Turbulent transport and heating of minority ions in hot, magnetised plasmas

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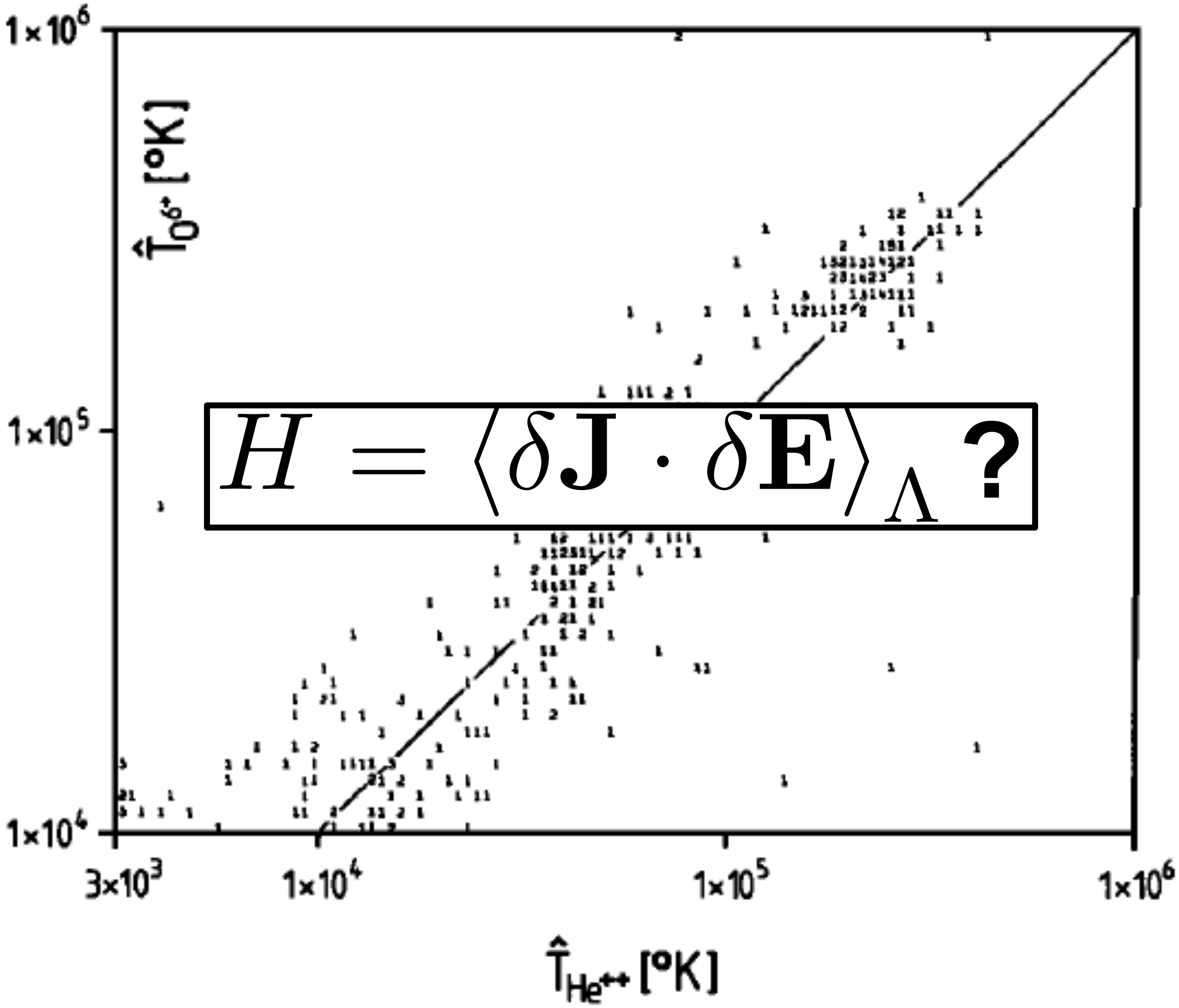
Heavy ions are preferentially heated

Minority ion temperatures in solar wind



$$\boxed{H = \langle \delta \mathbf{J} \cdot \delta \mathbf{E} \rangle_{\Lambda}\ \mathbf{?}}$$

$$\hat{T} \equiv T/A$$

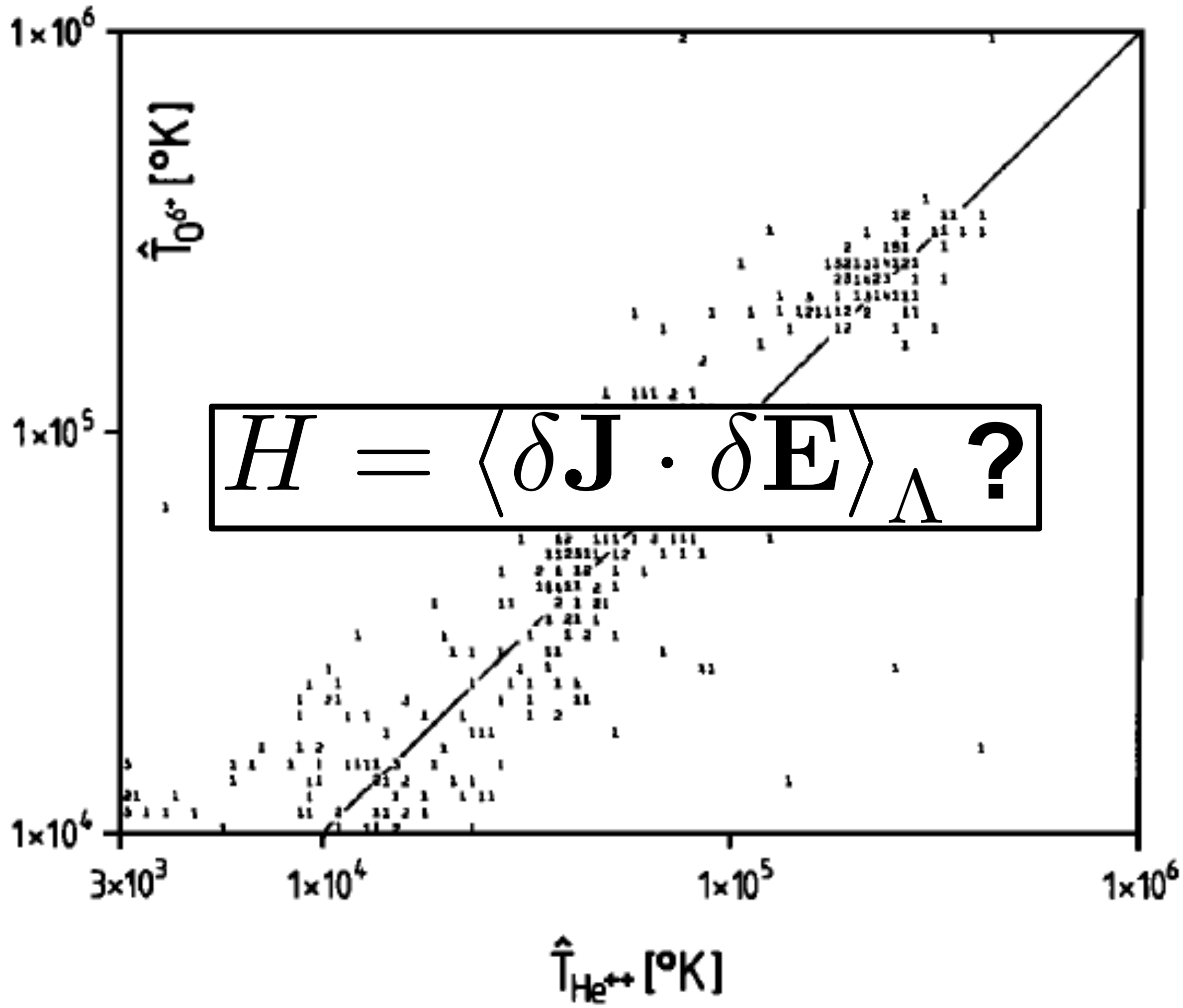
$$\hat{T}_{O^{6+}} = \hat{T}_{He^{++}}$$

