Statistical description of magnetized coronae above turbulent accretion disks

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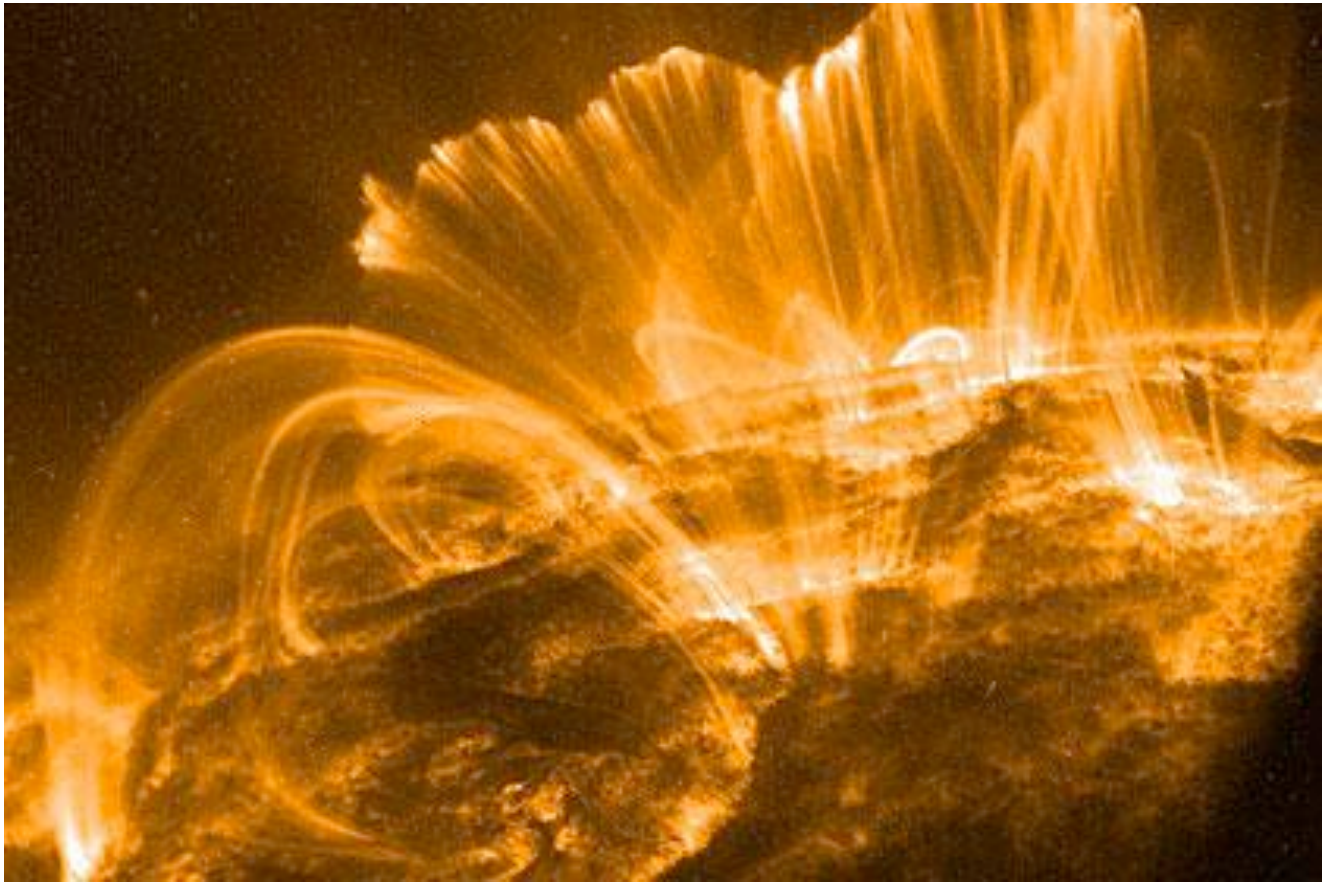
OUTLINE

- Introduction to astrophysical coronae
- Magnetic structure of coronae: a statistical approach:
  - Loop Kinetic Equation
  - Reconnection as a collision integral
  - Self-consistent magnetic pressure profile
  - Coronal Energetics and Angular Momentum Transport.
- Summary

Solar Corona



Typical parameters:

$L \sim 10^9 - 10^{10}$ cm $\quad$ $B \sim 100$ G

$n_e \sim 10^9 - 10^{10}$ cm^{-3} $\quad$ $T \sim 2 \times 10^6$ K $\gg T_{photosphere}$

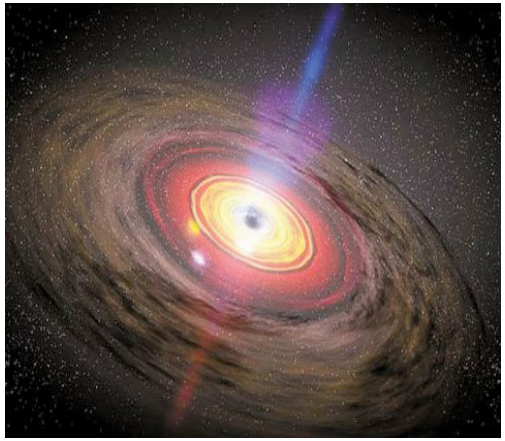
$\beta = 16\pi\, n_e\, k_B T / B^2 \ll 1$

