Homework #4, AST 203, Spring 2009

Due in class (i.e., by 4:20 pm), Thursday April 2

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in.
- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.
- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

100 total points

1. Transits of extrasolar planets (25 points)
A new NASA mission called "Kepler" was launched on March 6th to observe the transits of extrasolar planets. During a transit, a planet goes in front of its host star as seen from our vantage point on Earth. This blocks a little bit of light from the star, so an accurate measurement of the change in brightness of the star will show a temporary dimming. The main goal of the Kepler mission is to detect Earth-size planets in the habitable zone around stars like our Sun. In this problem, we will predict what the Kepler mission will observe. Consider a planet with the radius of the Earth, orbiting at 1 AU from a star that has parameters like our Sun. The orbital plane of this planet lies along our line of sight.

a) How long does a transit last (in hours)? (*10 points*)

Hint: the Earth is very far from this planetary system.

b) By how many percent does the brightness of the host star diminish in the middle of the transit? (*10 points*)

c) Consider now the planet the size of Jupiter, orbiting at 0.5 AU from the same star. By how many percent does the brightness of the host star diminish in the middle of the transit by this planet? (*5 points*)

2. White Dwarfs and Neutron Stars (30 points)
White dwarfs and neutrons stars are"compact objects" – stars whose large mass (near a solar mass) is concentrated in a small volume. Such stars possess exotic properties that

allow us to probe the behavior of matter at extreme densities. Consider a white dwarf (WD) with the mass of 1 $\mathrm{M}_\odot$ and radius 1 $\mathrm{R}_{\mathrm{Earth}}$, and a neutron star (NS) with a mass of $1.4M_\odot$ and radius 10 km.

a) Find how much mass is contained in one teaspoon ($1\mathrm{cm}^3$) of material from WD and NS. Express the answer in metric tons. (*7 points*)
Hint: the material can be taken to be at the mean density of these stars

b) Calculate the mass density of a neutron and compare it to the mean density of NS. A neutron can be considered as a sphere of radius 1 femto-meter (10^{-15} m). (*6 points*)

c) Calculate the mean distance between atoms in WD. Express your result in Ångstroms (1 Ångstrom = 10^{-10} meters). You may approximate the white dwarf to be made entirely of carbon atoms (12 times the mass of hydrogen). Compare with the typical sizes of atoms under normal conditions, $\sim$ 1 Ångstrom. *Hint: We are looking for an approximate answer here.* (*7 points*)

d) We can estimate the rate of rotation of compact objects by knowing that they are the end product of contraction of rotating main sequence stars after they exhaust their nuclear fuel. Each piece of gas in a star contracts in such a way that the product of its distance to the rotation axis times the velocity of rotation about the axis is a constant. This is known as conservation of "angular momentum," and is the same phenomenon that causes a spinning ice skater to turn faster up when she raises her hands. Consider a point on the equator of a main-sequence star with the radius of the Sun, and rotation period of 30 days. Now imagine that this point contracted with the star and ends up on the surface of a WD or a NS. Find the expected rotation periods of the WD and NS. Express them in minutes and milliseconds, respectively. (*10 points*)

3. Distance to a Supernova (25 points total)

a) The Crab Nebula, a supernova remnant, is roughly spherical, and is expanding. The angle it subtends on the sky increases at a rate of 0.23 arcseconds a year. Spectra taken of the glowing gases indicate, via the Doppler shift, that the gas is also expanding along the line of sight at 1200 km s^{-1} relative to the center. From this, deduce an approximate distance to the Crab Nebula, in light years. (*8 points*)

b) The angular size of the Crab nebula is currently 5 ± 1.5 arcminutes in diameter (the error reflects the fact that the nebula is not exactly spherical). Assuming that it has been expanding at a constant rate, calculate roughly the year at which the light from the initial supernova reached the Earth. Be sure to include the effects of the uncertainty in the angular size. Compare your result with the date at which the Chinese observed this supernova, AD 1054. Do these dates agree within the precision of your calculation? (*5 points*)

c) You are given the velocity of the gas above, and assume that the mass of the gas is twenty solar masses. Calculate the kinetic energy of the explosion; express your result in Joules. The star that exploded was probably an O star, with a main sequence luminosity 10^3 times that of the Sun. Calculate how long the O star has to shine to generate as much energy as is currently present in the kinetic energy of the expanding gases. (*7 points*)

d) Another way to estimate the date of the explosion is to study how fast the neutron star in the middle of Crab Nebula is slowing down. This neutron star is detected as a radio pulsar with period of $P = 33$ milliseconds. It is losing its rotational energy to a wind, and as a result is currently slowing down at the rate $r = 4.2 \times 10^{-13}$ seconds per second (the units of this rate are a bit funny – this is the number of seconds by which the period of the star is increasing every second). The characteristic age of a pulsar is given by the expression $P/(2r)$, where the factor of 2 accounts for the fact that the pulsar was spinning down faster in the past. Calculate the characteristic age of the Crab pulsar and compare the date of the historical explosion to the date inferred from pulsar age determination. (*5 points*)

4. True or False? (20 points)

Determine if the following statements are true or false, and give the reasoning to support your conclusion in a short paragraph.

a) If the Sun had been born as a high-mass star some 4.5 billion years ago, rather than as a low-mass star, the planet Jupiter would probably have Earth-like conditions today, while Earth would be hot like Venus. (*7 points*)

b) Globular clusters generally contain lots of white dwarfs. (*6 points*)

c) If a 3.5 $M_\odot$ main-sequence star is orbiting a 2.5 $M_\odot$ red giant, the red giant must have been more massive than $3M_\odot$ when it was a main-sequence star. (*7 points*)