$$\Rightarrow T \propto A$$

Schmidt et al., Geophys. Res. Lett. (1980).

Are similar effects possible in tokamaks?

Minority ion temperatures in solar wind



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Outline

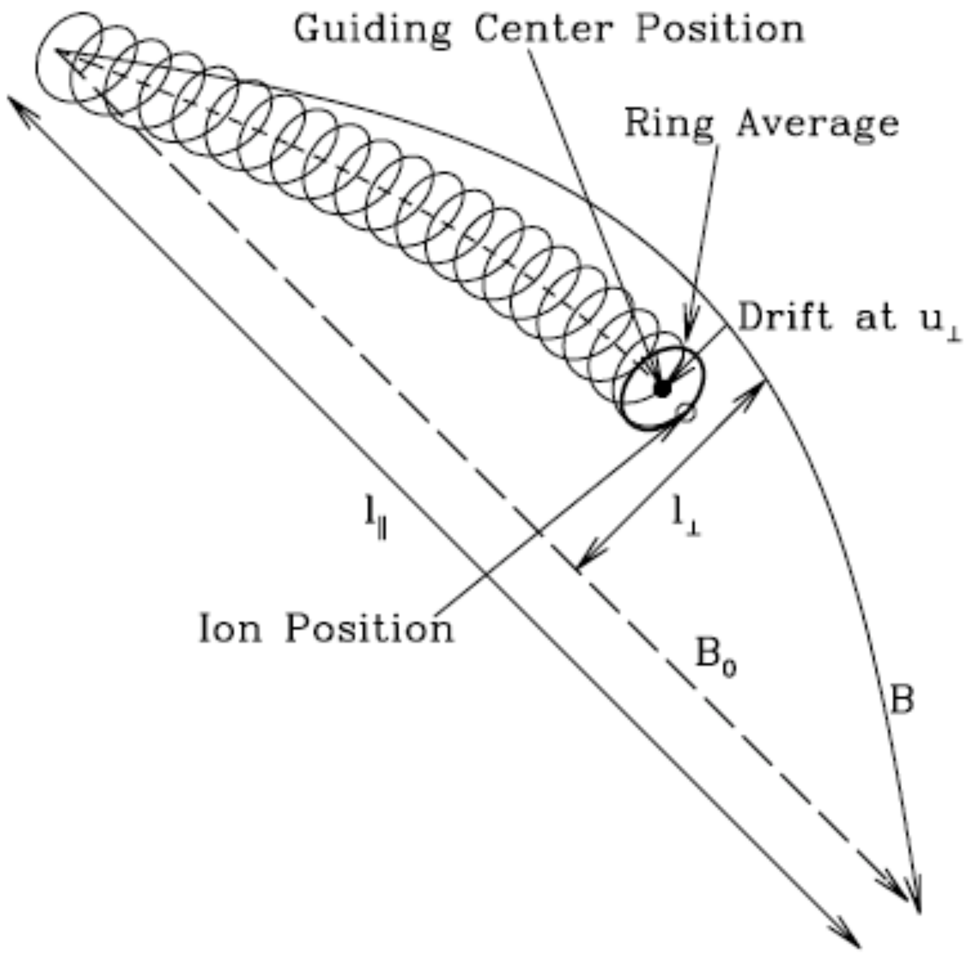
- Description of system and model equations (gyrokinetics)
- Scaling of turbulent transport and heating with A (mass number) and Z (charge number)
- Heating or cooling?
- Comparison with numerical results
- Implications and discussion

System description

- Focus on toroidal plasma with electrons, hydrogenic ions, and (trace) minority ions
- Include rotational shear, but neglect rotation itself
- Restrict attention to electrostatic fluctuations with wavelengths much larger than minority ion gyroradii
- Scalings are actually more general*: apply also to EM fluctuations in non-rotating, homogeneous plasma slab with $\beta \lesssim 1$

Barnes, Parra, and Dorland, Phys. Rev. Lett.* ***109**, 185003 (2012).*

*Gyro*kinetic description of dynamics



- Average over fast Larmor gyration and follow slower motion of charged rings
- Eliminates fast time scale and gyro-angle variable (6D ⟶ 5D)
- Assume:
  - Low-amplitude, anisotropic fluctuations
  - Collisions weak, but strong enough to give equilibrium Maxwellian

Gyrokinetic equation

$$f = F + \delta f$$

fluctuations

mean

$\langle \delta f_s \rangle_s$

fluctuating potential

$$\frac{Dg_s}{Dt} + \big(\overbrace{\mathbf{v}_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_\chi \rangle_s}^{\text{velocity of charged rings}}\big) \cdot \nabla \left(g_s + \frac{Z_s e \langle \chi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s$$

$$= - \langle \mathbf{v}_\chi \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\phi \frac{m_s v_\parallel}{T_s} F_{M,s} \right)$$

