaps.

Let's see if it makes any quantitative sense.

Emission modeling

Force-free vs vacuum field

1. Pick field (static dipole, retarded dipole [Deutch], force-free)
2. Find the polar cap (field lines touching LC)
3. Decide which (open) field lines emit
4. Assume uniform emissivity (with cuts in radius)
5. Trace field lines emitting photons along field line
6. Add aberration and time of flight effect
7. Bin photons on the sky -- > sky map + light curves
8. Repeat until satisfied

Choices that have freedom to choose parameters

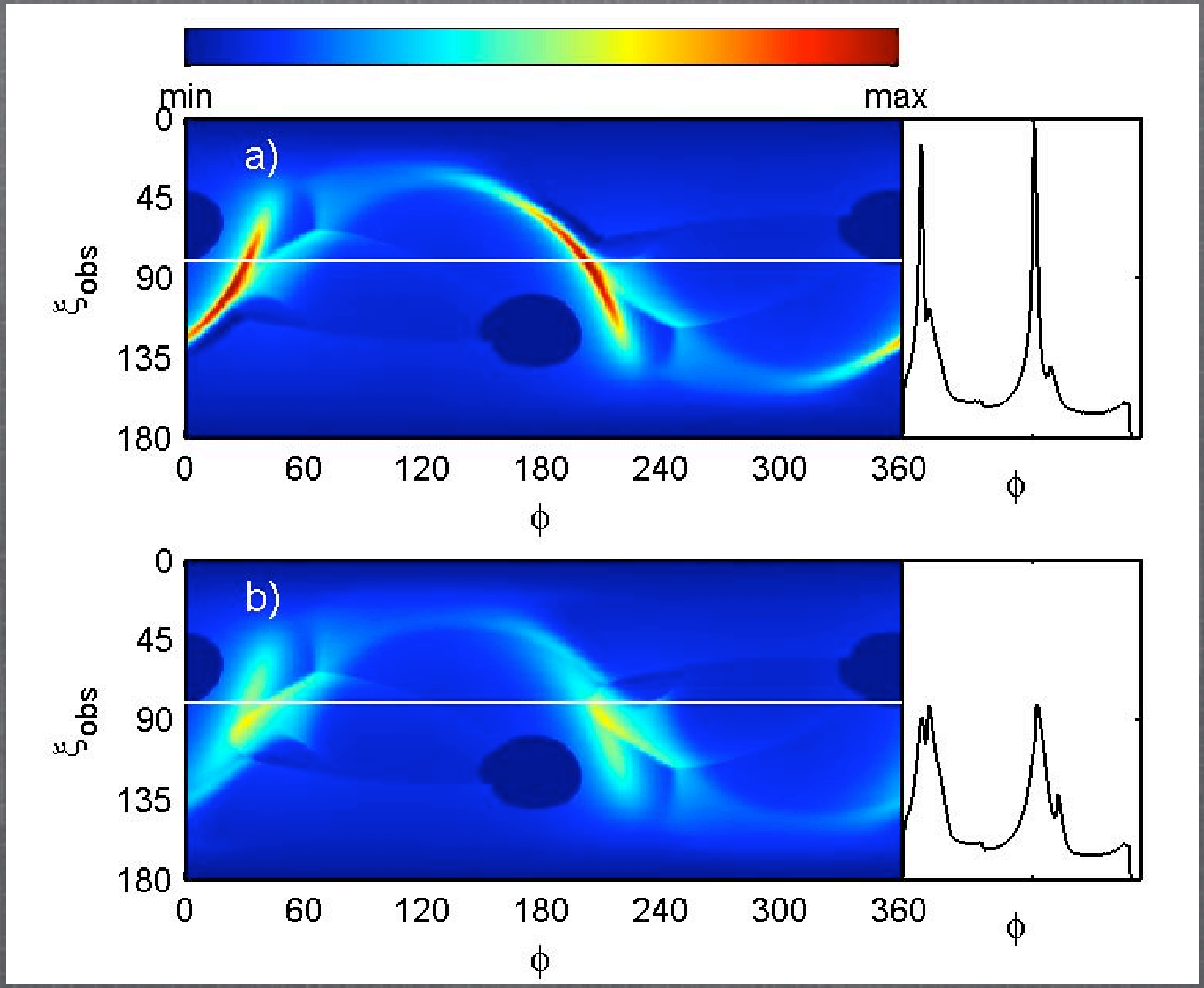
Extensive literature and examples by Romani, Harding, Dyks, Cheng, et al.

Emission modeling

Aberration is the crucial piece of the light curve modeling. Need to include it self-consistently. Deutch field is known in the Lab frame, not in corotating frame -- this changes the light curves

Before:

After:



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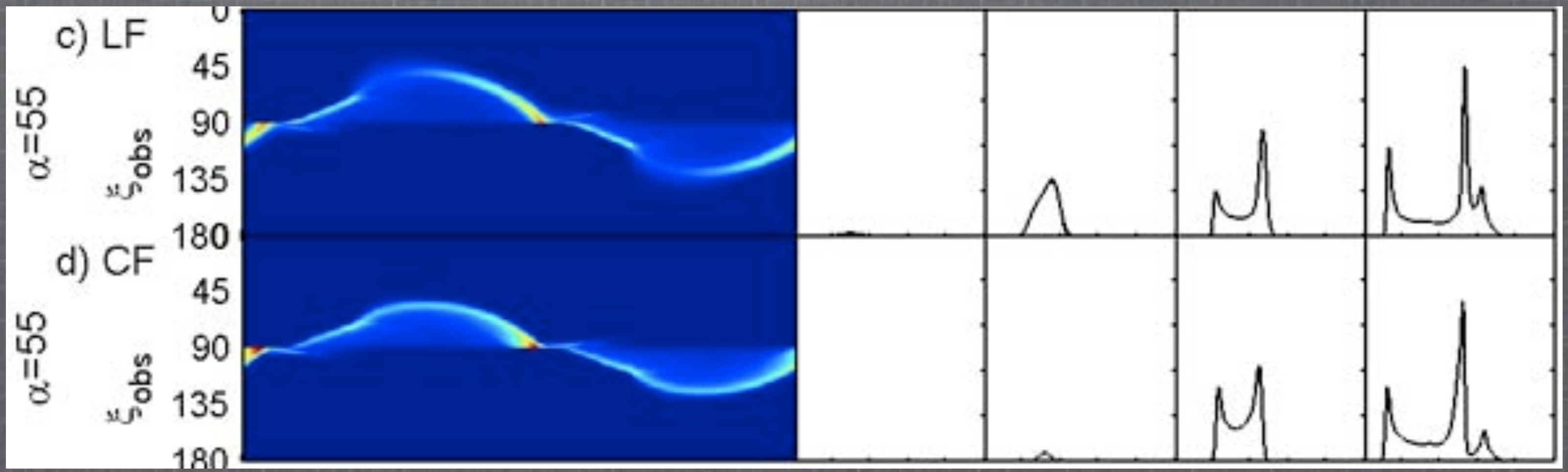
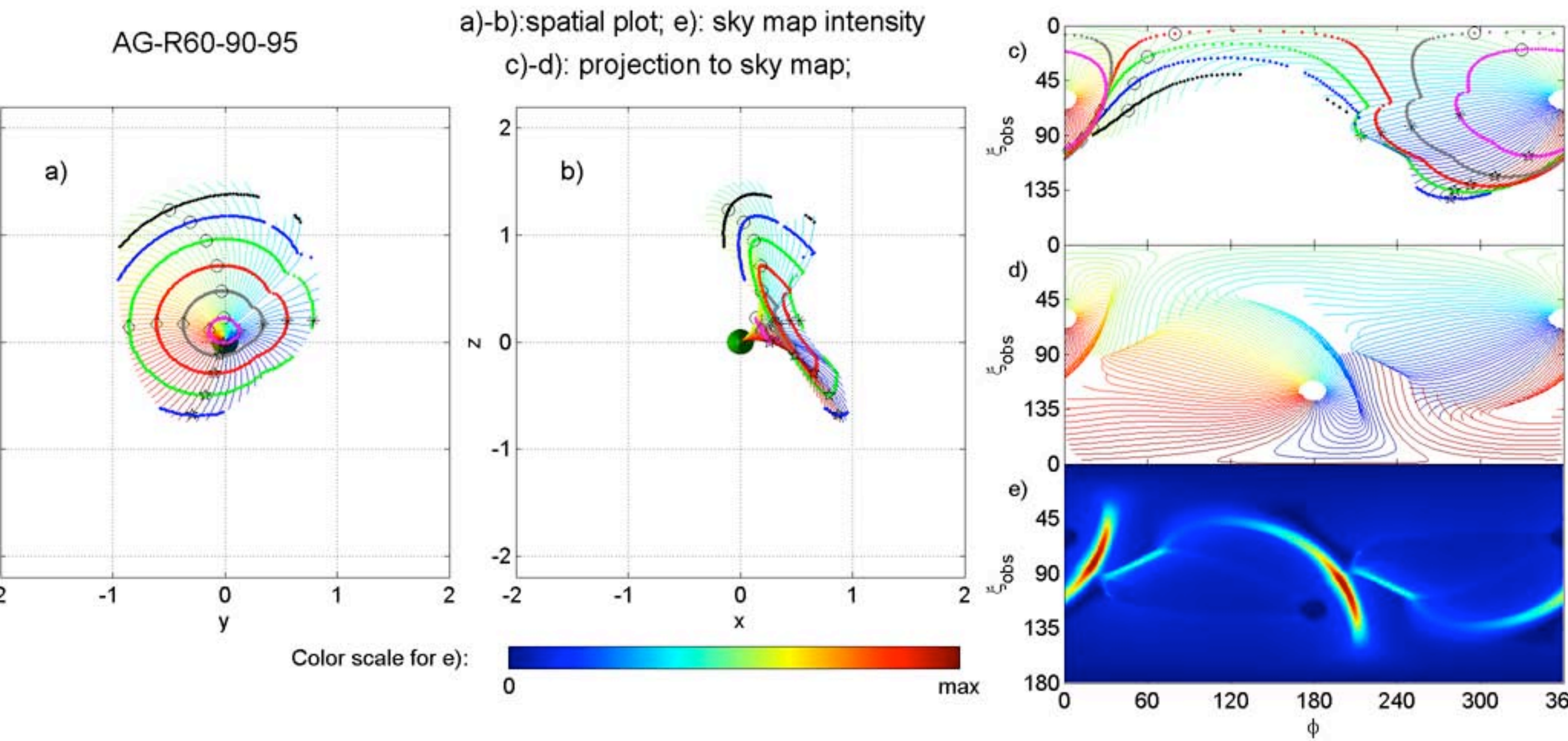


EMISSION MODELING

Force-free vs vacuum field

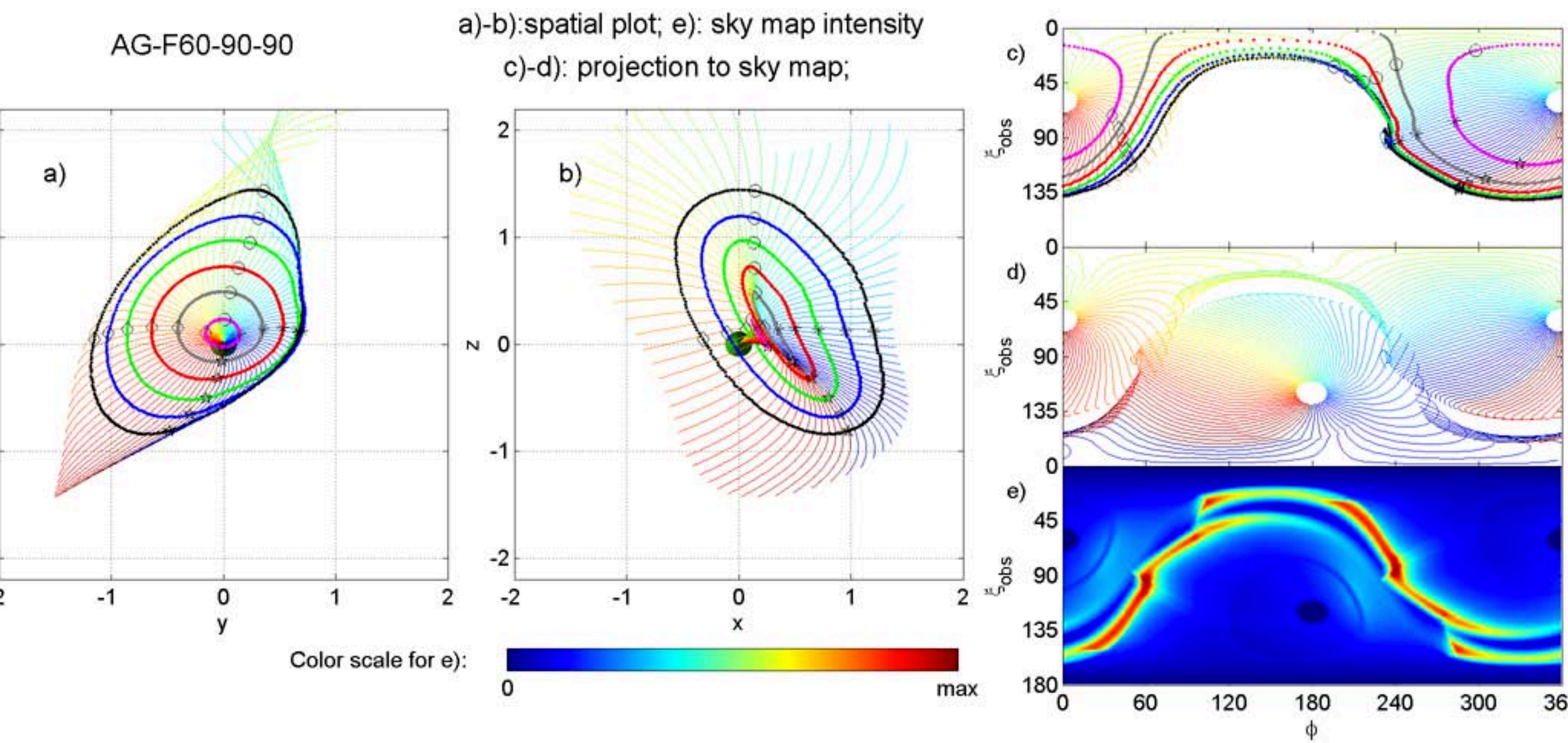


Vacuum (Deutch), 60 degree inclination, flux tube starting at 0.9 of the polar cap radius.

Color circles at every 0.25 Rlc along fieldlines.

Emission modeling

Force-free vs vacuum field



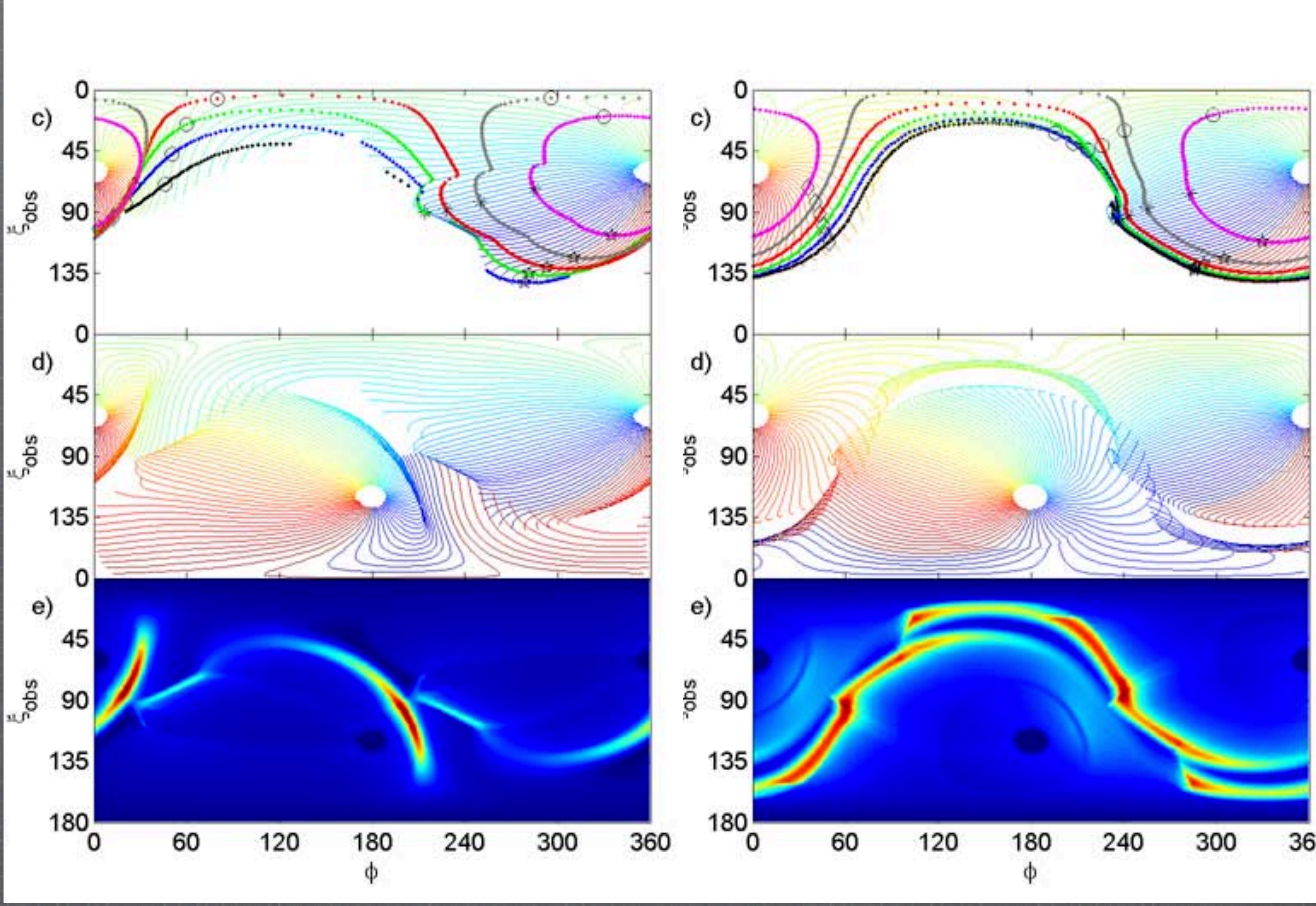
Force-free, 60 degree inclination, flux tube starting at 0.9 of the polar cap radius.

