# Homework #3, AST 203, Spring 2009

Due in class (i.e., by 4:20 pm), Thursday March 5

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in.
- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.
- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

## 100 total points

#### **1. Solar power in space** 25 total points

NASA's Mars Reconnaissance Orbiter (MRO) is currently in orbit around Mars. The spacecraft carries six instruments to study the Martian atmosphere and surface, including a ground-penetrating radar to search for water beneath the surface. You can find more information (not necessary for this problem) at http://www.nasa.gov/mro.

MRO communicates with Earth using a 10-foot diameter dish antenna and a transmitter powered by 100 square feet of solar panels. A very good modern conversion efficiency for solar panels is 30%, i.e. the panel converts 30% of the sunlight incident upon it into electrical power - the other 70% is lost.

- a) Take the Sun to be a blackbody with a surface temperature of 6000 K. The Sun's radius is  $7.0 \times 10^5$  km. Calculate the Sun's luminosity, in watts (Joules/second). (The luminosity you have calculated is called the "bolometric luminosity" because it sums the power emitted by the Sun at all wavelengths.) (6 points)
- b) Mars is 1.5 AU from the Sun. Calculate the brightness of the Sun at Mars' distance (i.e., the solar flux on Mars' surface) expressed in watts per square meter. (6 points)
- c) Using the approximation (good to about 10%) that 1 meter = 3 feet, calculate how much electrical power will be available to the MRO transmitter, assuming that the MRO solar panels are facing the Sun directly, and that they have a conversion efficiency of 30%. Express your result in watts. (6 points)

d) Suppose now that a spacecraft identical to MRO were launched (presumably from a bigger rocket!) to observe Saturn's moon Titan. How much power (in watts) would the MRO at Saturn have available to its transmitter? Saturn orbits at 9.6 AU from the Sun. For comparison, a typical light bulb in your home has a total power output of about 60 watts. What would be the ratio of MRO transmitter power at Saturn to that of a 60 watt light bulb? This is why missions to the outer Solar System so often rely on power generated from the radioactive decay of a plutonium isotope, rather than from solar panels. (7 points)

### **2. Human energetics** 25 total points

You are sitting down to study for the mid-term in the library. Let's find out what limits the time you are able to spend studying.

- a) Calculate how much power you are radiating, in watts. While your skin temperature is about 33° Celsius, the outside of your clothing is at about 28° Celsius. In the absence of perspiration, black body radiation at this temperature is your main radiation mechanism. You can take your surface area to be 2 m<sup>2</sup>. (5 points)
- b) What is the wavelength of the peak of your spectrum? Comment why we cannot see each other in a dark room. (5 points)
- c) Your body is constantly emitting thermal radiation and is also absorbing radiation from the surroundings. The library is unusually chilly today, and the ambient temperature is 15° Celsius. How much power are you absorbing from the room, in watts? You can use the black body formula at the temperature of the room. (5 points)
- d) What's your net energy loss rate, in watts? Compare this to a typical 60W light bulb. How much energy will you lose per day, in mega-Joules? (5 points)
- e) On a normal dorm diet, your daily energy intake from food should be close to 2000 kilo-calories (kcal), where one calorie is defined as 4.2 Joules. How long will you be able to study in this library before your daily energy intake is exhausted? (5 points)

#### **3. Extrasolar planet** 25 total points

There are currently 340 known extrasolar planets, i.e., planets around other suns. Most of them we infer only by their gravitational pull on their host stars, and a handful appear to dim the light of their host star as they transit in front of the star. In November of last year, however, astronomers announced the detection of a planet which not only was not only resolved (which is already amazing) but whose orbital motion was also confirmed by direct imaging. In other words, in two pictures made by Hubble Space Telescope, the planet was observed to have shifted position. This Jupiter-like planet is called Fomalhaut b, orbiting a star Fomalhaut, which is 25 light years from Earth (the name Fom al-Haut comes from Arabic and means 'the mouth of the fish', as the star is in the constellation Southern Fish – Piscis Austrinus). The host star has a mass of 2.1  $M_{\odot}$  and its radius is 1.8  $R_{\odot}$ . Its surface temperature is 8700 K. The planet orbits with the semimajor axis of 115 AU around the star, has small eccentricity, and the plane of the orbit is in the plane of the sky (in other words we are seeing the orbit face-on). If you want to see the picture of the star, the planet and the "debris disk" left over after the planet formation in this system, look at http://apod.nasa.gov/apod/ap081114.html

- a) Determine the angular separation between the star and the planet, in arc seconds. (5 points)
- b) Determine the period of the planet, in years. Hubble imaged the system in 2004 and 2006. What is the distance that the planet traveled between the images, in AU? (5 points)

*Hint: if you are reaching for your lecture notes to find the formula for the velocity of the planet, you are doing it the hard way.* 

- c) What is the angular separation between the position of the planet in 2004 and 2006, in arc seconds? Can the Hubble telescope, with the angular resolution of 0.04 arc seconds, detect this motion? (5 points)
- d) What is the equilibrium temperature on Fomalhaut b? You can ignore the effects of albedo and greenhouse effect. Can liquid water exist on Fomalhaut b? (5 points)
- e) At what distance from the star Fomalhaut would the planet have the same equilibrium temperature as on Earth? Again, ignore albedo and greenhouse effects. (5 points)

# 4. Spinning too fast 25 total points

The rate of rotation of astrophysical objects that are held together by gravity (e.g., stars or planets) cannot be larger than a certain maximum. Rotating faster than this rate will tear the star apart. Let's find the expression for this maximum rotation rate.

- a) You are sitting on the equator of a star of radius R that is spinning about its axis with period P. What is the rotation speed that you have on the equator? (3 points)
- b) Using your expression for the rotation speed, what is the centrifugal acceleration that you experience on the equator? (3 points)
- c) The star has mass M. What is the gravitational acceleration that you feel on the surface? (3 points)
- d) Now equate the centrifugal and gravitational acceleration, and find the period of rotation when they are equal. What happens if the star rotates faster than this period? (10 points)
- e) Calculate the limiting rotation periods for the Earth and for the Sun. Write them in the most appropriate units (e.g., seconds, minutes, hours, days, years, etc.). Find the ratio of the critical rotation period for the Earth to the current rotation period. The Sun is rotating with a period of 25 days. FInd this ratio for the Sun as well. (6 points)