Solutions to Homework #1, AST 203, Spring 2009

Due on Thursday February 12, 2009

*General grading rules: One point off per question (e.g., 2a or 2b) for egregiously ignoring the admonition to set the context of your solution. Thus take the point off if relevant symbols aren't defined, if important steps of explanation are missing, etc. If the answer is written down without *any* context whatsoever (e.g., for 2a, writing "50 nanoseconds", and nothing else), take off 1/3 of the points. One point off per question for inappropriately high precision (which usually means more than 2 significant figures in this homework). However, no points off for calculating a result to full precision, and rounding appropriately at the last step. No more than two points off per problem for overly high precision. Three points off for each arithmetic or algebra error. Further calculations correctly done based on this erroneous value should be given full credit. However, if the resulting answer is completely ludicrous (e.g., 10^{-30} seconds for the time to travel to the nearest star, 50 stars in the visible universe), and no mention is made that the value seems wrong, take three points further off. Answers differing slightly from the solutions given here because of slightly different rounding (e.g., off in the second decimal point for results that should be given to two significant figures) get full credit. One point off per question for not being explicit about the units, or for not expressing the final result in the units requested. Two points off for the right numerical answer with the wrong units. Leaving out the units in intermediate steps should be pointed out, but no points taken off. Specific instructions for each problem take precedence over the above. In each question, one cannot get less than zero points, or more than the total number the question is worth.*

100 total points

1. Scientific notation review *20 total points*

Write the following in proper scientific notation, giving the proper number of significant figures. Full sentences are not required here.

a) Fifty-six million, seven hundred thousand (*2 points*)
Solution: 5.67×10^7
No words needed here. 56.7×10^6, 567×10^5, *etc. get one point off. Numerically incorrect values, or incorrect number of significant figures get no credit.*

b) π (the number pi) times 0.3 (*3 points*)
Solution: $3.14 \times 0.3 = 0.9$, to a single significant figure.
No words needed here. Two significant figures (0.94) is fine, as are 9×10^{-1}, 9.4×10^{-1}. *Any other values get no credit.*

c) One two hundredth (*3 points*)
Solution: 5×10^{-3}
0.005, 0.5×10^{-2} *get one point off, anything else – no credit*

d) Six nanometers (expressed in meters) (*3 points*)
Solution: 6×10^{-9} meters or 6.0×10^{-9} meters.

Two points off if the meters are not written. No points for 3 or more significant figures.

e) Divide the number $N \times 10^x$ by number $M \times 10^y$. Simplify the symbolic result, and then substitute the following values for the variables, expressing the answer in scientific notation: $N = 2.1$, $M = 1.2$, $x = 3$, $y = 1$. (*5 points*)

Solution: $\frac{N\times10^x}{M\times10^y} = \frac{N}{M} \times 10^{x-y}$. Substituting the values, we get $\frac{2.1}{1.2} \times 10^{3-1} = 1.75 \times 10^2 \approx 1.8 \times 10^2$. Value of 1.7×10^2 is also acceptable.
3 points off for incorrect symbolic algebra. 1 point off for more than 2 significant digits.

f) The speed of light is 3.0×10^8 meters per second. There are thirty-one million, seven hundred thousand seconds in a year. What is the speed of light in units of kilometers per year? (*4 points*)

Solution: This is a matter of unit conversion. As we're given the speed of light to two significant figures, the final answer should have this number of significant figures, thus we have:

$$c = 3.0 \times 10^8 \frac{\text{meters}}{\text{second}} \times 3.17 \times 10^7 \frac{\text{seconds}}{\text{year}} \times \frac{\text{kilometer}}{10^3\,\text{meters}} = 9.5 \times 10^{12}\,\frac{\text{kilometers}}{\text{year}},$$

or about 10 trillion (10^{13}) kilometers per year. This is, of course, the distance of one light year.
2 points off for giving results to other than 2 significant figures.

2. Looking out in space and back in time *35 total points*

Because the speed of light is not infinite, we see objects as they were some time in the past. This usually doesn't matter in daily life on Earth as the distances (and therefore time delays) are small, but it is often important in astronomy and spacecraft engineering, or when considering communication among galactic civilizations.