Maxwellian equilibrium

average over Larmor gyration

toroidal plasma rotation

Focus on electrostatic

$$\frac{Dg_s}{Dt} + \left(\mathbf{v}_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_\chi \rangle_s\right) \cdot \nabla \left(g_s + \frac{Z_s e \langle \chi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s$$
$$= - \langle \mathbf{v}_\chi \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\phi \frac{m_s v_\parallel}{T_s} F_{M,s} \right)$$

$$\langle \chi \rangle_s = \left\langle \underset{\substack{\uparrow \\ \sim 1}}{\Phi} - \underset{\substack{\uparrow \\ \sim 1/A^{1/2}}}{\frac{v_\parallel \delta A_\parallel}{c}} - \frac{1}{Z_s e} \int_0^\mu \underset{\substack{\uparrow \\ \sim 1/Z}}{d\mu' \delta B_\parallel} \right\rangle_s$$

$\Rightarrow$ **Largely electrostatic**

Focus on electrostatic

$$\frac{Dg_s}{Dt} + \left(v_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_E \rangle_s\right) \cdot \nabla \left(g_s + \frac{Z_s e \langle \phi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s$$

$$= - \langle \mathbf{v}_E \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\zeta \frac{m_s v_\parallel}{T_s} F_{M,s} \right)$$

$$\langle \chi \rangle_s = \left\langle \underset{\sim 1}{\Phi} - \underset{\sim 1/A^{1/2}}{\frac{v_\parallel \delta A_\parallel}{c}} - \frac{1}{Z_s e} \int_0^\mu \underset{\sim 1/Z}{d\mu' \delta B_\parallel} \right\rangle_s$$

~1 **~1/A$^{1/2}$** **~1/Z**

➡ **Largely electrostatic**

Turbulent fluxes and heating

radial flux of toroidal angular momentum

$$\{\Gamma_s, \Pi_s, Q_s\} = \underbrace{\left\langle g_s \left\{1, m_s R\mathbf{v}\cdot\hat{\zeta}, \frac{m_s v^2}{2}\right\} \mathbf{v}_E \cdot \nabla r \right\rangle_\Lambda}_{\text{phase space average}}$$

particle flux

energy flux

$$H_s = \underbrace{-Z_s e \left\langle g_s \left(\mathbf{v}_{\|} + \mathbf{v}_{M,s}\right) \cdot \nabla\phi \right\rangle_\Lambda}_{\text{Joule heating}} \underbrace{- \Pi_s \frac{d\omega_\zeta}{dr}}_{\text{viscous heating}}$$

turbulent heating

Dependences on charge and mass

Electrostatic potential and turbulence space-time scales
determined by bulk (main ion/electron) turbulence

$$\frac{Dg_s}{Dt} + \left(v_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_E \rangle_s\right) \cdot \nabla \left(g_s + \frac{Z_s e \langle \phi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s$$

$$= -\langle \mathbf{v}_E \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\zeta \frac{m_s v_\parallel}{T_s} F_{M,s} \right)$$

No A or Z dependence

Z is the charge number

A is the mass number

Dependences on charge and mass

$$v_\parallel \sim v_{th} = \sqrt{\frac{2T_s}{m_s}} \qquad \mathbf{v}_E = \frac{c}{B^2}\hat{\mathbf{b}} \times \nabla\phi$$

$$\mathbf{v}_{M,s} = \frac{\hat{\mathbf{b}}}{\Omega_s} \times \left(v_\parallel^2 \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}} + \frac{v_\perp^2}{2} \frac{\nabla B}{B} \right) \sim \frac{T_s}{Z_s}$$

$$\frac{Dg_s}{Dt} + \left(v_\parallel + \mathbf{v}_{M,s} + \langle \mathbf{v}_E \rangle_s\right) \cdot \nabla \left(g_s + \frac{Z_s e \langle \phi \rangle_s}{T_s} F_{M,s} \right) - \langle C[\delta f_s] \rangle_s$$

$$= - \langle \mathbf{v}_E \rangle_s \cdot \left(\nabla F_{M,s} + R \nabla \omega_\zeta \frac{m_s v_\parallel}{T_s} F_{M,s} \right)$$

~1/A$^{1/2}$ ~1/Z ~A$^{1/2}$

Z is the charge number A is the mass number

Expansion in A ~ Z large

Expand g in powers of $A^{1/2}$. At lowest order:

$$\frac{Dg_0}{Dt} + \langle \mathbf{v}_E \rangle \cdot \nabla g_0 = -\left(\hat{b} \cdot \nabla Ze \langle \Phi \rangle + mR \langle \mathbf{v}_E \rangle \cdot \nabla \omega_\phi \right) \frac{v_\|}{T} F_M$$

$$\Rightarrow \quad g_0 \propto \boxed{\frac{Z}{A^{1/2}}}, \ \boxed{A^{1/2}}$$

$$\frac{Dg_1}{Dt} + \langle \mathbf{v}_E \rangle \cdot \nabla g_1 = \boxed{-\mathbf{v}_\| \cdot \nabla g_0}$$

$$\boxed{- \langle \mathbf{v}_E \rangle \cdot \nabla F_M - \mathbf{v}_M \cdot \nabla \frac{Ze \langle \phi \rangle}{T} F_M}$$

$$\Rightarrow \quad g_1 \propto \boxed{\frac{g_0}{A^{1/2}}}, \ \boxed{1} \quad \Rightarrow \quad g_1 \propto \frac{Z}{A}, \ 1$$

Symmetry property

$$\frac{Dg_0}{Dt} + \langle \mathbf{v}_E \rangle \cdot \nabla g_0 = -\left(\hat{b} \cdot \nabla Ze \langle \Phi \rangle + mR \langle \mathbf{v}_E \rangle \cdot \nabla \omega_\phi \right) \frac{v_{||}}{T} F_M$$

Symmetry of equation: $g_0(v_{||}) = -g_0(-v_{||})$

$$\Longrightarrow \overline{\int_{-\infty}^{\infty} dv_{||} \Phi g_0} = 0$$

$$\{\Gamma_0, Q_0\} = \left\langle g_0 \, \{1, \frac{mv^2}{2}\} \, \mathbf{v}_E \cdot \nabla r \right\rangle_\Lambda = 0$$

Momentum flux and heating scalings

$$\frac{Dg_0}{Dt} + \langle \mathbf{v}_E \rangle \cdot \nabla g_0 = -\left(\hat{b} \cdot \nabla Ze \langle \Phi \rangle + mR \langle \mathbf{v}_E \rangle \cdot \nabla \omega_\phi \right) \frac{v_{||}}{T} F_M$$

