Revisiting Quasi-Keplerian Flow at Large Reynolds Numbers in the Laboratory

Hantao Ji, Eric Edlund, and Jeremy Goodman

Department of Astrophysical Sciences & Plasma Physics Laboratory
Princeton University

Center for Momentum Transport and Flow Organization (CMTFO)
Center for Magnetic Self-Organization (CMSO)

PCTS Plasma Meeting
April 8-12, 2013
Princeton

Outline

- A summary of laboratory results in water relevant to angular momentum transport in accretion disks

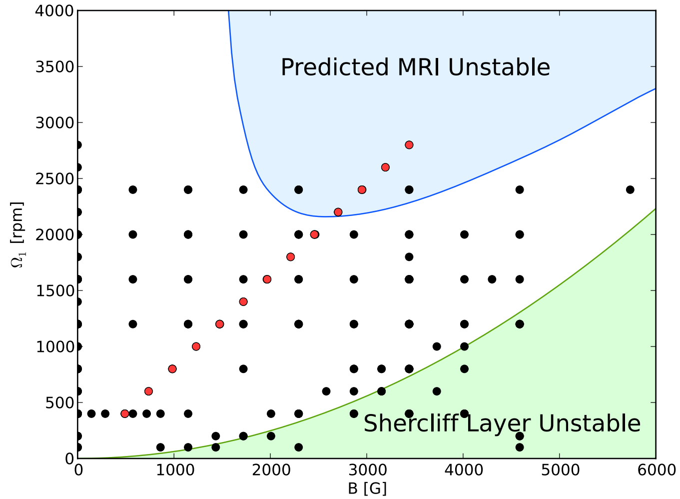
- Results from a new water experiment (HTX, Hydrodynamic Turbulence Experiment)

  - Robust nonlinear stability up to 10^6 shear Reynolds #
  - We offer a potential explanation of the reported conflicted results

- Summary and future work

Two Main Candidate Mechanisms to Generate Turbulence for Fast Accretion

- **Magnetorotational Instability (MRI) in hot highly conducting disks**
  - **Velikhov (1959) and Chandrasekhar (1960)**
  - **Shakura & Sunyaev (1973)**
  - **Balbus & Hawley (1991)**
  - **Theory/simulations**

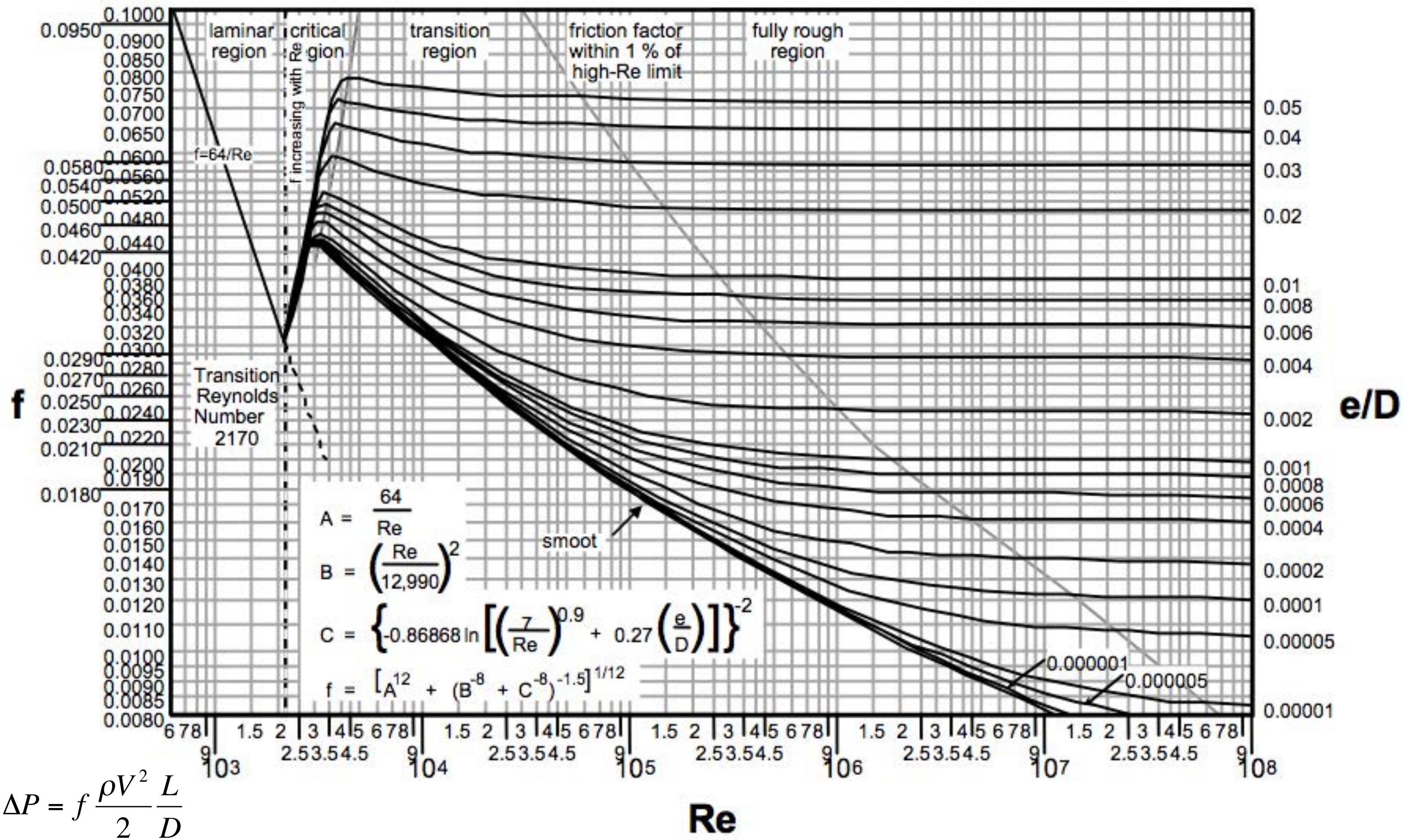


Magnetic field can destabilize otherwise stable flows.

- **Nonlinear hydrodynamic instabilities in cool, poorly ionized disks**
  - **Zeldovich (1981)**
  - **Richard & Zahn (1999) based on Wendt (1933) and Taylor (1936)**
  - **Theory/simulations**

Terrestrial flows (e.g. pipe flows) are often nonlinearly unstable if $Re > 10^2$-10^4 despite linear stability.

Moody Diagram (1944): Pipe Flow Friction Nonlinear (Subcritical) Transition to Turbulence



$$\Delta P = f\,\frac{\rho V^2}{2}\,\frac{L}{D}$$

Q: Does subcritical transition exist at a sufficiently large Reynolds #, if so, how does the turbulence transport angular momentum in Keplerian flows?

- Direct astronomical observations or direct numerical simulations of accretion disk turbulence are still not yet possible.
- How about laboratory experiments?

The Basic Experimental Idea

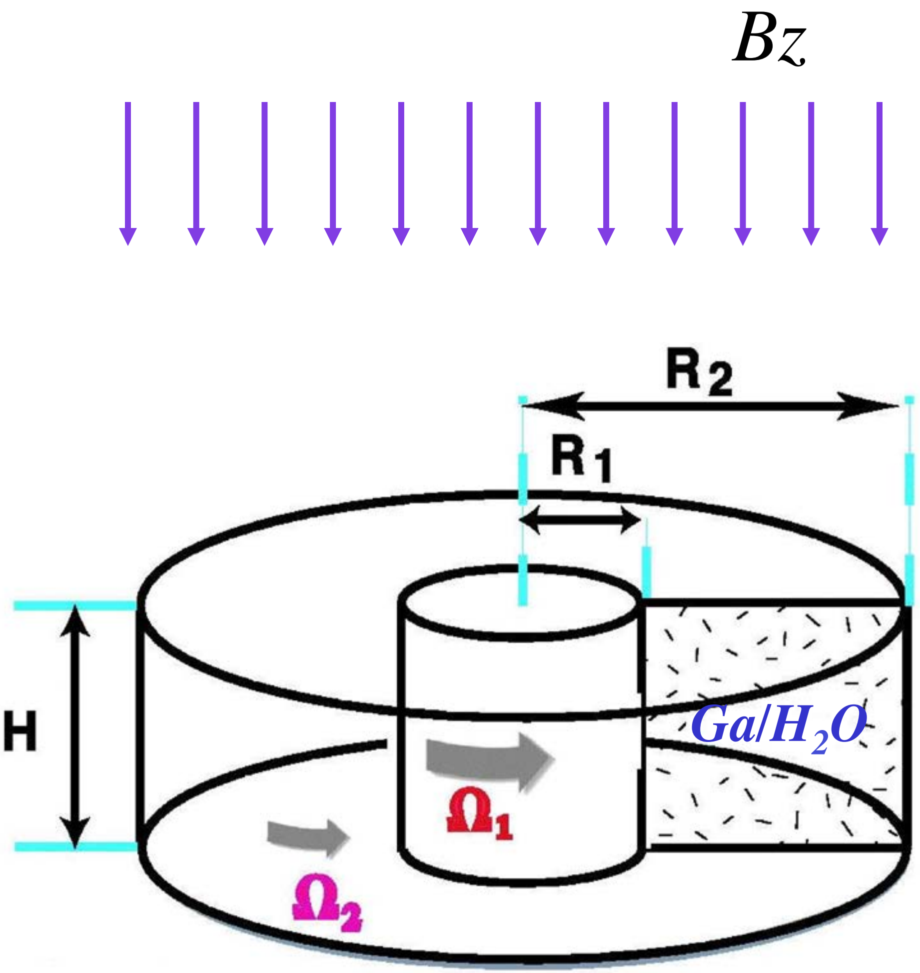
- **Couette flow geometry to realize quasi-Keplerian flows:**

$$\Omega_1 > \Omega_2$$
$$R_1^2\Omega_1 < R_2^2\Omega_2$$

- **Centrifugal force balanced by pressure force from the outer wall**

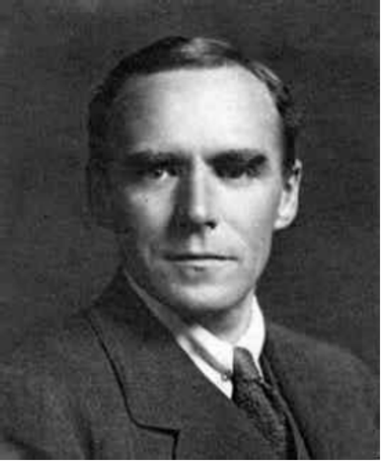
- **Use water for nonlinear hydro instabilities**

- **Use liquid gallium for MRI, unstable with appropriate Ω_1, Ω_2 and Bz on a table-top size.**



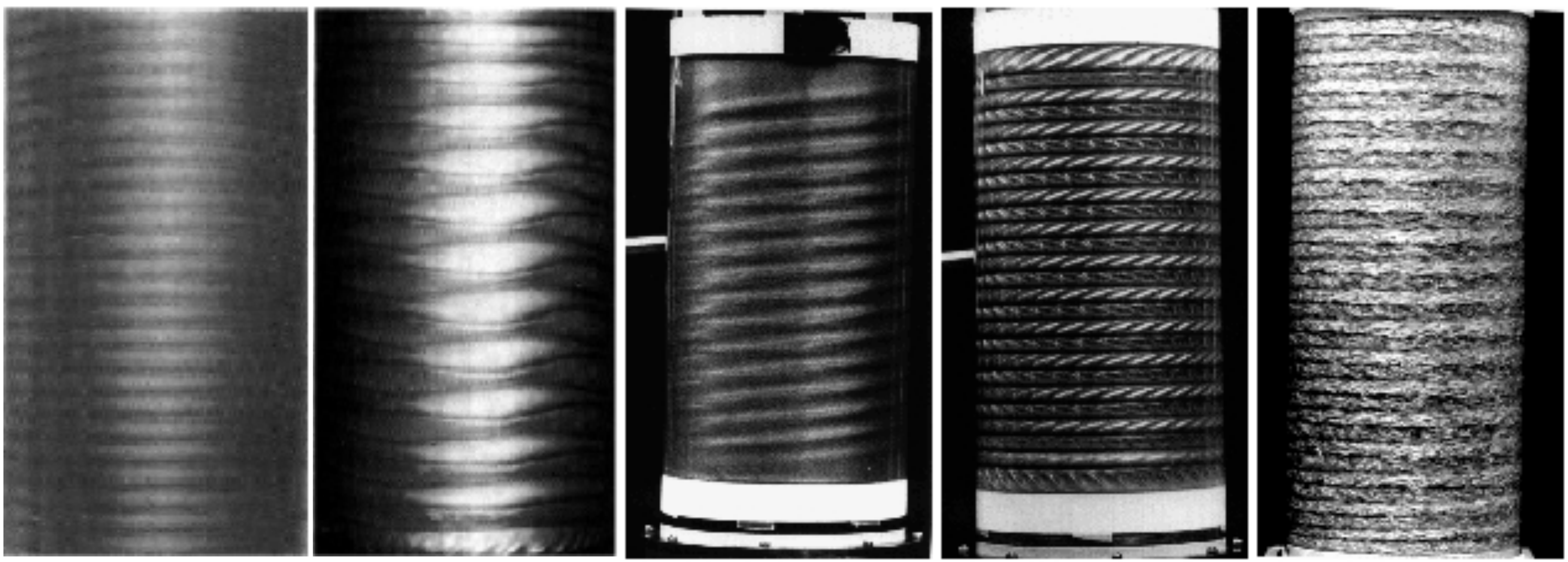
Ji, Goodman & Kageyama, MNRAS (2001); Goodman & Ji, JFM (2002)

Taylor-Couette Flows (Between Rotating Cylinders)