The speed of light is 3.0×10^5 km/sec. The distance between the Earth and the Sun (one astronomical unit, or AU) is 150 million kilometers. Give all answers to the correct number of significant figures. Remember that all electromagnetic radiation (including radio waves) travels at the speed of light.

a) You look at the clock on the lecture room wall, which is 45 feet (15 meters) away from you. How far back in time are you seeing it? If you wanted to know the current time more precisely, how should you correct the reading shown by the clock? The light entering your eyes from the clock is ambient light in the surroundings that has reflected off the clock. (*5 points*)

Solution: We are given the distance to two significant figures, which allows us to carry out our calculations to two significant figures. Light travels at 3.0×10^8 meters per second, so the time for light to travel 15 meters is simply:

$$\frac{15\,\text{meters}}{3.0 \times 10^8\,\text{meters/sec}} \approx 5 \times 10^{-8}\,\text{sec} \times 10^9\,\frac{\text{nanosec}}{\text{sec}} = 50\,\text{nanosec}.$$

That is, it takes light 50 nanoseconds to travel 15 meters (or $\sim$ 45 feet); the speed of light is about 1 foot per nanosecond (to one significant figure), a useful number to remember.
The clock reading you see is 50 nanoseconds in the past, so you may want to add those 50 nanoseconds to the clock reading to correct it to the current time. Obviously, unless the wall clock is an atomic clock with more than 10 significant digits, this correction would not do much...
1 point off for subtracting 50 nanoseconds from the clock's reading; Answer of 45 nanoseconds also accepted

b) A good internet connection is characterized by large bandwidth (amount of data you can push through at once) and low latency (how long it takes to reach the remote server). However, some things are beyond the control of your Internet Service Provider. Since internet signals propagate close to the speed of light, calculate the minimum possible latency (round trip travel time of signal) from Princeton to a server in:
1) Manhattan (50 miles from Princeton);
2) San Francisco (2600 miles);
3) Beijing (6800 miles).
Express your answers in milliseconds. (*10 points*)
Solution The minimum latency is achieved if the server does not introduce any delay between receiving the query signal and responding with the web page. The latency then is just the round trip time at the speed of light. If the distance to the server is d, then the round-trip travel time is $t_{round\ trip} = 2d/c$, where $c = 3 \times 10^8$ m/sec is the speed of light. One mile is 1.6 km = 1.6×10^3 m. So, we now have
1) Manhattan: d = 50 miles,

$$\mathrm{t_{round\ trip}} = (2\times 50\ \mathrm{miles}\times 1.6\times 10^3\ \mathrm{m/mile})/3\times 10^8\ \mathrm{m/sec} = 5.3\times 10^{-4}\ \mathrm{sec} = 5\times 10^{-1}\ \mathrm{msec}$$

to one significant figure, where we abbreviated milliseconds as "msec," and used 1 msec = 10^{-3} sec. The answer is half a millisecond. Pretty fast!
2) San Francisco: d = 2600 miles,

$$\mathrm{t_{round\ trip}} = (2\times 2.6\times 10^3\ \mathrm{miles}\times 1.6\times 10^3\ \mathrm{m/mile})/3\times 10^8\ \mathrm{m/sec} = 2.7\times 10^{-2}\ \mathrm{sec} = 27\ \mathrm{msec}.$$

28 msec is also accepted, if you round 27.7 to 28.
3) Beijing: d = 6800 miles,

$$\mathrm{t_{round\ trip}} = (2\times 6.8\times 10^3\ \mathrm{miles}\times 1.6\times 10^3\ \mathrm{m/mile})/3\times 10^8\ \mathrm{m/sec} = 7.2\times 10^{-2}\ \mathrm{sec} = 72\ \mathrm{msec}.$$

73 msec is also accepted, if you round 72.5 to 73.
For reference, the actual latency to San Francisco from Princeton is about 80msec, which includes the finite time for server response, and delays along the line as the signal is routed through the servers on the internet. However, you can see that more than a third of this is due to the finite speed of light!