Symmetry of equation: $g_0(v_{||}) = -g_0(-v_{||})$

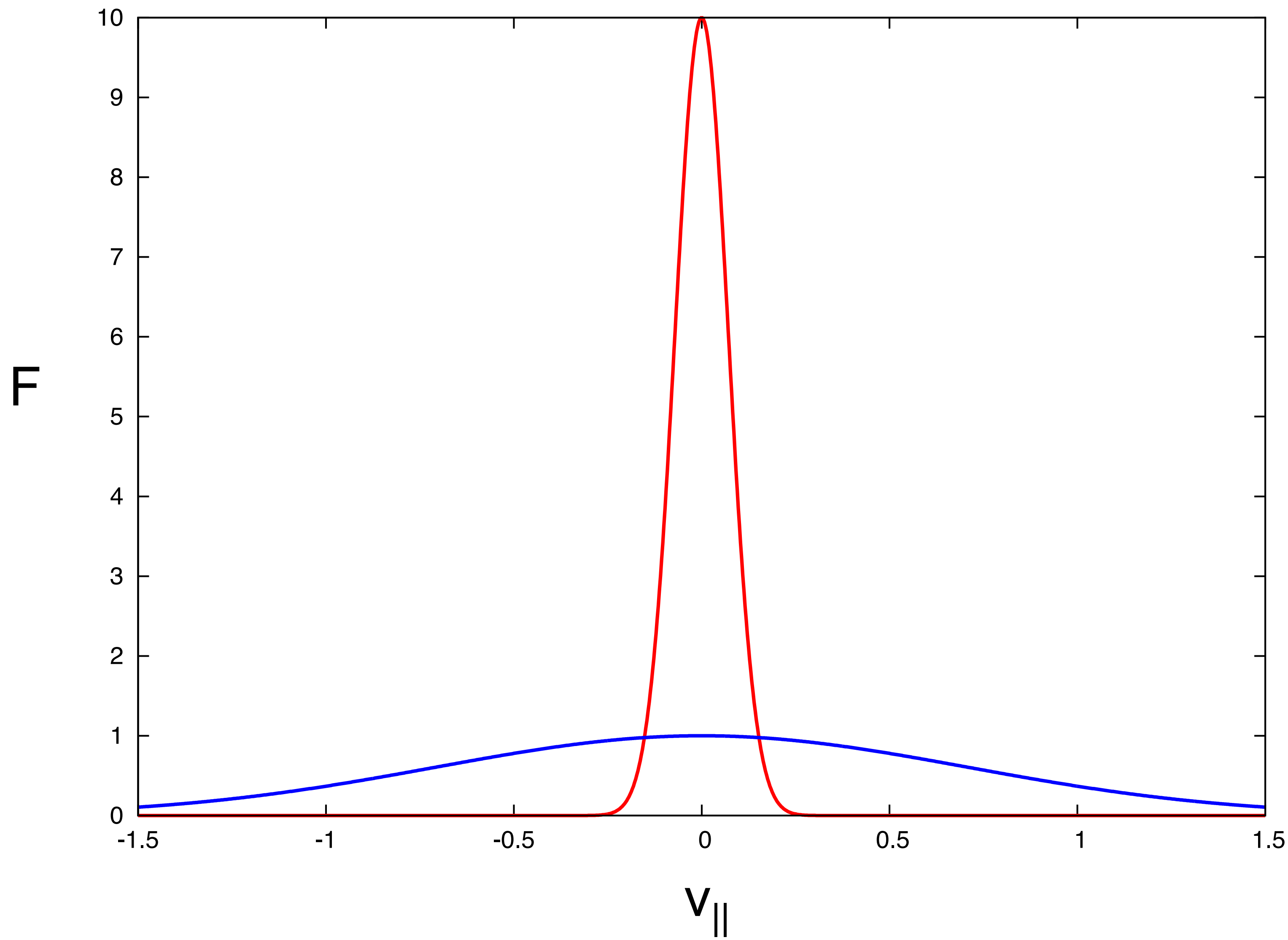
$$\Longrightarrow \overline{\int_{-\infty}^{\infty} dv_{||} \Phi g_0} = 0$$

$$\Pi_0 = \left\langle g_0 \left(mR\mathbf{v} \cdot \hat{\zeta} \right) \mathbf{v}_E \cdot \nabla r \right\rangle_\Lambda \propto Z, \; A$$

$$H_0 = -Ze \left\langle g_0 \mathbf{v}_{||} \cdot \nabla \phi \right\rangle_\Lambda - \Pi_0 \frac{d\omega_\zeta}{dr} \propto \frac{Z^2}{A}, \; Z, \; A$$