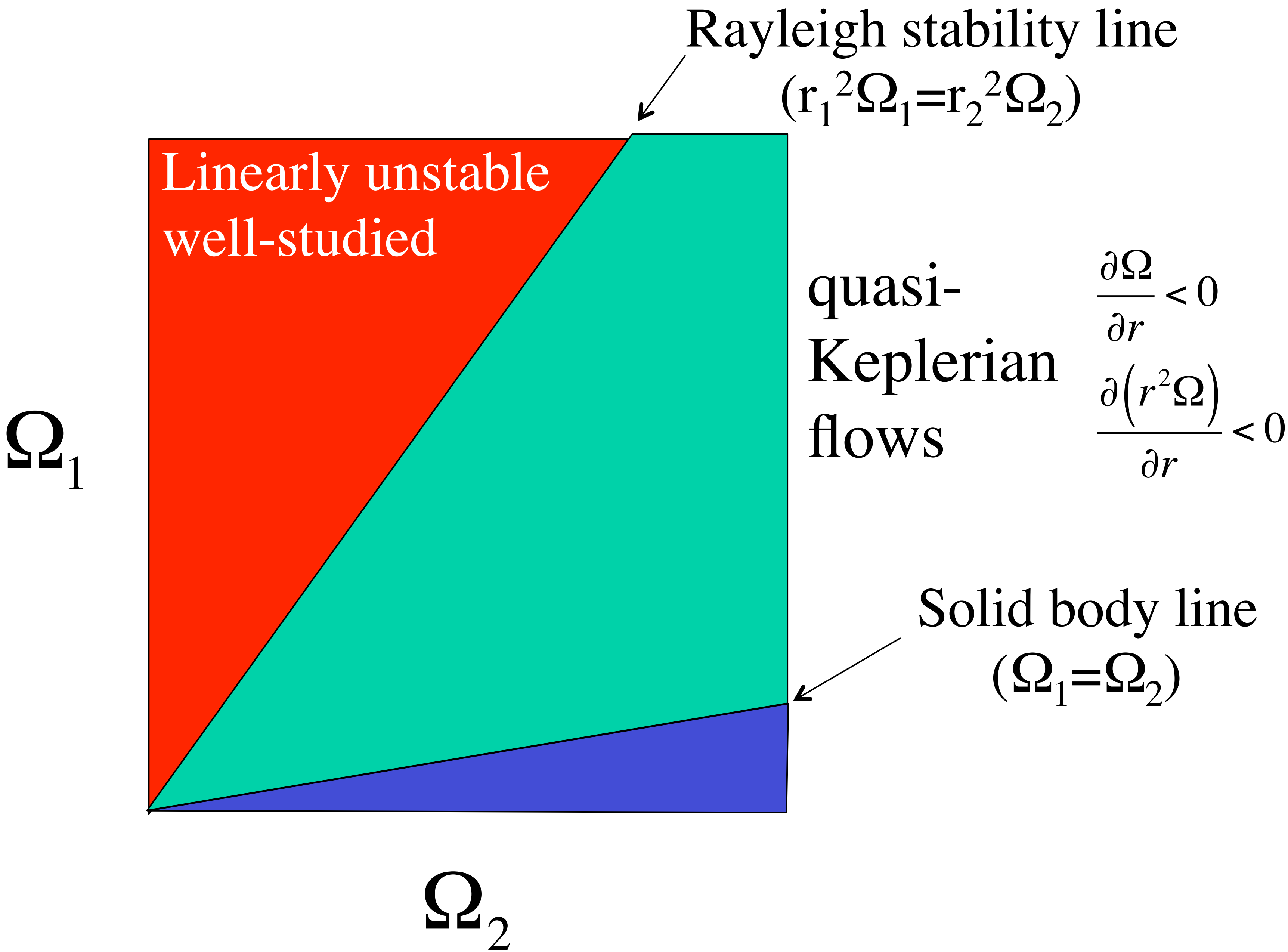
1923

1890

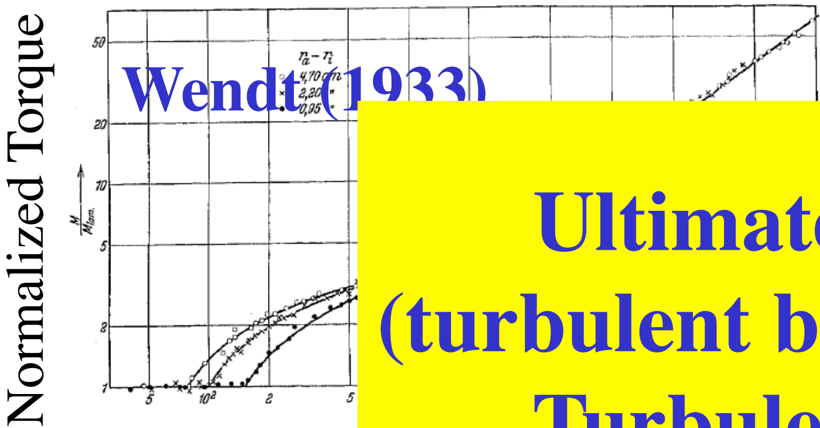


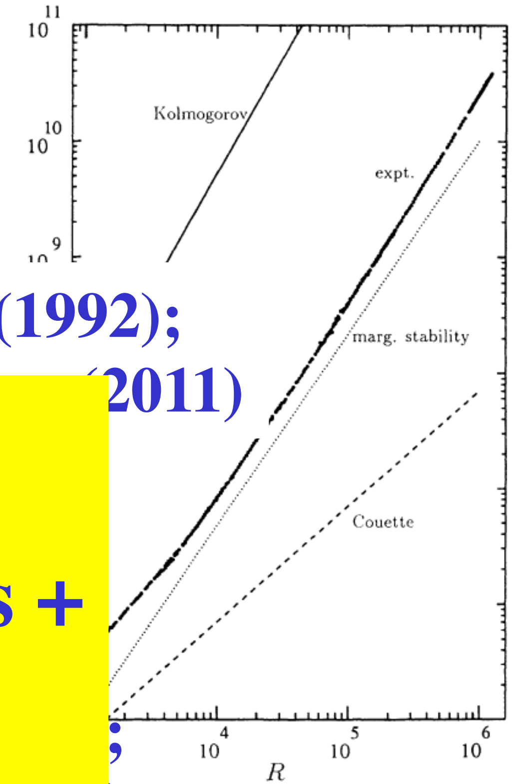
- **Rich nonlinear dynamics: bifurcations and transition to turbulence**

Stability Diagram of Taylor-Couette Flow



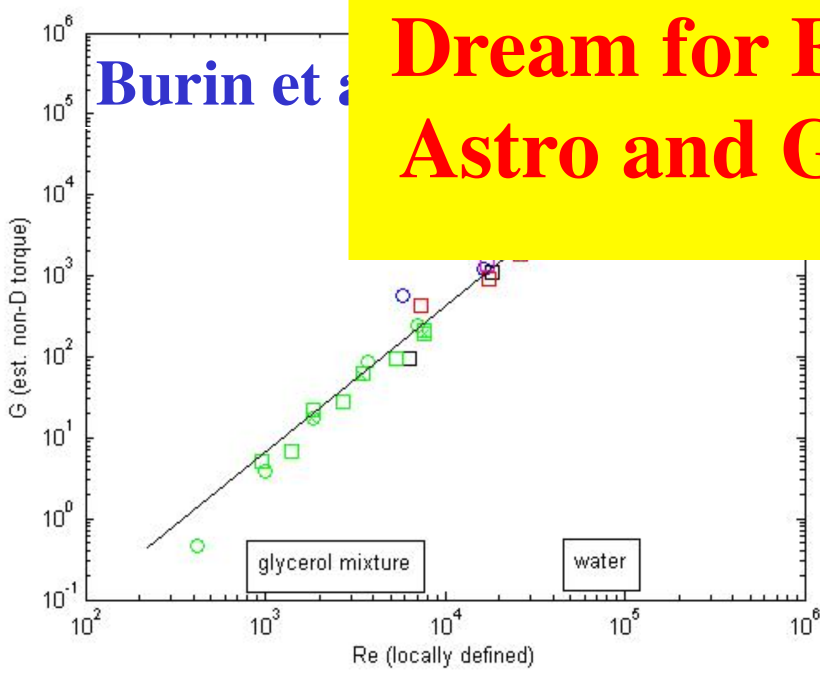
Supercritical Turbulence and Angular Momentum Transport

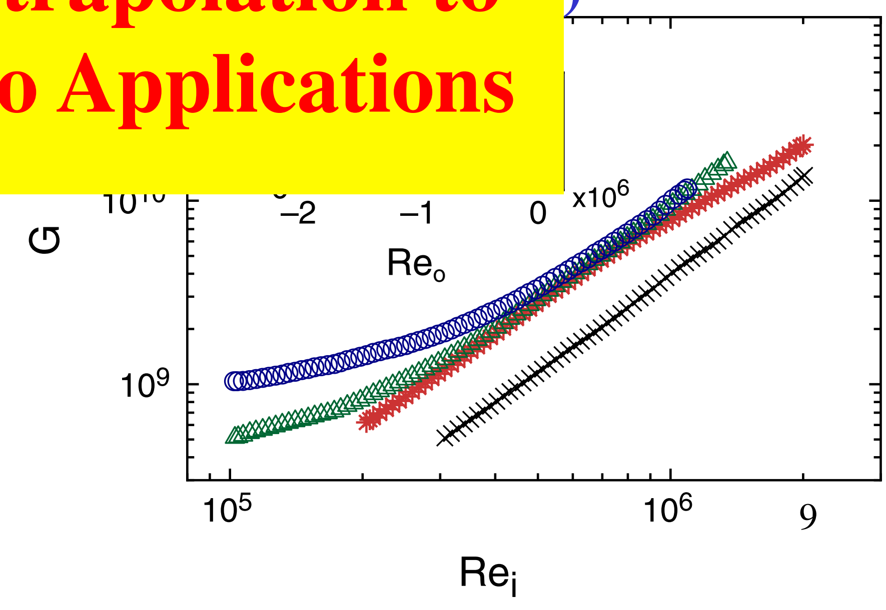




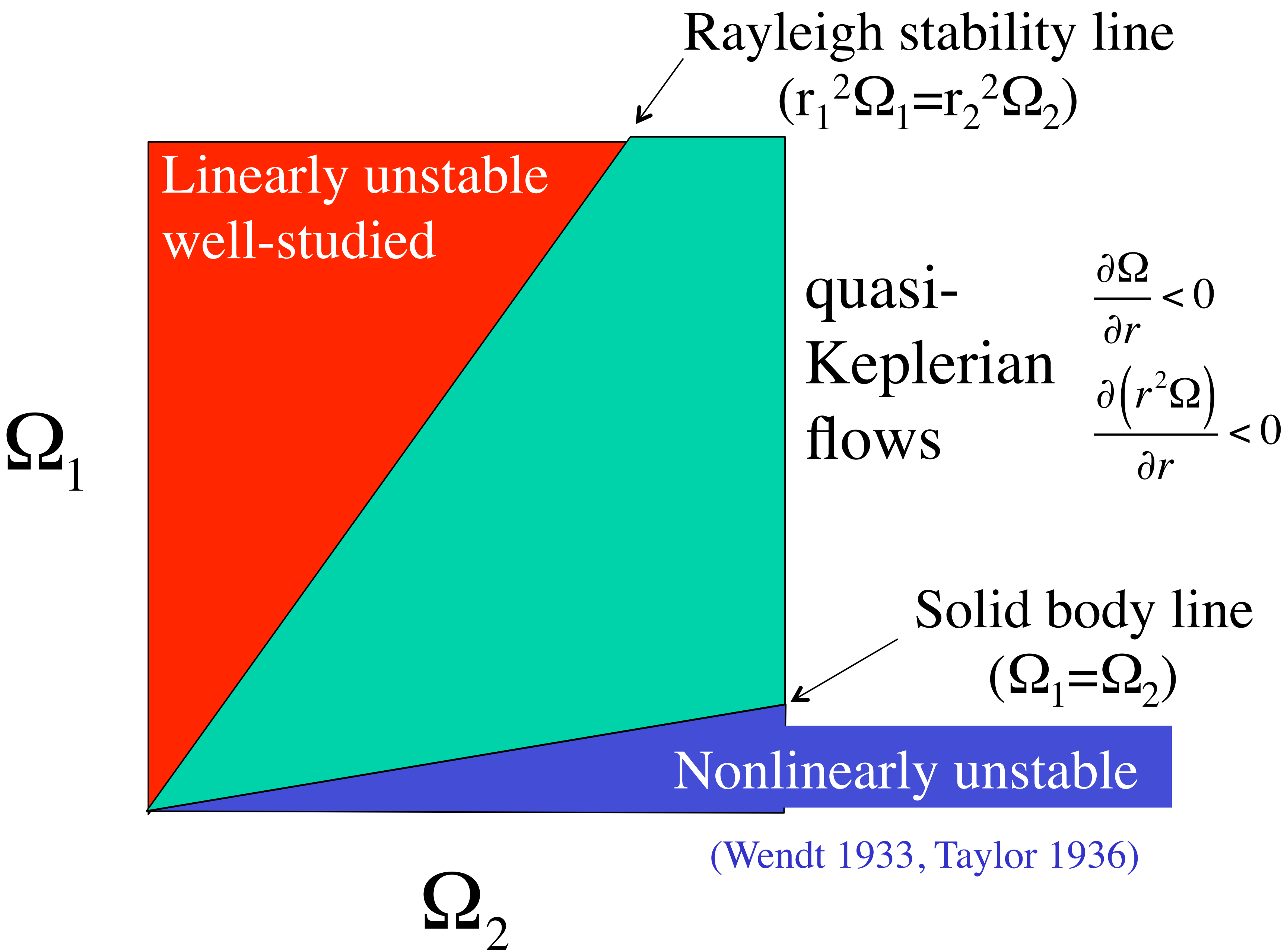
Ultimate turbulence (turbulent boundary layers + Turbulent Interior):

Dream for Extrapolation to Astro and Geo Applications

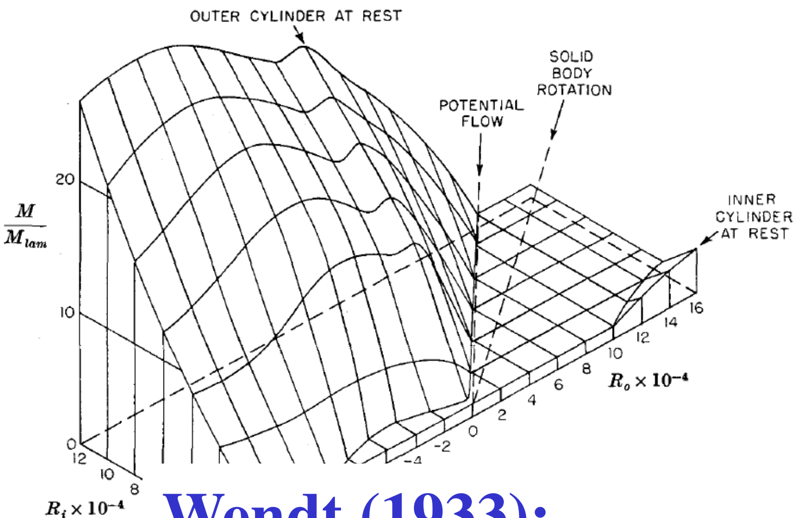




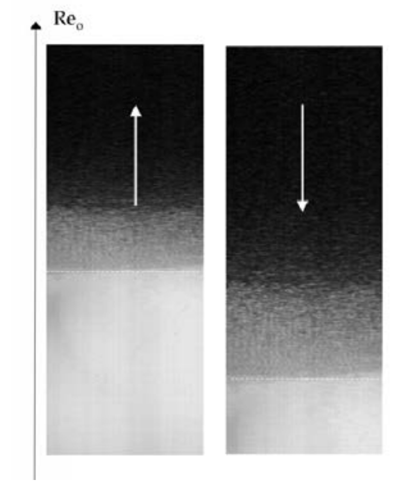
Stability Diagram of Taylor-Couette Flow