3 points off for missing the factor of 2 in the logic. 2 points off for extra precision (for the whole problem).

c) Approximately, what is the farthest distance from the Earth at which an alien spaceship has any chance of detecting human-generated radio waves? (*Hint:* Humanity's first radio broadcast, Marconi's transmission to communicate across the Atlantic, took place on December 12, 1901). Express your answer in light years and in kilometers. (*5 points*)

Solution

Due to the finite speed of light, all human-generated radio waves today must be within a sphere of radius R = (time since first broadcast) × (speed of light), centered on the Earth. The time since the first broadcast to February of 2009 is 107 years (or 107 years and 2 months to be exact). Therefore, the radius of this sphere is 107 light years, or, approximately, 100 light years. The distance in kilometers is 100 light years $\times 10^{13}$ km/year $= 10^{15}$ km. We used here the result of problem 1f).
Answers with more than 2 significant figures get 2 points off. 2 points off for not giving both the light years and kilometers

Mars rovers Spirit and Opportunity are programmed to stop and ask for instructions when they run into trouble. After this, they wait for commands from mission controllers before they can start moving again. This is called "safe mode." If Spirit suddenly hits the Martian equivalent of a quicksand patch and enters "safe mode," what is the minimum time it has to survive before the controllers back on Earth can free it?

Mars and Earth are on nearly circular coplanar orbits around the Sun with radius 1.5 AU and 1 AU respectively. Consider two cases:

d) Mars is farthest from Earth in the two planet's orbits. Give your answer in minutes. (*10 points*)

Solution At this point, Mars and Earth are 2.5 AU apart (if it is not clear why this is, please come to office hours!). Note that because we know the speed of light to only two significant figures, we can use two significant figures in this calculation of distance as well. The time for light to travel that distance is:

$$t = \frac{2.5\,\mathrm{AU}}{3\times 10^5\,\mathrm{km/sec}} \times 1.5\times 10^8\,\frac{\mathrm{km}}{\mathrm{AU}} = \frac{2.5}{2}\times 10^3\,\mathrm{sec} = 1.25\times 10^3\,\mathrm{sec}.$$

Note that we are not throwing away extra digits in the intermediate result. The minimum time for the rover to survive before the commands come back is the *round-trip* time between the Earth and Mars, because the rover has to let the controllers know it's in trouble, and then wait to receive the response. So, the answer is then $2\times 1.25\times 10^3$ sec $= 2.5\times 10^3$ sec ≈ 42 minutes.
3 points off for missing the factor of 2 in the round trip time. 4 points off for missing the distance at the largest separation. Full credit for one significant figure, e.g. 40 min.

e) Mars is nearest to the Earth in the two planet's orbits. Give your answer in minutes. (*5 points*)

Solution Again, the Sun, Earth, and Mars lie along a straight line, but now the Earth is between the Sun and Mars. Thus the distance from the Earth to Mars is 0.5 AU. The same calculation gives the answer which is 5 times smaller: 8.3 minutes.
8 minutes also acceptable

3. Planetary Atmospheres *20 total points*

In this problem we will calculate the mass of the atmospheres of three planets: Earth, Venus and Mars. These are all "rocky" planets, which have held on to their thin (relative, e.g., to Jupiter!) atmospheres. The mass contained in an atmospheric column with a footprint of 1 square meter on each of these planets is: Earth (1.0×10^4 kg/m^2), Venus (1.0×10^6 kg/m^2), Mars (2.2×10^2 kg/m^2). To appreciate these numbers, think about how many tons of air are over your head when you walk outside. Each planet is roughly a sphere, of radius $R_{Earth} = 6371$ km, $R_{Venus} = 6051$ km, and $R_{Mars} = 3396$ km. Calculate the total mass contained in each of these atmospheres (in kilograms), and find the ratio between the most and least massive one.

