Homework #2, AST 203, Spring 2009

Due in class (i.e., by 4:20 pm), Tuesday February 24

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in.
- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.
- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

100 total points

1. Dropping out of the sky *30 total points*

MIR was a Soviet (and later Russian) space station that operated from mid-1980s in circular low Earth orbit, at an altitude of 400 km above the surface of the Earth. In 2001 MIR was successfully deorbited and crashed in a remote region of the Pacific Ocean. In this problem we will calculate

a) Let's find out how fast MIR was moving in its orbit. Derive an algebraic expression for the speed v of an object in a circular orbit around the Earth, in terms of the mass of the Earth $M_\oplus$, orbital radius r, gravitational constant G, and numerical constants. You can do this either by starting with Newton's form of Kepler's Third Law, or by using Newton's Law of Gravitation and his second law of motion. (*10 points*)

b) Now plug in the numbers and calculate MIR's orbital speed, in kilometers per second. *Hint: Think hard what you want to use for the radius of the orbit!* (*5 points*)

c) The space station had the mass of 125 tons. What was its orbital kinetic energy, in Joules?
Hint: A joule is the MKS unit of energy, and is equal to one kilogram meter^2/second^2. (*5 points*)

d) The kinetic energy of the station was dissipated during the burning up in thick layers of the atmosphere and at final impact in the ocean. Let's get a sense for how much energy that was. The energy per kilogram of the high explosive TNT is 1.5×10^7

joules/kg. Calculate how many tons of TNT release the same amount of energy as the deorbiting of MIR. For comparison, a typical aircraft bomb is 500 pounds of explosives, and the first nuclear test was equivalent to 18 kilotons of TNT. Which one is more destructive: a kilogram of TNT or a kilogram thrown with MIR's orbital velocity? Calculate the ratio of the released energies. (*10 points*)

2. Solar eclipse *40 total points*

a) What would the orbit of the Moon have to be in order to have one total solar eclipse each month? Explain and sketch a diagram. (*5 points*)

b) After the orbital change in part a), how often would lunar eclipses be visible from the Earth? Explain. (*5 points*)

In the following we will calculate how long a typical solar eclipse can last on Earth. During the eclipse, the shadow of the Moon sweeps over the face of the Earth. The motion of the shadow is due to Moon's orbital motion. Let's find out how fast the shadow is moving.

c) You know that the period of rotation of the Moon about the Earth is approximately 27 days. Using Newton's version of the Kepler's third law, find the orbital velocity of the Moon about the Earth, in kilometers per second (you already derived part of the answer in problem 1a). The Mass of the Earth is 6.0×10^{24} kg, and you can neglect the eccentricity of Moon's orbit. (*15 points*)

d) Since the Moon is very close to the Earth compared to the distance to the Sun (check this!), the shadow of the Moon is a circle with the Moon's diameter, and the shadow moves over the Earth with the orbital velocity of the Moon. How long does it take for the shadow of the Moon to completely sweep across the diameter of the Earth (in hours)? Earth's radius is $\sim$ 6400km, Lunar radius is $\sim$ 1700km.
Hint: you are looking for the time between the moment the Moon's shadow begins to cross the Earth to the time the shadow completely leaves the Earth. Draw a sketch to help you along. (*15 points*)

3. Radioactive decay *20 total points*

a) Cesium-137 (^{137}Cs; Cs has atomic number 55), is unstable isotope of Cesium which decays through the "beta-decay," or the reaction that turns one of the neutrons in the nucleus into a proton and also emits an electron ("beta-particle") and an antineutrino. After beta-decay, what type of atom has ^{137}Cs become? Give the symbol of the resulting element, its atomic mass and atomic number. You will need to refer to the periodic table, e.g. in Appendix D of the text, to answer this problem. (*10 points*)

b) The Chernobyl nuclear disaster was a notorious nuclear accident that happened in 1986 in Ukraine. A fission reactor melted, and part of its core, containing nuclear fuel was vaporized and contaminated a very large area (from Ukraine to Sweden). Significant amount of the fallout consisted of Cesium-137, whose half-life is 30 years. Due to its location in the periodic table, Cesium has very similar chemical properties to

Potassium, and is readily taken up by the human body, where it damages the tissues as it emits electrons (and gamma rays) during the decay. This element is one of the main health hazards of the Chernobyl fallout. What is the fraction of Cesium-137 that has decayed away by now? *Hint: The mass of the material undergoing radioactive decay follows the following formula with time:* $M(t) = M_0(\frac{1}{2})^{(t/t_{1/2})}$*, where* M_0 *is the initial mass of the material, and* $t_{1/2}$ *is the half-life.* (*10 points*)

4. Light and power *10 total points*

Choose the answer, and explain why this answer is correct. Provide formulas that support your choice.

1) (*5 points*)
   A star has the same radius as the Sun, but its surface temperature is twice as large. The luminosity of this star is:
   a) twice as large as the Sun's;
   b) four times as large as the Sun's;
   c) eight times as large as the Sun's;
   d) sixteen times as large as the Sun's;

2) (*5 points*)
   The wavelength of the peak of black body emission from this star is:
   a) two times longer than the Sun's;
   b) same as the Sun's;
   c) half that of the Sun's;
   d) one quarter that of the Sun's;