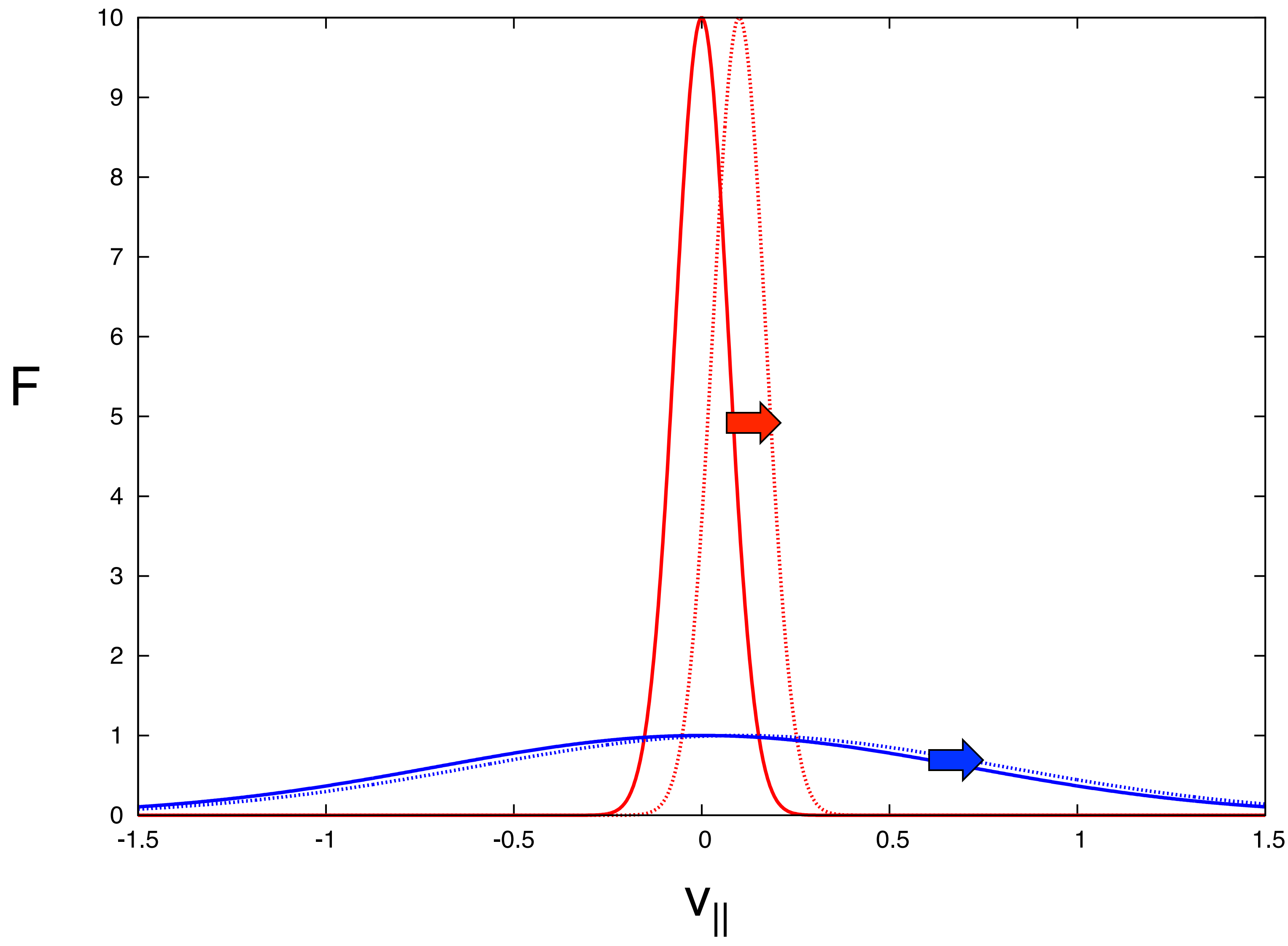
Simple physical picture

$$H_0 = -Ze \left\langle g_0 \mathbf{v}_{\|} \cdot \nabla\phi \right\rangle_{\Lambda} - \Pi_0 \frac{d\omega_\zeta}{dr} \propto \frac{Z^2}{A},\ Z,\ A$$



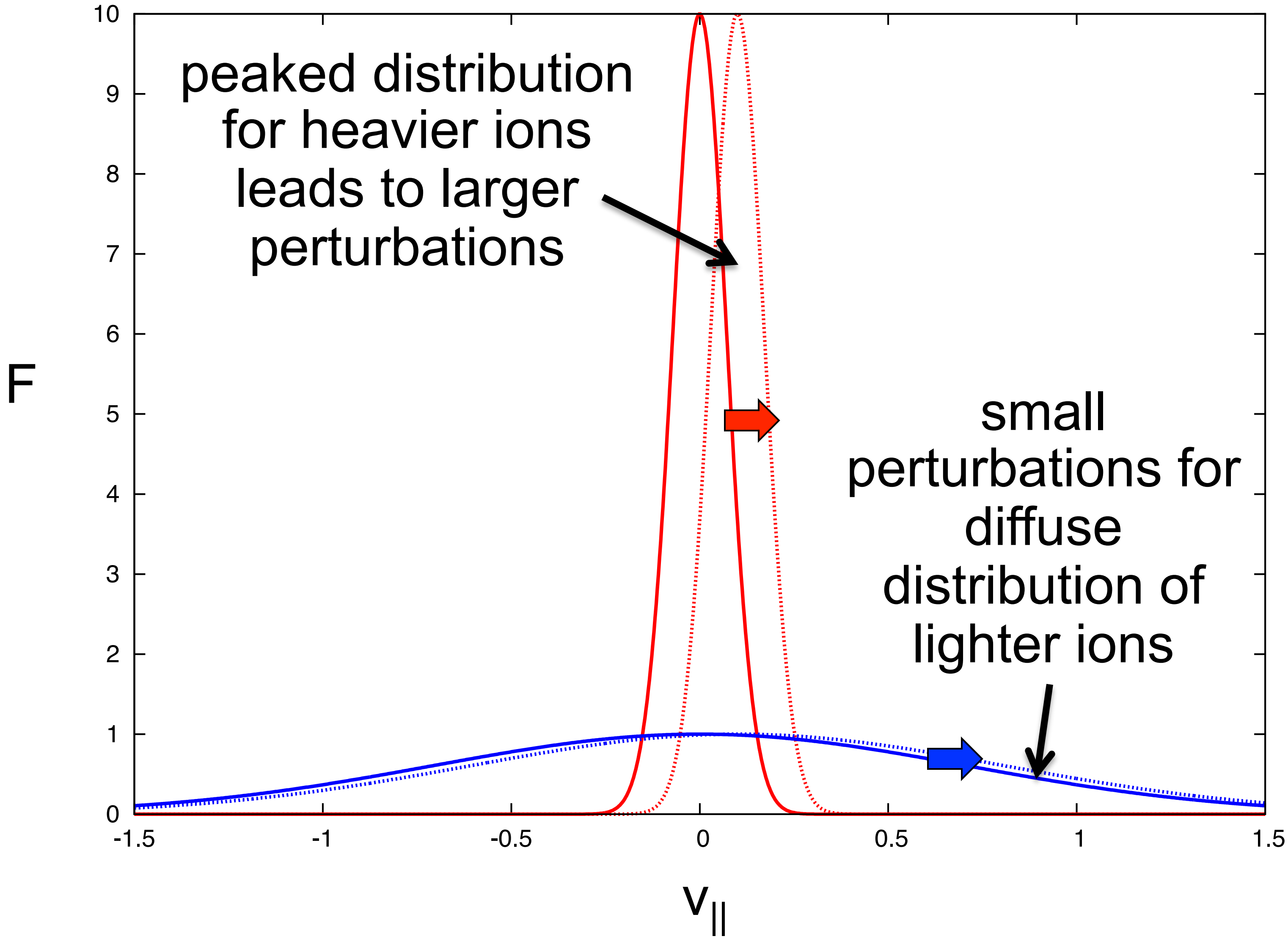
Simple physical picture

Fluctuating electric accelerates particles from background distribution along the mean field



Simple physical picture

$$H = \delta \mathbf{J} \cdot \delta \mathbf{E} \propto \frac{Z}{A^{1/2}} \delta n$$