Solution: First, let's do this symbolically. We know the mass in an atmospheric column of a given footprint, call it Σ kg/m^2. The total mass of the atmosphere is then the surface area of the planet multiplied by the atmospheric mass contained over a unit of the surface area. You can think of this as knowing the mass in each thin column of the atmosphere, and asking how many such columns will fit onto the planet side by side. Mathematically, $\mathrm{M_{atm}} = \Sigma \times 4\pi \mathrm{R^2_{planet}}$, where we used the familiar formula for the surface area of a sphere. Now we are ready to plug in the numbers.
Earth: $\mathrm{M_{atm,\ Earth}} = 1.0 \times 10^4\ \mathrm{kg/m^2} \times (4 \times 3.14 \times (6371 \times 10^3\ \mathrm{m})^2) = 5.1 \times 10^{18}$ kg.
Venus: $\mathrm{M_{atm,\ Venus}} = 1.0 \times 10^6\ \mathrm{kg/m^2} \times (4 \times 3.14 \times (6051 \times 10^3\ \mathrm{m})^2) = 4.6 \times 10^{20}$ kg.
Mars: $\mathrm{M_{atm,\ Mars}} = 2.2 \times 10^2\ \mathrm{kg/m^2} \times (4 \times 3.14 \times (3396 \times 10^3\ \mathrm{m})^2) = 3.2 \times 10^{16}$ kg.

We now see that Venus has the heaviest atmosphere (by far!), while Mars is the lightest. The ratio is: 1.4×10^4. So, the atmosphere on Venus is ten thousand times more massive than on Mars. Venus' atmosphere is also almost 100 times more massive than the atmosphere of Earth – and the two planets have almost the same size! Note that the answer is given to two significant figures.

4. Heliocentric orbits *25 total points*

Let's see how we can use Kepler's laws to learn some properties of the Solar System. The orbital periods of the major planets are: Mercury (87.97 days), Venus (224.7 days), Earth (365.26 days), Mars (687 days), Jupiter (11.86 years), Saturn (29.45 years), Uranus (84.02 years) and Neptune (164.8 years).

a) Using Kepler's third law, calculate the average distance from the Sun for all the planets (in AU). (*5 points*)
Solution Kepler's third law states that the planet's period, P (measured in years), is related to the planet's semimajor axis a (measured in AU) as: $P^2 = a^3$. Therefore,

the semimajor axis is

$$a(\text{AU}) = P^{2/3}(\text{years}).$$

The semimajor axis is the average distance between the Sun and the orbiting planet. Let's convert the periods given above into years: $\text{P}_{\text{Mercury}} = 87.97$ days/365.26 days = 0.24yrs; $\text{P}_{\text{Venus}} = 224.7$ days/365.26 days = 0.62 yrs; $\text{P}_{\text{Mars}} = 687$ days/365.26 days = 1.9 yrs. Other periods are in years already, and the period of the Earth is, obviously, 1 year. Now, we can plug in these numbers to Kepler's law to get the semimajor axis:

$$\text{a}_{\text{Mercury}} = 0.24^{2/3}\text{AU} = 0.39\text{ AU};\ \text{a}_{\text{Venus}} = 0.62^{2/3}\text{AU} = 0.73\text{ AU};$$

$$\text{a}_{\text{Earth}} = 1\text{AU};\ \text{a}_{\text{Mars}} = 1.9^{2/3}\text{AU} = 1.5\text{ AU};\ \text{a}_{\text{Jupiter}} = 11.86^{2/3}\text{AU} = 5.2\text{ AU};$$

$$\text{a}_{\text{Saturn}} = 29.45^{2/3}\text{AU} = 9.5\text{ AU};\ \text{a}_{\text{Uranus}} = 84.02^{2/3}\text{AU} = 19\text{ AU};$$

$$\text{a}_{\text{Neptune}} = 164.8^{2/3}\text{AU} = 30\text{ AU}.$$

2 points off if the answer for the Earth is not 1 AU. This means that the student is not thinking about the problem, but is just number crunching. More than four significant figures get 2 points off

b) The orbits of these planets have low eccentricity, which means that their orbits are nearly circular. Calculate the orbital speed of each planet in kilometers per second. (*5 points*)