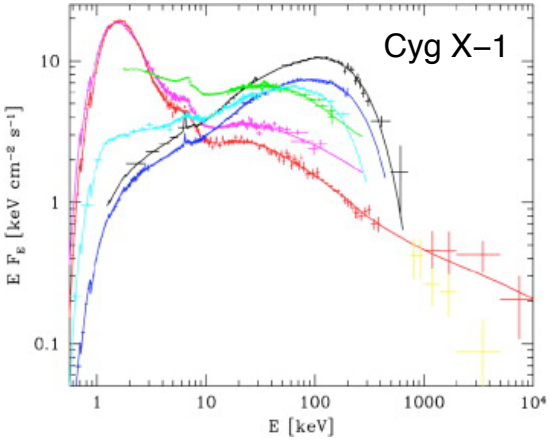
Accretion Disk Coronae (ADC)

Observational evidence for black-hole ADC:

significant high-energy (hard X-ray) component in observed X-ray spectra:

$\varepsilon >> k_B T_{disk} \sim 0.01\text{-}1$ keV



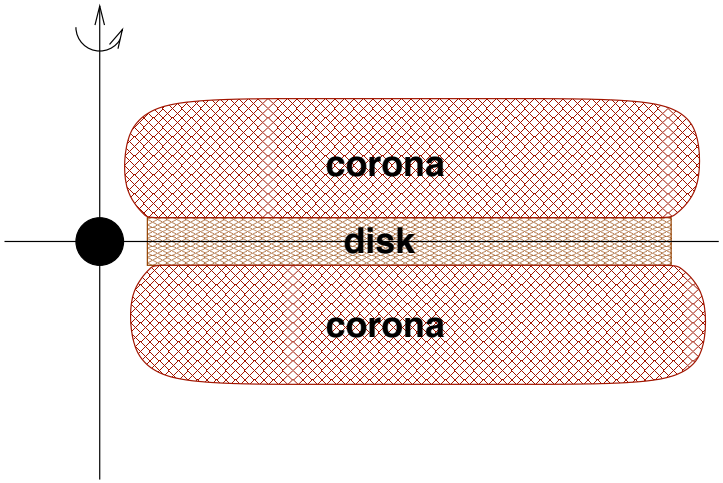


Zdziarski et al. 2002

Popular explanation:

Comptonization of soft disk photons (UV, soft X-rays) by energetic (~100 keV) electrons in a thick, hot, tenuous corona above the disk.

(e.g., Bisnovatyi-Kogan & Blinnikov 1976; Liang & Price 1977).



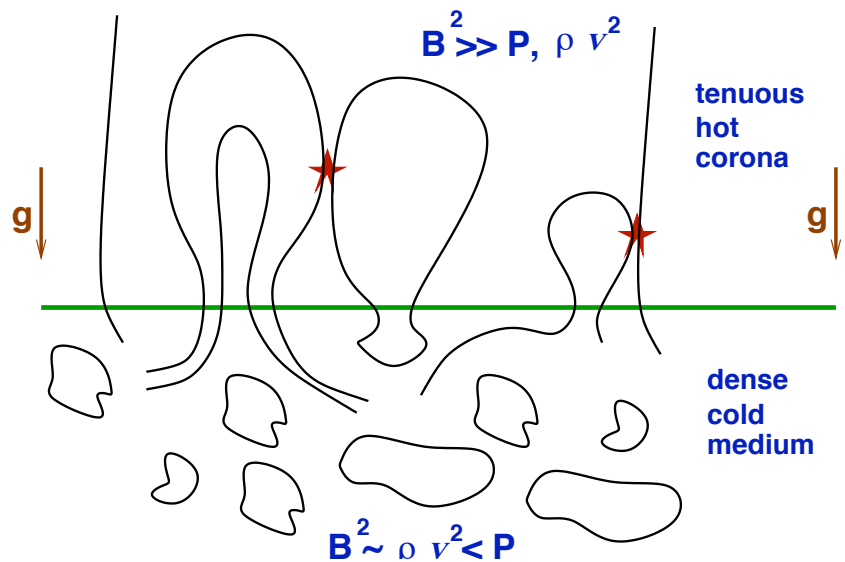
Sandwich Model *(Liang & Price 1977; Galeev, Rosner, & Vaiana 1979)*

Generic Corona Formation

Analogy with the Solar Corona:

(Galeev, Rosner & Vaiana 1979)

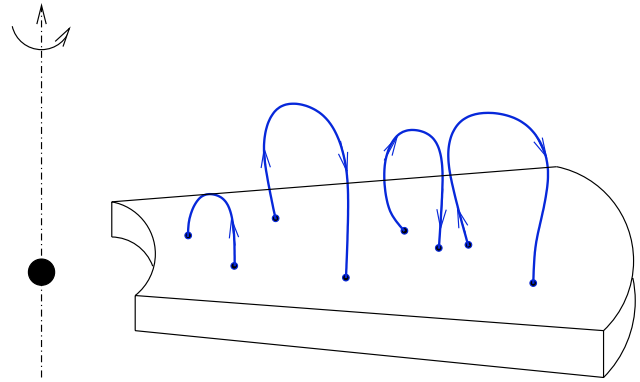
- Dense turbulent medium in external gravitational field.
- Magnetic field is generated by MHD dynamo
- Gravitational stratification ⇒ Parker instability ⇒ Buoyant flux tubes emerge into overlying low-density region.
- Nearly force-free coronal magnetic field is formed.
- Part of turbulent energy goes up as Poynting flux and accumulates as corona's free magnetic energy.
- Magnetic energy is dissipated in reconnection episodes (flares) ⇒ coronal heating.