Particle and energy flux scalings

$$\frac{Dg_1}{Dt} + \langle \mathbf{v}_E \rangle \cdot \nabla g_1 = -\mathbf{v}_{||} \cdot \nabla g_0$$

$$- \langle \mathbf{v}_E \rangle \cdot \nabla F_M - \mathbf{v}_M \cdot \nabla \frac{Ze \langle \phi \rangle}{T} F_M$$

Symmetry opposite that of g_0 equation: $g_1(v_{||}) = g_1(-v_{||})$

$$\Rightarrow \overline{\int_{-\infty}^{\infty} dv_{||} \Phi g_1} \neq 0$$

$$\{\Gamma_1, Q_1\} = \left\langle g_1 \ \{1, \frac{mv^2}{2}\} \ \mathbf{v}_E \cdot \nabla r \right\rangle_\Lambda \propto \frac{Z}{A}, \ 1$$

Minority ions heated (not cooled)

$$H_s = -\left\langle \frac{T_s g_s}{F_{M,s}} C[g_s] \right\rangle_\Lambda + \cancel{\left(Q_s - \frac{3}{2}\Gamma_s \right) \frac{\partial \ln T_s}{\partial r} + \Gamma_s \frac{\partial \ln n_s}{\partial r}}$$

~ Z²/A, Z, A

Independent of Z, A

$$C = \nu \times (\text{diffusive piece} + \text{integral piece})$$

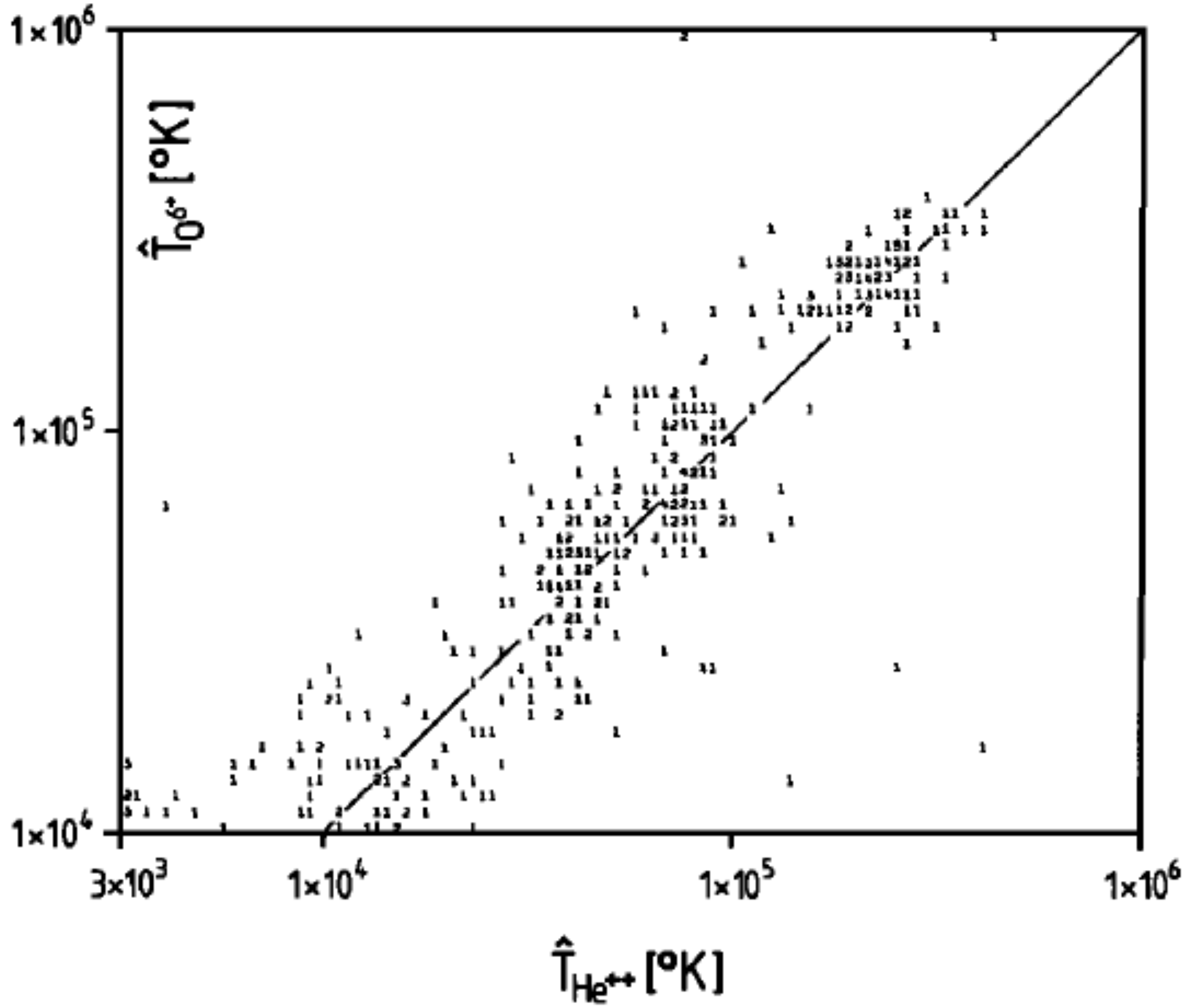
Dominant for small ν

➡ Collisions increase entropy for each species separately, so heavy ions are heated instead of cooled

Heating shut off when the heavy ion temperature becomes large enough to interfere with our expansion, i.e, T_{heavy}/T_{light} ~ A