EMISSION MODELING

Force-free vs vacuum field

Near caustic gone

All caustics come from near LC

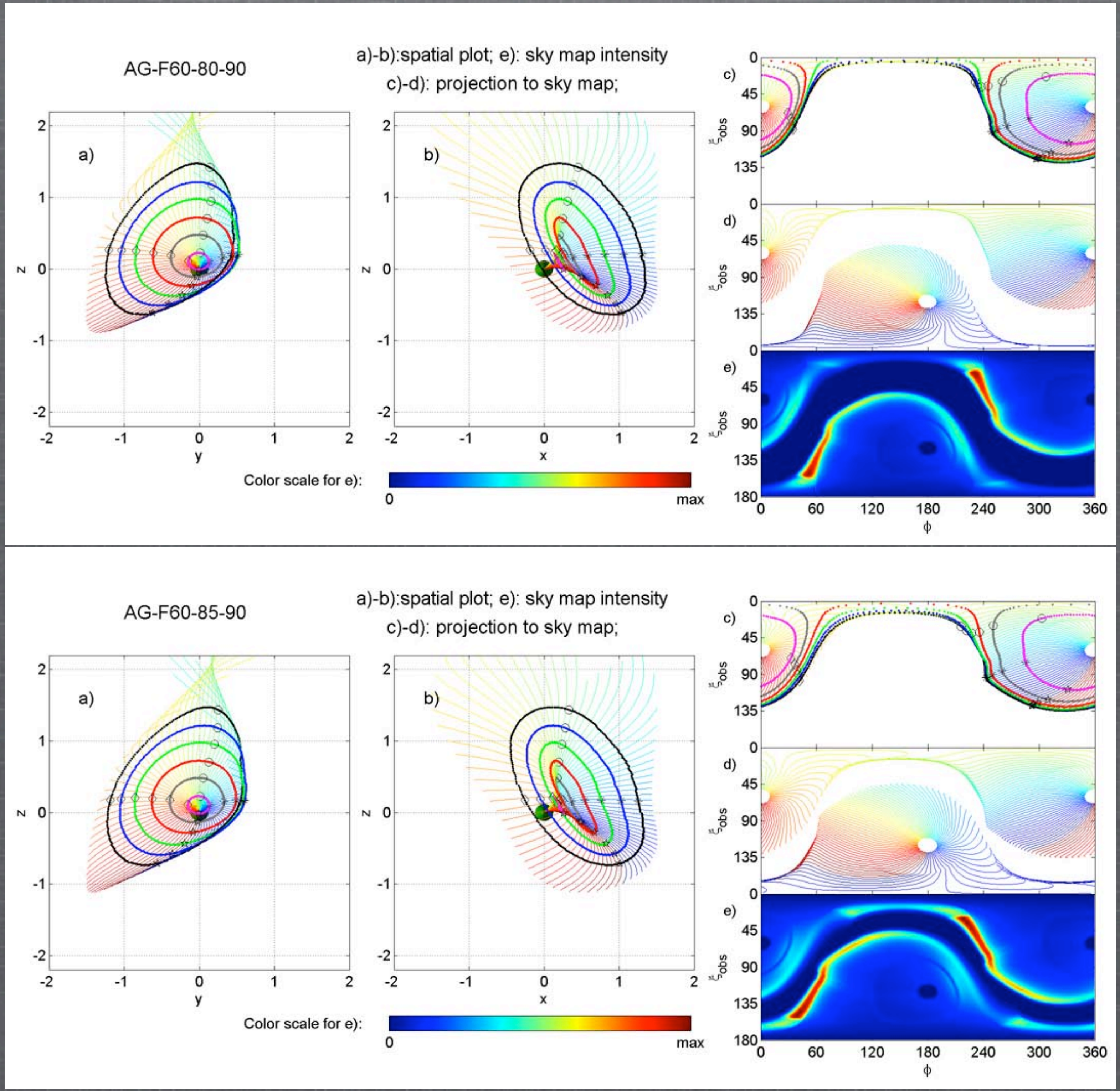


Vacuum

Force-free

EMISSION MODELING

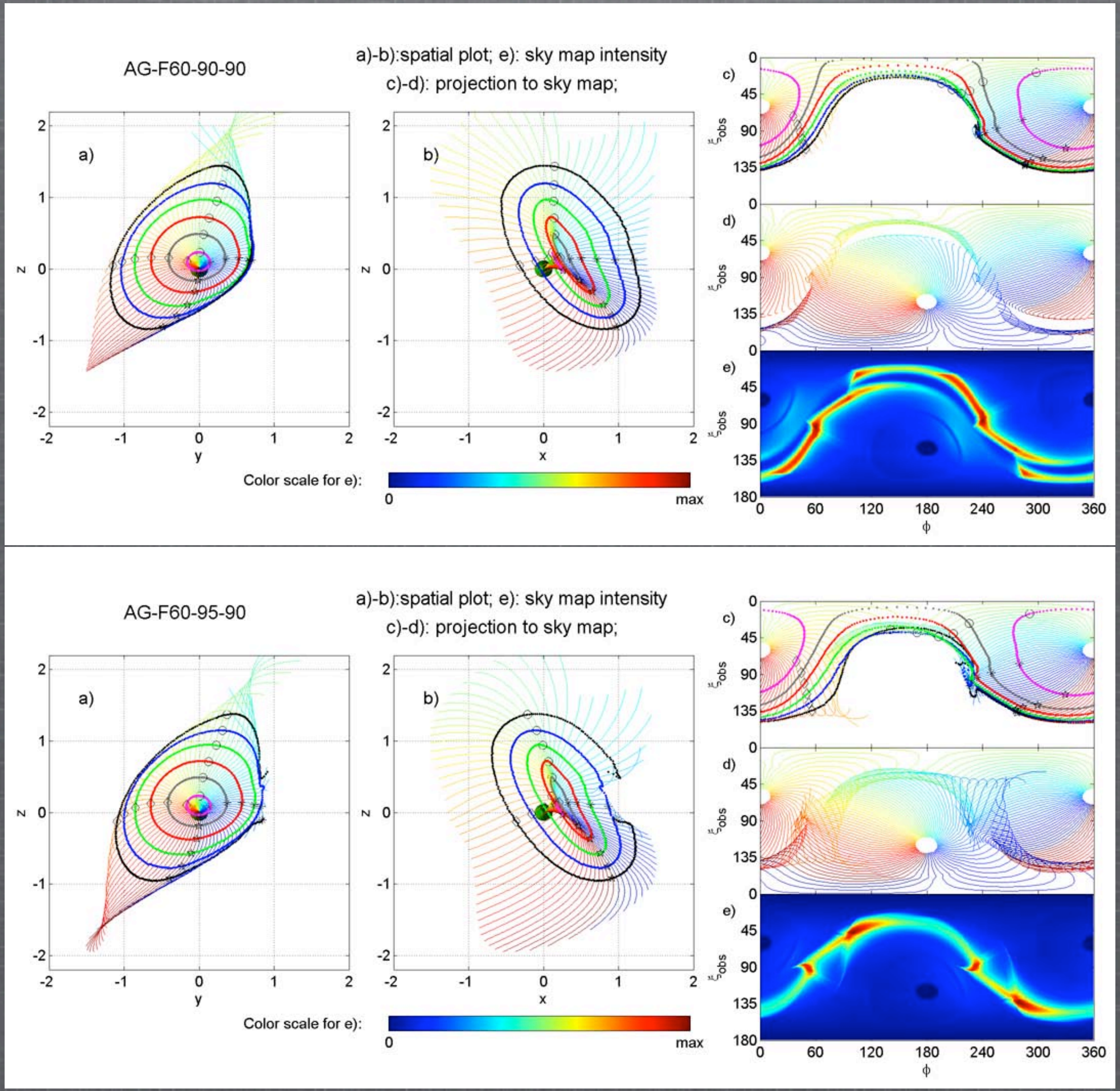
Force-free vs vacuum field



Force-free, 60 degree inclination, flux tube starting at 0.8 and 0.85 of the polar cap radius.

EMISSION MODELING

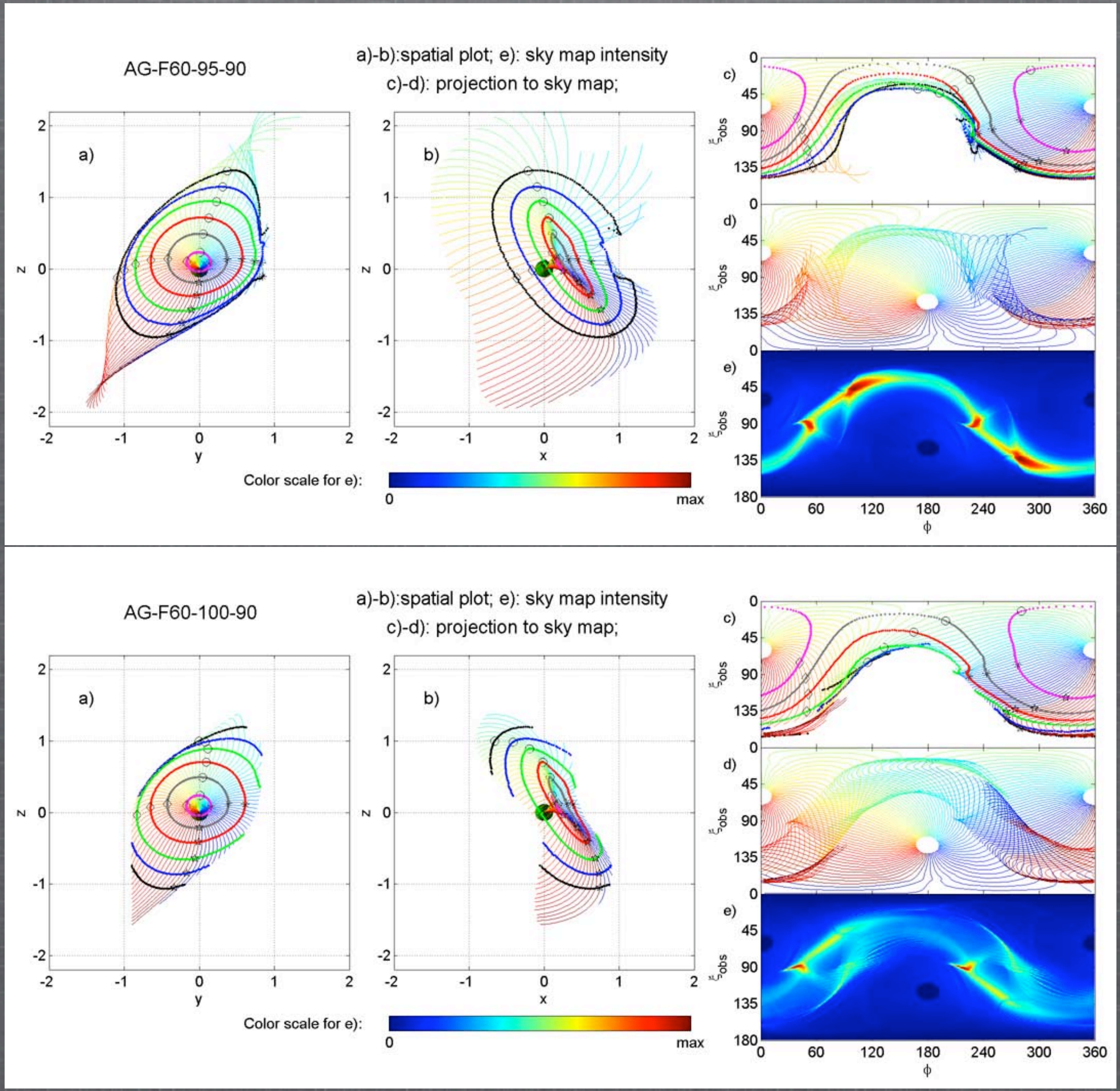
Force-free vs vacuum field



Force-free, 60 degree inclination, flux tube starting at 0.9 and 0.95 of the polar cap radius.

Emission modeling

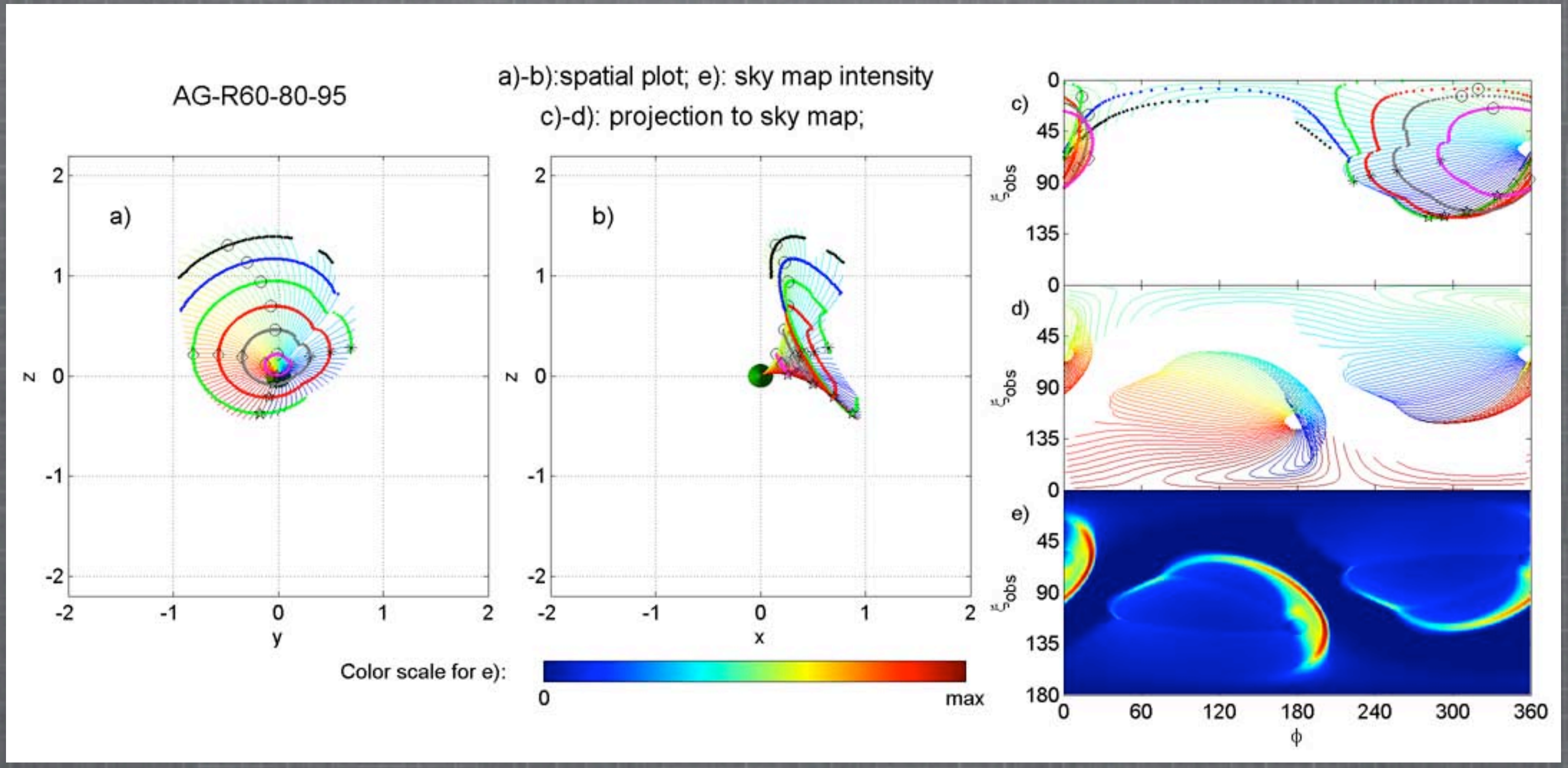
Force-free vs vacuum field

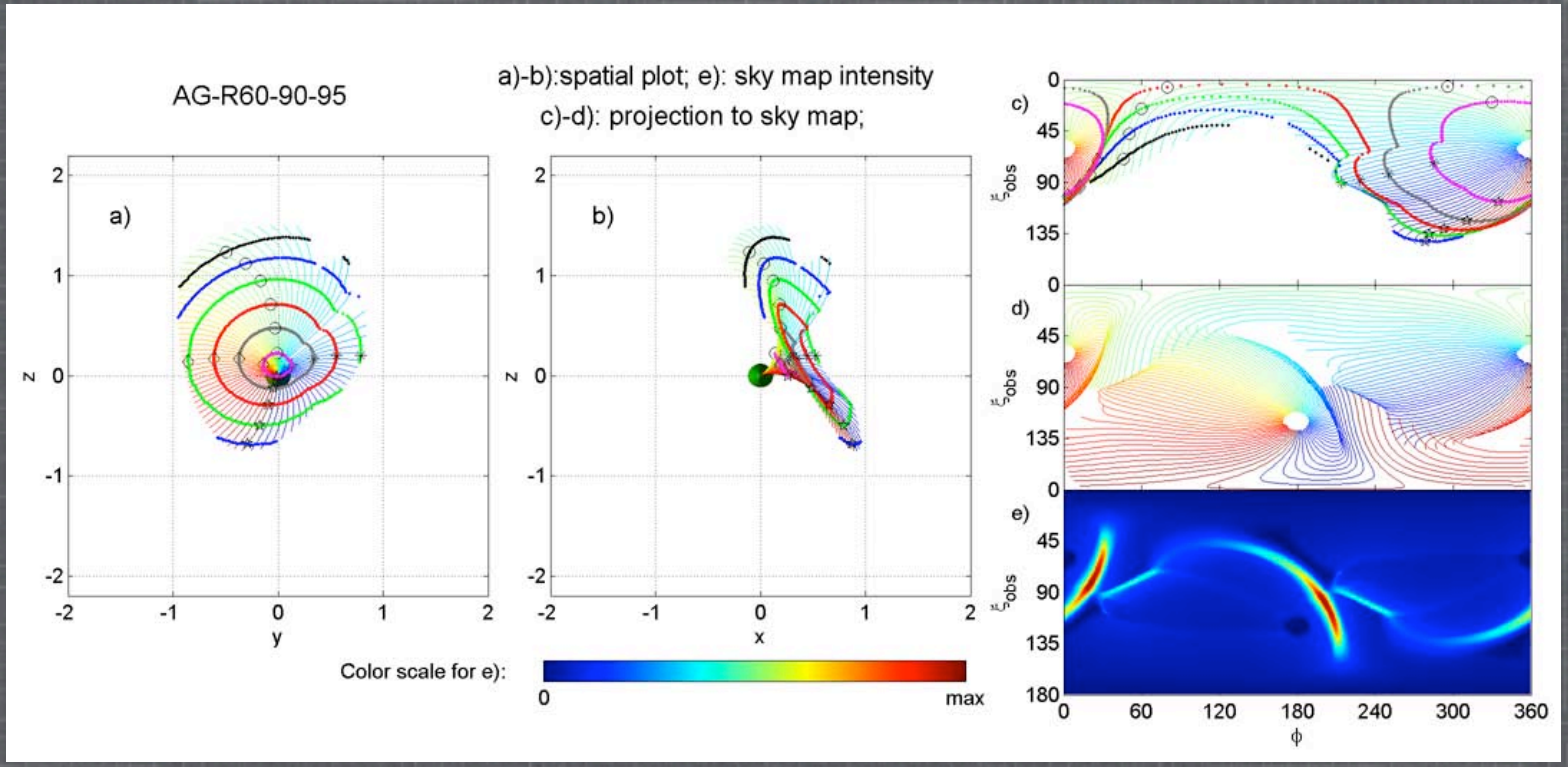


Force-free, 60 degree inclination, flux tube starting at 0.95 and 1.0 of the polar cap radius.

Emission modeling

Force-free vs vacuum field

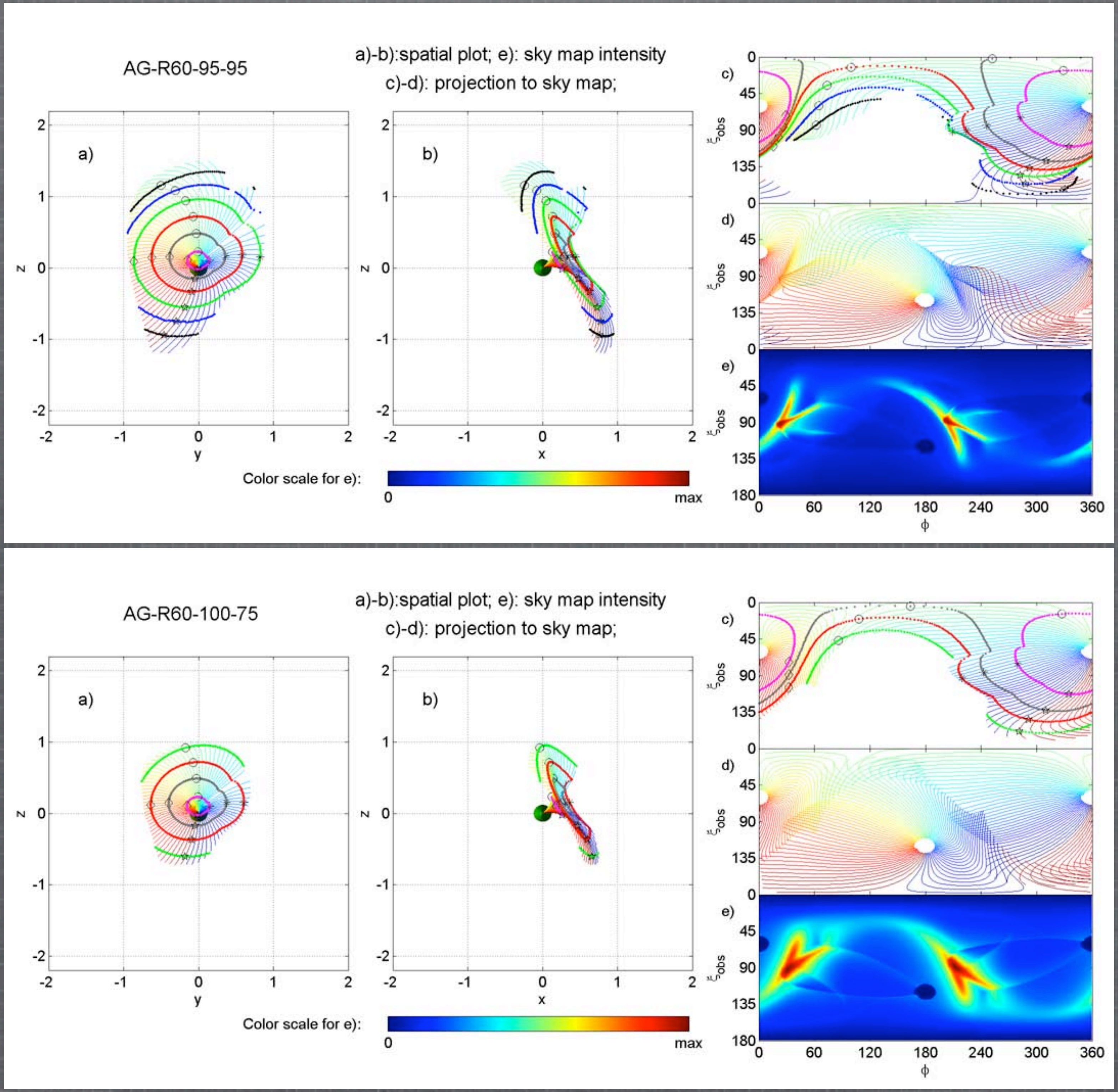




Vacuum, 60 degree inclination, flux tube starting at 0.8 and 0.9 of the polar cap radius.