Physical Motivation for studying ADC magnetic structure

- Most ADC research focuses on gas/radiation/spectra — because this is what we observe directly.
- Corona is magnetically-dominated ⇒ need to understand magnetic field structure before addressing coronal heating/ radiation.
- Hence our interest in magnetic interaction between a turbulent accretion disk and its force-free corona.

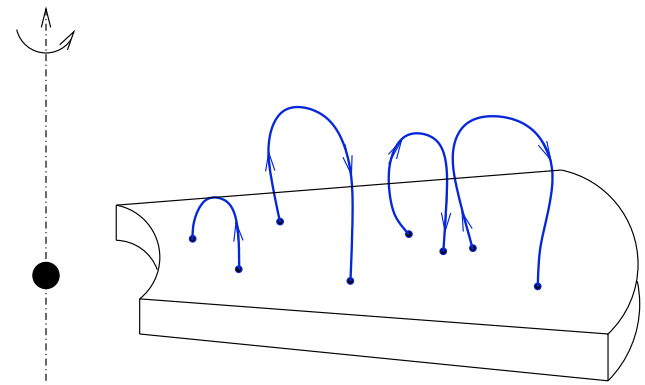
(Tout & Pringle 1996; Miller & Stone 2000)



MAIN QUESTIONS

Motivation:

Corona is magnetically-dominated ⇒
need to understand coronal magnetic field
before addressing gas heating/radiation.



Questions:

- How are coronal magnetic loops distributed in sizes, magnetic fluxes, etc.? Is there a coronal "inverse cascade" ?
- How is coronal magnetic energy density distributed with height *z* above the disk ?
- How is **magnetic energy dissipation** distributed with height ?
- How big is **angular momentum transfer** by coronal loops ?
- What fraction of magnetic flux is **open** at any given time ?
- How do all of these depend on efficiency of **reconnection** ?

Differences between Solar and ADC

- Nature of turbulence is different: convection vs. MRI.
- Strong rotation ($\Omega^{-1} \sim \tau_{turb}$) and Keplerian shear in disks.
- Strong magneto-centrifugal and radiation-driven winds in disks.

Thermally-driven solar wind is weak and less important.

- Thin disks: $l_{turb} \sim H \ll R$, emerging loops are small, $r \sim H$.

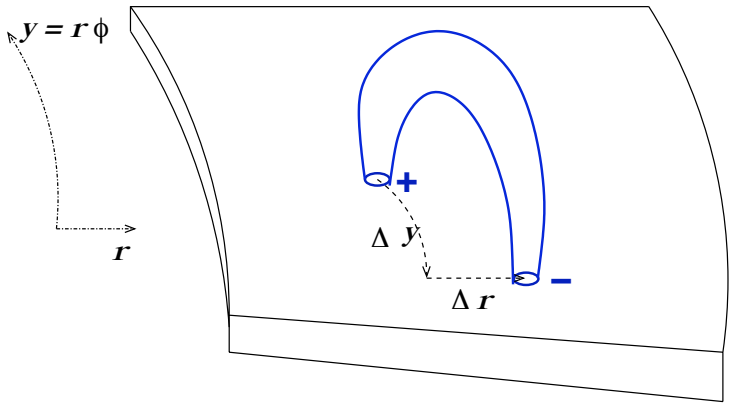
Sun: convection zone is ~ 0.3R; emerging loops in active regions may be large, $r \sim 10^{10}$ cm.

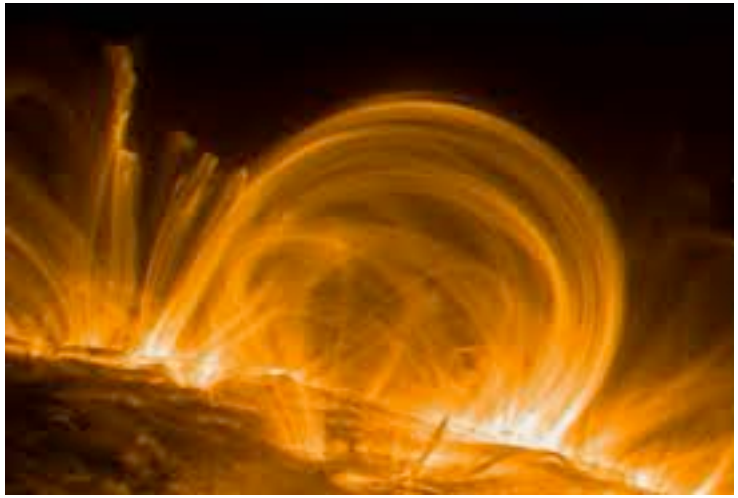
- Strong radiative (e.g., Compton) cooling of ADC in black-hole systems.
- We have only one Sun but many different disks.
- Cultural difference: due to different observational capabilities.

Sun is close ⇒ individual coronal events and structures (flares, loops, prominences, CMEs) are resolved.

Disks are far, spatially unresolved, only overall spectrum is available ⇒ focus on spatially-integrated, averaged quantities.

Theoretical Model: magnetic loops





- Elementary magnetic structures: closed loops
- Primary parameters:

r and φ footpoint separations: Δr, $\Delta y = r\,\Delta\varphi$.

(other possibilities: flux, internal twist, etc. ...)

APPROACH

(Uzdensky & Goodman ApJ 2008, 682, 608)

- **Goal:** build a **statistical** description of the coronal magnetic field above a turbulent accretion disk.
- **Program:** *(c.f. Tout & Pringle 1996)*

– Represent corona by ensemble of elementary magnetic loops characterized by footpoint separations: Δr, $\Delta y = r\,\Delta\varphi$.