Nonlinear Instabilities Observed With Only Outer Cylinder Rotating

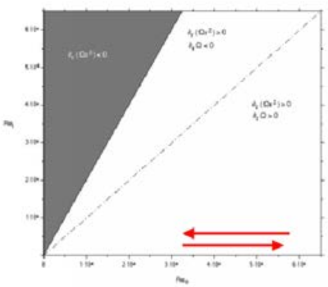


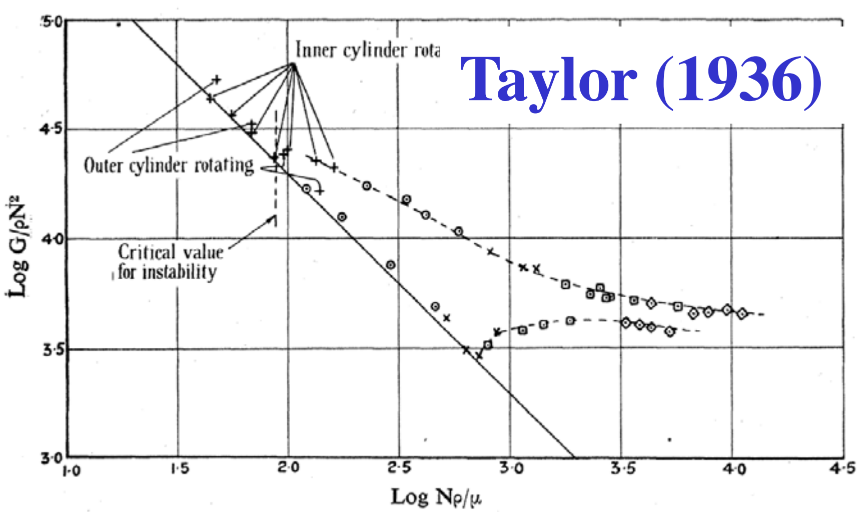
Wendt (1933); re-plotted by Coles (1965)



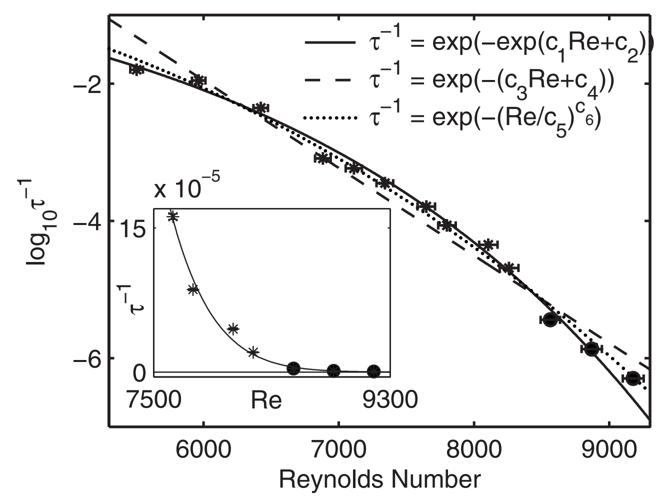
Borrero, Schatz, & Tagg (2010)

Richard (2001)



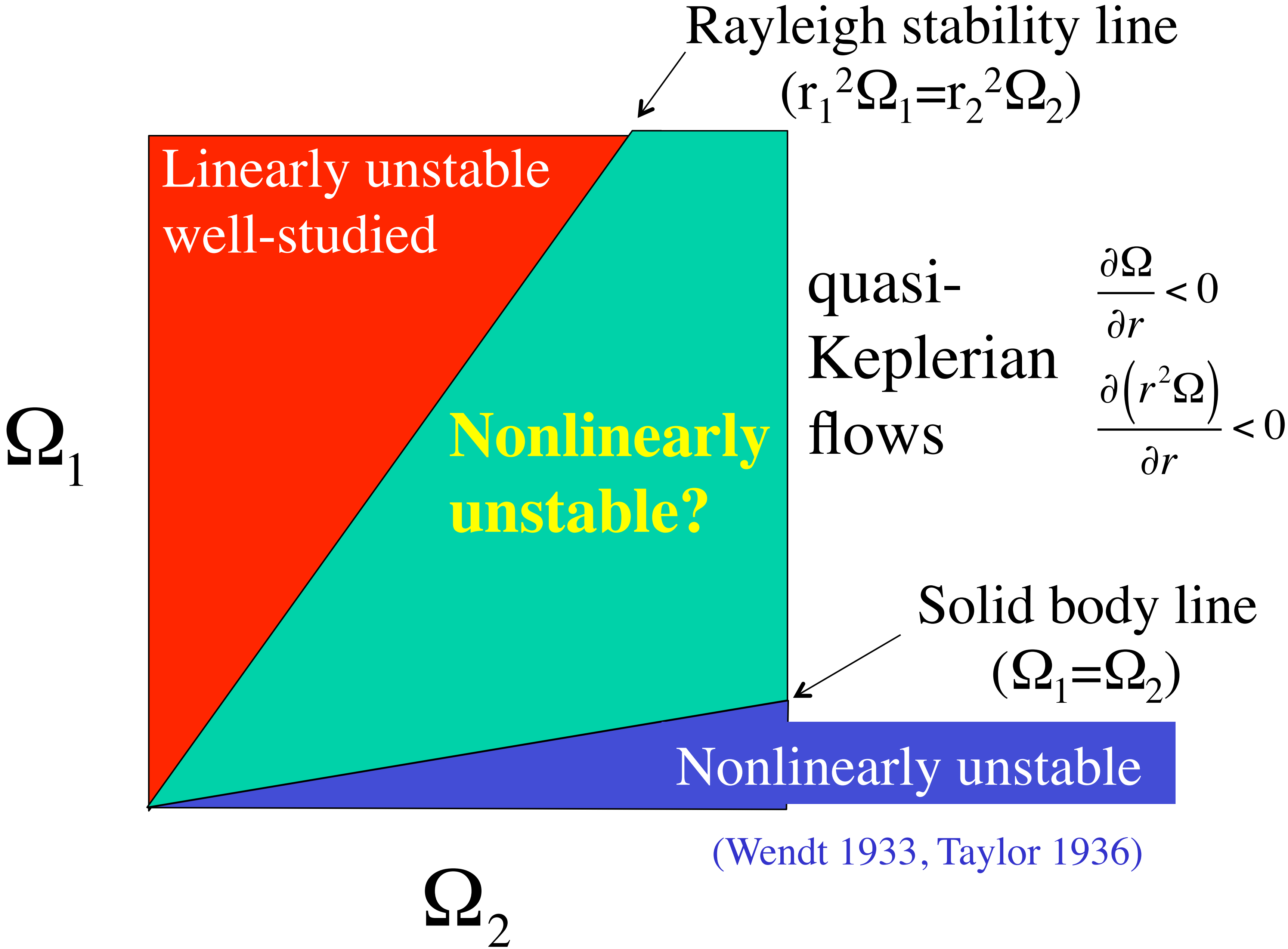


Taylor (1936)



Burin & Czarnocki (2012)

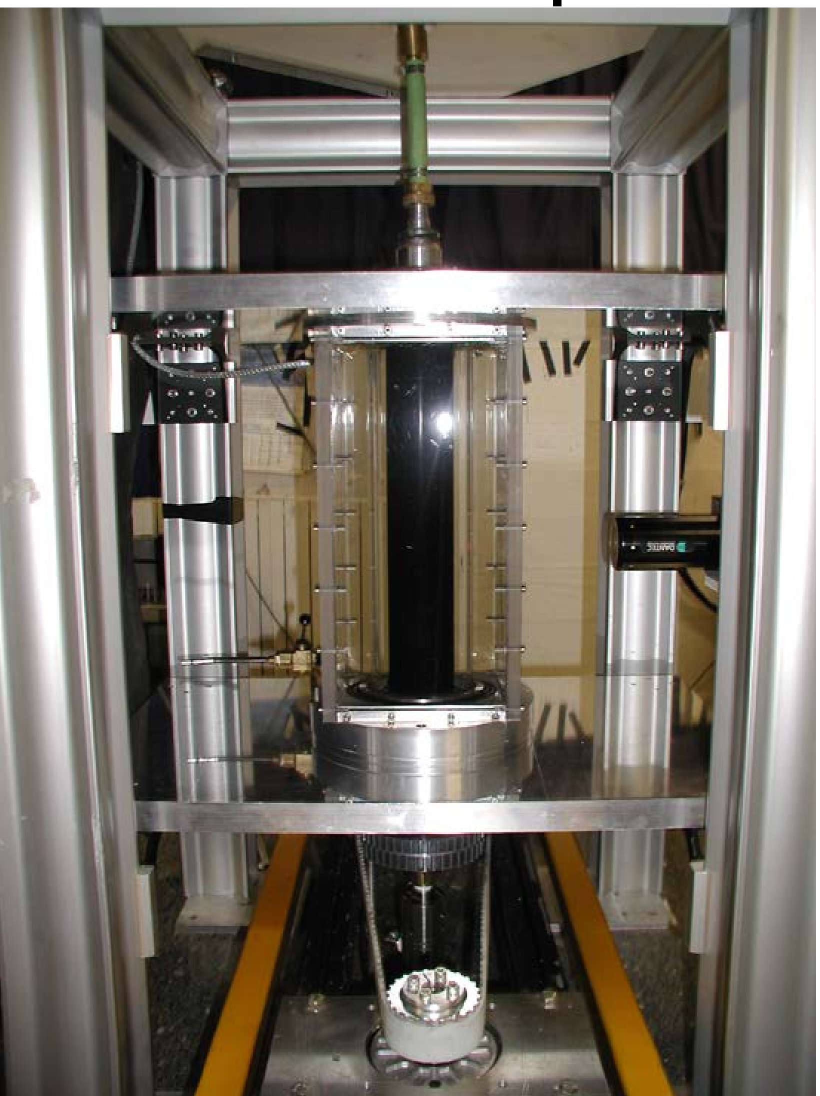
Stability Diagram of Taylor-Couette Flow



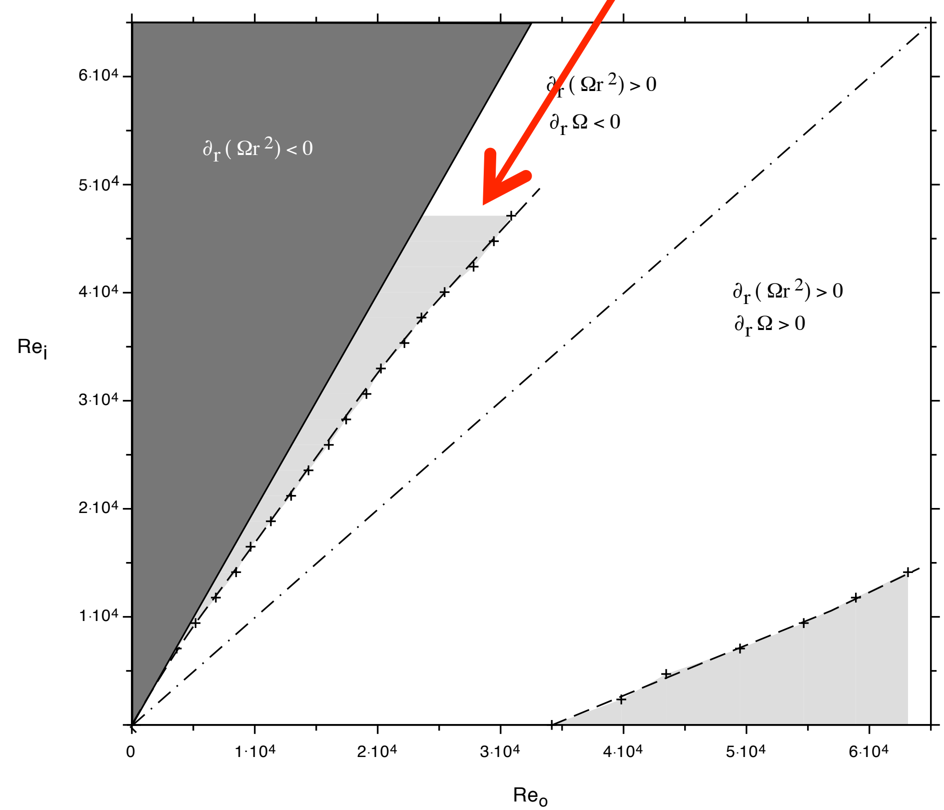
Richard (2001)

r_1=3.5cm, r_2=5cm, h=38cm

η=0.7, Γ=25.3, Re<10^5



"Wavy activity" observed through flow visualization in qK regime

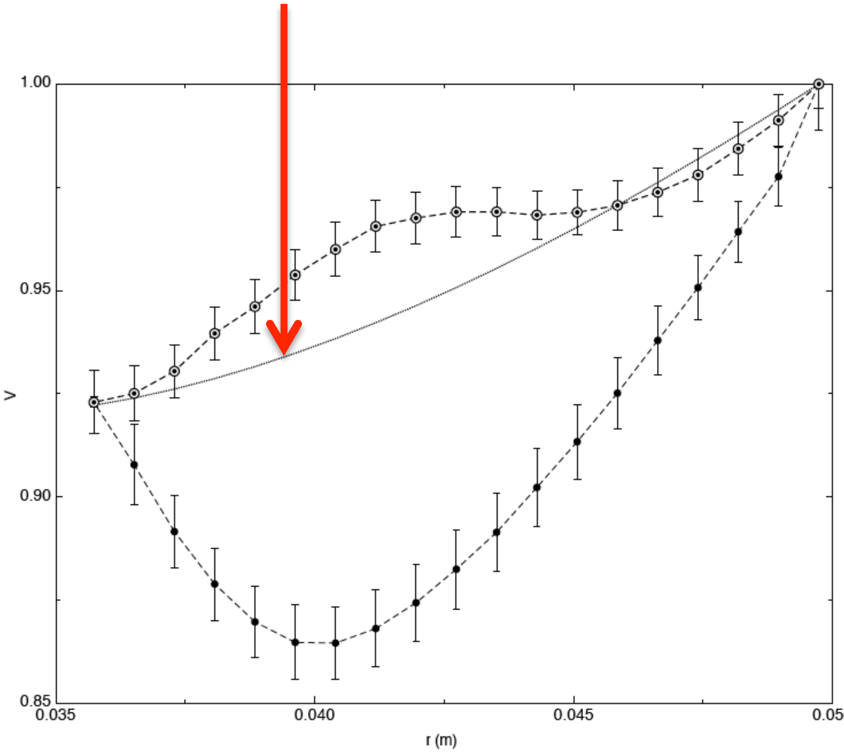


Richard (2001)

r_1=3.5cm, r_2=5cm, h=38cm
η=0.7, Γ=25.3, Re<10^5

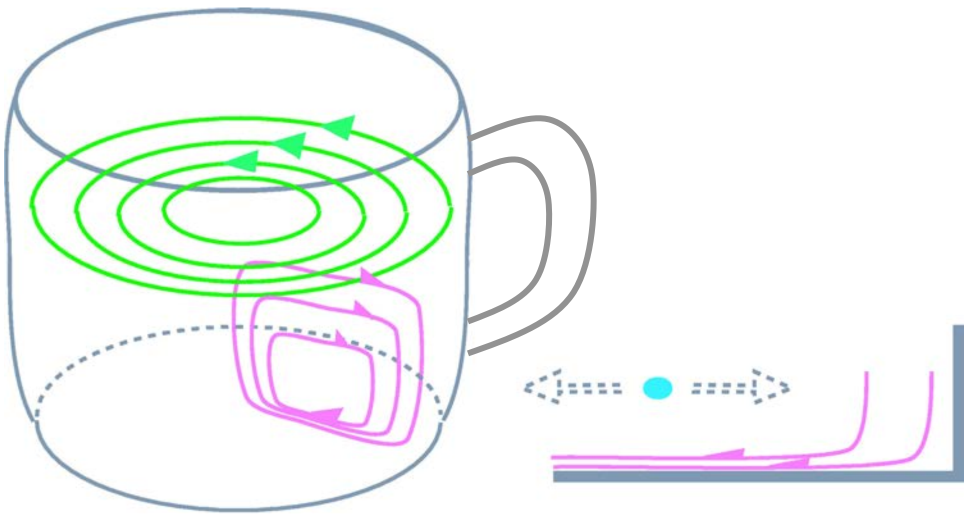
- **Sensitivities to axial boundaries:**
  - "Ekman" configuration: end caps connected to OC
  - "Split" configuration: inner half connected to IC and outer half to OC