EMISSION MODELING

Force-free vs vacuum field



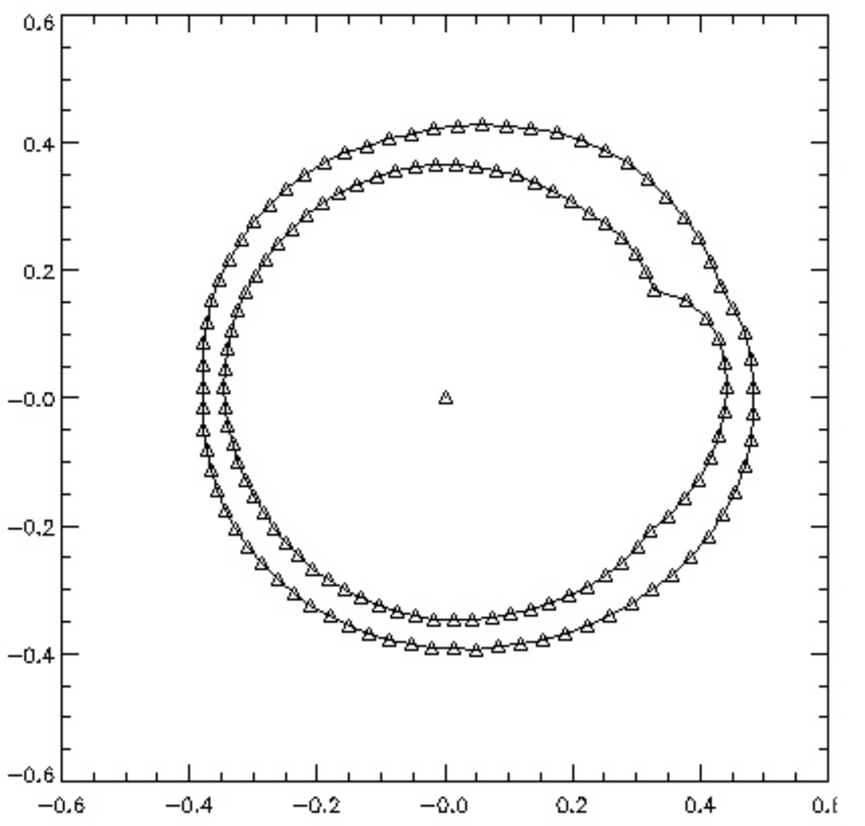
Vacuum, 60 degree inclination, flux tube starting at 0.95 and 1.0 of the polar cap radius.

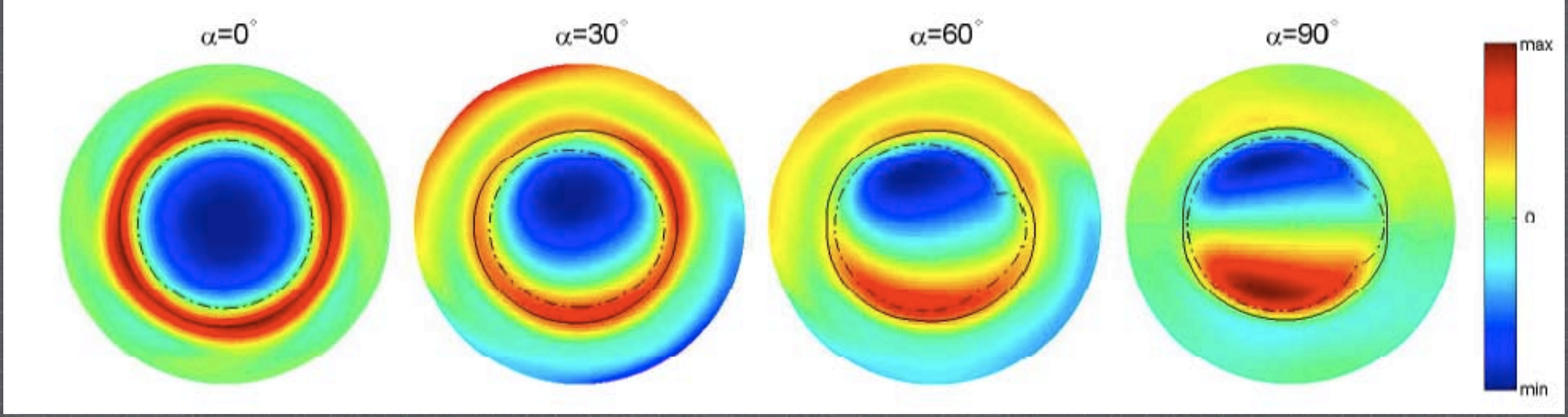
EMISSION MODELING

Force-free vs vacuum field

Why such a big difference if the field is close to dipolar near the star?

1) Larger polar cap
2) Weakening of vacuum caustics with increasing flux
3) Caustics form near LC due to the "monopole pileup".





EMISSION MODELING

Force-free vs vacuum field

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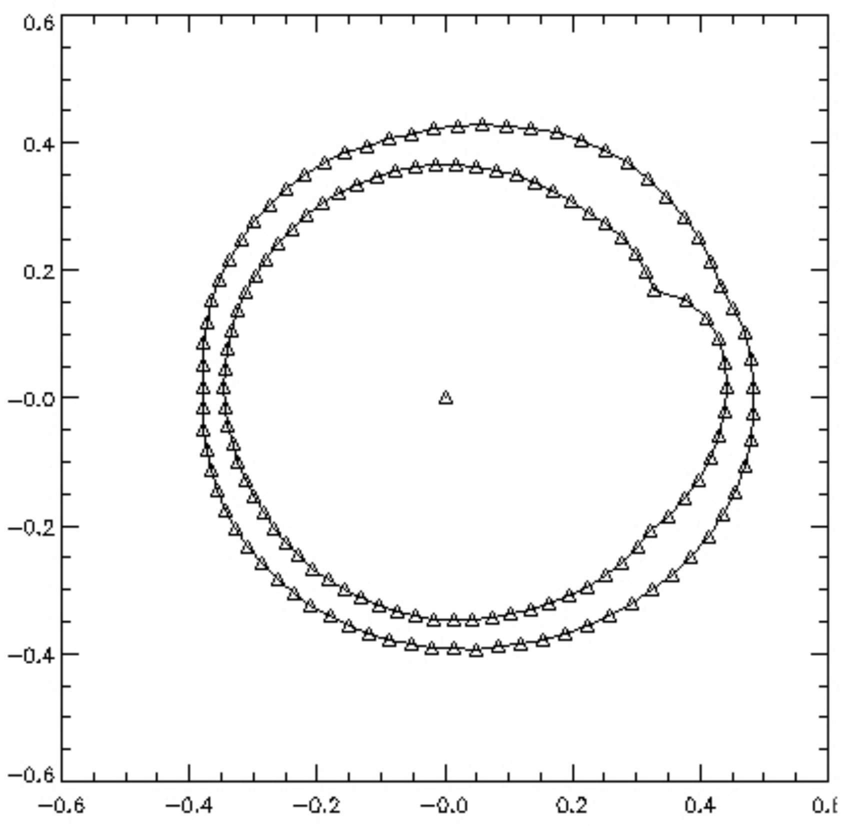
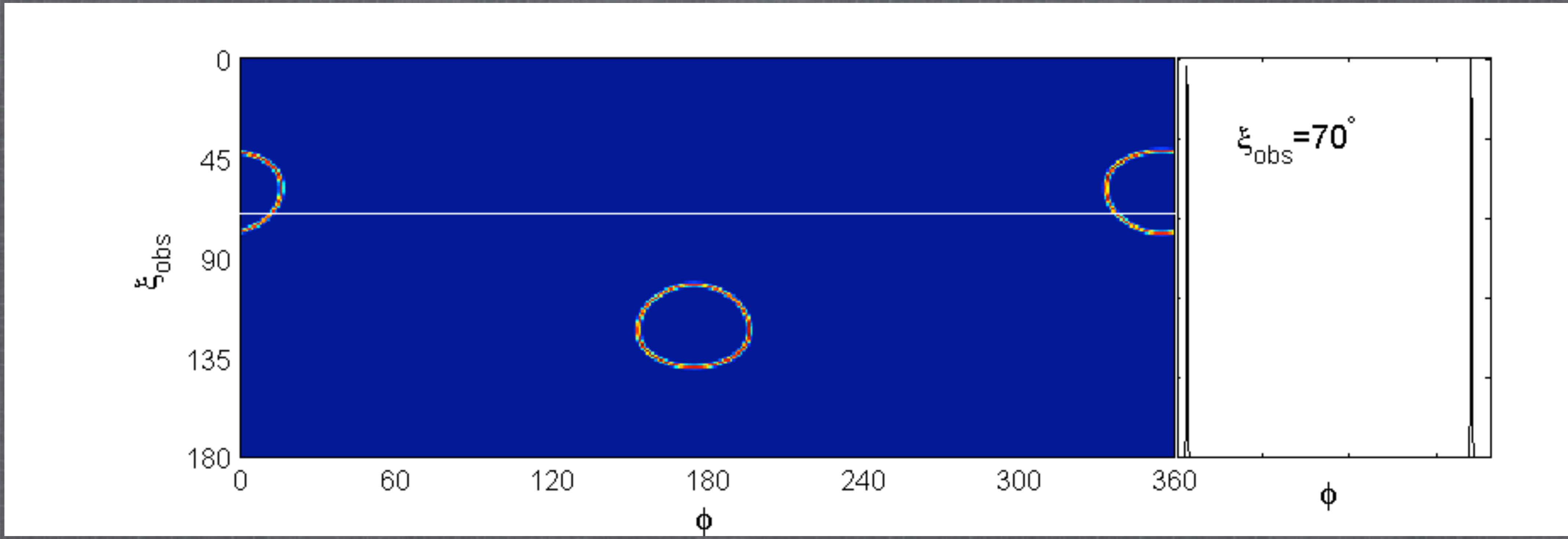
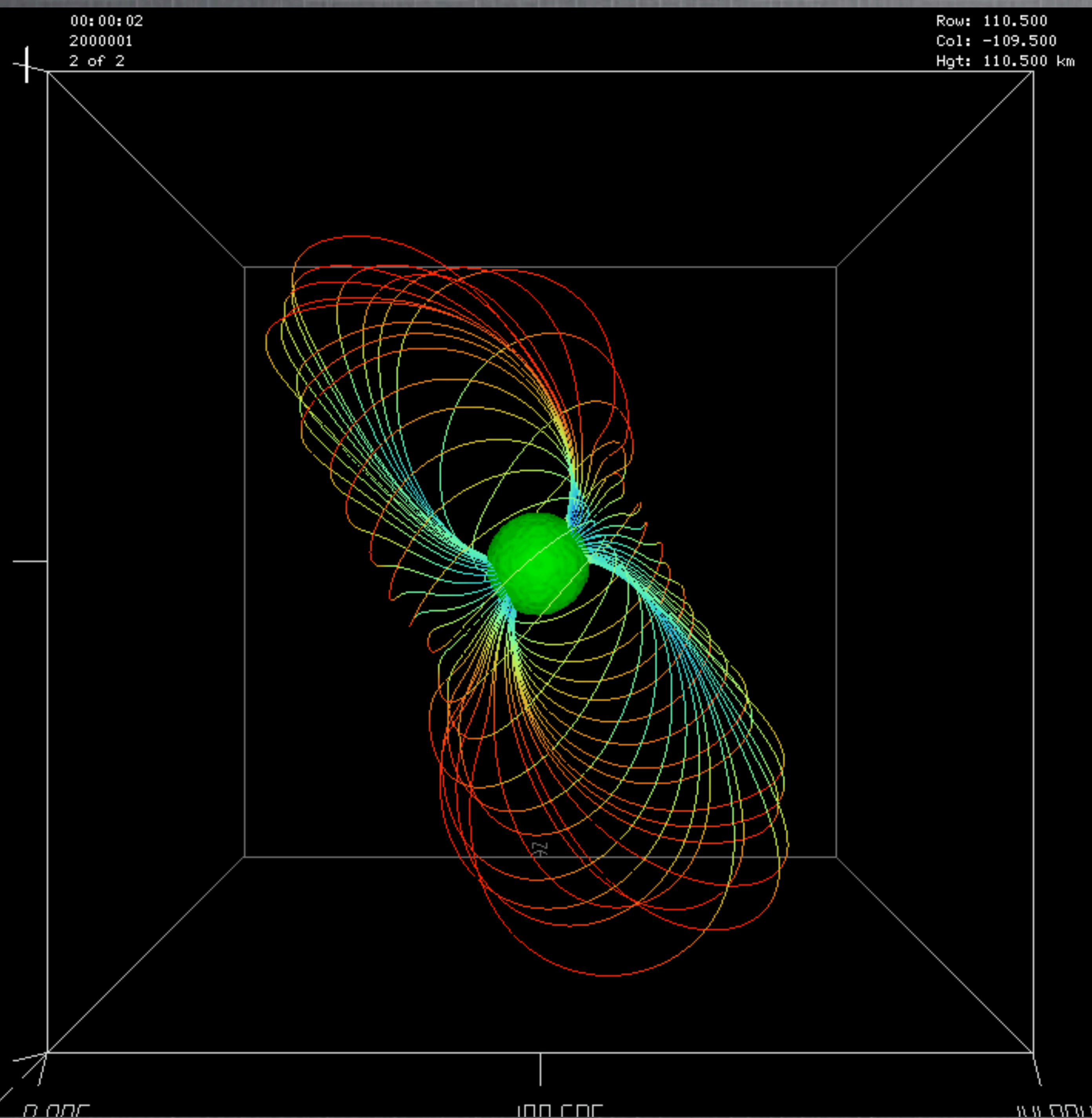


Skymap for split monopole field. Fieldlines maps onto itself

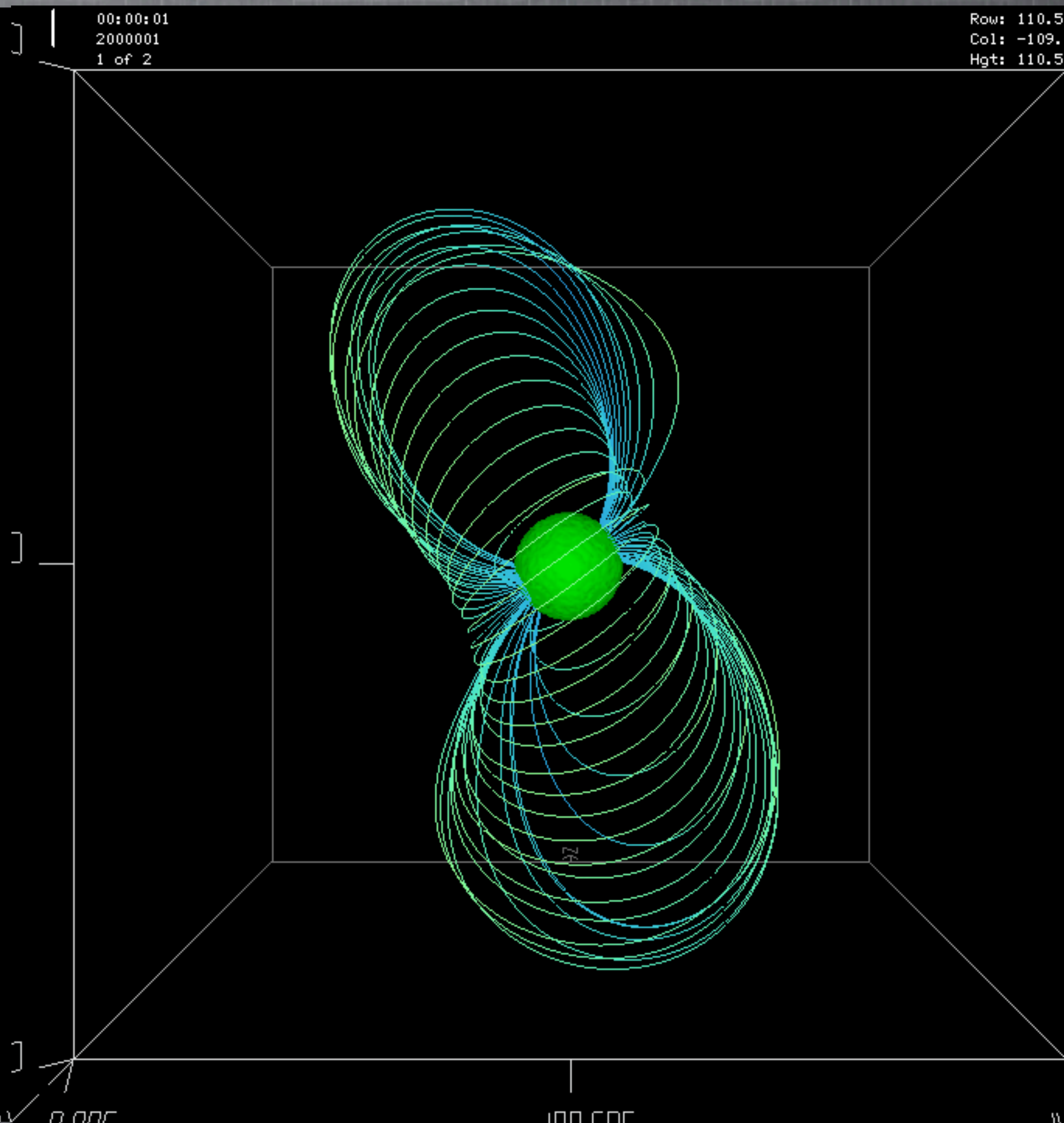


Emission modeling

Force-free vs vacuum field: last open lines



Last closed fieldline for force-free



Last closed fieldline for vacuum

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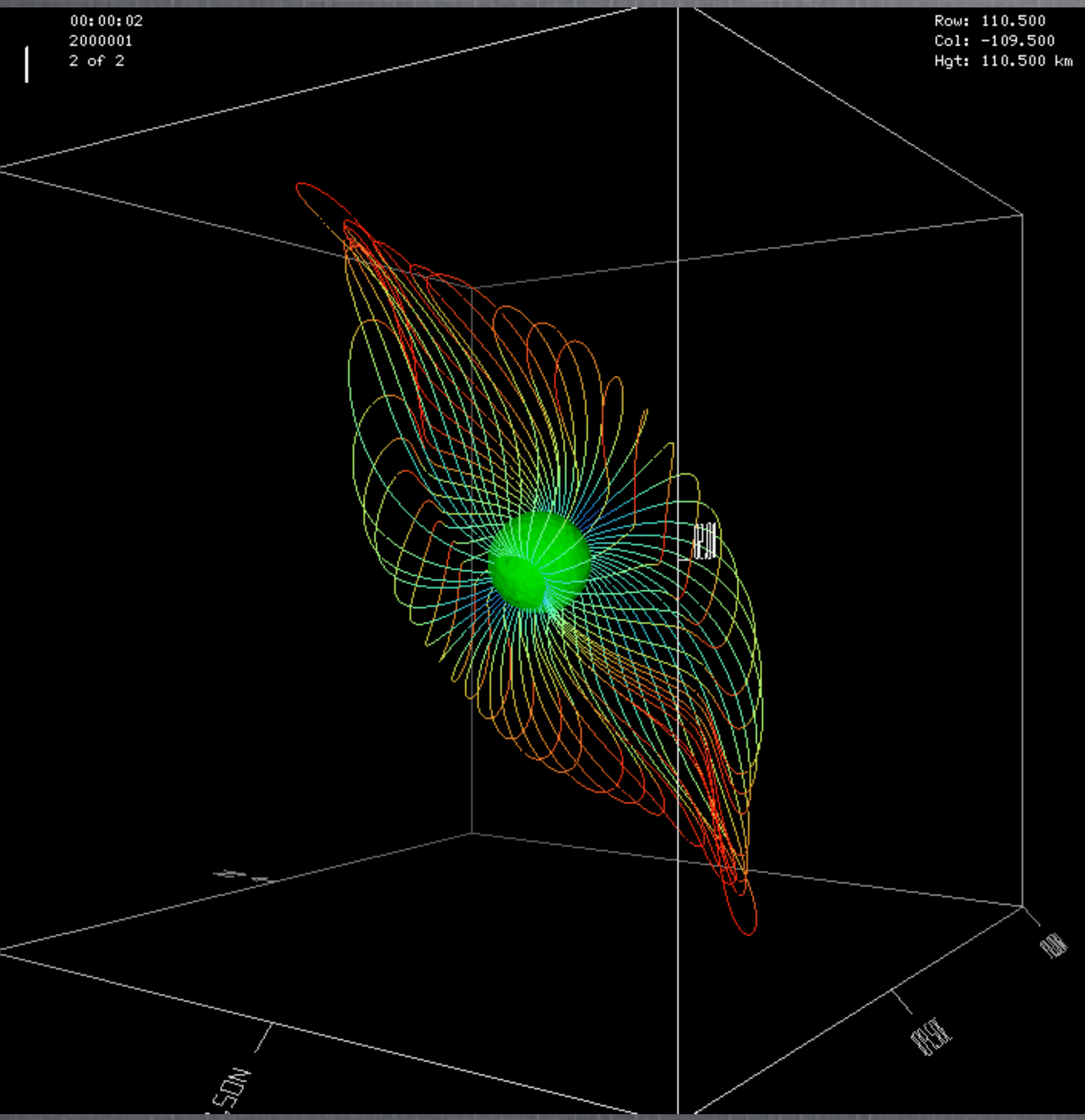