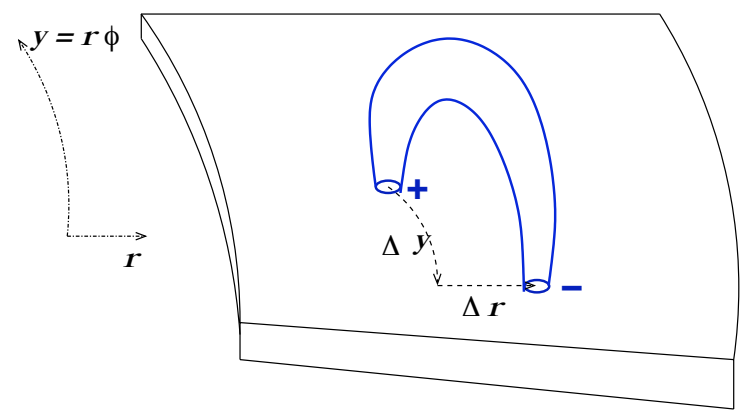
– Formulate equations of motion for loops in this parameter space.

– Introduce **Loop Distribution Function** $F(\Delta r, \Delta y)$.

– Derive the **Loop Kinetic Equation** for F.

– Obtain a **Statistical Steady State**.

– Study Energetics and AMT.



Scales of Interest:

– spatial: $H \ll l \ll R$

– temporal: $\Omega^{-1} < \tau \ll T_{accr}$

PHYSICS OF THE ADC

Processes governing individual loop evolution:

- **stressing:**
(increasing magnetic energy and magnetic scale-height H_B)

• emergence (and submergence) of loops into corona;

• stretching by Keplerian shear;

• random footpoint walk due to disk turbulence.

- **relaxation:**
(dissipating magnetic energy & bringing the field closer to potential)

• reconnection between loops (manifested as flares)

⇒ inverse cascade: formation of larger structures.

Overall, a magnetically-active corona can be described as

A BOILING MAGNETIC FOAM !

Loop Kinetic Equation

Loop Kinetic Equation for the distribution function *F*:

(c.f., Norman et al. 1996, Fermo et al. 2010 in other contexts)

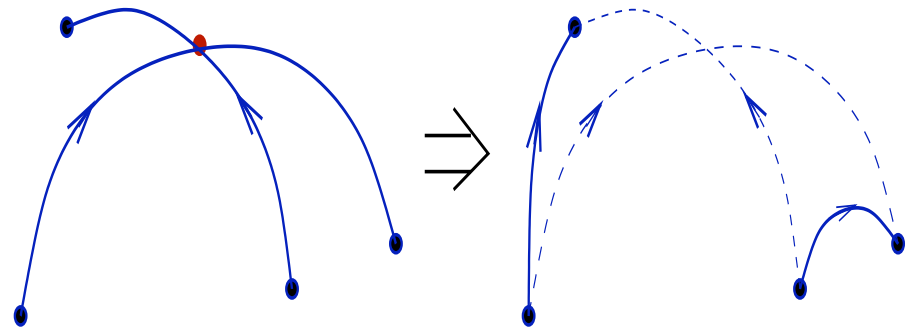
$$\frac{\partial F}{\partial t}(\mathcal{A},t) = \underbrace{S(\mathcal{A})}_{(1)} + \underbrace{\left(D_r \frac{\partial^2}{\partial \Delta r^2} + D_y \frac{\partial^2}{\partial \Delta y^2}\right) F(\mathcal{A})}_{(2)} - \underbrace{\frac{\partial}{\partial \Delta y}(F\dot{\Delta y})}_{(3)} + \underbrace{\dot{F}_{\text{rec}}(\mathcal{A})}_{(4)}$$

$A=(\Delta r, \Delta y)$

1) Flux Emergence ⇒ source term $S(A)$
2) Random footpoint motions ⇒ diffusion
3) Keplerian shear ⇒ advection term $-\partial_{\Delta y}(F\,\dot{\Delta y})$, with $\dot{\Delta y} = -(3/2)\,\Omega\,\Delta r$.
4) Reconnection between loops ⇒ $\dot{F}_{rec}$

Reconnection as a Collision Integral

- Reconnection acts as a binary collisions between particles in a gas:
- Footpoint positions are preserved during interaction ⇒ reconnection rules (similar to conservation laws).
- Reconnection is represented by a nonlinear integral operator ~ collision integral in Boltzmann's eqn:



$$\dot{F}_{\text{rec}}(\mathcal{A}) = \dot{F}_{\text{coll},-}(\mathcal{A}) + \dot{F}_{\text{coll},+}(\mathcal{A})$$

$$\dot{F}_{\text{coll},-}(\mathcal{A}) = -\int d\mathcal{B}\; Q_{\mathcal{AB}}\, F(\mathcal{A})\, F(\mathcal{B})$$

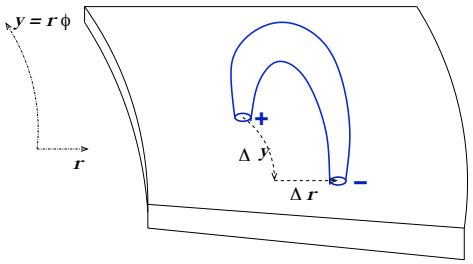
$$\dot{F}_{\text{coll},+}(\mathcal{A}) = \frac{1}{2}\int\int d\mathcal{C}\, d\mathcal{D}\; Q_{\mathcal{CD}\to\mathcal{A}}\, F(\mathcal{C})\, F(\mathcal{D})$$

$Q_{AB} = q\,\Omega\,\sigma_{AB}$ is the rate of reconnection events, σ_{AB} is reconnection cross-section

Reconnection cross-section

- Rate of reconnection events: $Q_{AB} = q\,\Omega\,\sigma_{AB}$

(q = dimensionless rate of reconnection events)



- Reconnection cross-section:

$$\sigma_{AB} = \int \sigma_{AB,y}(b_r)\, db_r$$

(Integration is over impact parameters b_r for which intersection between two given loops is possible.)