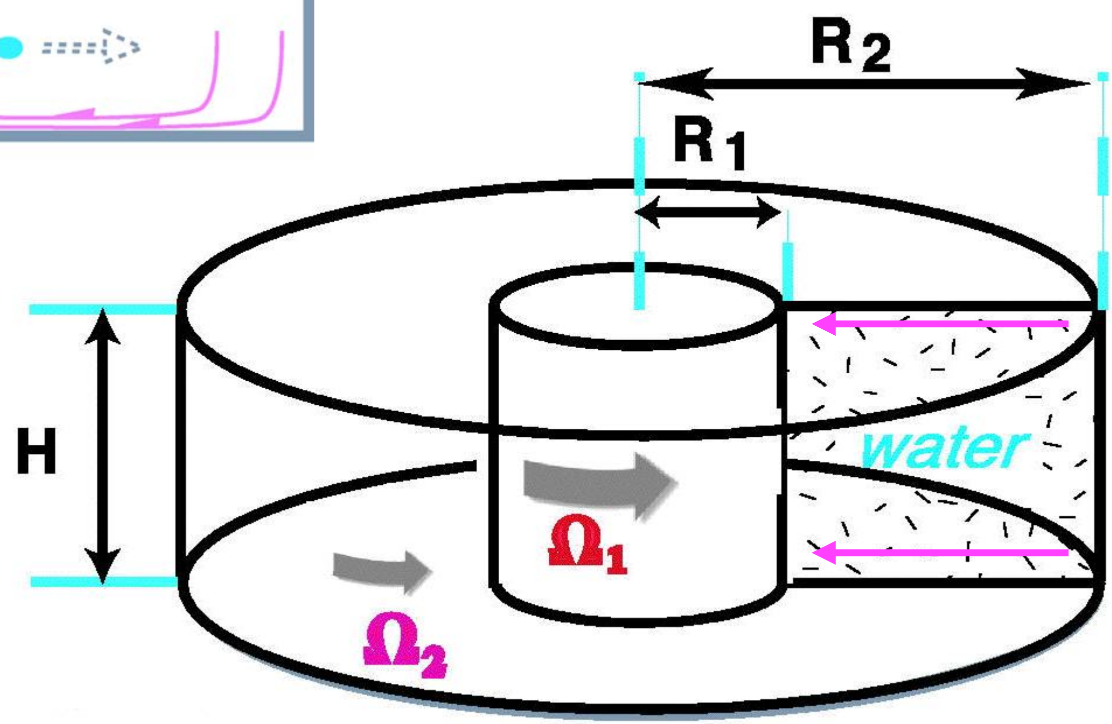
"ideal Couette" profile for infinitely long cylinders



Importance of the axial boundaries: Ekman effects

Ekman Effects due to Imperfect Axial Boundaries are Significant

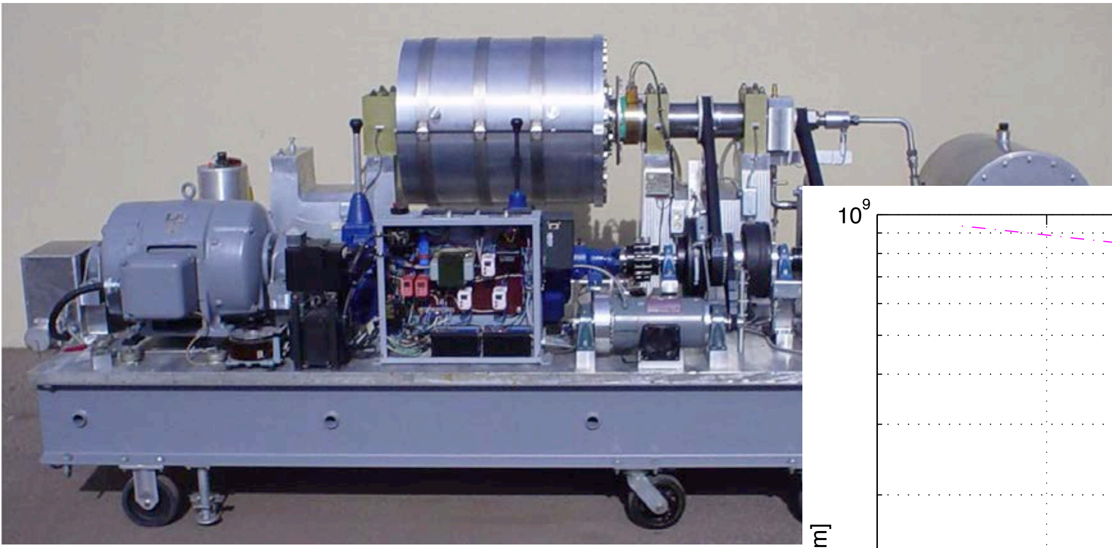




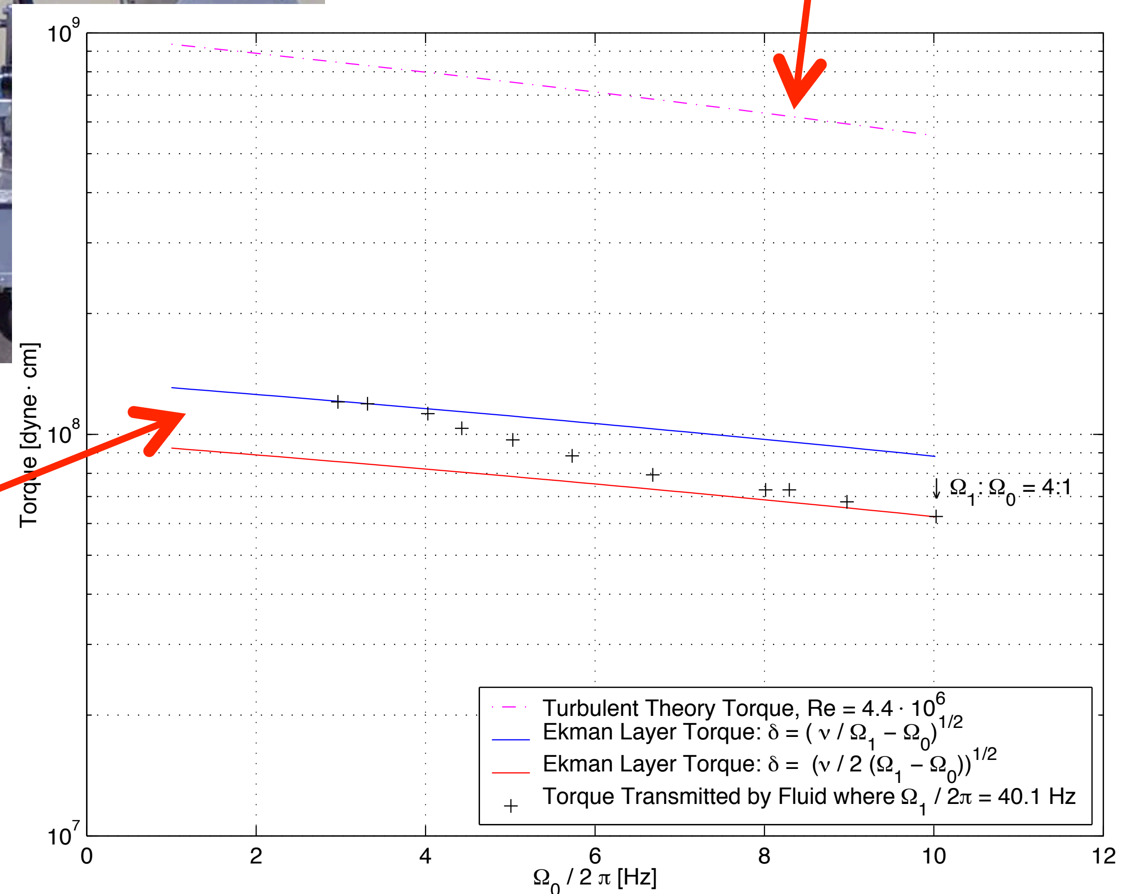
Beckley (2002)

r_1=15.25cm, r_2=30.50cm, h=30.50cm

η=0.5, Γ=2, Re<4.4×10^6



Predicted torque due to Pipe flow turbulence

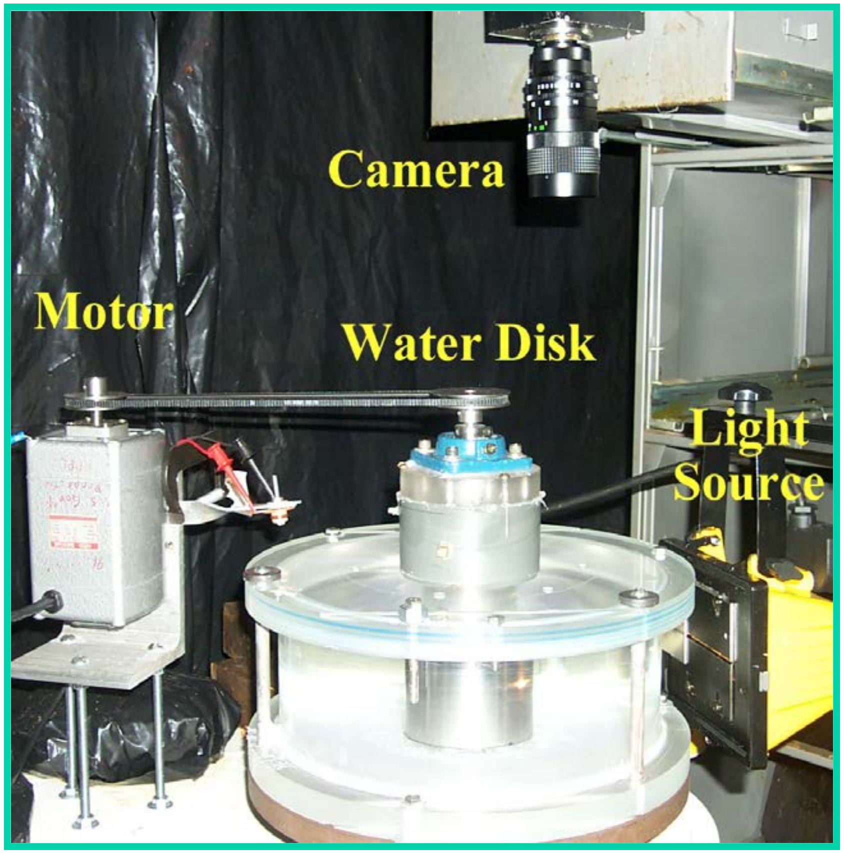


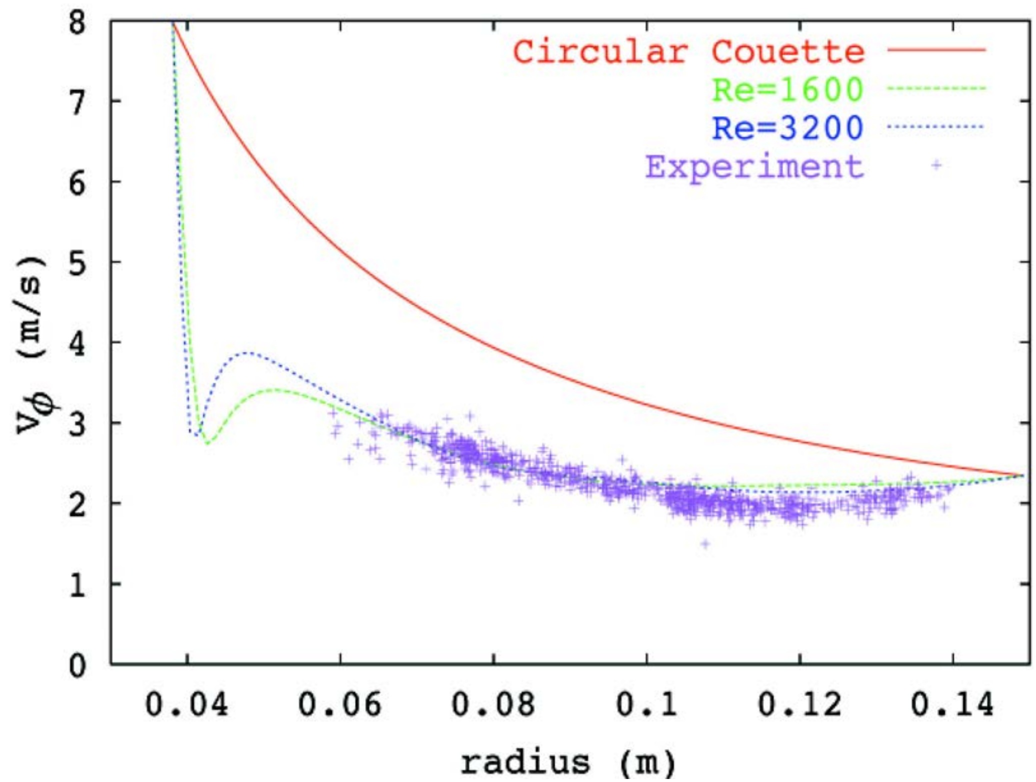
Predicted torque due to Ekman effects

Kageyama, Ji, Goodman, Chen, Shoshan (2004)