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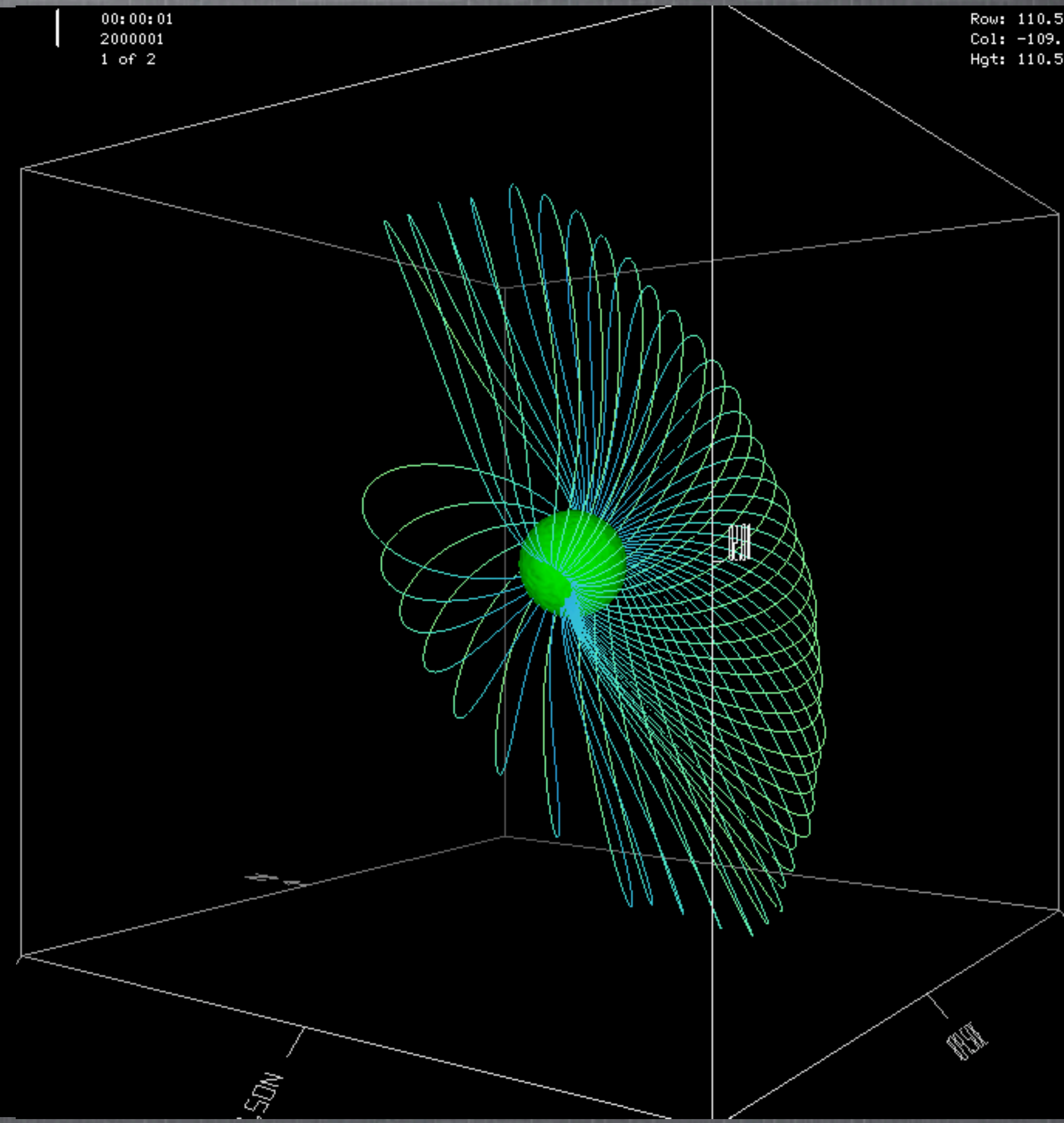

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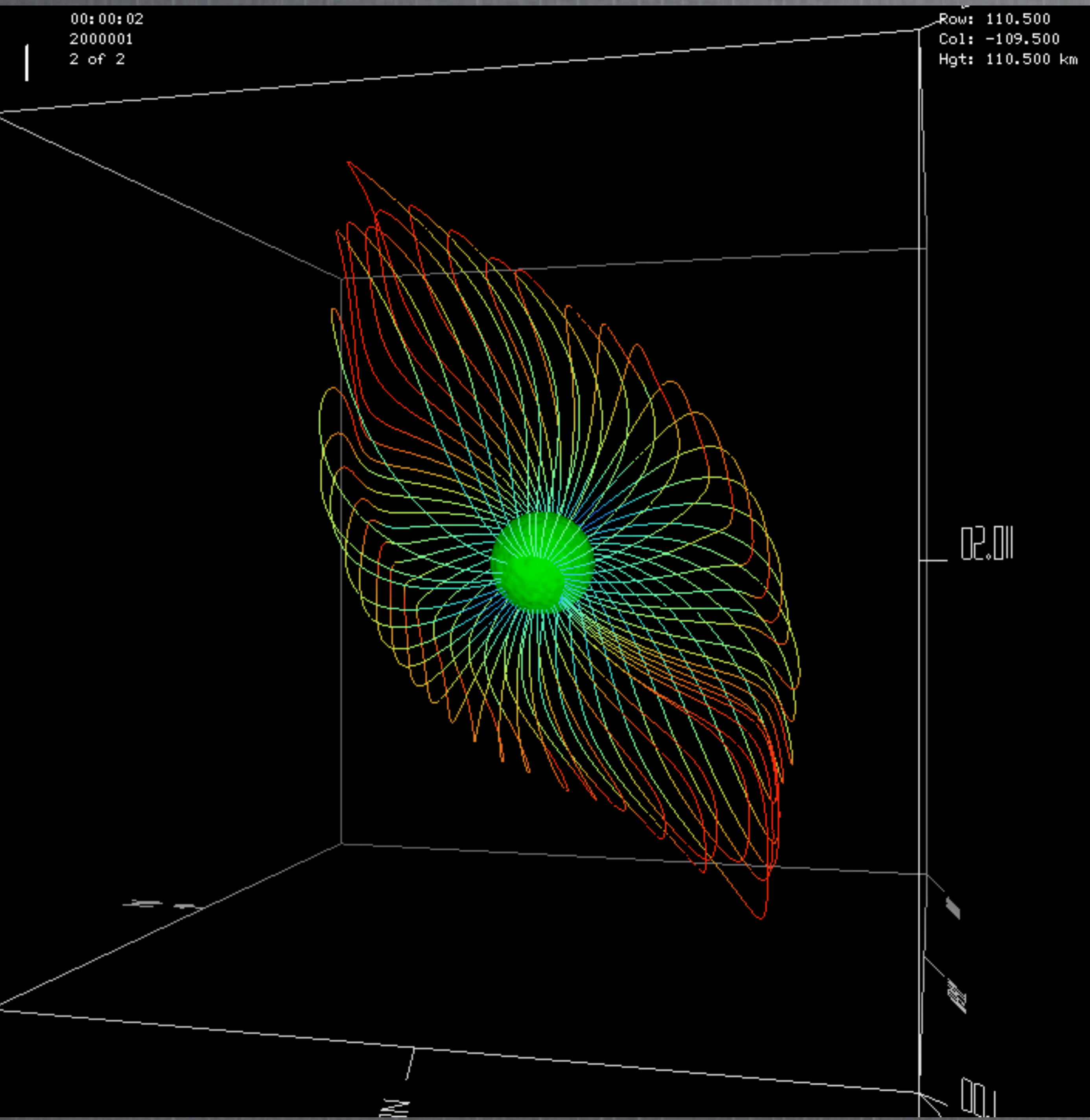

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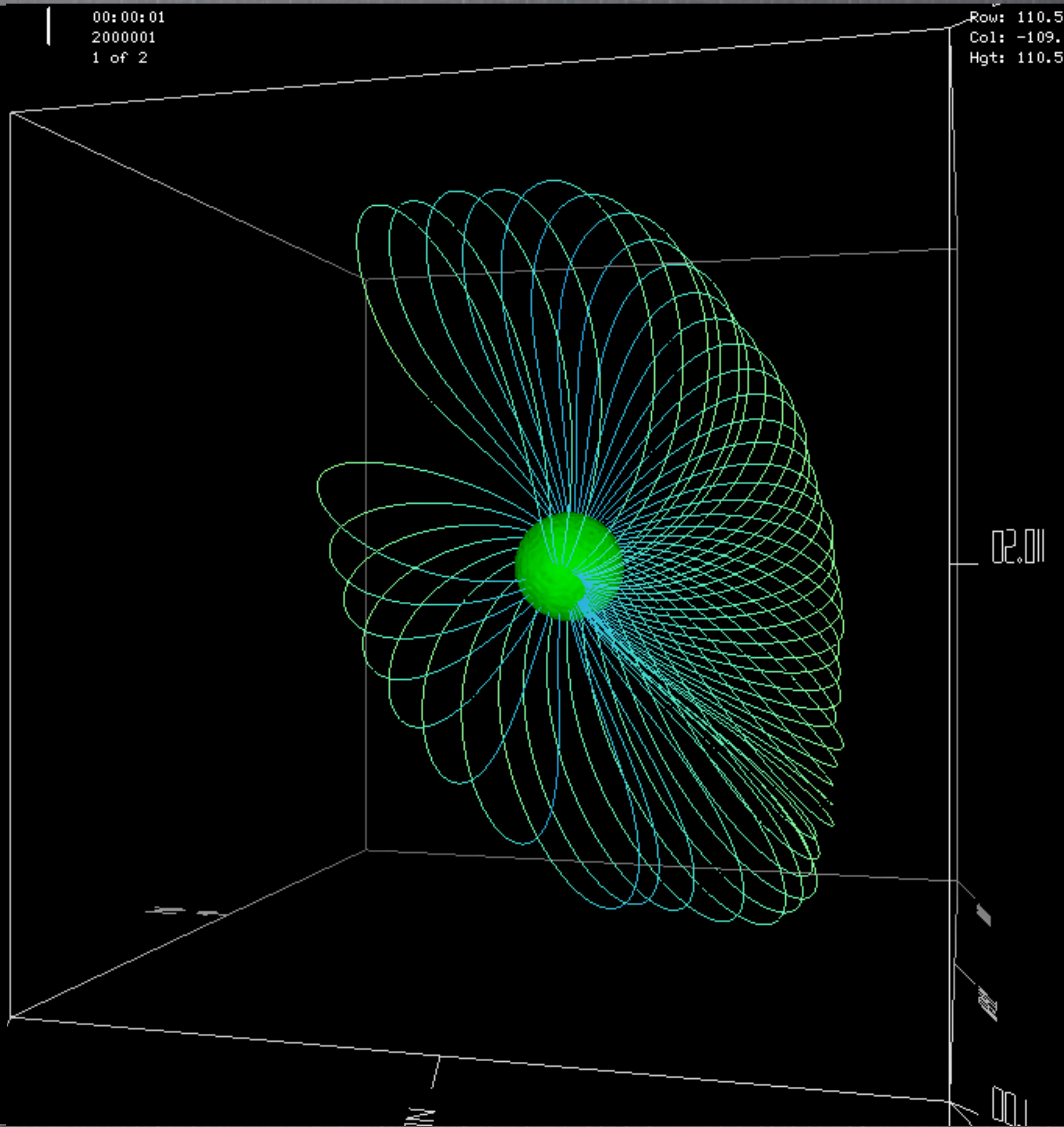

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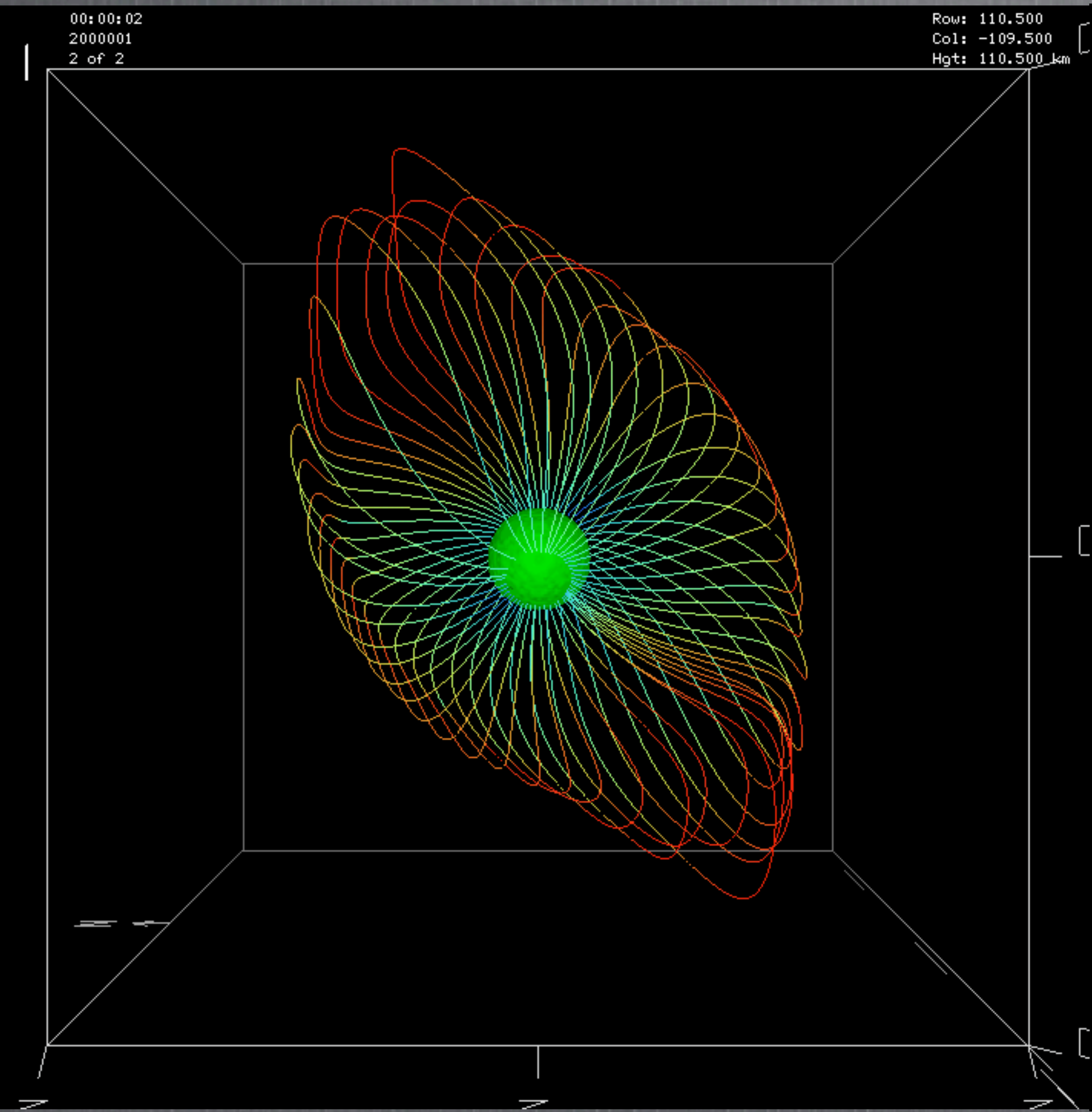

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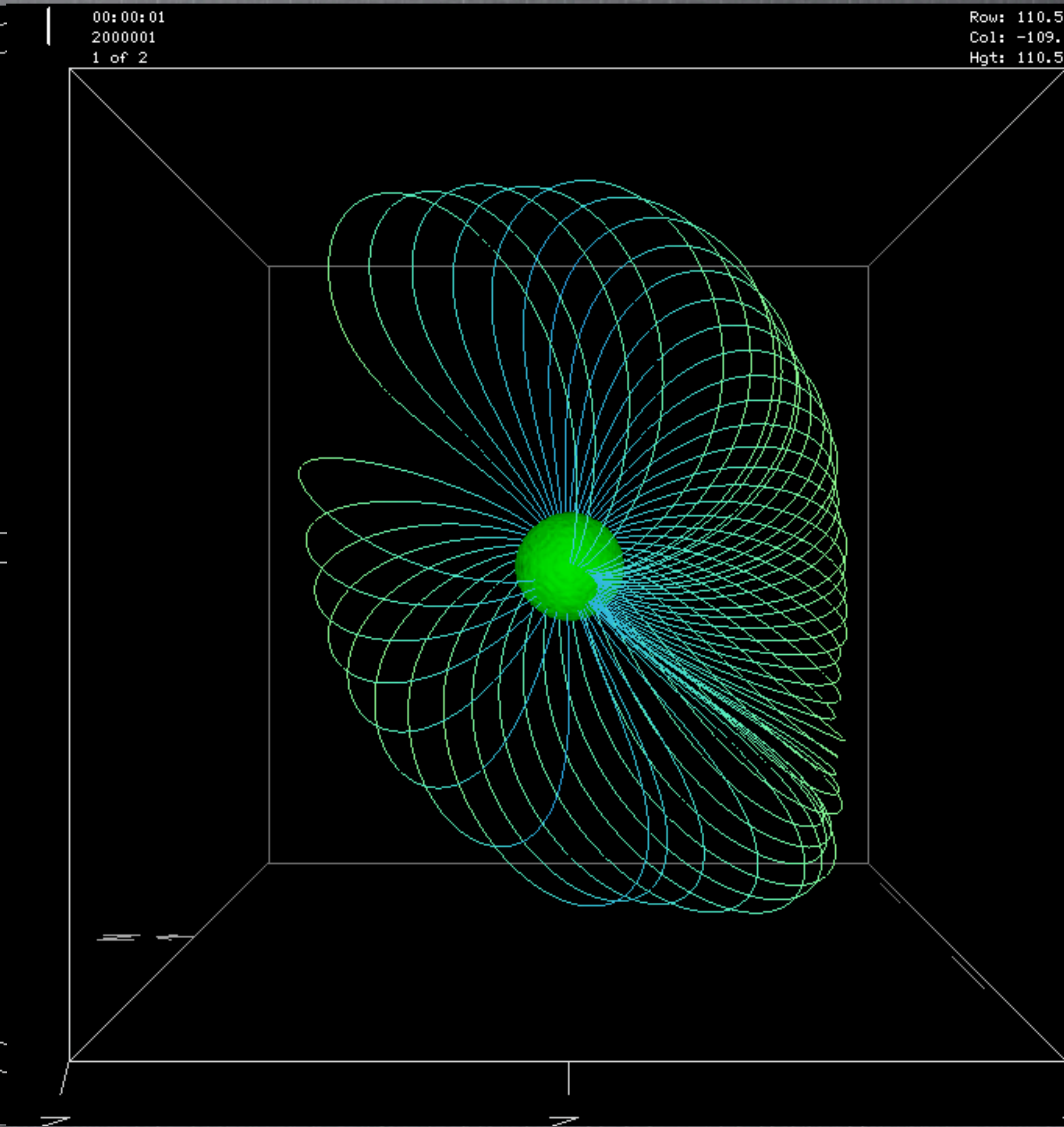