Consistent with solar wind observations

Minority ion temperatures in solar wind



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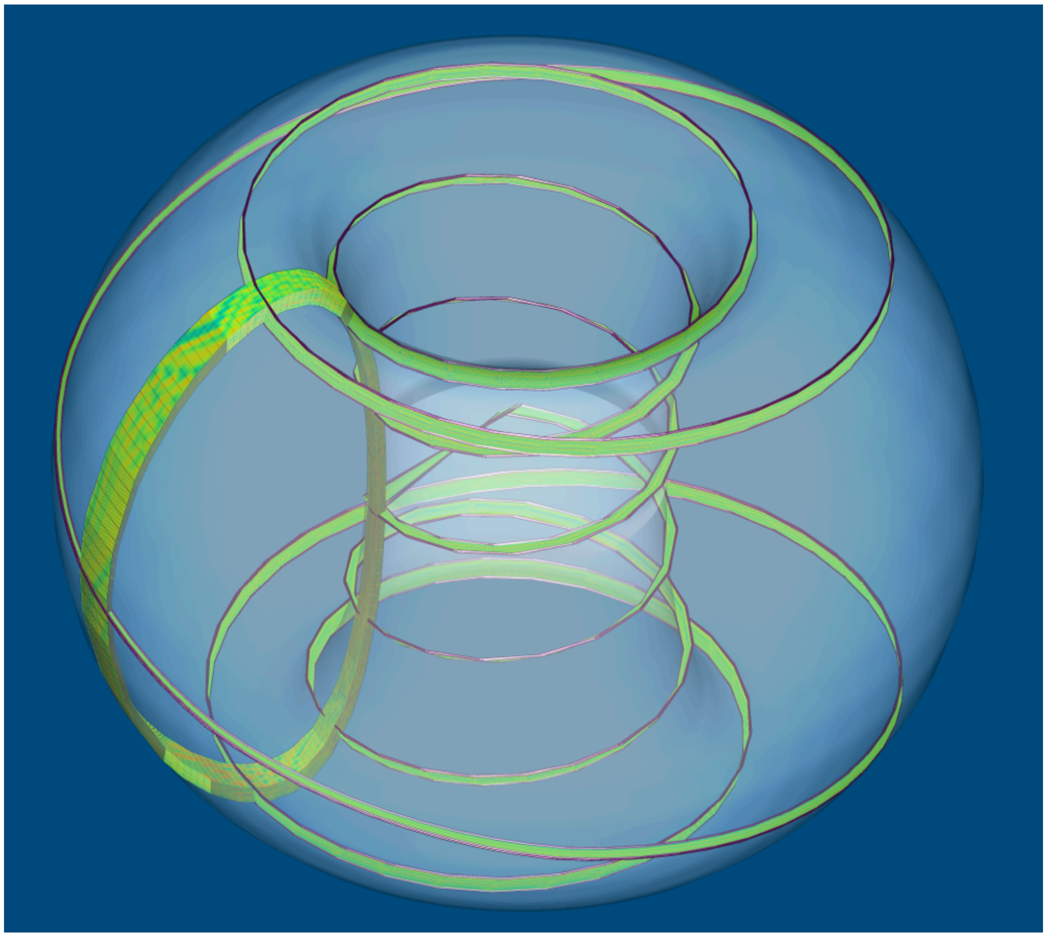
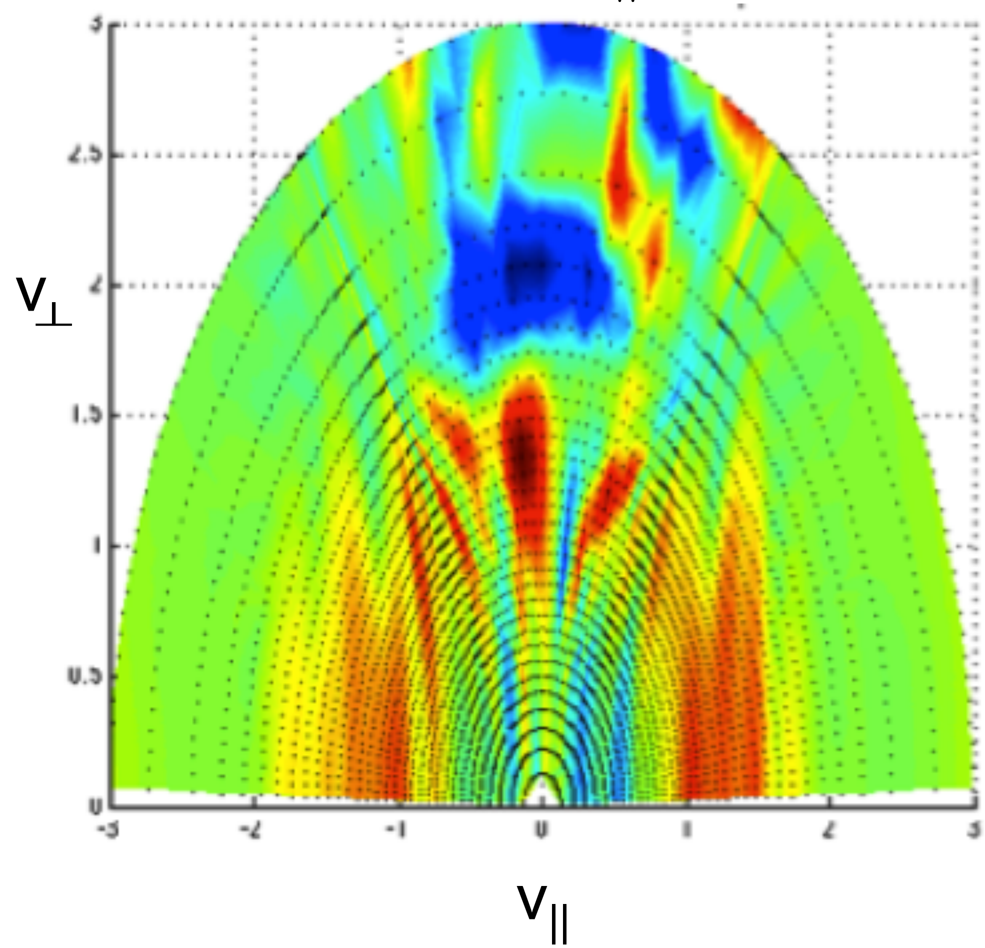
Gyrokinetic plasma turbulence simulations