r_1=3.8cm, r_2=14.9cm, h=10cm

η=0.255, Γ=0.9, Re<~10^6



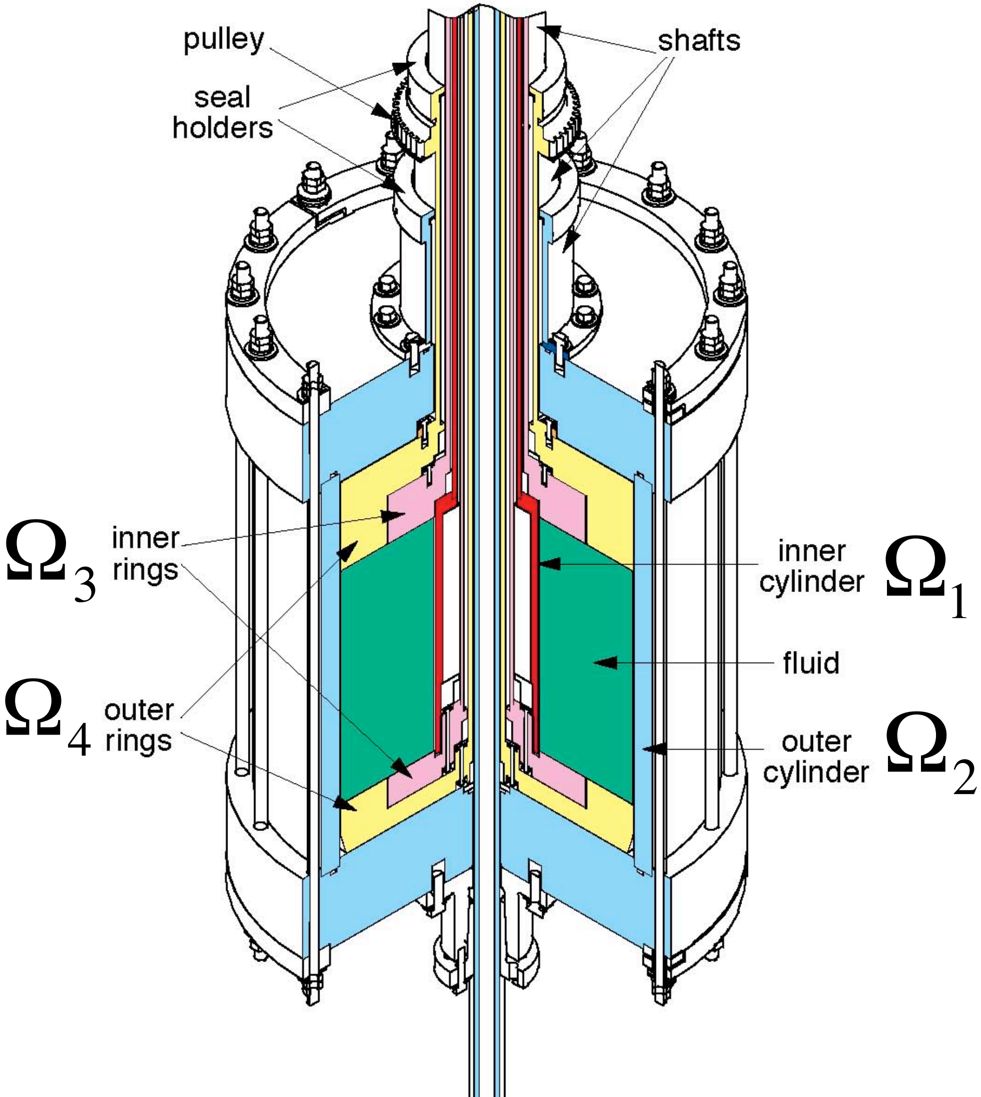


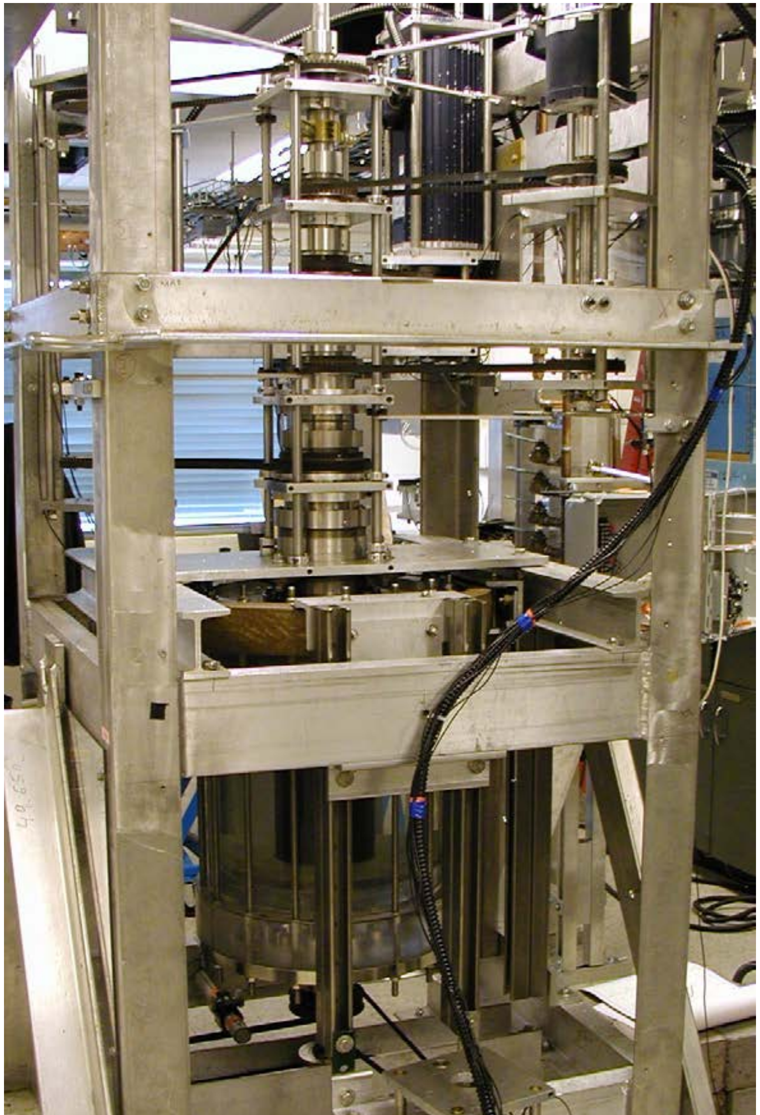
Ekman effects confirmed at a prototype experiment at Princeton

Ji, Burin, Schartman, Goodman (2006)

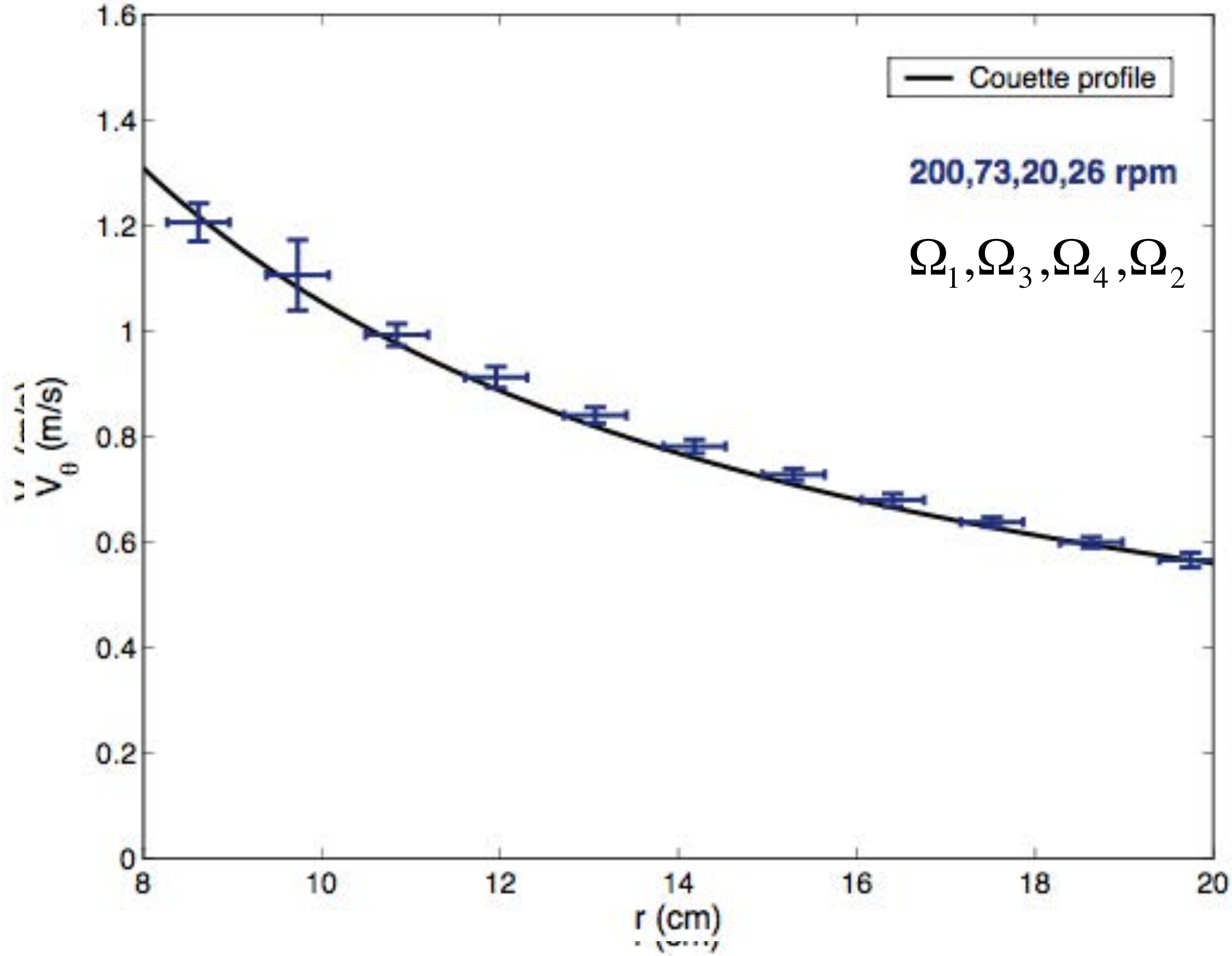
r_1=7.06cm, r_2=20.30cm, h=27.86cm

η=0.348, Γ=2.10, Re<2×10^6 (now 2×10^7 in liquid gallium)