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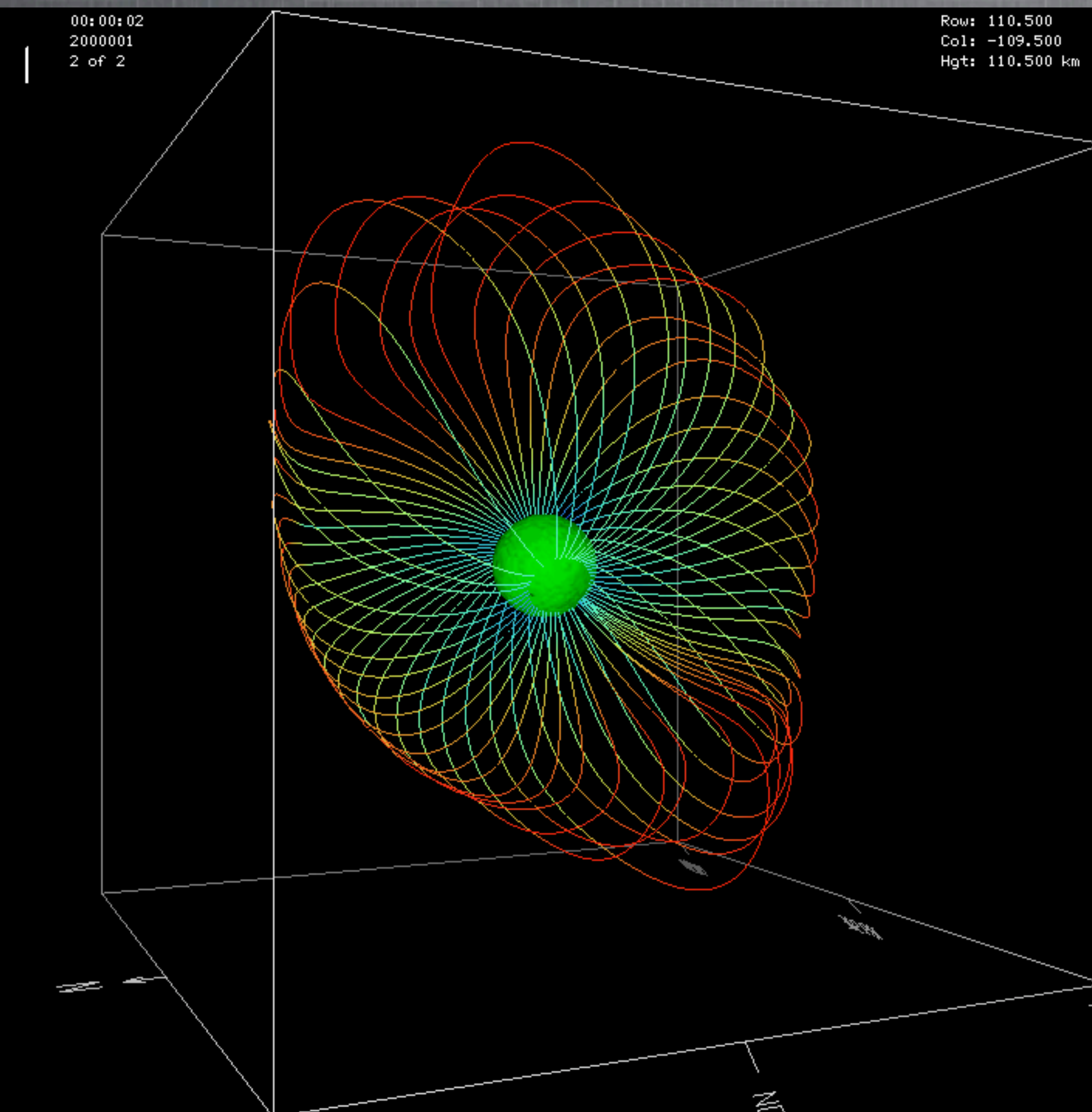

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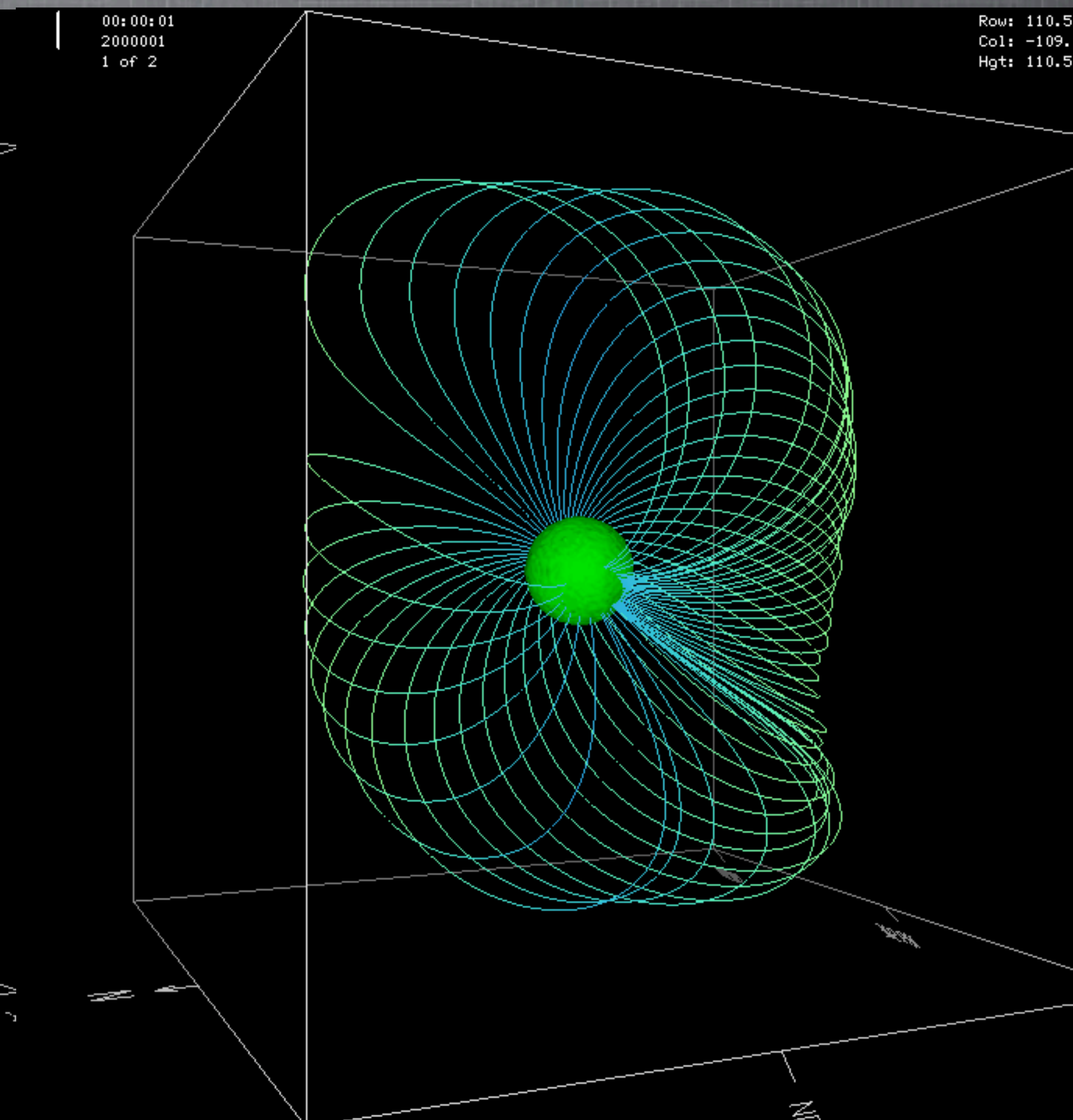

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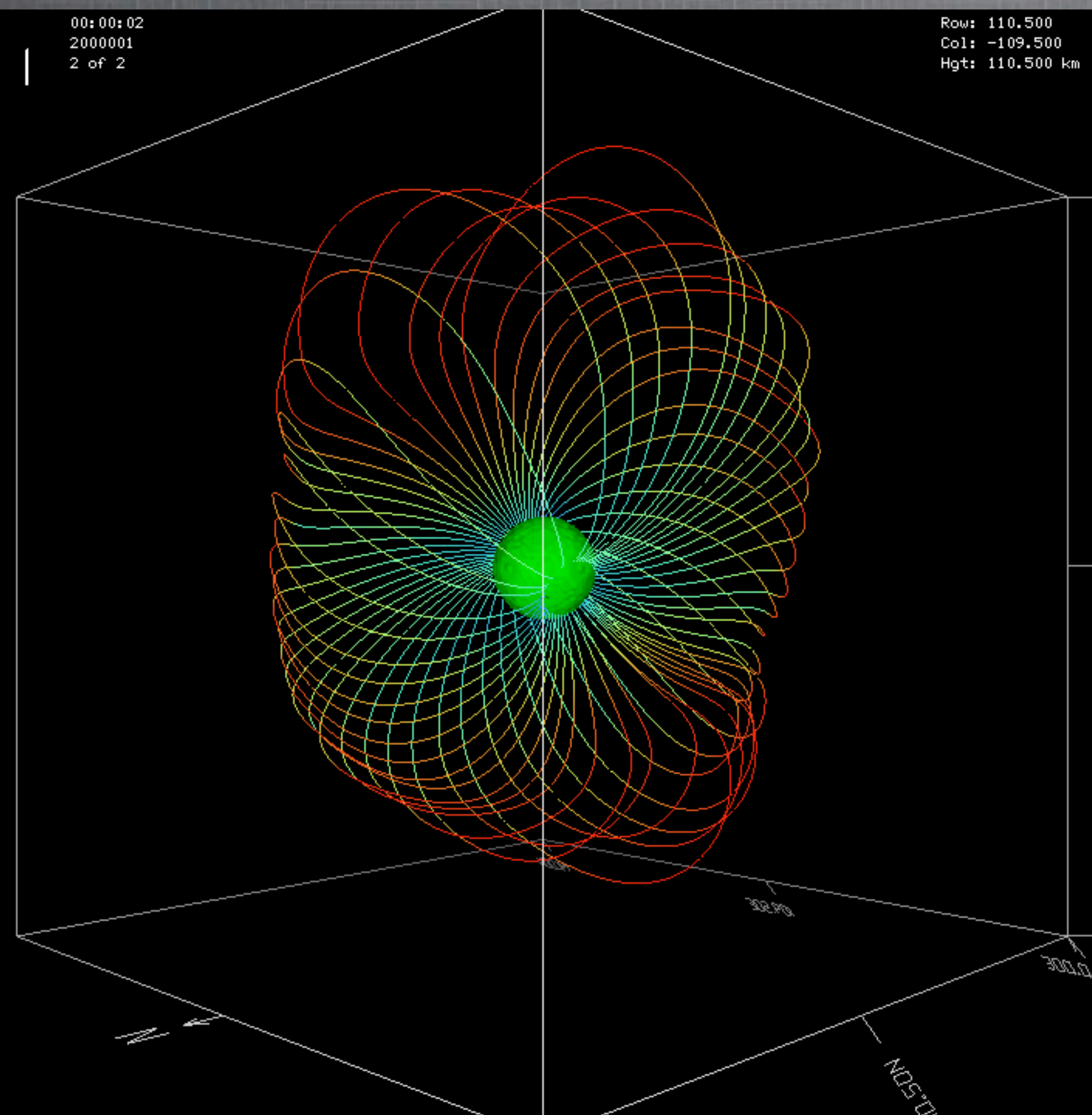

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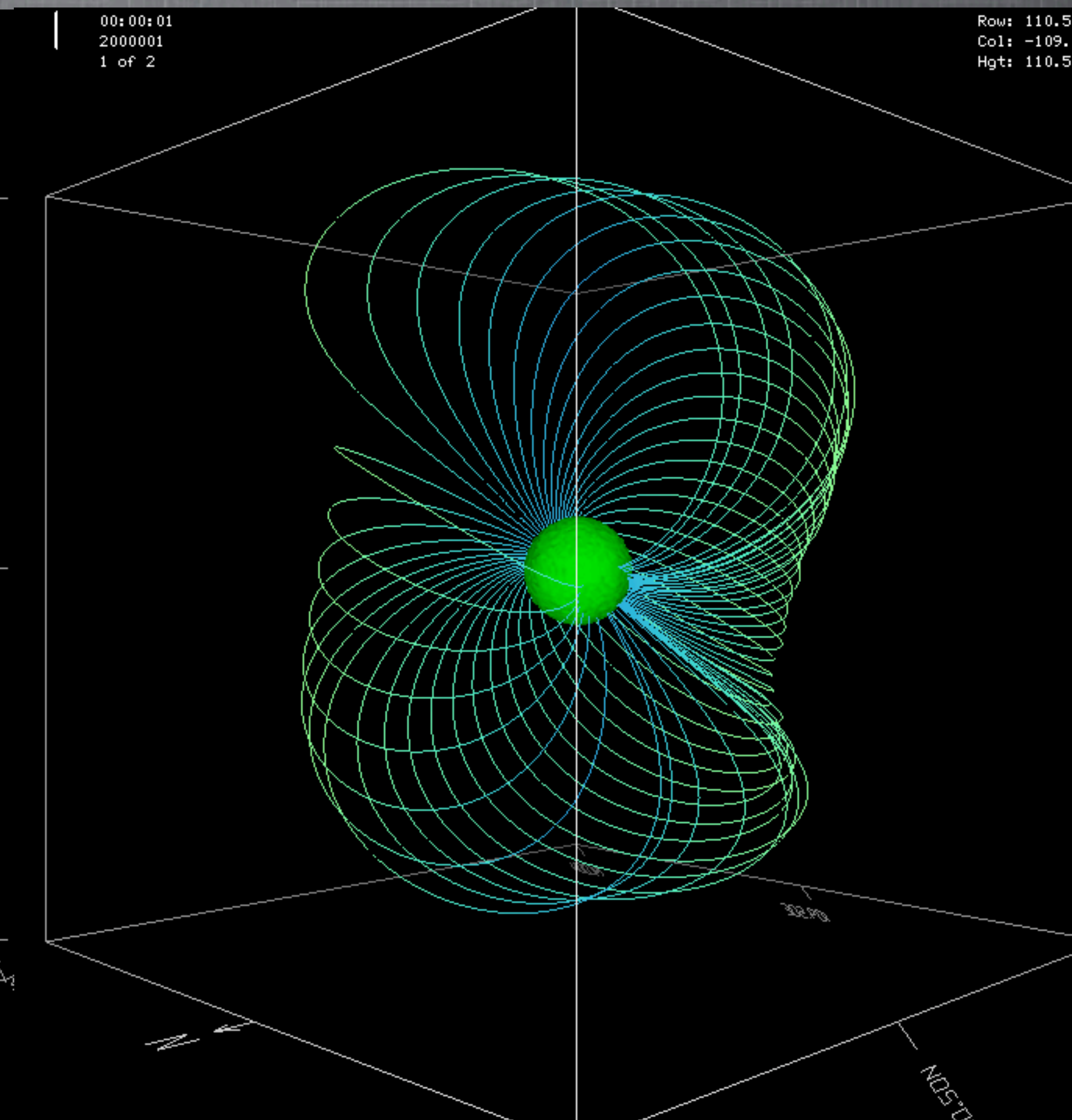

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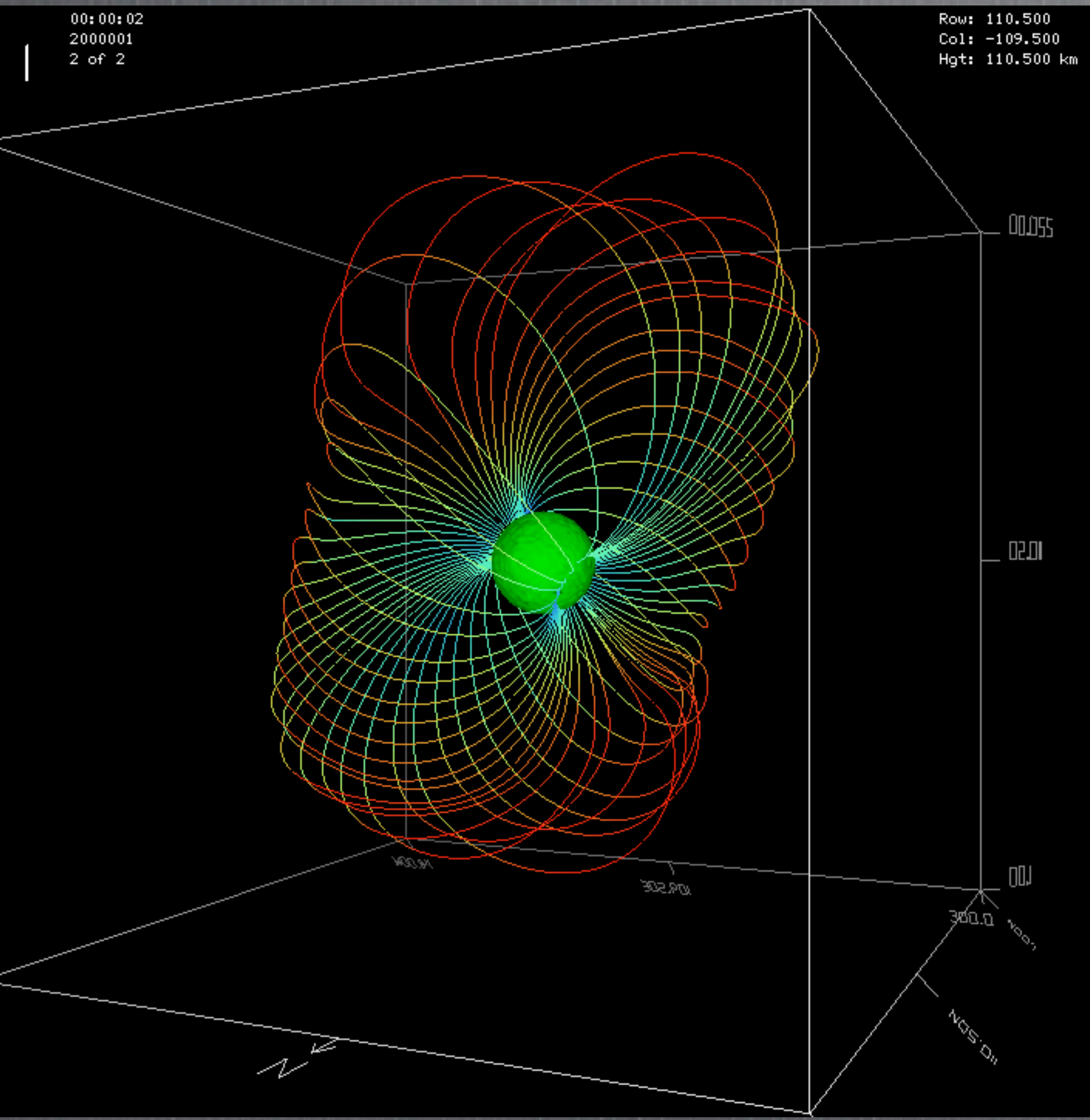