Solution The orbital speed is the circumference of the orbit divided by the period of the orbit. Hence, $v = 2\pi a/P$. If we plug in the values in AU and years, this will give the speed in units AU per year, so we need to convert this to kilometers per second. The conversion factor is 1 AU/yr = 1.5×10^8 km/3.2×10^7 sec = 4.7 km/sec. Now we can calculate the velocity in km/s:

$$\text{v}_{\text{Mercury}} = 2\pi{\times}0.39\text{ AU}/0.24\text{ yrs} = 48\text{km/sec};\ \text{v}_{\text{Venus}} = 2\pi{\times}0.73\text{ AU}/0.62\text{ yrs} = 35\text{ km/sec};$$

$$\text{v}_{\text{Earth}} = 2\pi \times 1\text{ AU}/1\text{ yrs} = 30\text{ km/sec};\ \text{v}_{\text{Mars}} = 2\pi \times 1.5\text{ AU}/1.9\text{yrs} = 23\text{ km/sec};$$

$$\text{v}_{\text{Jupiter}} = 2\pi{\times}5.2\text{ AU}/11.86\text{ yrs} = 13\text{ km/sec};\text{v}_{\text{Saturn}} = 2\pi{\times}9.5\text{ AU}/29.45\text{ yrs} = 9.5\text{ km/sec};$$

$$\text{v}_{\text{Uranus}} = 2\pi{\times}19\text{ AU}/84.02\text{ yrs} = 6.7\text{ km/sec},\ \text{v}_{\text{Neptune}} = 2\pi{\times}30\text{ AU}/164.8\text{ yrs} = 5.4\text{ km/sec}.$$

Note, that in all formulas above we had to multiply by the conversion factor to get the final answer in km/sec.
2 points off if any of the velocities are faster than the speed of light. This means that the student is not thinking about the problem, but is just number crunching.

c) Halley's comet is on a heliocentric elliptical orbit. Its last closest approach to the Sun (perihelion) was in 1986 and the next one will be in 2061. At perihelion it comes to 0.6 AU from the Sun. Find the largest distance (aphelion) between the Sun and the comet over its orbit. Express your answer in AU. The orbit of which planet is closest to Halley's comet at aphelion? (*15 points*)

Solution Knowing two subsequent times when the comet went through the same point in the orbit gives us the orbital period, 2061-1986=75 years. Moving on an elliptical orbit, the distance when the comet is farthest from the Sun is $\mathrm{R_{aphelion}} = \mathrm{a} \times (1 + \mathrm{e})$, where a is the semimajor axis, and e is the eccentricity. So, in order to find the aphelion distance we need to know what the semimajor axis and eccentricity are. How can we find them? We can determine the semimajor axis from the Kepler's third law. Since we know the period of the orbit, the semimajor axis is $\mathrm{a_{Halley}} = \mathrm{P}^{2/3} = 75^{2/3}\mathrm{AU} \approx 17.8$ AU (we are keeping extra digits here, because this is an intermediate result. We will cut them later). We also know the perihelion distance of $\mathrm{R_{perihelion}} = 0.6$ AU. For an elliptical orbit, the perihelion distance is related to the semimajor axis and eccentricity as: $\mathrm{R_{perihelion}} = \mathrm{a} \times (1 - \mathrm{e})$. So, from this expression we can find the eccentricity – the last remaining unknown. Simple algebra tells us that:

$$\mathrm{e} = 1 - \mathrm{R_{perihelion}}/\mathrm{a} = 1 - 0.6\ \mathrm{AU}/17.8\ \mathrm{AU} = 0.97.$$

Now, that's a very eccentric orbit! (Recall that e=1 is the maximum possible eccentricity, and corresponds to such an elongated orbit that it appears as a straight line.) With the eccentricity and semimajor axis at hand, we and can go ahead and calculate the aphelion distance:

$$\mathrm{R_{aphelion}} = \mathrm{a} \times (1 + \mathrm{e}) = 17.8\ \mathrm{AU}(1 + 0.97) = 35\ \mathrm{AU}.$$

From section a) or from the tables in the book, we see that the orbit of Halley's comet goes all the way out to beyond the orbit of Neptune! At perihelion, it ends up between Mercury and Venus. Such highly eccentric orbits allow transport of material between the outer and inner Solar System.

The grade is split as 10 points for understanding conceptual solution of the problem and 5 for correct calculation. 3 points off for getting nonsensical values, like $e > 1$, or aphelion in thousands of AU.