GS2 simulation
domain

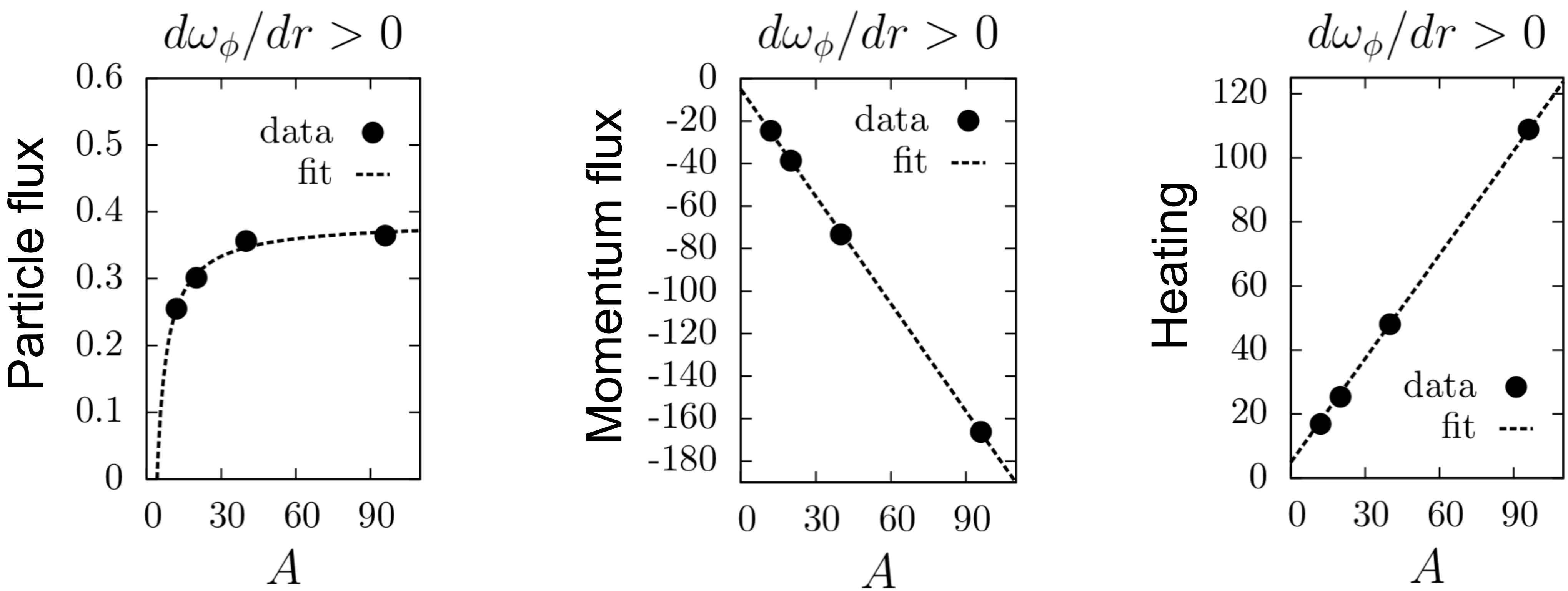


$$g = g(\mathbf{R}, v_{\|}, v_{\perp}, t)$$



Comparison with turbulence simulations

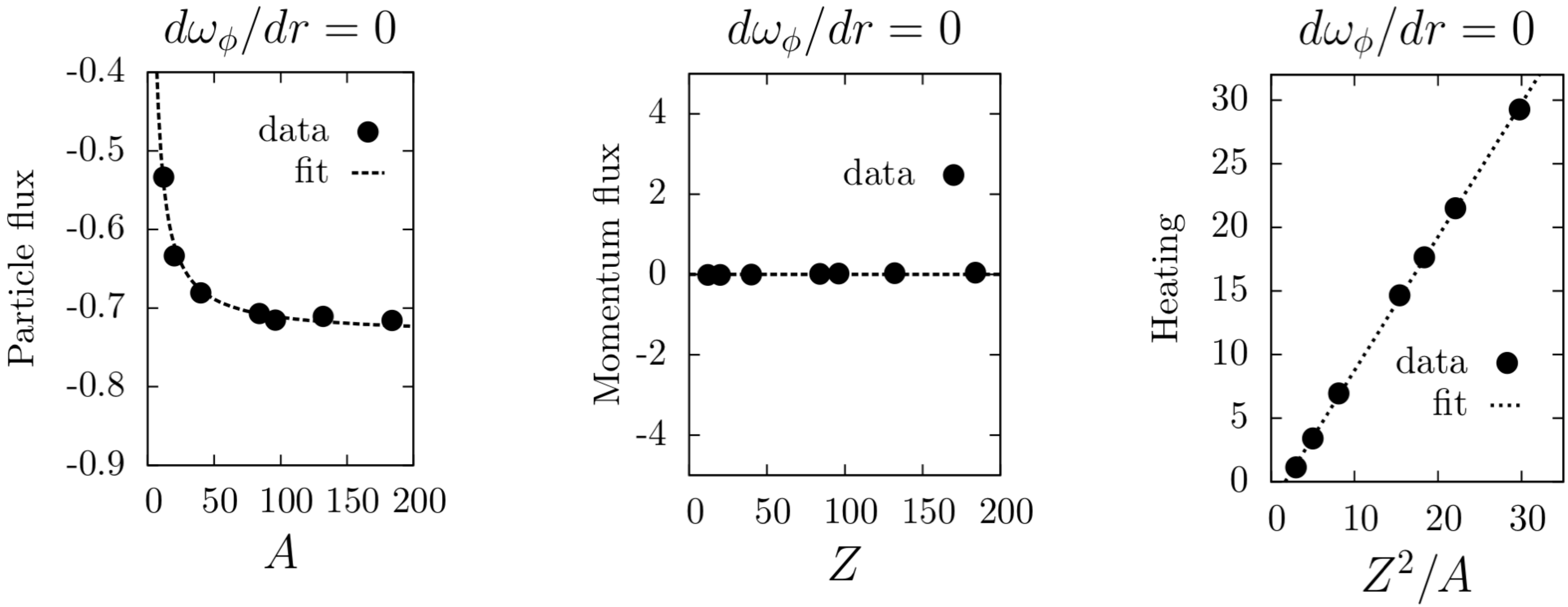
Nonlinear gyrokinetic simulations with GS2



Predictions: $\Gamma \propto \dfrac{Z}{A},\ 1$ $\quad \Pi \propto Z,\ A$ $\quad H \propto Z,\ A$

Comparison with turbulence simulations

Nonlinear gyrokinetic simulations with GS2



Predictions: $\Gamma \propto \frac{Z}{A},\ 1$ $\qquad \Pi = 0$ $\qquad H \propto \frac{Z^2}{A}$

Turbulent heating vs. collisional temperature equilibration

$$\frac{3}{2}\frac{\partial p_s}{\partial t} + \ldots = H_s + n_s \sum_u \nu^{\epsilon}_{su} \left(T_u - T_s\right)$$

$$H_s \sim S_s n_s T_i \left(\frac{\delta n_i}{n_i}\right)^2 \omega_t \qquad \mathcal{E}_s \sim \frac{Z_s^2}{A_s} n_s \Delta T_s \nu_{ii}$$

$$H_s \sim \mathcal{E}_s \Rightarrow \frac{\Delta T_s}{T_i} \sim S_s \frac{A_s}{Z_s^2} \left(\frac{\delta n_i}{n_i}\right)^2 \frac{\omega_t}{\nu_{ii}}$$

Summary

- Heavy ions preferentially heated by turbulence
- Turbulent heating shut off when T_{heavy}/T_{light}~A if collisional equilibration small
- Heavy tokamak impurities may be hotter than deuterium in limited circumstances (rotation, partial ionization, very weak collisions)
- Momentum flux of impurities increases with mass: can this affect bulk plasma rotation?