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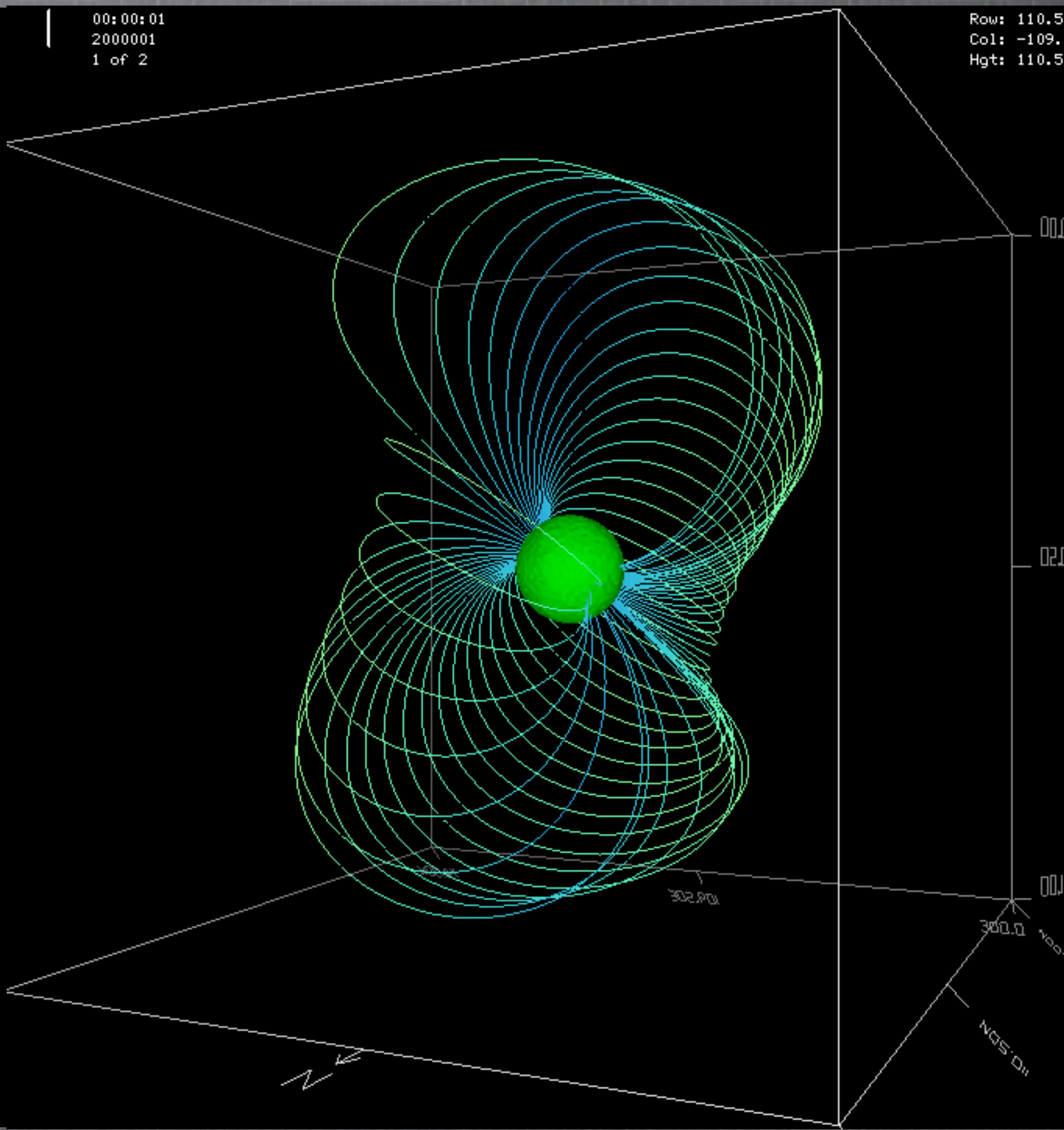

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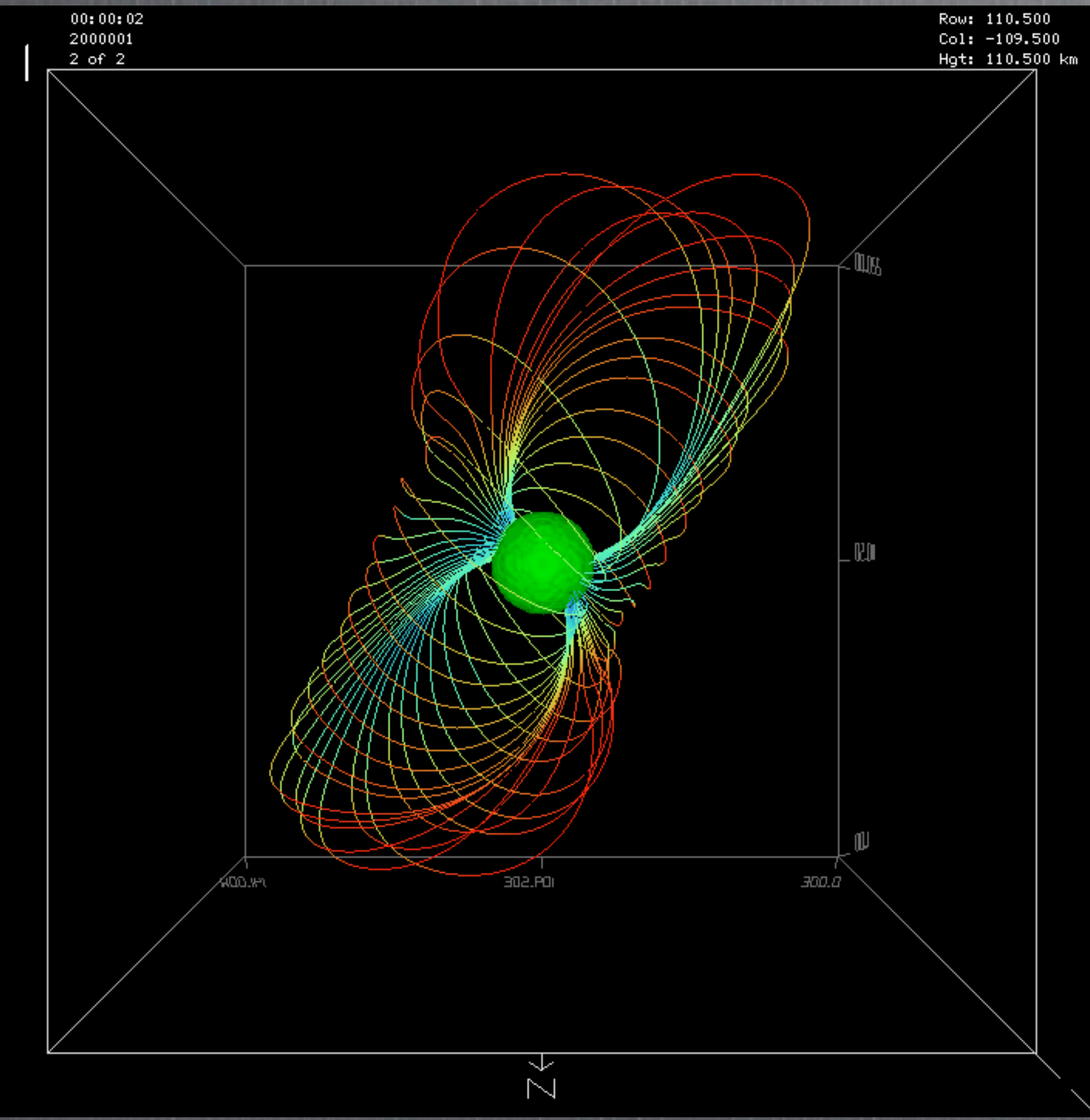

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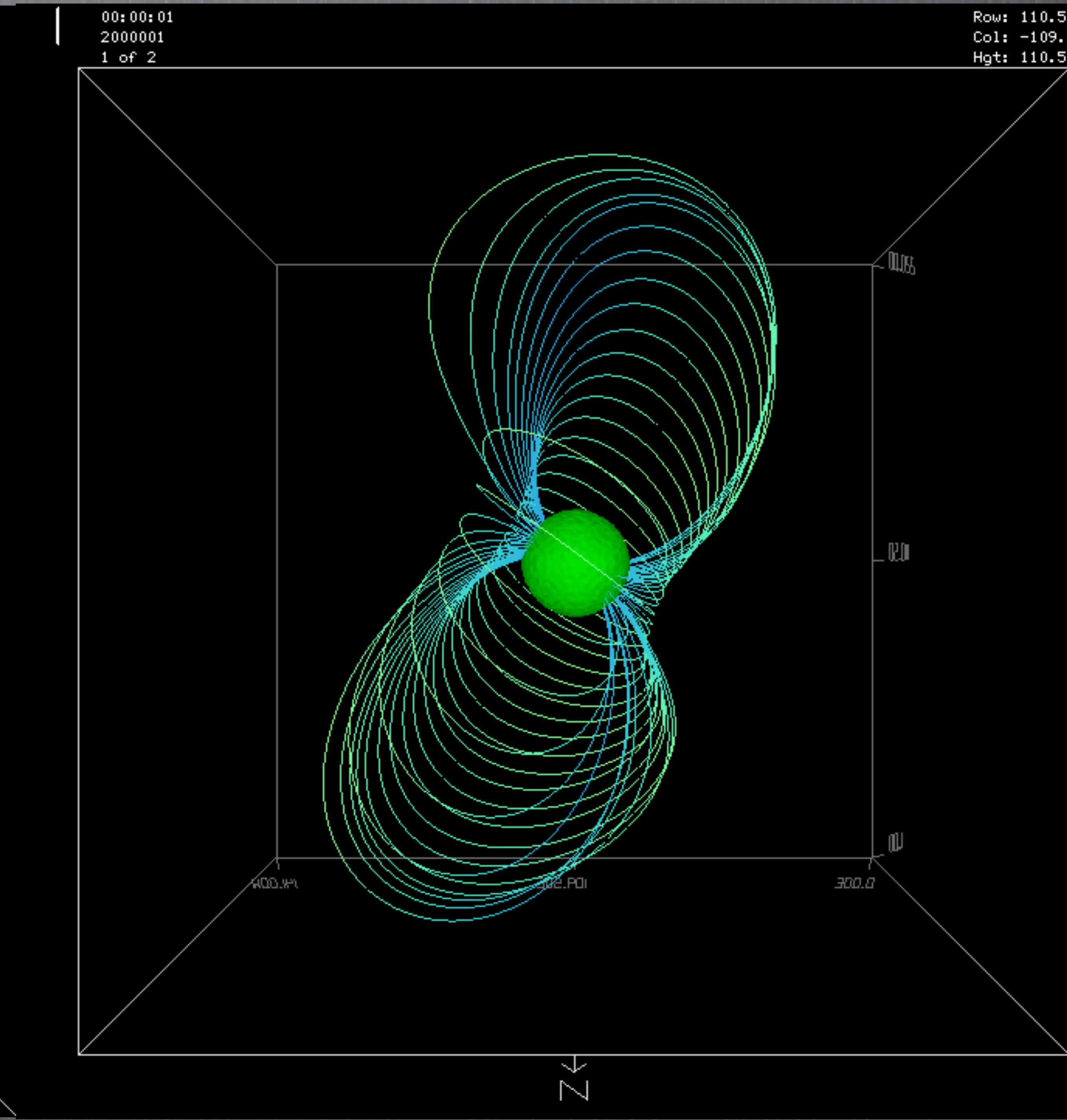

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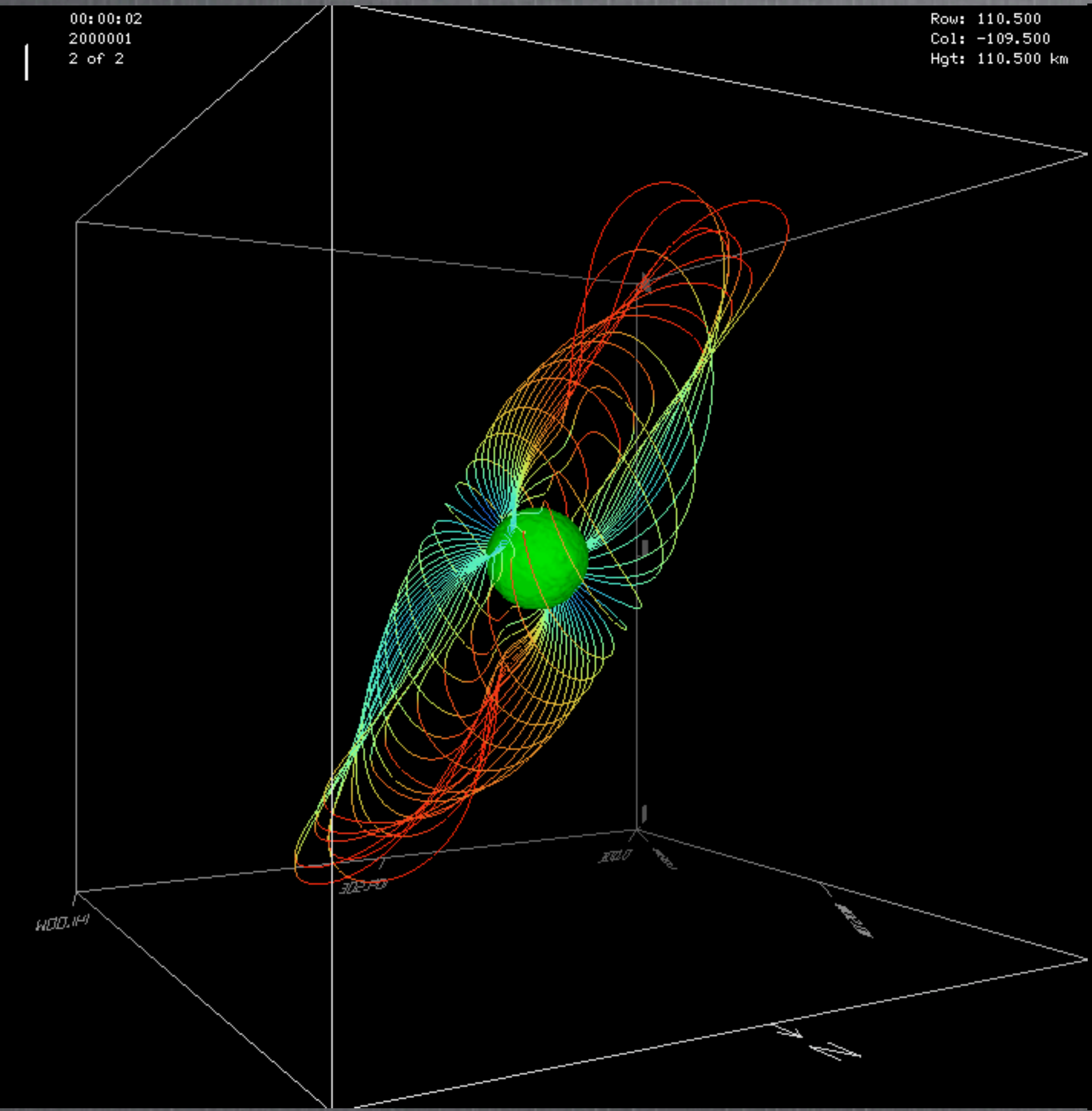
Emission modeling

Force-free vs vacuum field: last open lines



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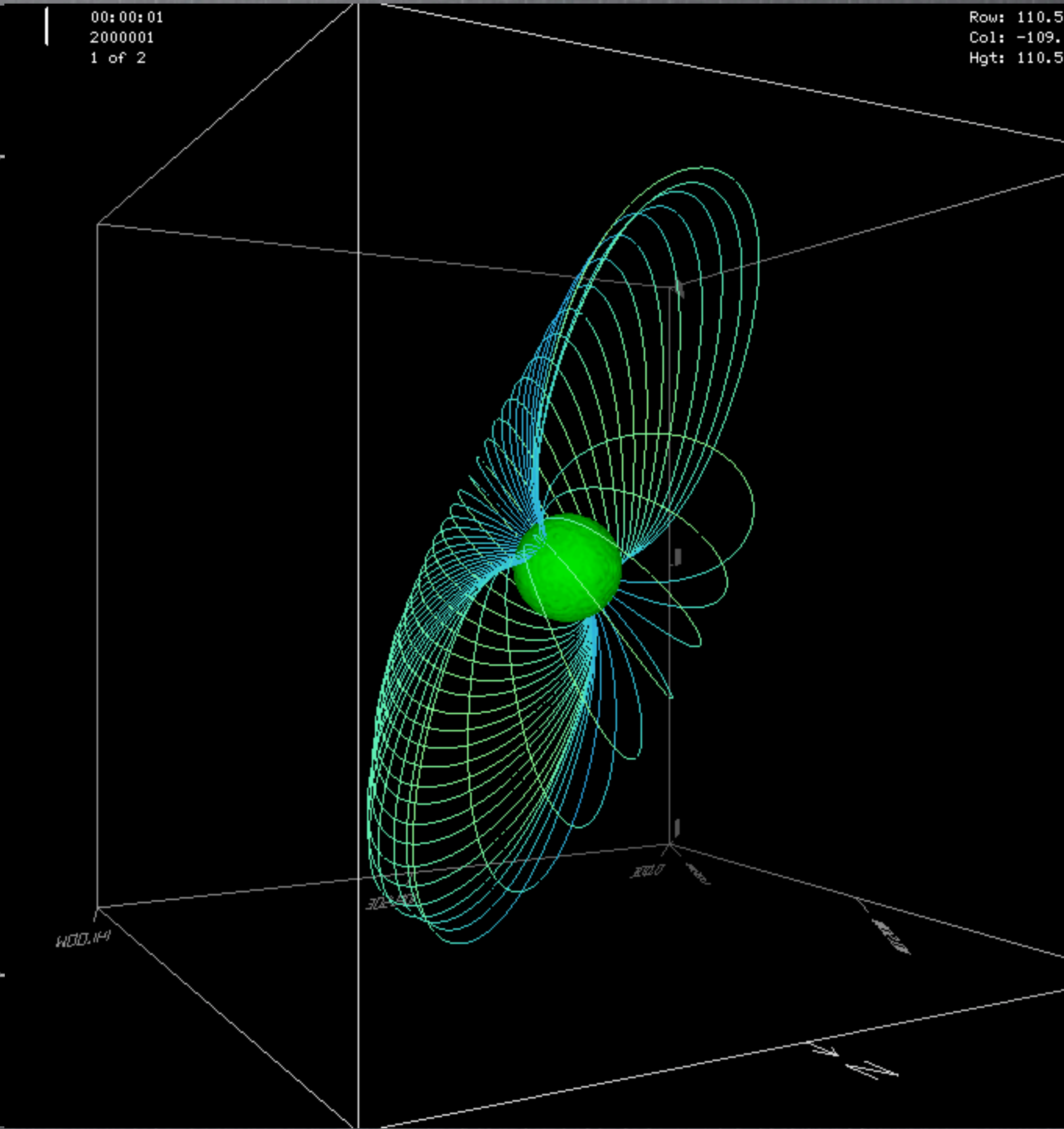

