Correlations and the outer scale of turbulence in fast solar wind.

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Abstract

We show that the scaling of structure functions of magnetic and velocity fields in a mostly highly Alfvénic fast solar-wind stream depends strongly on the joint distribution of the dimensionless correlation measures cross helicity and residual energy. Fluctuations that are both more balanced (cross helicity ~ 0) and equipartitioned (residual energy ~ 0) have steep structure functions at very low frequencies. Fluctuations that are magnetically dominated (residual energy < 0) or imbalanced (cross helicity > 0), and have closely aligned magnetic and velocity vectors, have small outer scales typical of fast solar wind. We conclude that the strength of non-linear interaction of individual fluctuations within a stream, diagnosed by the degree of correlation in direction and magnitude of magnetic and velocity fluctuations, determines the size of the outer scale of the turbulence.

Background

Solar wind: hot, tenuous, plasma traveling at supersonic, super-alfvénic speeds away from the Sun. β ~ 1 at the Earth, turbulence transports and dissipates energy via a wide MHD inertial range.

The energy containing scales in the solar wind are thought to be made from superpositions of coronal structures and Alfvén waves, this results in a "1/f" spectrum for fluctuations measured at large scales, particularly in fast wind.

Recent results (Podesta et al., 2009, Hnat et al., 2011, Wicks et al., 2013) have shown that the average alignment of velocity and magnetic field fluctuations changes over scales corresponding to this 1/f range and that the scaling of structure functions of Elsasser fields changes depending on their alignment over this range.

Here we pose the questions: is the 1/f range made of passively convected coronal structures, and what affect does changing correlation of fluctuations have on the turbulence, and vice-versa?

Data Analysis

We want to measure two characteristics of the fluctuations in the solar wind:

1. Geometrical correlation
2. Turbulent properties

The geometry of fluctuations in velocity (V) and magnetic field (B, in Alfvén units) can be fully characterized by two dimensionless parameters, normalized cross helicity (σ_c) and normalized residual energy (σ_r).

Fluctuations:

$$\delta\mathbf{v}(t,\tau) = \mathbf{V}(t) - \mathbf{V}(t+\tau) \qquad \delta\mathbf{b}(t,\tau) = \frac{\mathbf{B}(t) - \mathbf{B}(t+\tau)}{\sqrt{4\pi\rho_0(t,\tau)}} \qquad (1)$$

Local means:

$$\mathbf{B}_0(t,\tau) = \frac{1}{\tau}\int_{t'=t}^{t'=t+\tau}\mathbf{B}(t') \qquad \rho_0(t,\tau) = \frac{1}{\tau}\int_{t'=t}^{t'=t+\tau}\rho(t') \qquad (2)$$

Take projection perpendicular to B to remove compressible part

$$\delta\mathbf{b}_\perp(t,\tau) = \delta\mathbf{b}(t,\tau) - \left(\delta\mathbf{b}(t,\tau)\cdot\hat{\mathbf{B}}_0(t,\tau)\right)\hat{\mathbf{B}}_0(t,\tau) \qquad (3)$$

Correlation measures:

$$\sigma_c(t,\tau) = \frac{2\delta\mathbf{v}_\perp(t,\tau)\cdot\delta\mathbf{b}_\perp(t,\tau)}{|\delta\mathbf{v}_\perp(t,\tau)|^2 + |\delta\mathbf{b}_\perp(t,\tau)|^2} \qquad (4)$$

$$\sigma_r(t,\tau) = \frac{|\delta\mathbf{v}_\perp(t,\tau)|^2 - |\delta\mathbf{b}_\perp(t,\tau)|^2}{|\delta\mathbf{v}_\perp(t,\tau)|^2 + |\delta\mathbf{b}_\perp(t,\tau)|^2} \qquad (5)$$

The properties of the turbulence are investigated using structure functions. The differences calculated as in (3) are squared and averaged to create a scale dependent measure of the amplitude of fluctuations in the time series.

$$S_2(\delta\mathbf{b}_\perp,\tau) = \left\langle |\delta\mathbf{b}_\perp(t,\tau)|^2 \right\rangle \qquad (6)$$

References:

B. Bavassano, E. Pietropaolo, R. & Bruno, J. Geophys. Res., 103, A4, 6521-6529, 1998.
B. Bavassano, & R. Bruno, Ann. Geophys., 24, 3179-3184, 2006.
M. Dobrowolny, A. Mangeney, & P. L. Veltri, Phys. Rev. Lett., 45, 144, 1980.
B. Hnat, S. C. Chapman, G. Gogoberidze, & R. T. Wicks, Phys. Rev. E, 84, 065401, 2011.
J. J. Podesta, B. D. G. Chandran, A. Bhattacharjee, D. A. Roberts, & M. L. Goldstein, J. Geophys. Res., 114, A01107, 2009.
R. T. Wicks, A. Mallet, T. S. Horbury, C. H. K. Chen, A. A. Schekochihin, & J. J. Mitchell, Phys. Rev. Lett., 110, 025003, 2013.