- Estimate for $\sigma_{AB,y}(b_r)$:

$$\sigma_{AB,y}(b_r) \sim d(z) \sim \sqrt{\frac{\Delta\Psi}{\bar{B}(z)}} \sim d_0 \sqrt{\frac{B_0}{\bar{B}(z)}}$$

$d(z)$ = loop thickness at interaction height $z \sim Z_{top,B}$; $d_0 = d(0)$.

⇒ Larger loops have larger reconnection cross-sections !

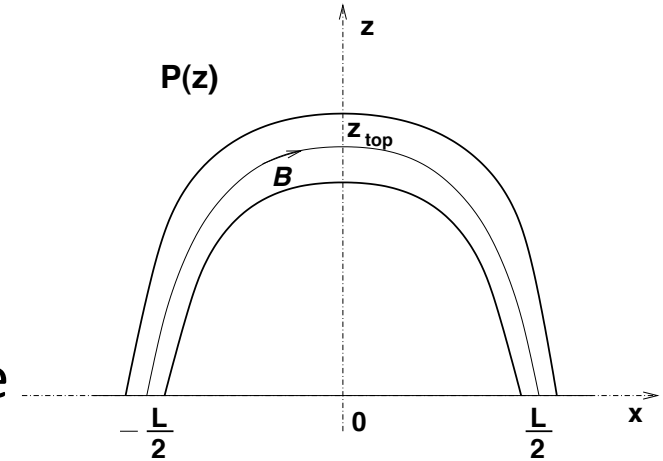
- Reconnection source term:

$$\dot{F}_{coll,+}(A) = \frac{q\Omega}{2} \int dC \int db_r\, \sigma_{CD,y}(b_r)\, F(C) F[D_{A-C}(b_r)].$$

Self-consistent corona: paradigm

- Collective magnetic pressure of loops as a **self-consistent mean field,** $\bar{B}(z)$:

$$\bar{B}(z) \sim \Delta\Psi \int_{L(Z_{top}=z)}^{\infty} F(L')\, dL'$$



- Each loop is confined by stratified pressure atmosphere $[P(z) = \bar{B}^2 / 8\pi]$ ⇒ **equilibrium loop shape**: $B_x(l;L) = const = B_{top}(L)$.

$$x(z;L) = -\, b_{top} \int_{z}^{Z_{top}} \frac{dz'}{\sqrt{b^2(z') - b_{top}^2}}$$

where $b(z) \equiv \bar{B}(z) / B_0$ and $B_0 \equiv \bar{B}(z=0)$

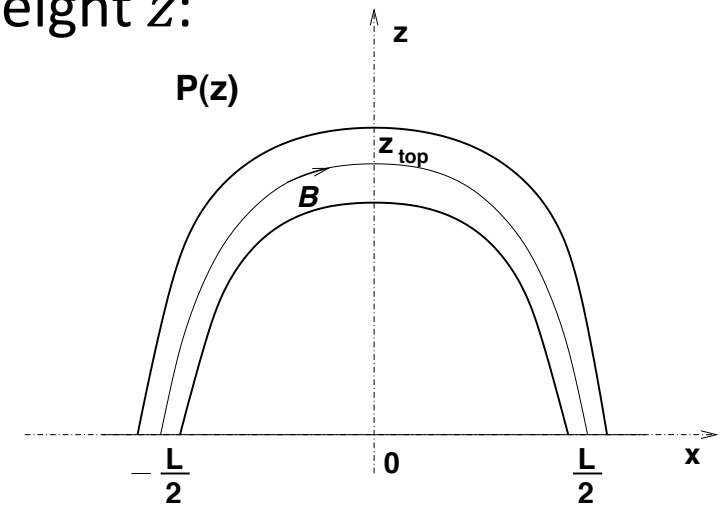
- **Goal:** self-consitently determine $\bar{B}(z)$, $Z_{\text{top}}(L)$, etc., for given $F(L)$.

Self-consistent Corona: Solution

- Mean magnetic pressure as a function of height z:

$$\bar{B}(z) = \pi\,\Delta\Psi \int_{L(z)}^{\infty} F(L')\,dL' \quad \Rightarrow \quad L = L(b)$$

$$b(z) \equiv \bar{B}(z)/B_0 \qquad B_0 \equiv \bar{B}(z=0)$$



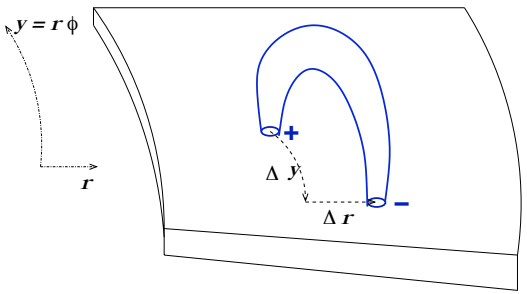
- Loop height as a function of b_{top}:

$$Z_{top}(b_{top}) = \frac{1}{\pi} \int_{b_{top}}^{1} \frac{L(b')\,db'}{\sqrt{b'^2 - b_{top}^2}}$$