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Emission modeling

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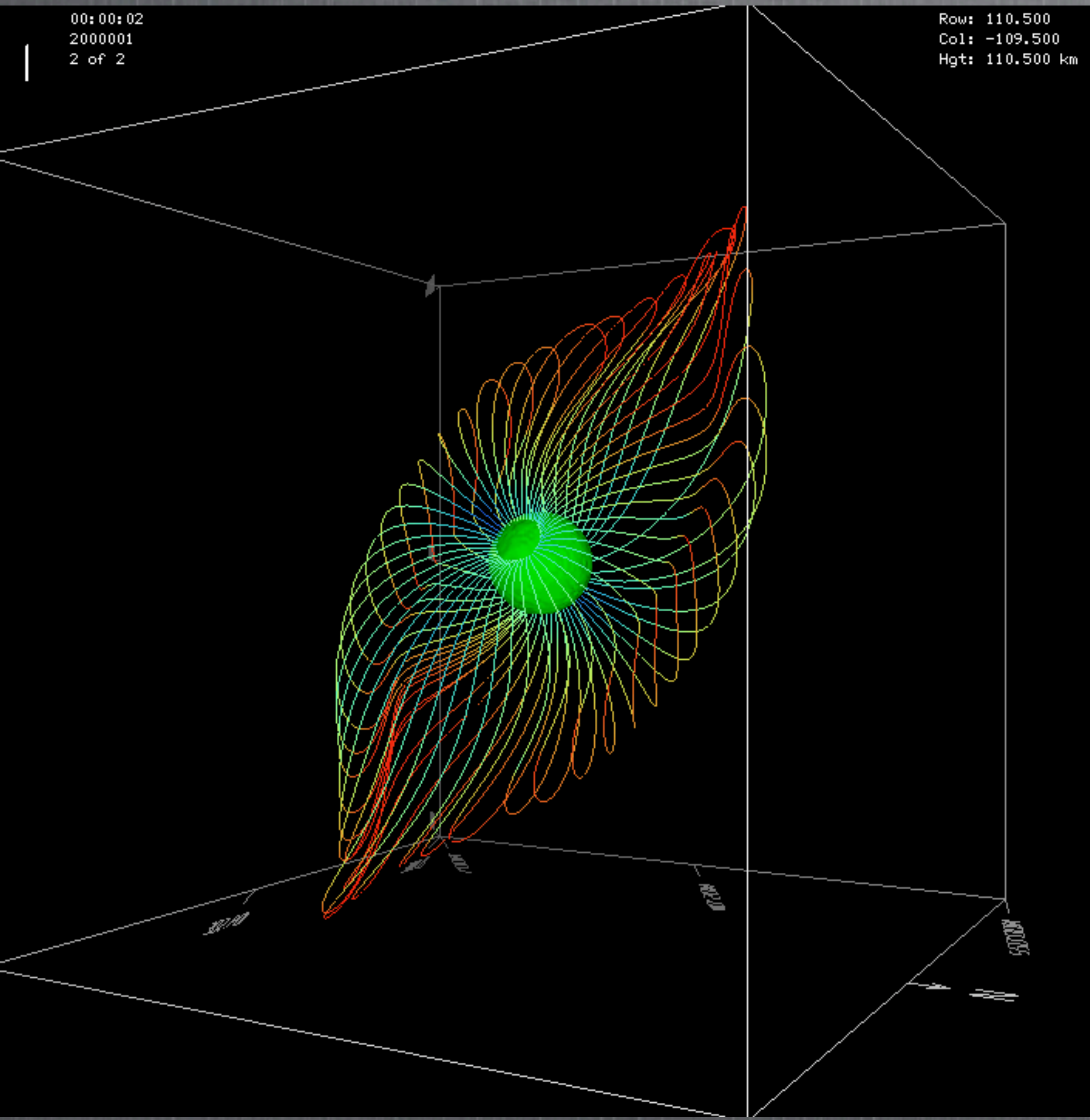

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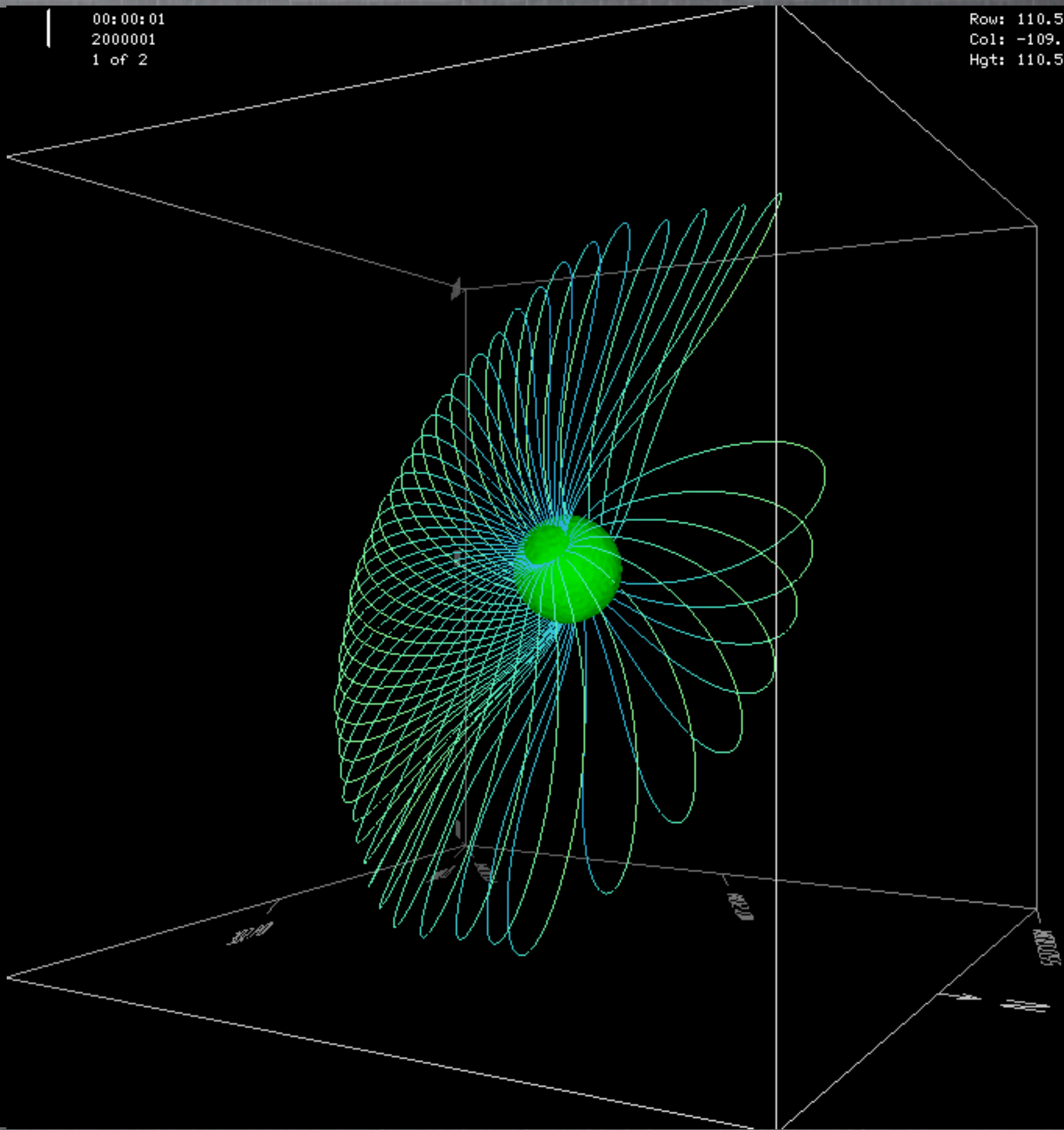

Last closed fieldline for vacuum

Emission modeling

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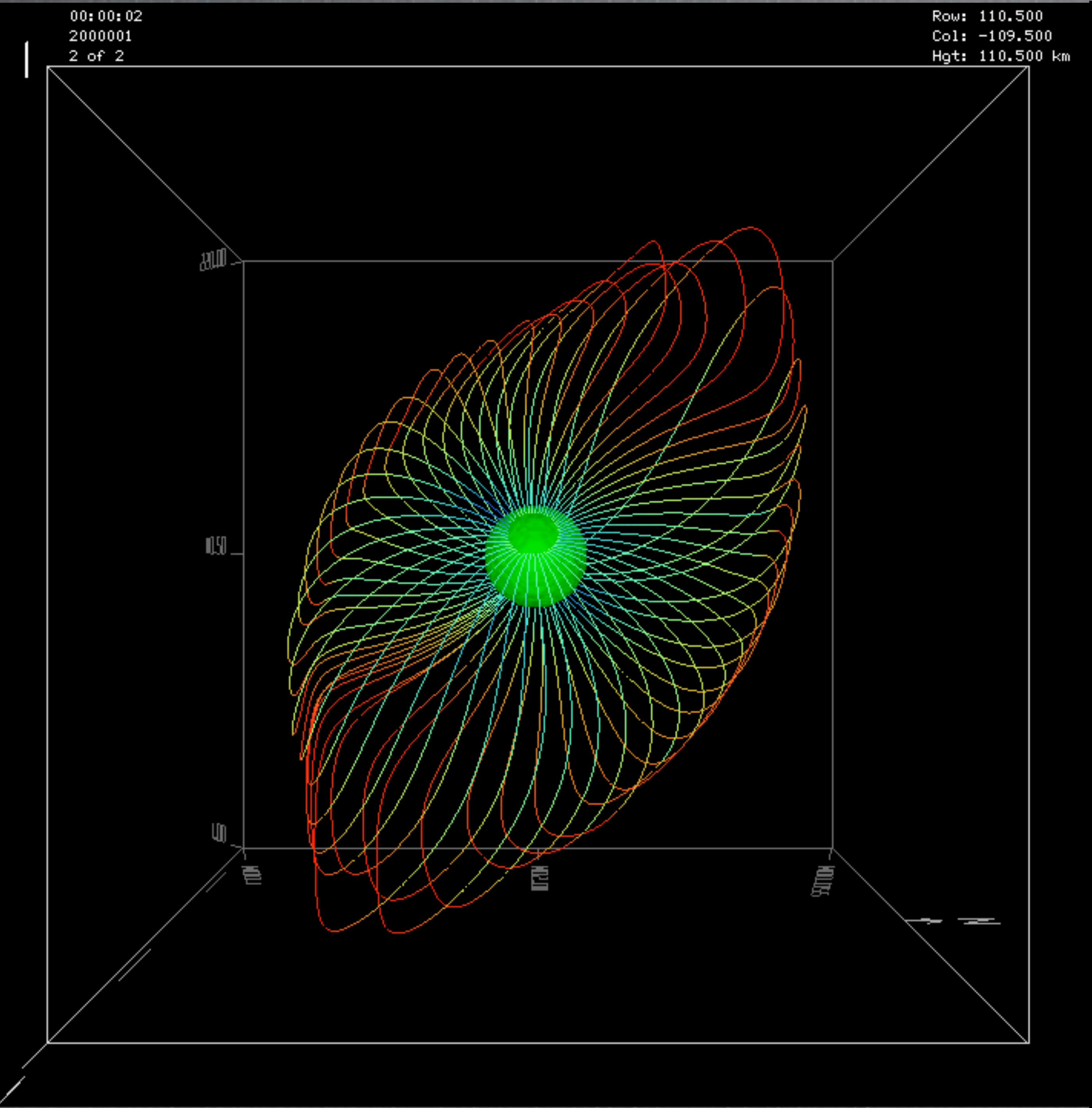

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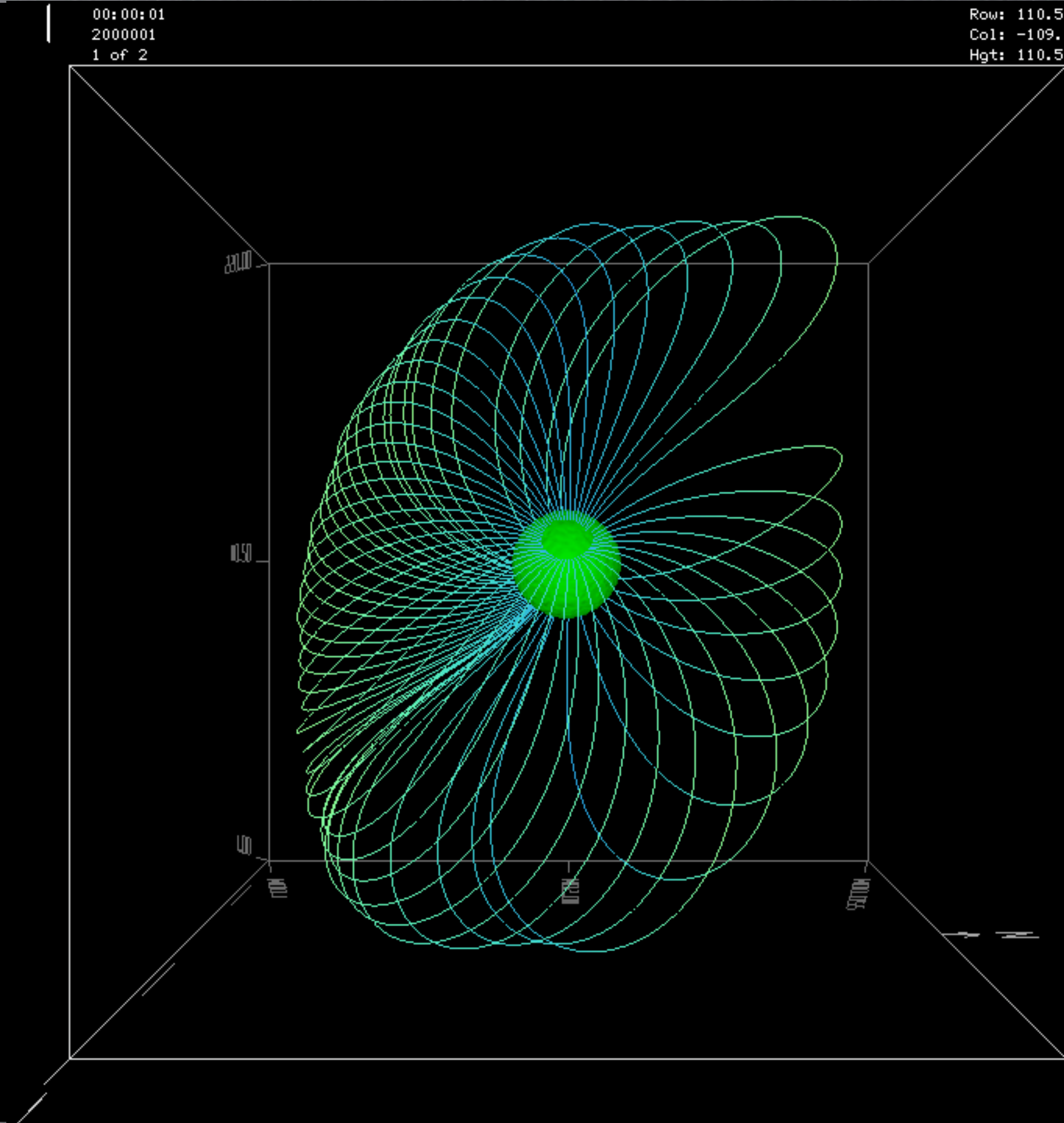
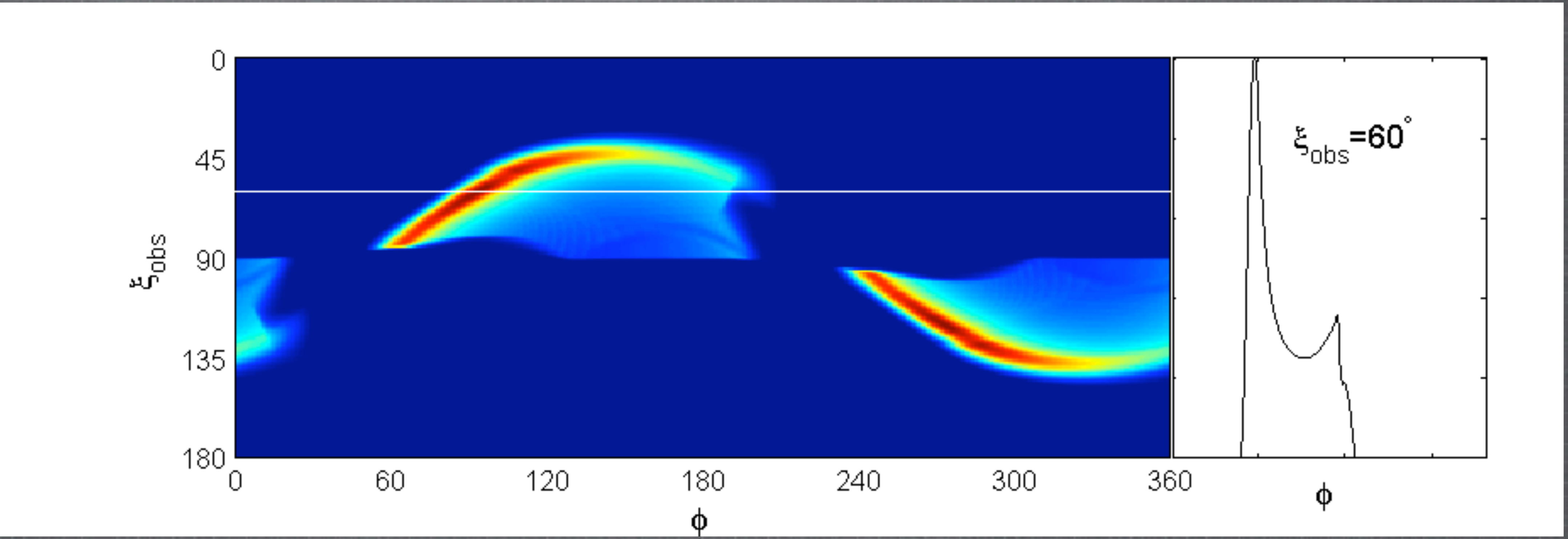


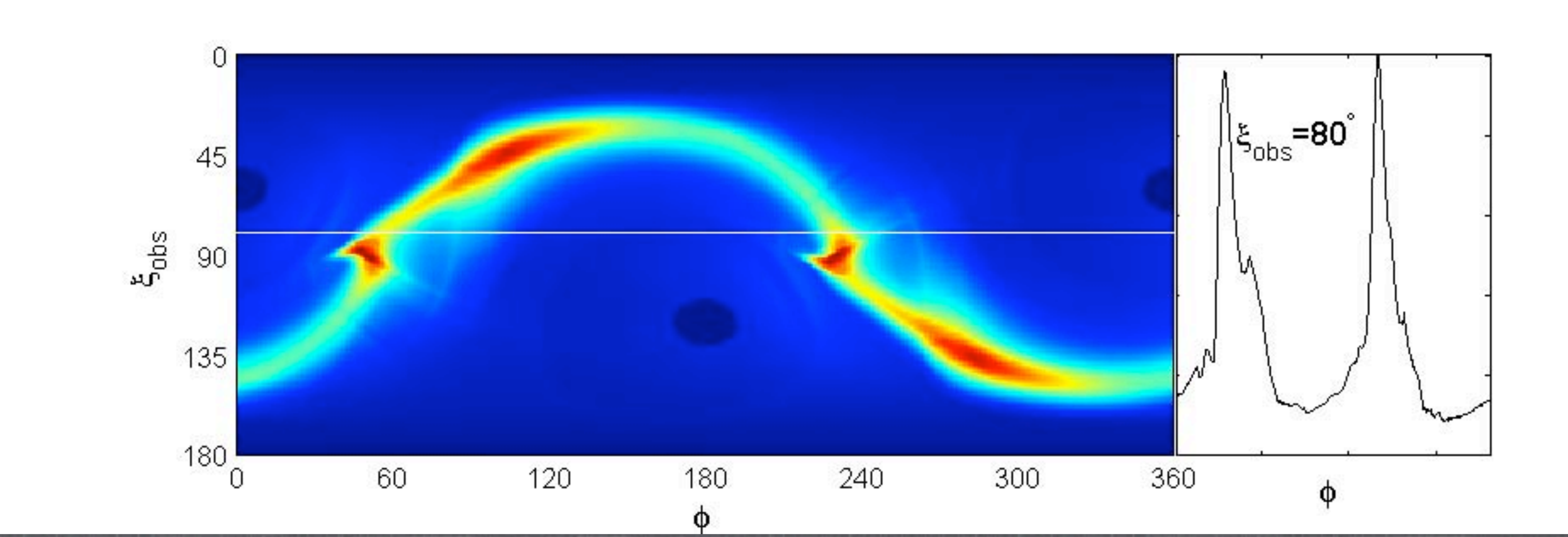
Last closed fieldline for vacuum

EMISSION MODELING

Force-free lightcurves

Other possibilities: outer gap model? In force-free know the charge density, so can predict the null charge surface accurately.