Fine Control of Ekman Effects by Rings



Guess #0

Guess #1

Guess #2

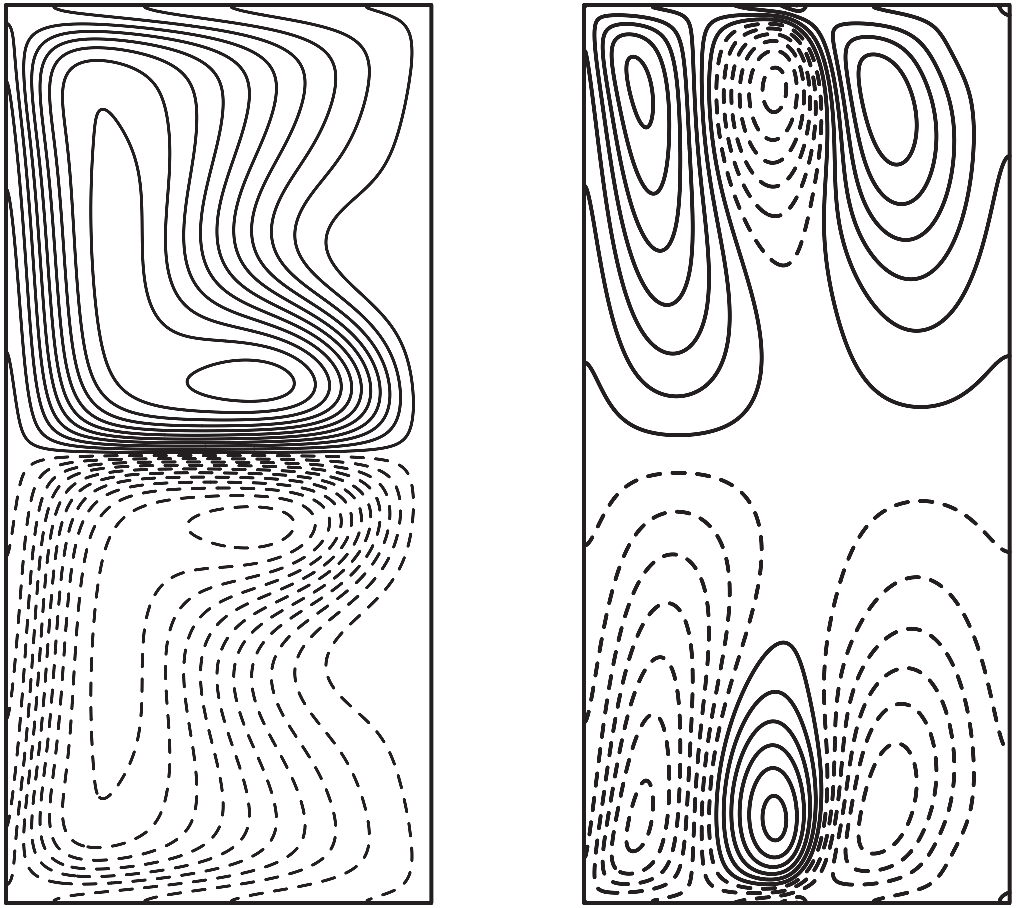
Guess #3

Guess #4

Guess #5

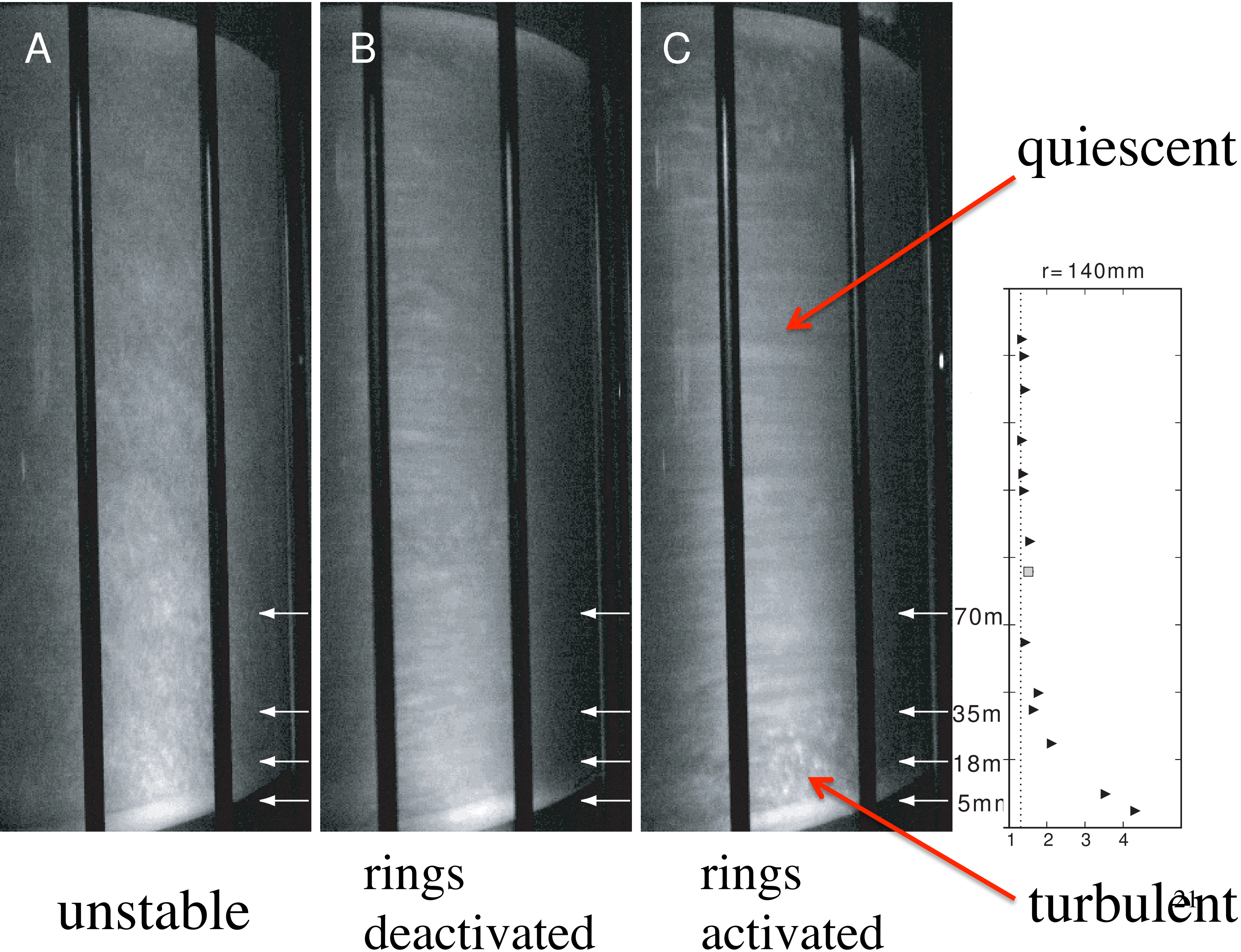
Guess #6

What Happened:
Rings Break Large Scale Ekman Circulations into Smaller Eddies Near Each End



(a) Lids (b) Rings

A. Obabko, F. Cattaneo, P. Fischer (2008)


A
B
C
quiescent
r=140mm
70m
35m
18m
5m
1
2
3
4
turbulent
unstable
rings
deactivated
rings
activated

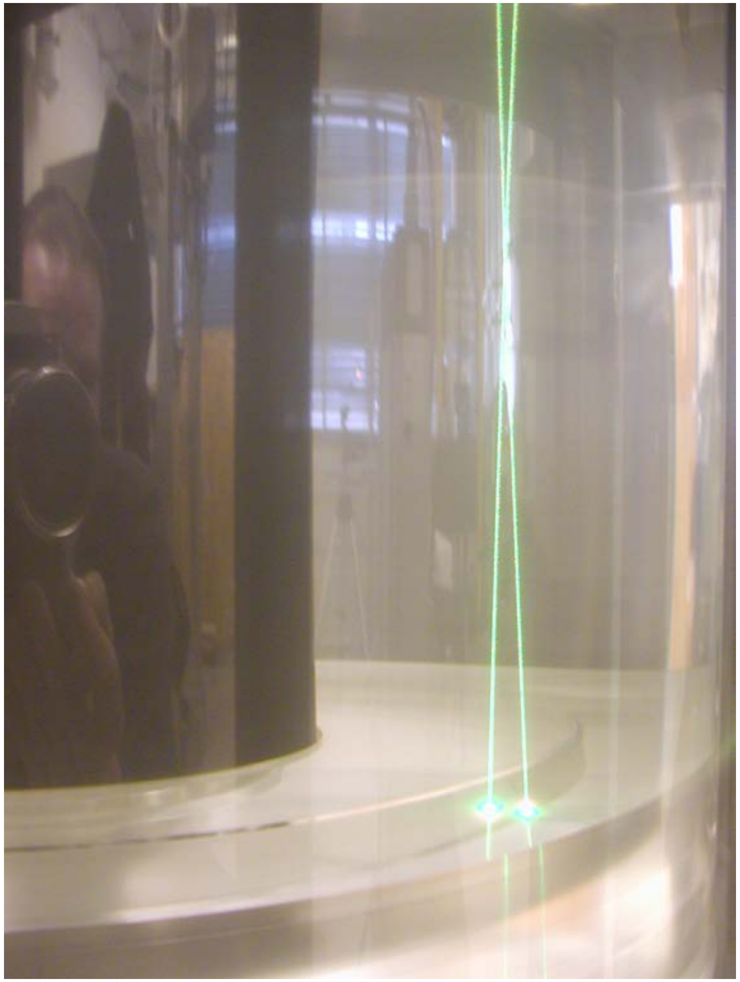
Direct Measurement of Reynolds Stress

- Quantifying transport:

$$\nu_{turb} = \beta R^3 \left| \frac{\partial \Omega}{\partial R} \right| \qquad \beta \equiv \frac{\langle \tilde{V}_r \tilde{V}_\theta \rangle}{q^2 \langle V_\theta \rangle^2}$$

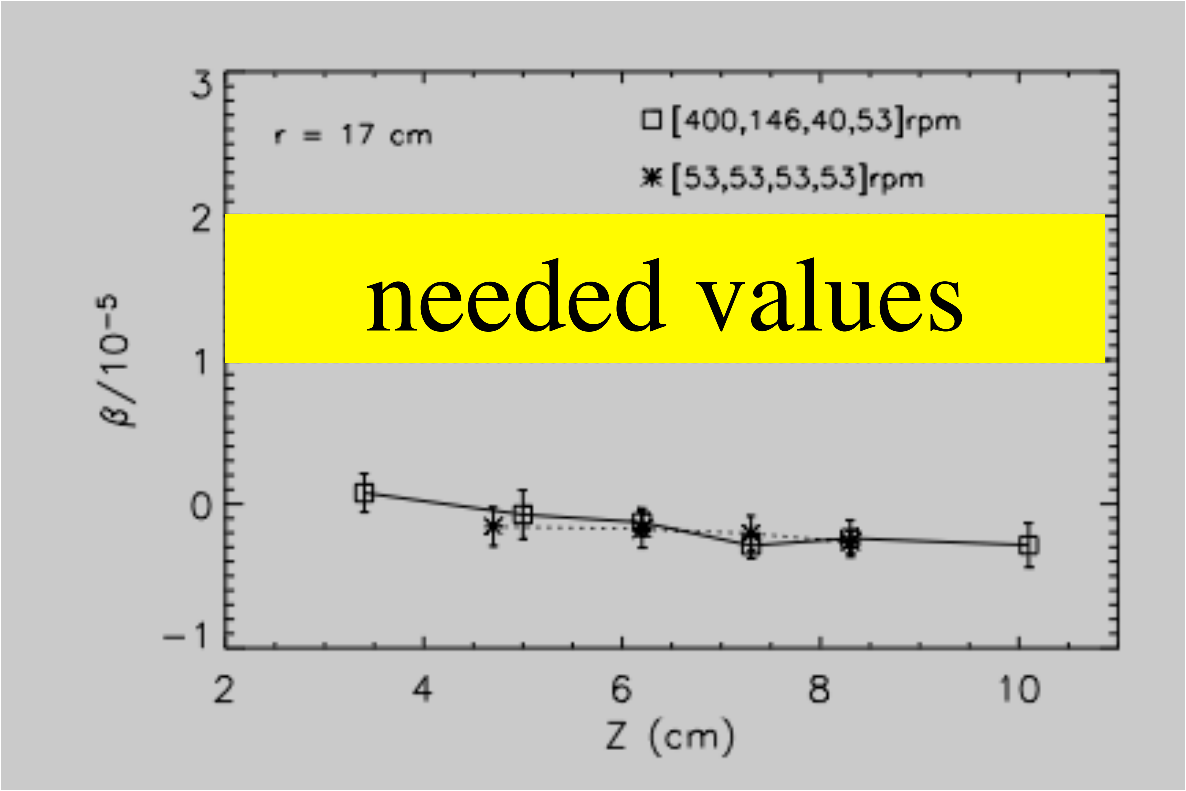
Value needed to explain observation $\beta = (1-2)\times 10^{-5}$

- Simultaneous measurement of V_r and V_θ by a dual synchronized Laser Doppler Velocimetry
  - Random errors are reduced by large number statistics
  - Systematic errors are removed by comparing with solid-body flows
- Benchmarked in hydrodynamically unstable cases



V_r measured by a pair of lasers

Results: Negligible Angular Momentum Transport in qK Flows!



indistinguishable from solid body flows

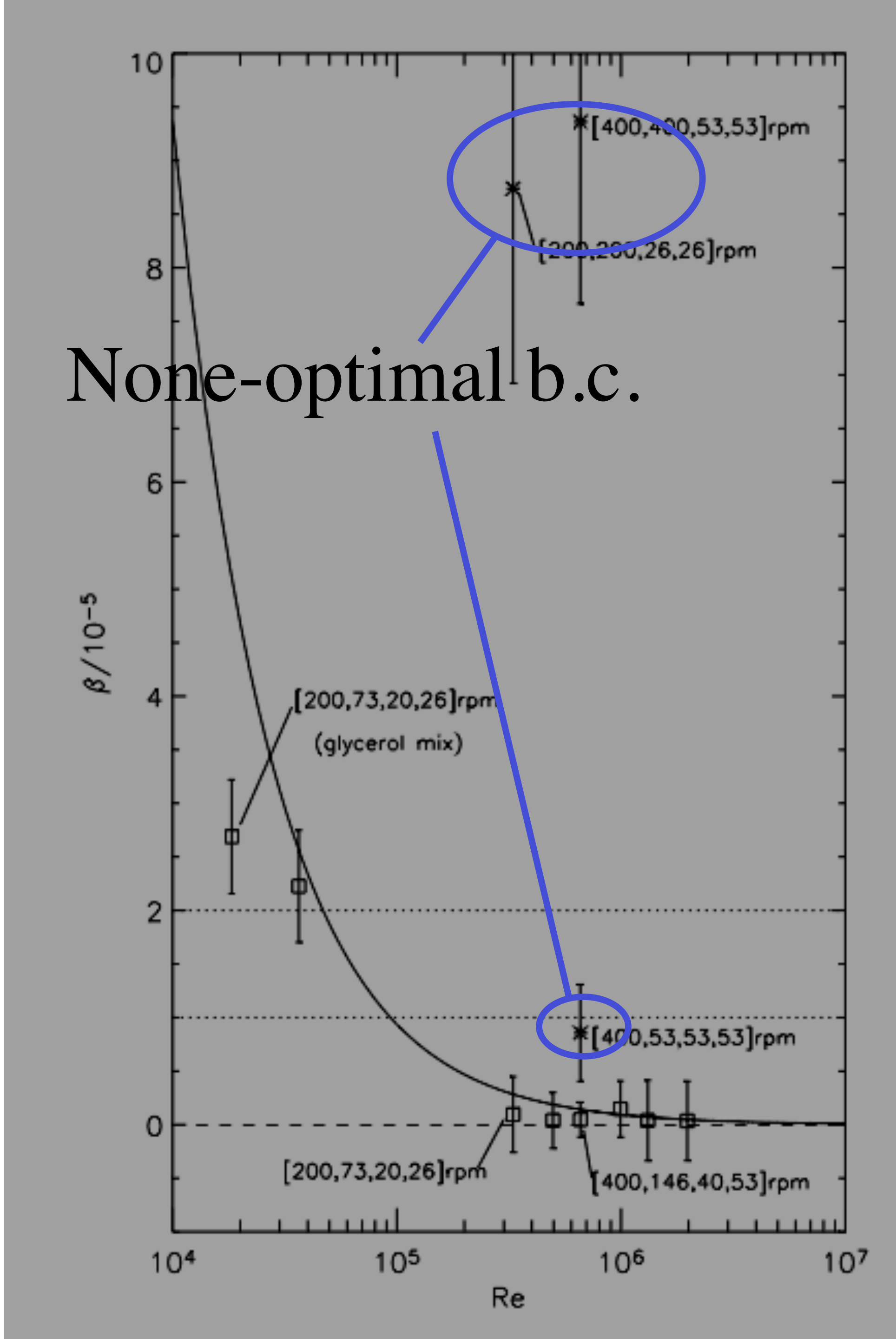
No Signs of Turbulence up to Re=2×10^6

- Large Reynolds stress detected if
  - Boundary conditions not optimum, or
  - Even with optimum boundary conditions, but at smaller Re's