Results

Fluctuations are measured by differencing time series of observations from the Wind spacecraft over different time scales τ. The cross helicity and residual energy are calculated for each difference. Example time series of velocity, density and magnetic field are plotted below, along with the associated cross helicity and residual energy at different scales τ.

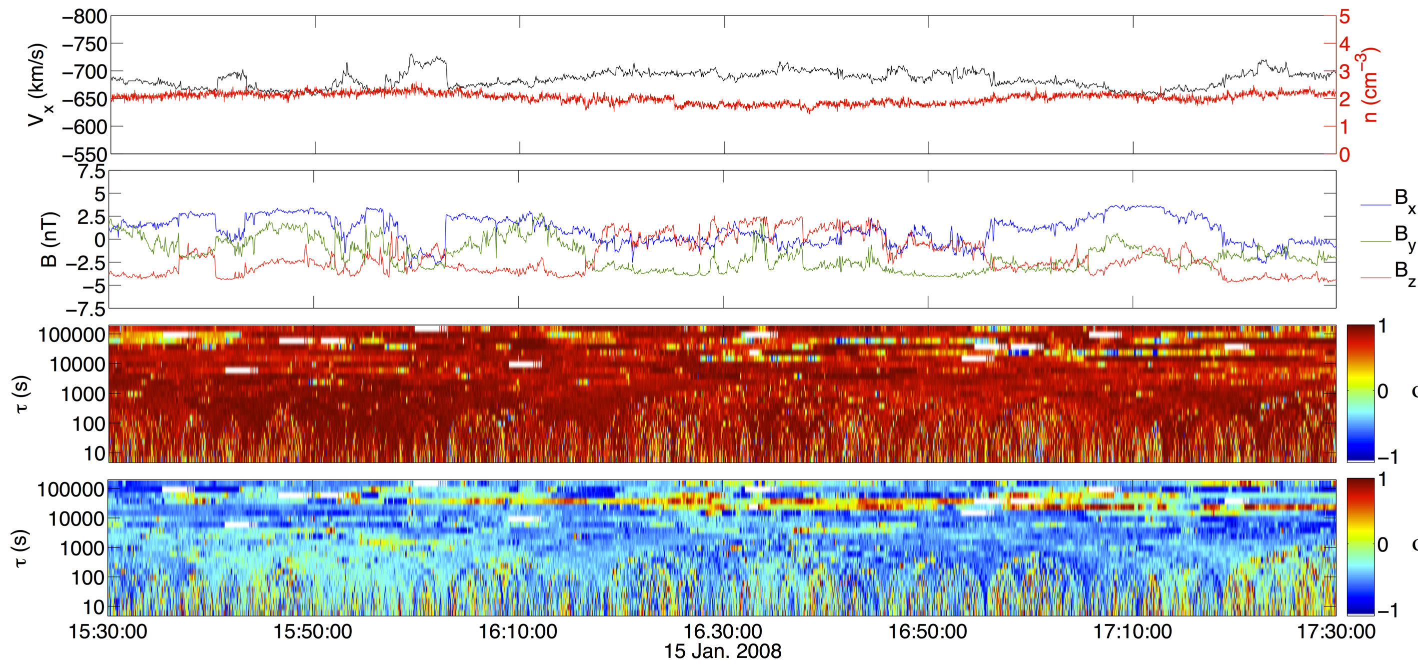


FIGURE 1: Time series of observations from two hours of a 7-day long fast stream measured by the Wind spacecraft. Bottom panels show the normalized cross helicity and residual energy calculated over different scales as a function of time.

S_2 are plotted against time scale τ in Fig. 2. Three scales are indicated by vertical lines:

T_O is the outer scale
T_A is the largest time Alfvén waves can have interacted over.
T_S is the solar wind travel time from the Sun.