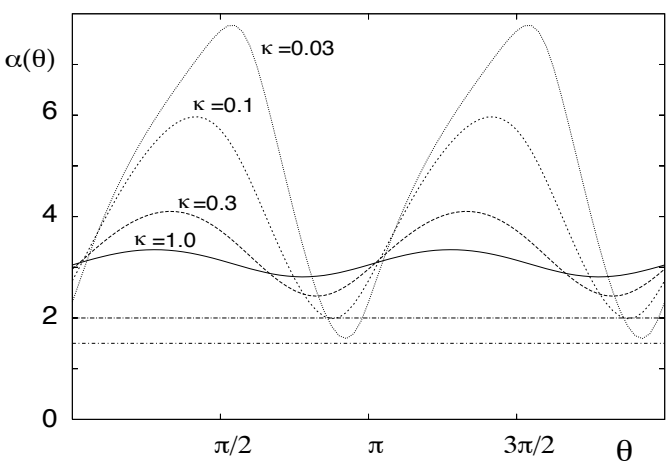
Niels Abel (1802-1829)

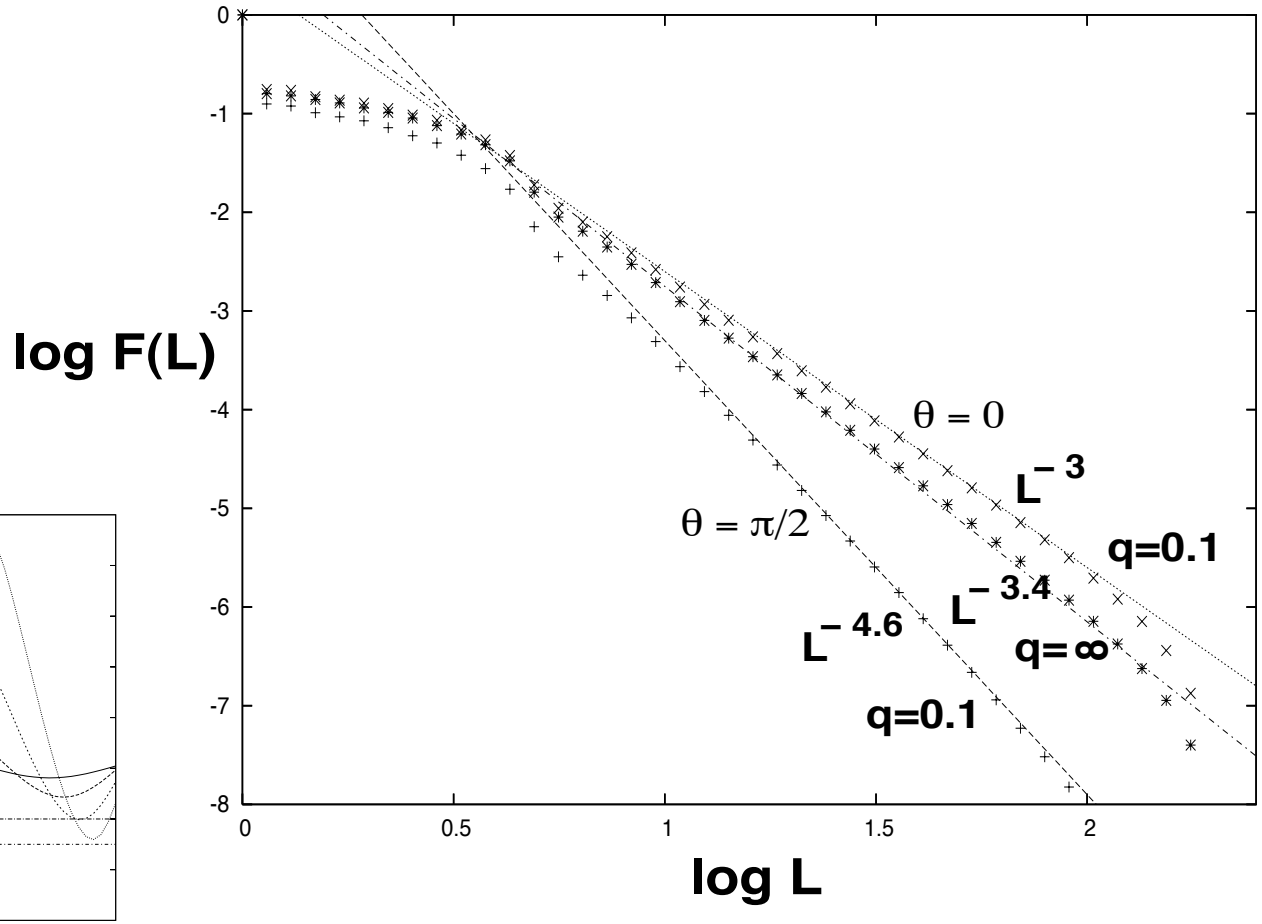
Results: Statistical steady state

- projected loop length: $\mathbf{L^2 = \Delta r^2 + \Delta y^2}$
- orientation angle with respect to toroidal (φ) direction: θ
- $q = \infty$ – no Keplerian shear; $q \ll 1$ – strong shear
- semi-circular loops