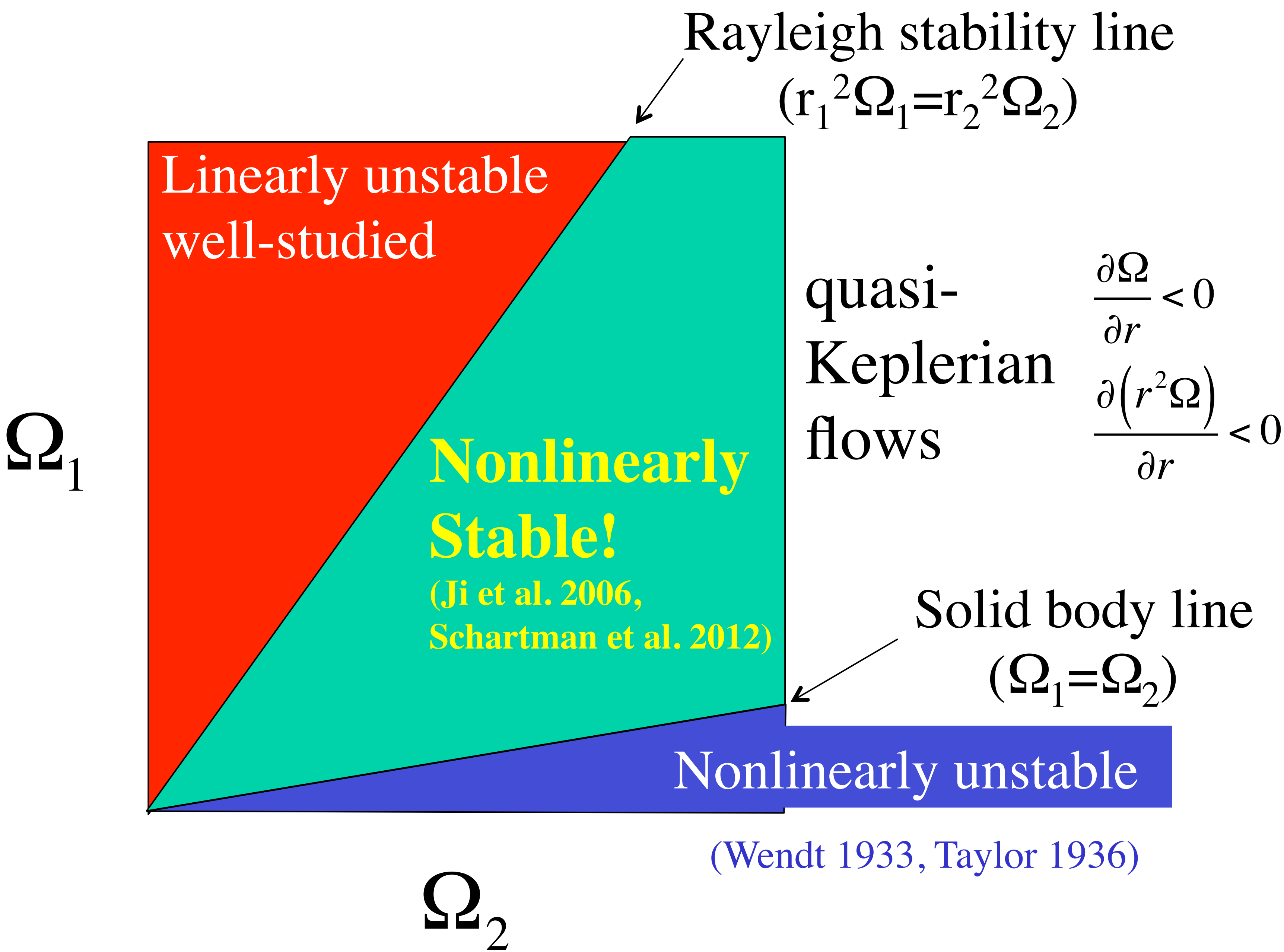
- $\beta=(1.13\pm1.15)\times10^{-6}$, or $<3.4\times10^{-6}$ with 98% confidence.

- Remarkable since no other terrestrial examples are known

Ji, Burin, Schartman, Goodman (2006)
Schartman, Ji, Burin, Goodman (2012)



Stability Diagram of Taylor-Couette Flow

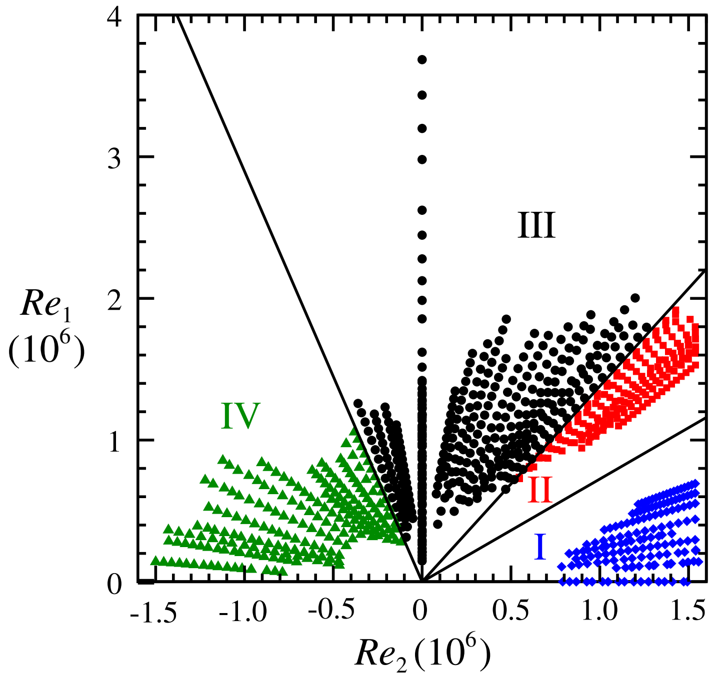


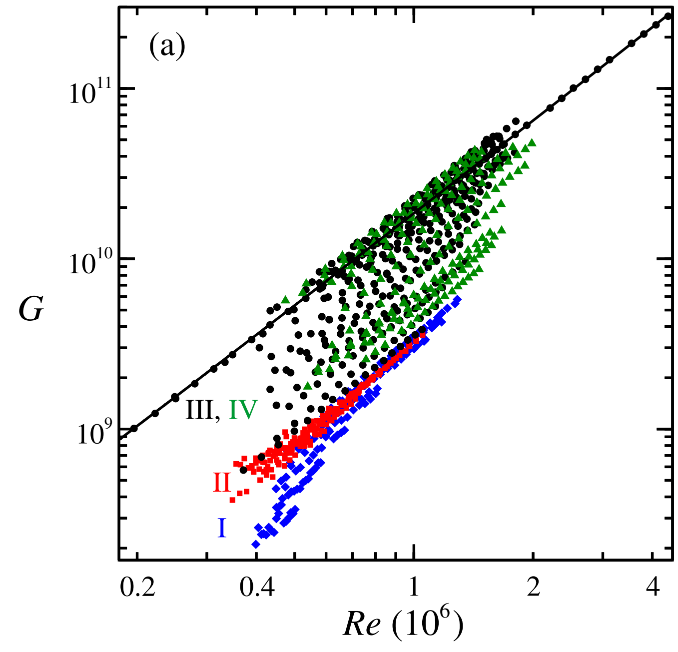
Paoletti & Lathrop (2011)

r_1=16.00cm, r_2=22.085cm, h=69.50cm

η=0.7245, Γ=11.47, Re<2×10^6

endcaps attached to OC





Enhanced torque from a sleeve mounted on middle 1/3 of IC:

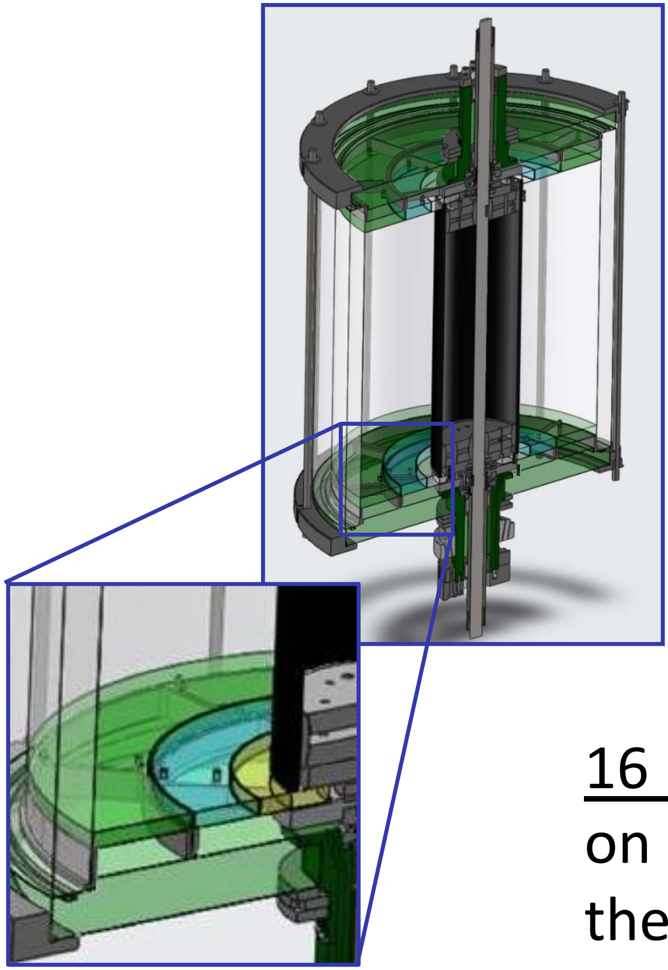
- significant if applicable to accretion disks
- but is this scheme really immune from Ekman effects?

Edlund, Ji, Goodman (2013)