Key features:

- $10 < \tau < T_O$ inertial range scaling
- $T_O < \tau < T_A$ flat S_2 => 1/f scaling in B and Z^+
- $T_A < \tau < T_S$ flat scaling in all variables => uncorrelated

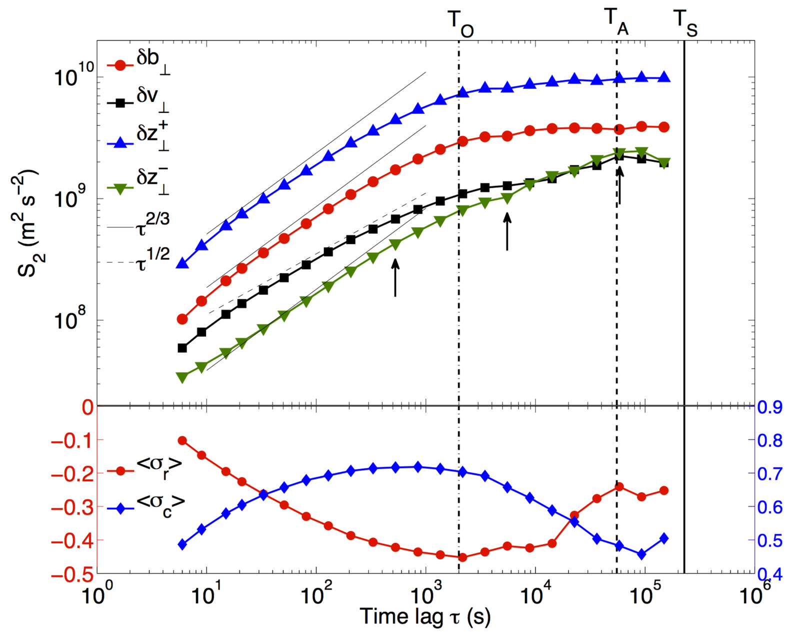


FIGURE 2: Structure functions of all variables (top panel), mean residual energy and cross helicity (bottom panel).

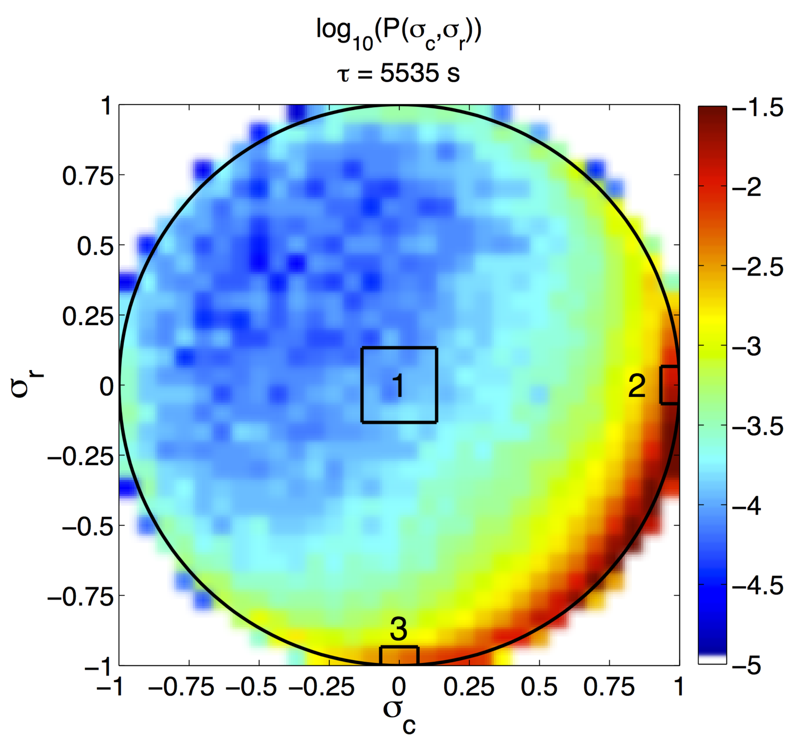
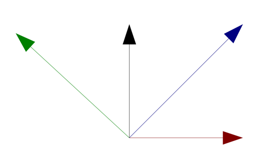
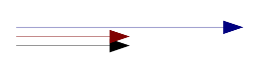


FIGURE 3: Joint probability distribution of cross helicity and residual energy. Three regions with different properties selected for Fig. 5 are shown.

Residual energy and cross helicity are correlated by the geometry of the fluctuations. The joint probability distribution (c.f. Bavassano et al., 1998 Bavassano & Bruno, 2006) is shown in Fig. 3. Different regions within the circle have different geometries.