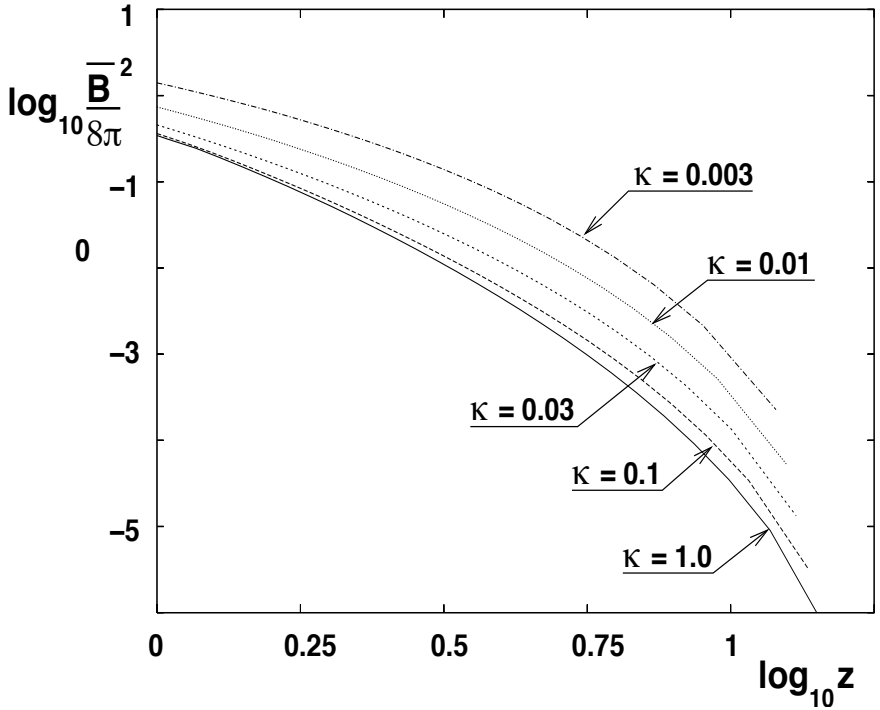


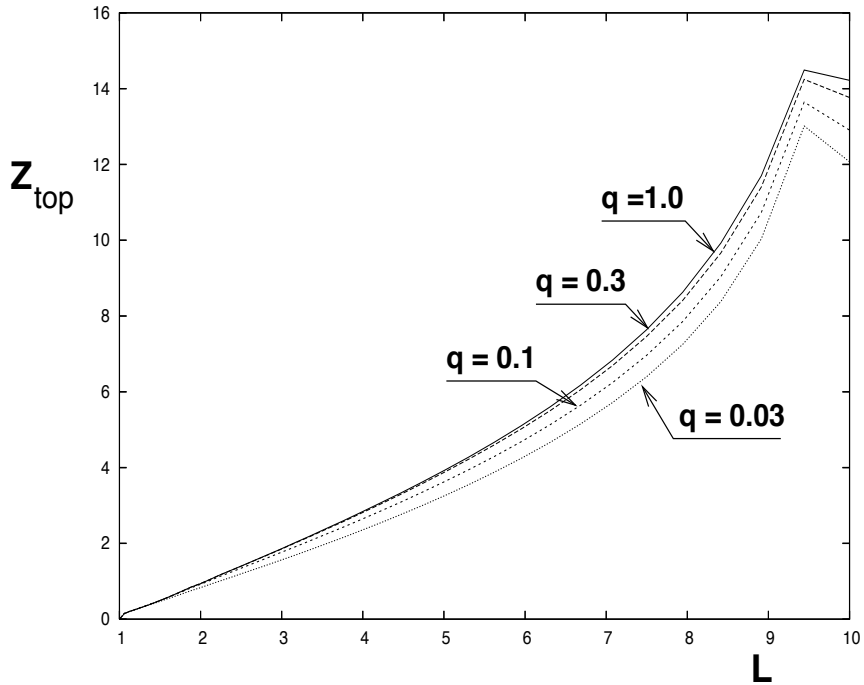
$F(L,\theta) \sim L^{-\alpha(\theta)}$





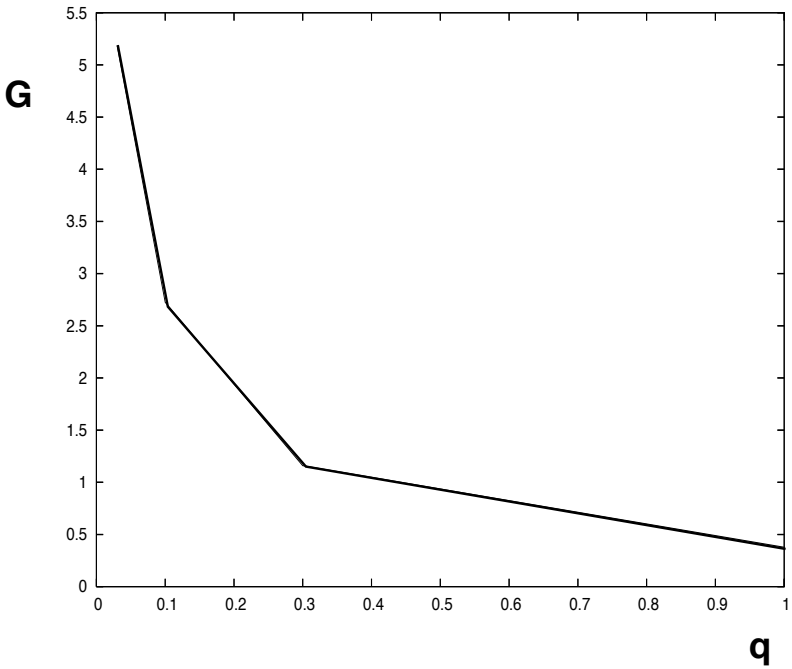
Magnetic Pressure Profile and Loop Height