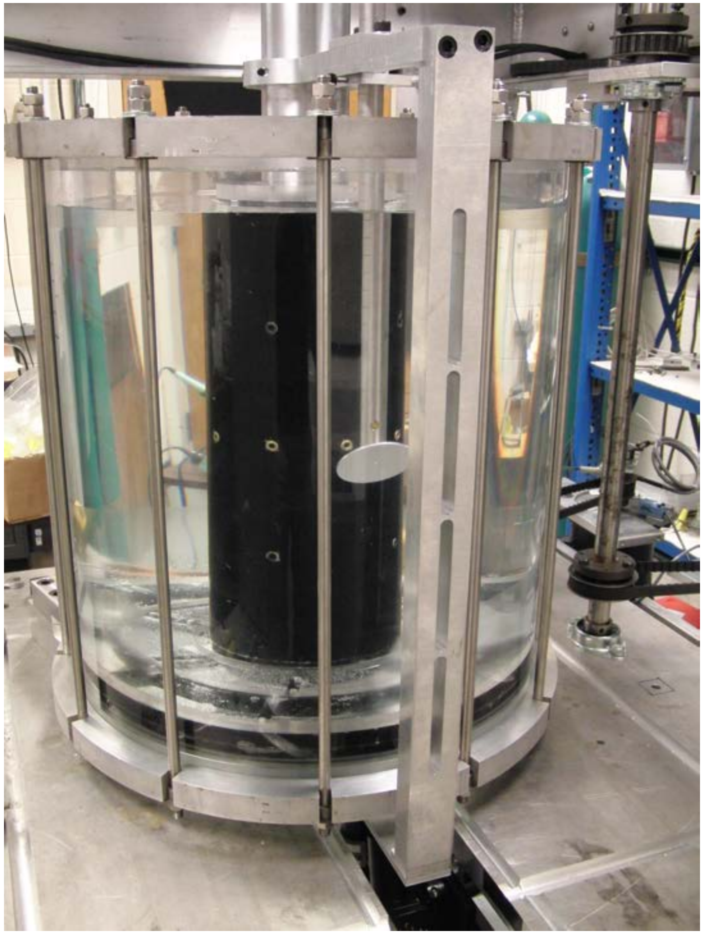
R_1=6.9cm, r_2=20.3cm, h=39.7cm

η=0.34, Γ=2.96, Re<2×10^6

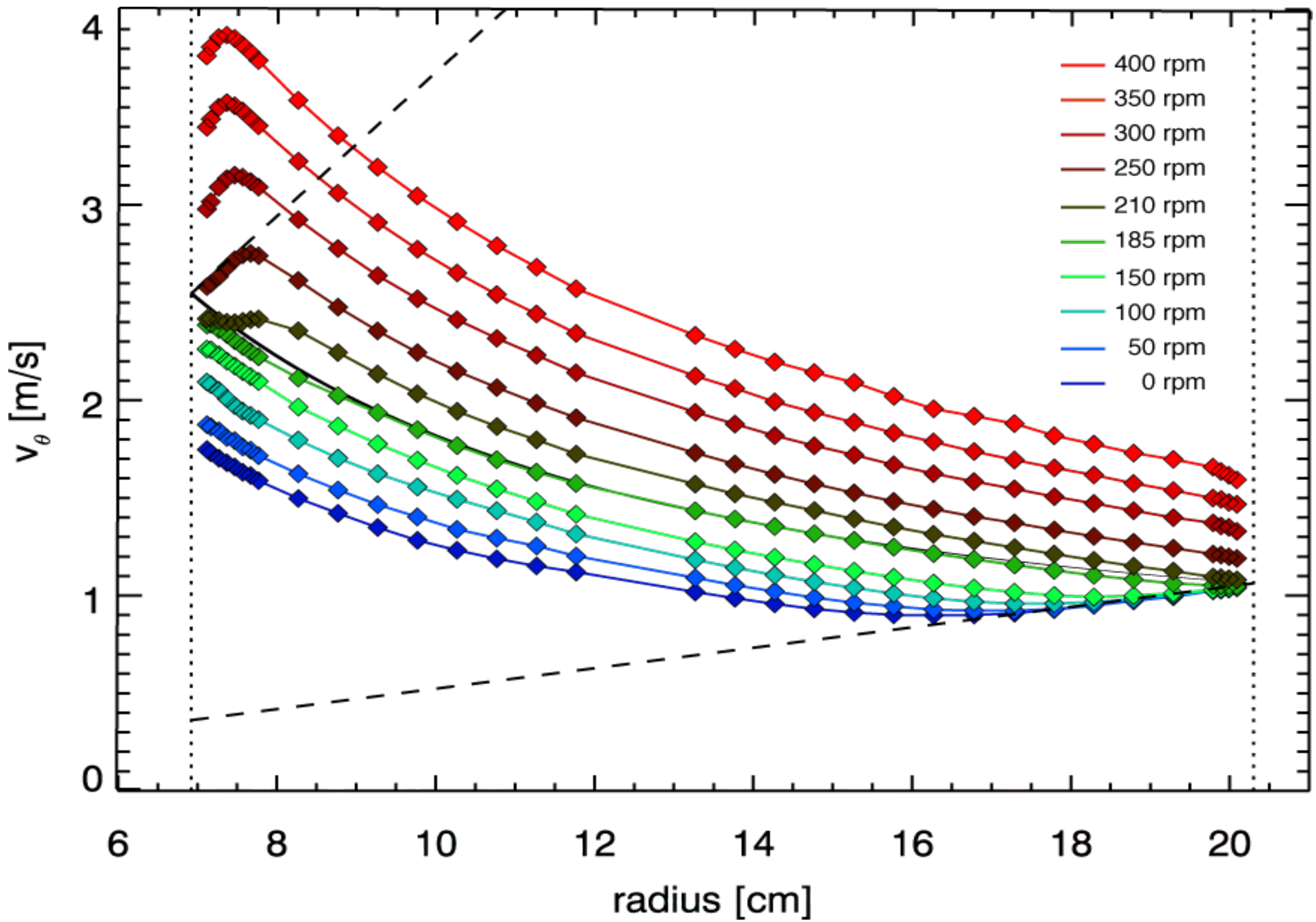
one ring (Ω_3) w/ rims on IC & OC



<u>16 nozzles</u>
on IC to perturb
the flow

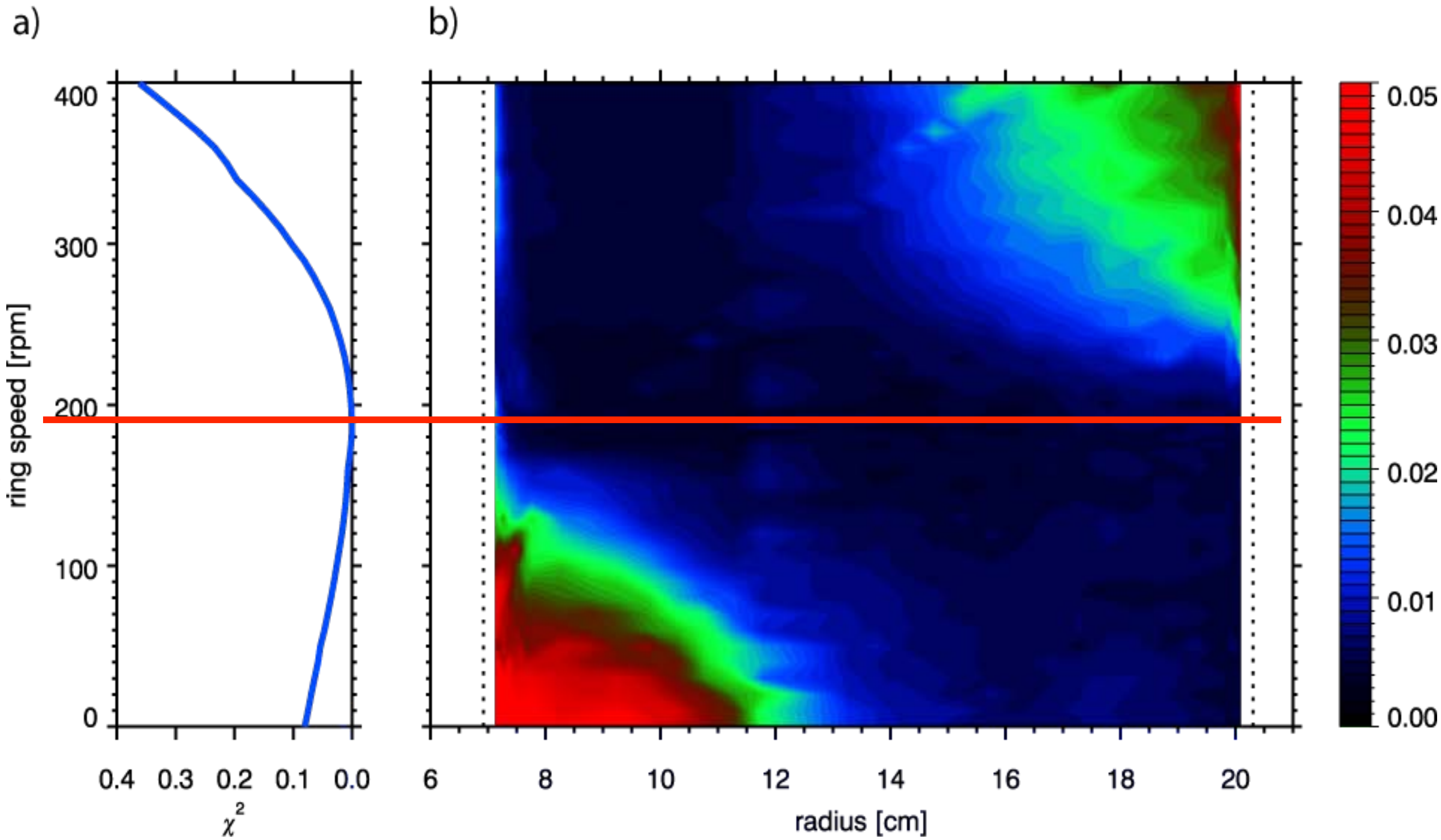


One Ring Can Do a Good Job As Well!



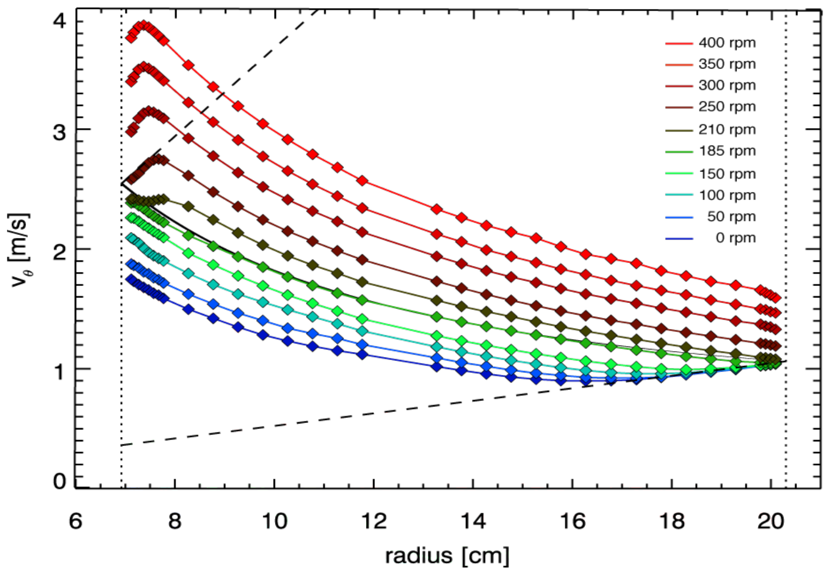
“Optimal Ring Speed” Minimizes Turbulence!

Optimal ring speed

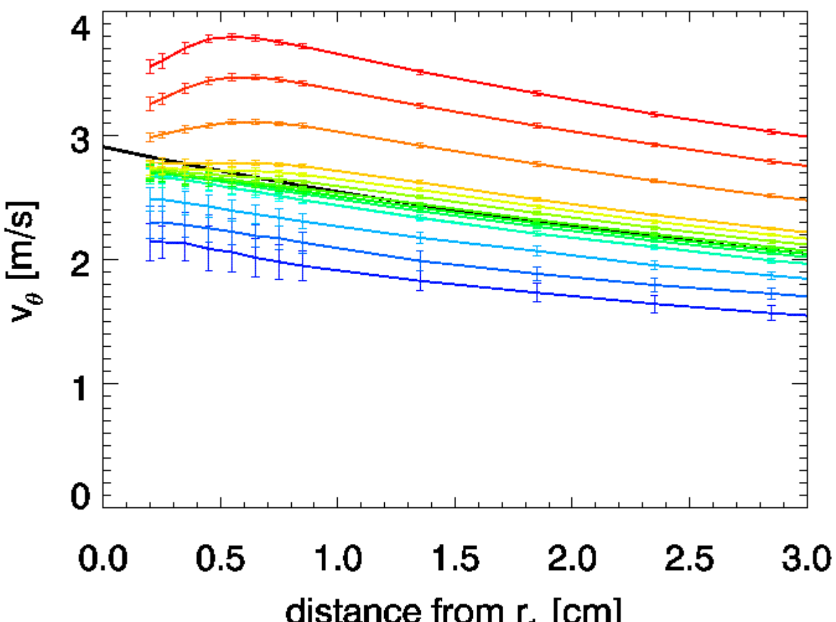


Why?

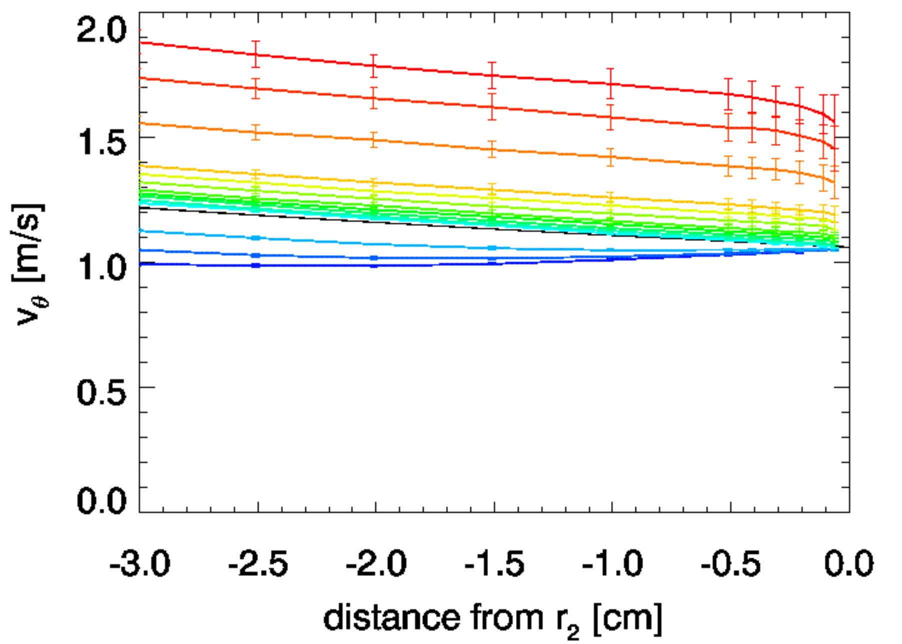
Turbulent Boundary Layers ?!



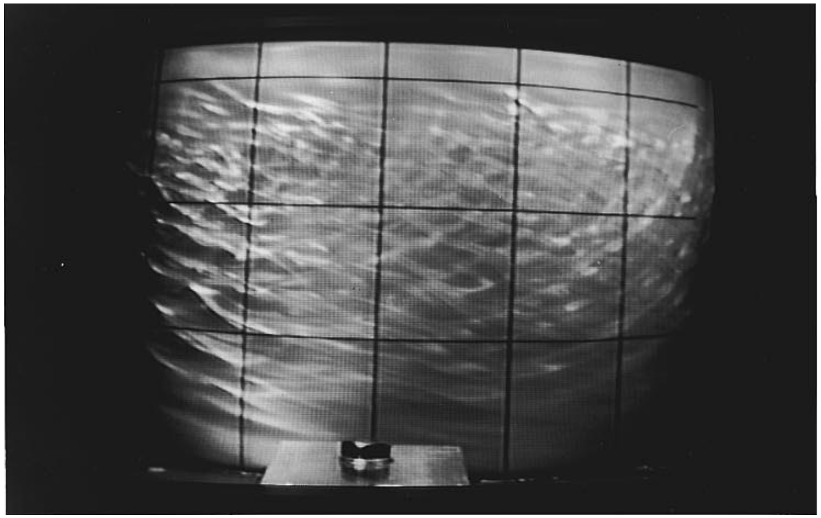
Inner 3 cm

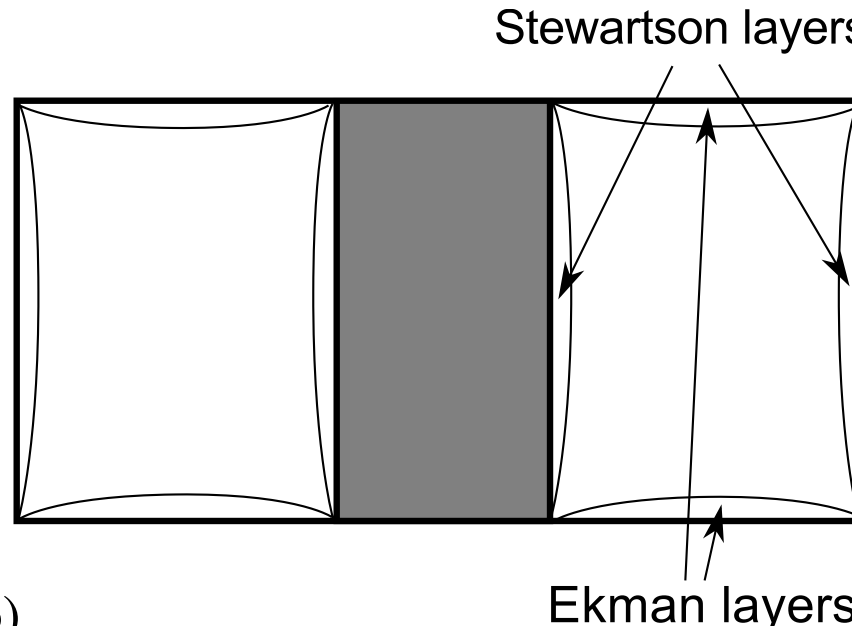


Outer 3 cm

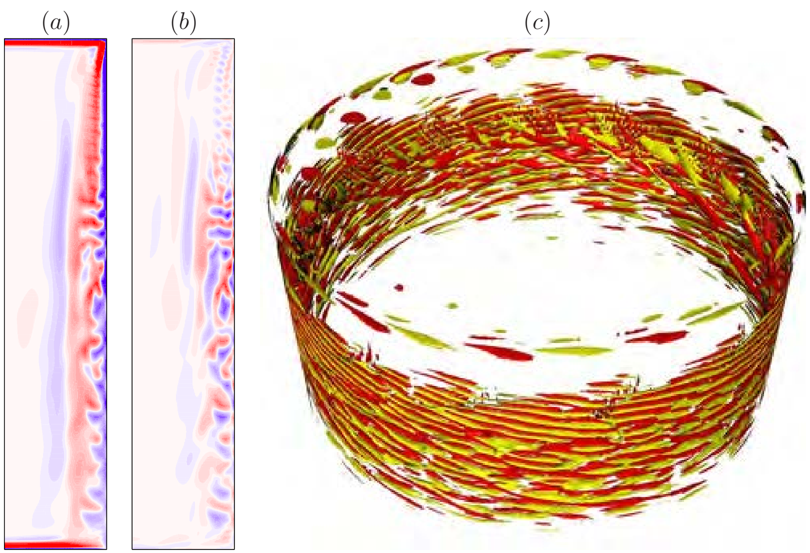


Unstable Stewartson Layer on IC or OC

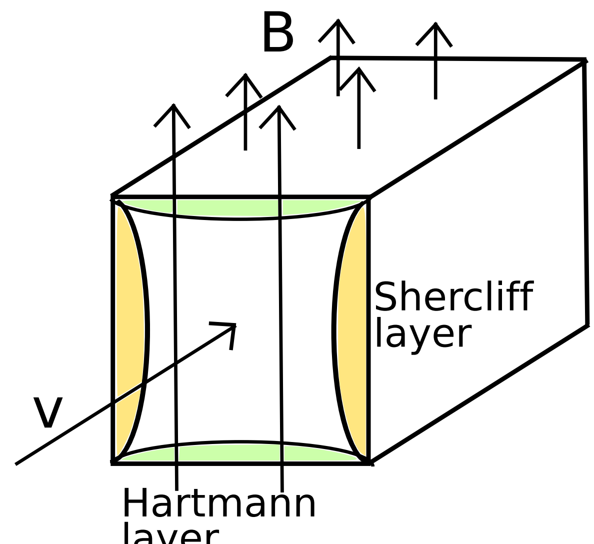




Hart & Kittelman (1996)



Lopez & Marques (2010)



Stability Diagram of Taylor-Couette Flow