Three regions are selected based on geometrical properties:

Region 1  $\delta\mathbf{z}_\perp^+ \sim \delta\mathbf{z}_\perp^-$, $\delta\mathbf{b}_\perp \sim \delta\mathbf{v}_\perp$

Region 2  $\delta\mathbf{z}_\perp^+ \gg \delta\mathbf{z}_\perp^-$, $\delta\mathbf{b}_\perp \sim \delta\mathbf{v}_\perp$

Region 3  $\delta\mathbf{b}_\perp \gg \delta\mathbf{v}_\perp$, $\delta\mathbf{z}_\perp^+ \sim \delta\mathbf{z}_\perp^-$

Structure functions are selected if they fall into one of these regions. Conditioned structure functions are plotted in Fig. 4.

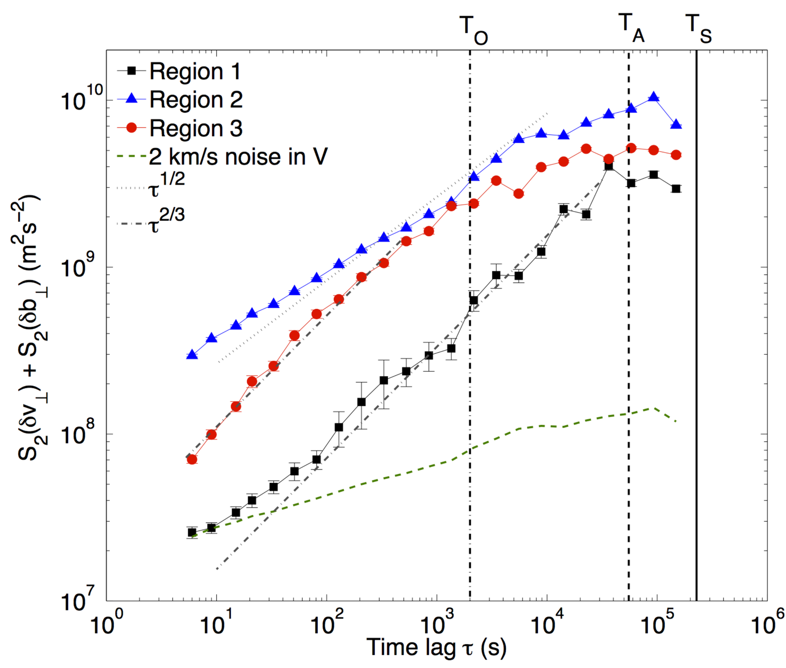


FIGURE 4: Conditioned second order structure functions of fluctuations with different geometry.

Key results:

Balanced fluctuations (Region 1) have a much extended inertial range.

Fluctuations with correlated geometries (Regions 2 and 3) have reduced inertial ranges and extended 1/f ranges.