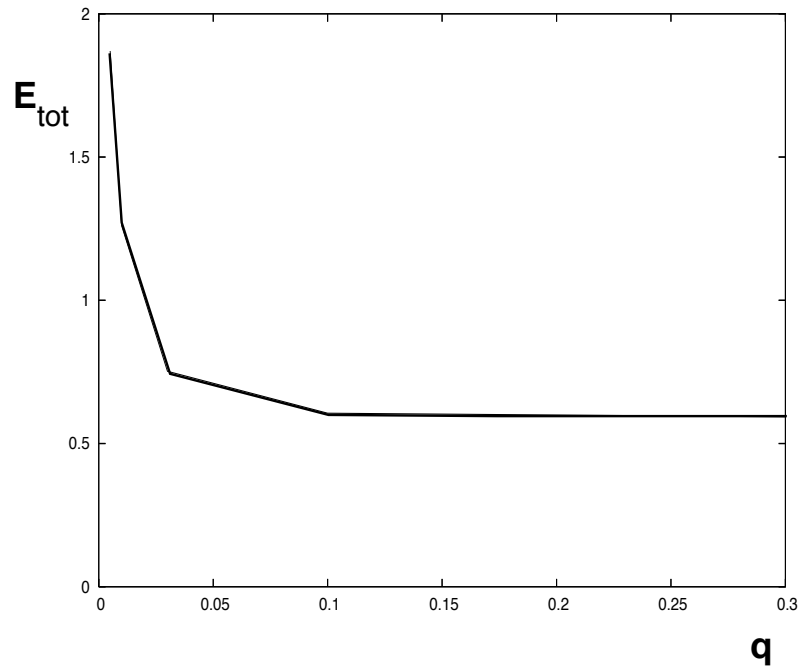


Results: magnetic energy and torque

Coronal Angular Momentum Transport



Total Coronal Magnetic Energy

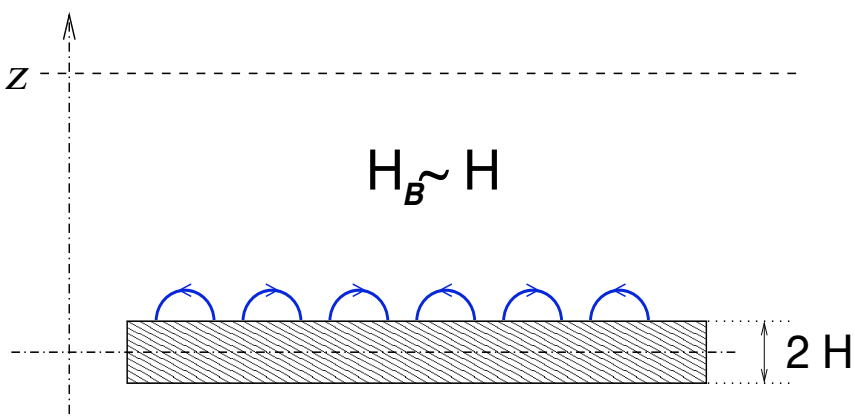


Role of Reconnection in ADCe

Reconnection is efficient:

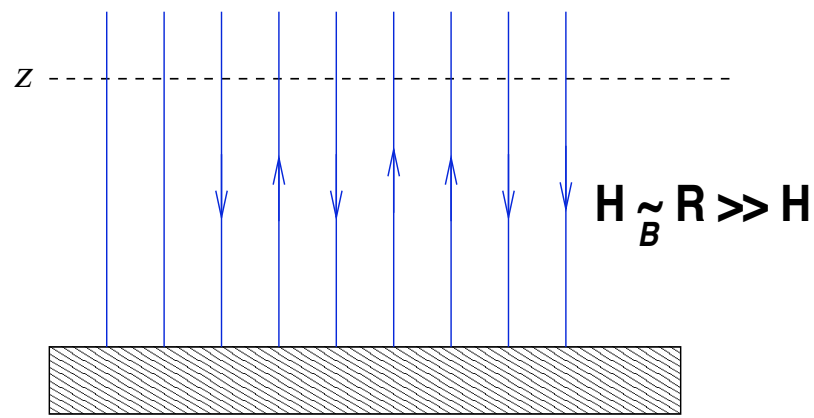
magnetic field is close to potential, $E_{magn} \simeq E_{pot}$;

free magnetic energy and magnetic dissipation are small.



Reconnection is forbidden:

forest of open field lines, $E_{magn} \simeq E_{open} \gg E_{pot}$; free magnetic energy is large but dissipation is not allowed.



SUMMARY

- Accretion Disk Corona (ADC) is magnetically-dominated, so one needs to understand the coronal magnetic field first.
- Statistical description of coronal magnetic field as an ensemble of loops of various sizes.
- Kinetic equation for Loop Distribution Function
  (includes Keplerian shear, reconnection between loops, etc.).
- Preliminary results: steady-state distribution is a power-law with orientation angle-dependent exponent: $F(L,\theta) \sim L^{-\alpha(\theta)}$.
- Each loop is confined by collective magnetic pressure of other loops, described by self-consistent mean field $\bar{B}(z)$.
- More things can be done:
  - Energy statistics of flares and angular momentum transfer, etc.
  - comparison with numerical simulations
  - Applications to solar corona?