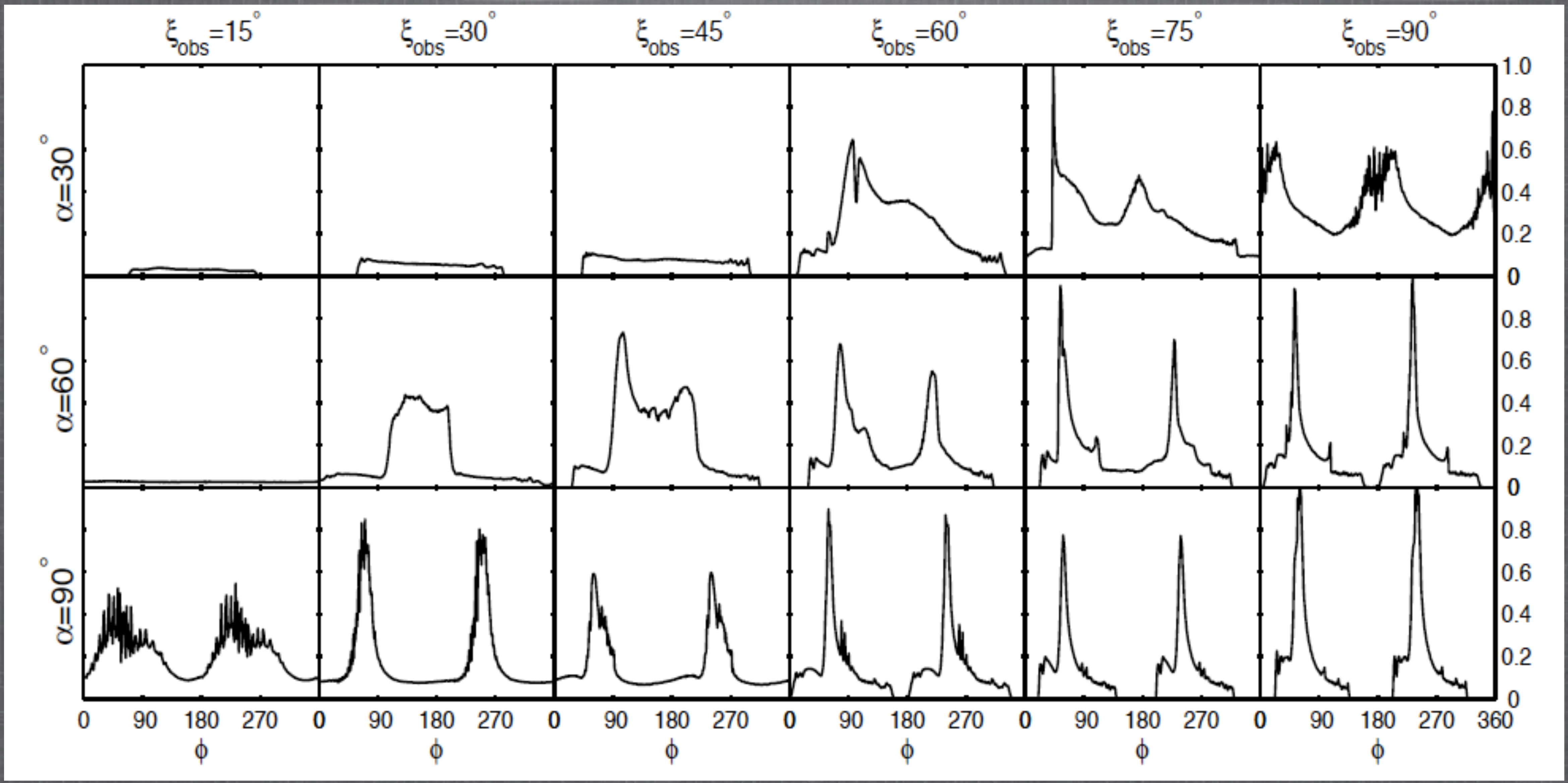




Adding pitch angle near LC

EMISSION MODELING

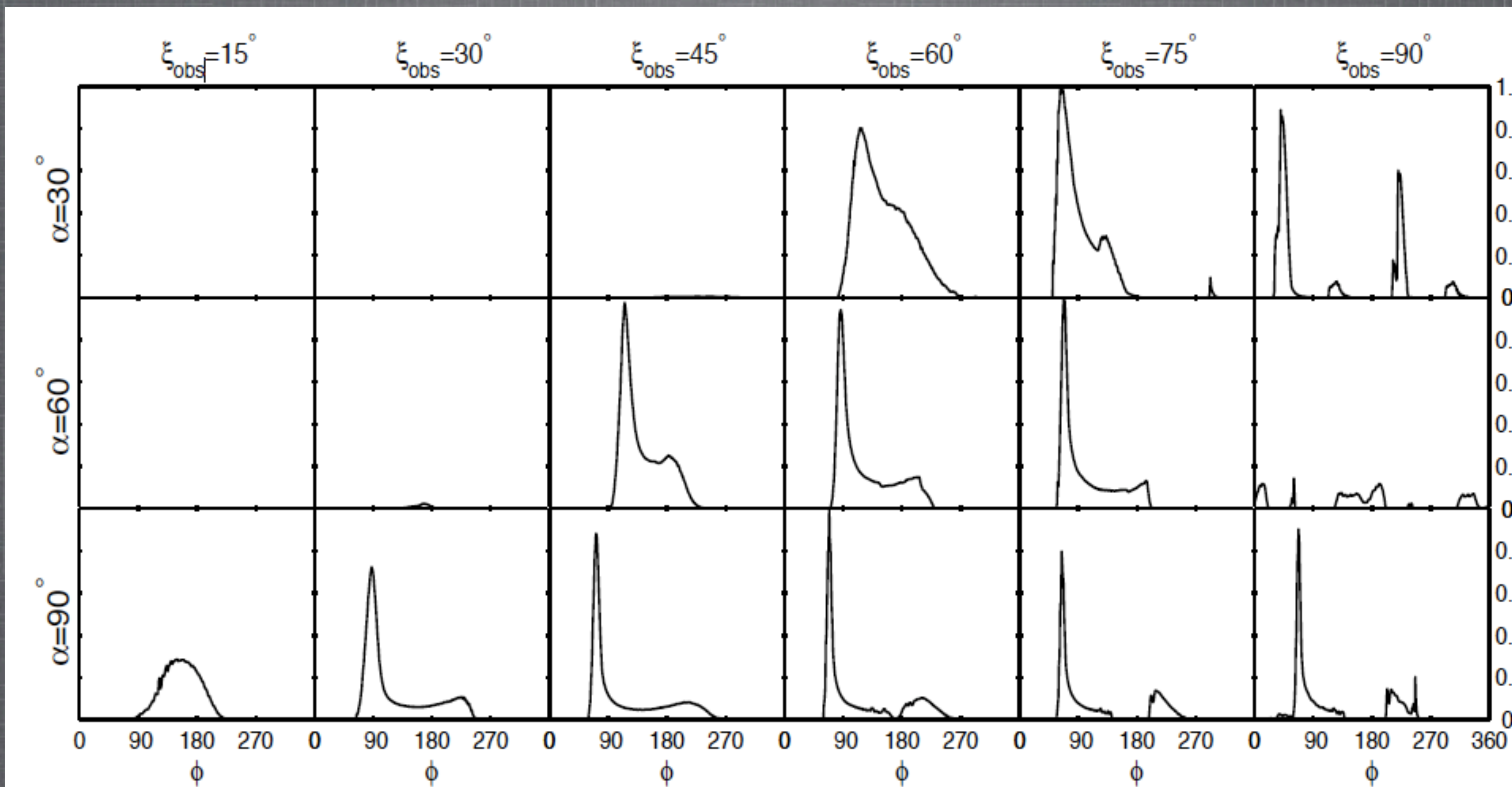
Force-free Lightcurves



"Annular gap" light curves
without pitch angle

EMISSION MODELING

Force-free Lightcurves



"Outer gap" light curves with force-free field

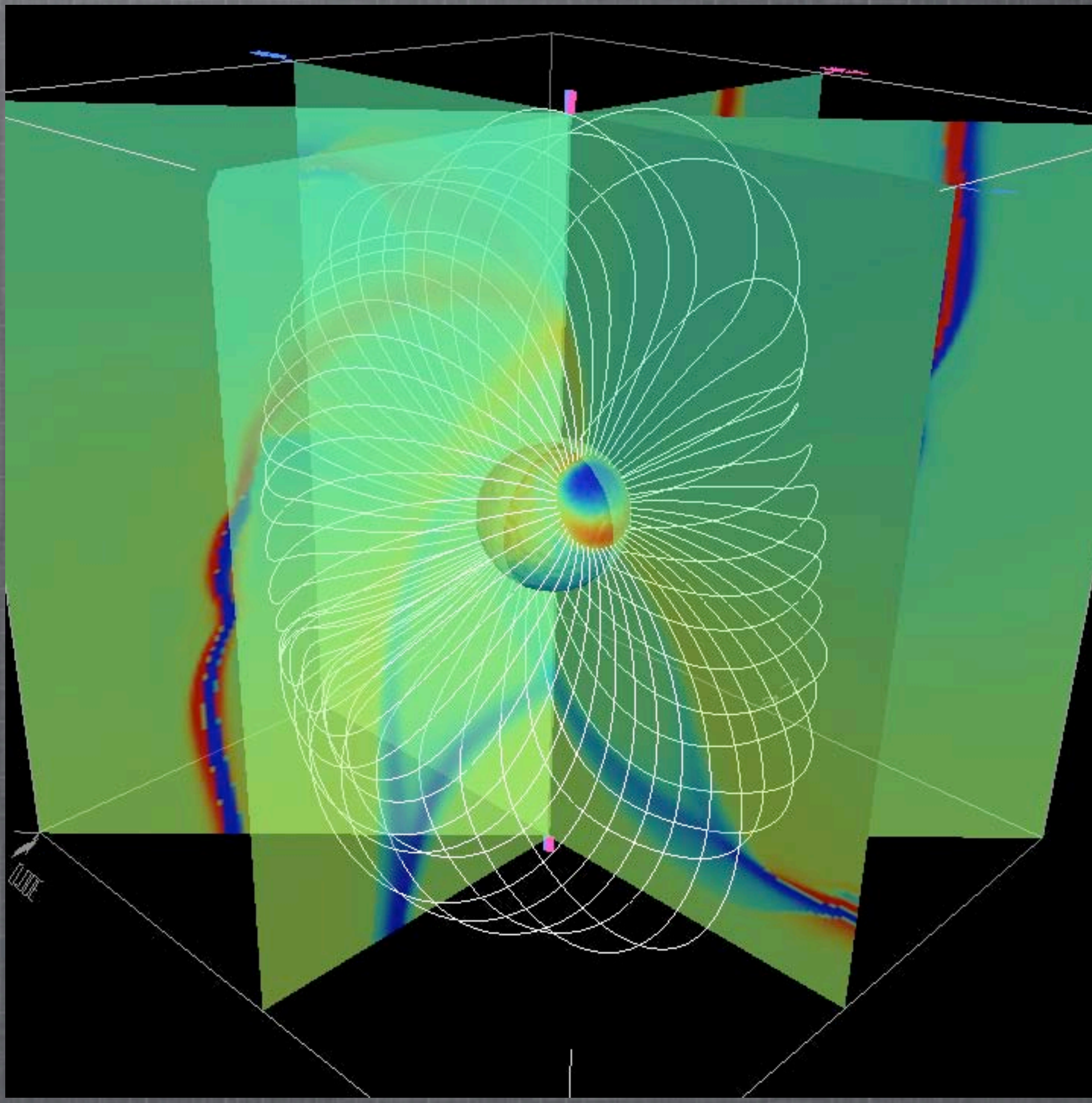
Emission modeling

Force-free vs vacuum field

Another idea: we are forced to pick the emission flux surface. What if we pick the emission location based on the current in the magnetosphere?

This eliminates one degree of freedom, but introduces another: need to understand the relationship between current and light.

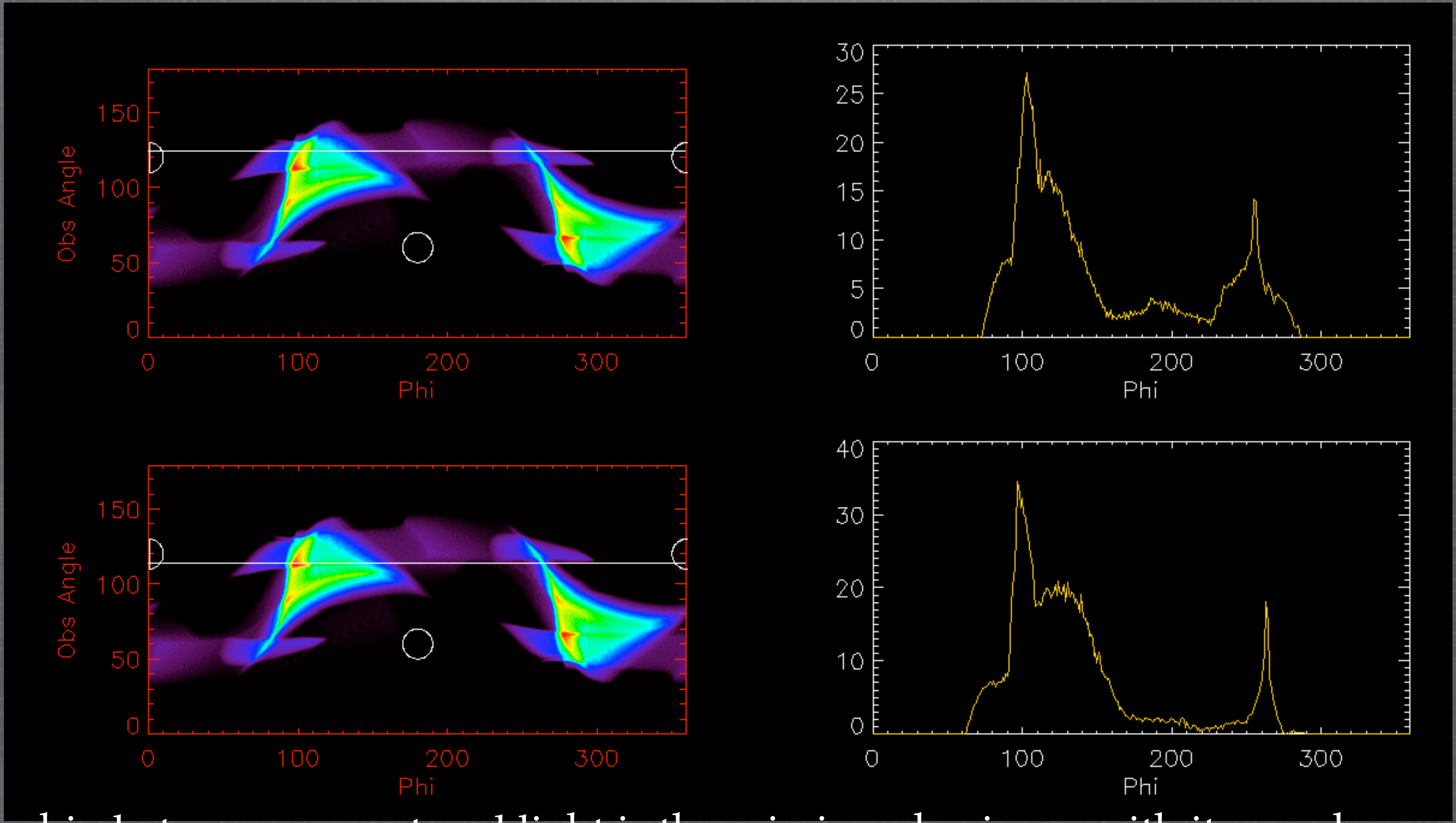
We use lambda as a proxy for the current.



EMISSION MODELING

Force-free vs vacuum field

We use lambda (current) as a proxy for the emission.



Relationship between current and light is the missing physic s -- with it, can show emission unambiguosly. Alternatively, can try to constrain the locaton of the emission from the light curves only.

Emission modeling

Force-free vs vacuum field

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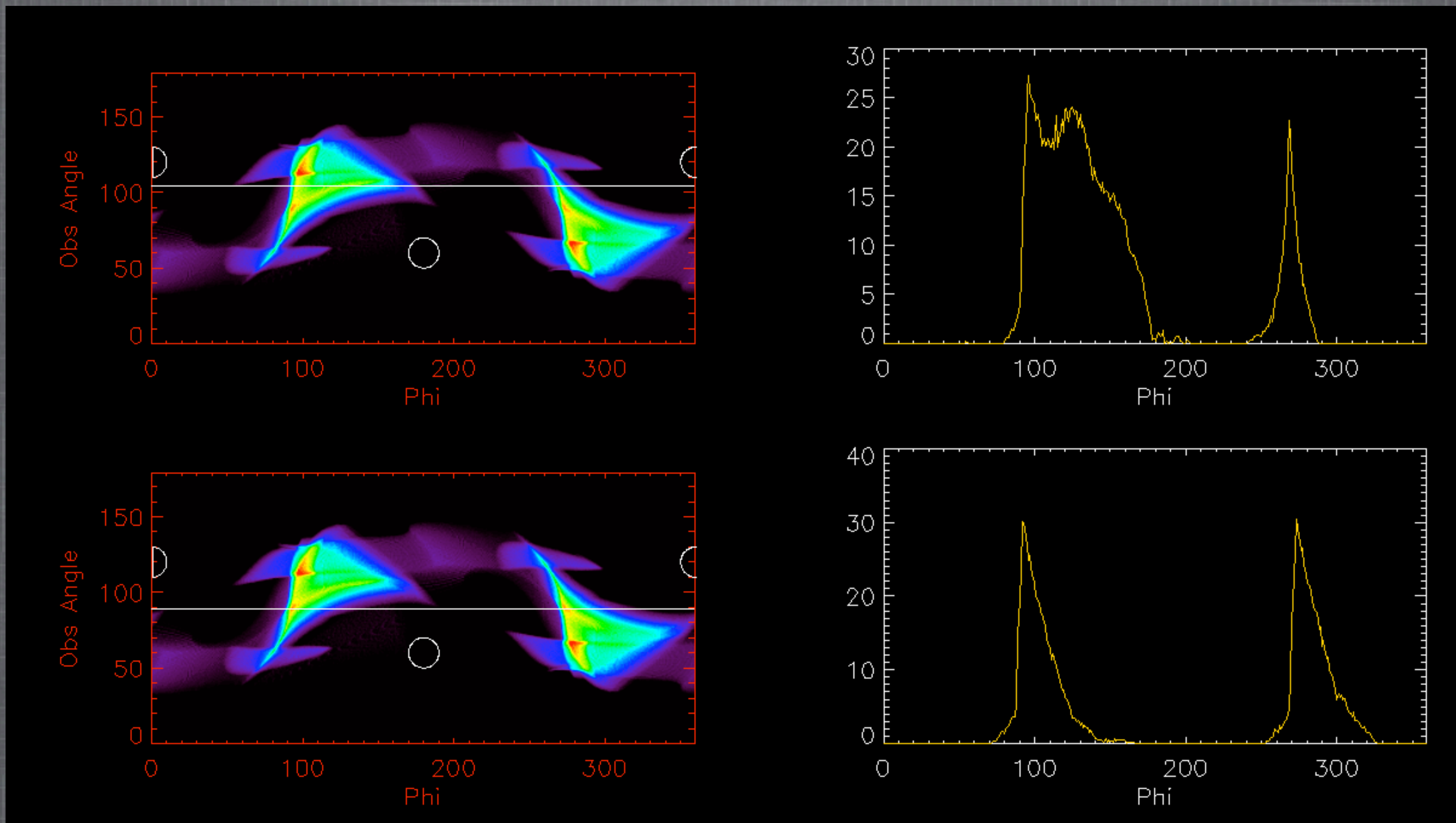

Relationship between current and light is the missing physic s -- with it, can show emission unambiguosly. Alternatively, can try to constrain the locaton of the emission from the light curves only.

Conclusions

Magnetospheric shape with plasma effects is now known under the force-free framework.

Spin-down of arbitrary inclination rotators can be calculated. Braking index still 3. Torques are alignment-causing. Is there signal in the data?

Light curves with force-free field show different caustics than vacuum field.

Force-free models can only say where the currents are, not what happens to them -- need more physics. Need prescription for emission.

Is time-dependence important?