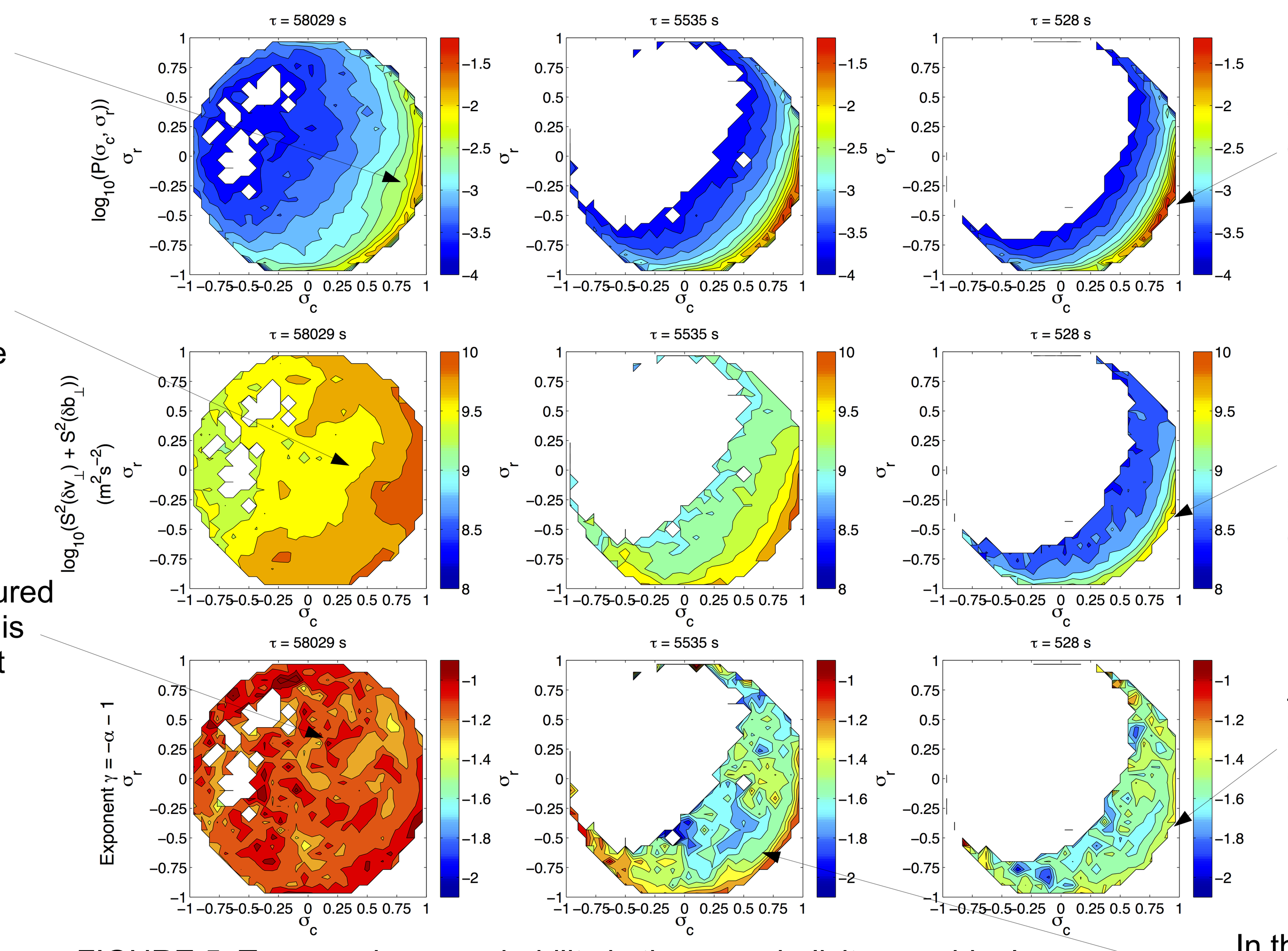
"Full Circle"

We extend the analysis of Fig. 4 to each bin in the cross helicity – residual energy phase space. We generate maps of the probability, average amplitude and scaling exponent in this space at the three scales indicated by arrows in Fig. 2.

Outward propagating magnetically dominated fluctuations at largest scales

Less than on order of magnitude ratio in amplitude between edge and central fluctuations

Scaling exponent measured from structure functions is close to -1 i.e. 1/f almost everywhere.



Probability is concentrated at the edge of the distribution over the 1/f and inertial ranges.

Largest amplitudes are also found at the edge. More than an order of magnitude ratio between edge and the most central observations

Inertial range scaling found everywhere, although edge flatter than center.

FIGURE 5: Top row shows probability in the cross helicity – residual energy space. Middle row shows the average amplitude of the structure functions. Bottom row shows the scaling exponent γ calculated from the exponent of the structure functions α, where $S_2(\tau) \propto \tau^\alpha$.

In the range $T_A > \tau > T_O$ fluctuations at the edge of the phase space have 1/f scaling but those in the middle have -5/3 scaling.

Phenomenological model

The results above show that the geometry of the fluctuations, parameterized in the residual energy – cross helicity phase space is important in determining the probability and the amplitude of the fluctuations in both the 1/f and inertial ranges. One reason for this could be that the correlations of the fluctuations are affecting the amount of energy that can be transferred between scales by non-linear interactions. Following Dobrowolny, Mangeney, & Veltri, 1980:

Homogenous Ideal MHD:

Nonlinear time: $\tau^\pm \sim l/\delta\mathbf{z}^\mp$

Energy transfer rate: $\Pi^\pm(l) \sim \delta\mathbf{z}^\mp(\delta\mathbf{z}^\pm)^2/l$

Hydrodynamic turbulence:

Nonlinear time: $\tau^v \sim l/\delta\mathbf{v}$

Energy transfer rate: $\Pi^v(l) \sim \delta\mathbf{v}^3/l$

A second MHD model:

The angle between oppositely propagating Elsasser fluctuations is important in their non-linear interaction. If we assume that the gradient of the fluctuation is perpendicular to the fluctuation vector (as for ideal Alfvén waves) then:

Nonlinear time: $\tau^\pm \sim l/\delta\mathbf{z}^\mp\sin(\phi)$

Energy transfer rate: $\Pi^\pm(l) \sim \delta\mathbf{z}^\mp(\delta\mathbf{z}^\pm)^2\sin(\phi)/l$

Different combinations of the above terms may occur depending on the history of the solar wind stream.

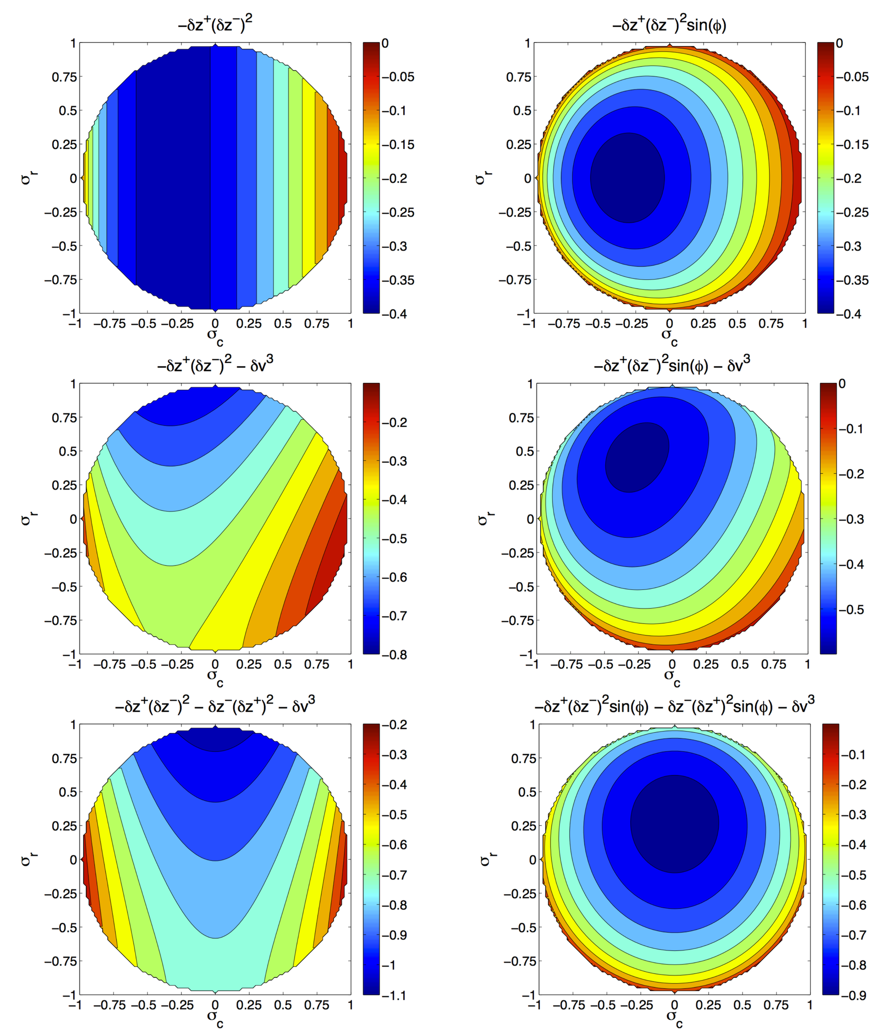


FIGURE 6: Energy transfer rates for combinations of different ideal MHD and velocity shear terms in the residual energy – cross helicity phase space.

Conclusions

The probability, amplitude and scaling exponent of fluctuations in the solar wind are dependent on their correlation properties. Highly aligned fluctuations (found at the edge of the residual energy – cross helicity phase space) have extended 1/f scaling ranges and shortened inertial ranges. The range of scales considered to be the energy containing scales is therefore dependent on the characteristics of the fluctuations. The outer scale of turbulence observed in structure functions or Fourier spectra is thus determined by the most probable fluctuations. The high frequency end of the 1/f range is therefore not made entirely of a superposition of non-interacting structures.

The turbulent cascade takes a path through this phase space. Using a combination of 3rd order / Yaglom laws and the energy transfer rates may help determine the path any particular packet of plasma takes and therefore the average correlation properties of the resulting small-scale fluctuations.