Energy Partitions in Dissipative Mode Space for Instability-Driven Plasma Turbulence

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Instability-driven plasma turbulence has array of damped collective modes that remove energy at largest scales

- Damped modes are active but "invisible" in initial value simulations
- •They are nonlinearly driven
- •They dominate dissipation and saturation
- Brief tutorial follows with diverse examples

Issues to address:

- •Advantages/disadvantages of different mode decompositions
- Dissipation rate of damped modes
- •Nature of energy transfer in mode space
- •Labeling damped modes consistent with energy transfer properties
- •Energy distribution in mode space
- •Scaling properties?

Historically, damped modes ignored; turbulent saturation was treated as a hydrodynamic cascade process



Instability driven by flow shear, gradients in current, density, temperature

- Directions of homogeneity Fourier transform to get k space
- Direction of instability gradients spanned by eigenmodes that are functions of k
- OK energy cascades in k space, eigenmode space
- But eigenmodes have characteristic frequency
 - If one frequency is complex for instability, other frequencies are complex
 - •Typically many modes are damped

For ITG Cyclone Base Case, 1 unstable mode >10⁴ damped modes for typical resolution in gyrokinetic simulations



Sampling of 300 modes at wavevector of maximum growth

Idea that damped modes decay to negligible amplitude, energy cascades only in *k* space is invalid generally

Damped modes excited nonlinearly

Growing modes -> Large amplitude in linear regime

Damped modes -> Small amplitude in linear regime

=> Universal parametric instability with 2 x linear growth rate

Many modes excited

Damped modes introduce damping at large wavenumber-space scales

=> Cascade is very lossy



ITG growth rate at infinitesimal amplitude and finite amplitude

Example: Saturation of electron temperature gradient instability by secondary Kelvin-Helmholtz

Model: 3D fluid model for pressure, potential, and magnetic flux

Geometry: slab

- y, z directions: periodic
- -> wavenumer space k_v, k_z
- x direction: magnetic shear
 - -> 100 Orthogonal polynomials in code
- 3 fields x 100 polynomials => 300 modes for
 - each value of k_y , k_z
- Evolution: ETG instability growths, secondary Kelvin-Helmholtz breaks up primary structure





Secondary KH is not conservative – It has strong decomposition onto damped modes at largest scales



ITG turbulence excites a large number of damped modes

For gyrokinetic Cyclone Base Case, 1 mode is unstable ~10⁴ modes are damped

Modes span space of (z, $v_{||}$, μ)

Coordinates x, y – Fourier transform -> $k_{\perp}^2 = k_x^2 + k_y^2$ Modes at largest perpendicular scales span scales in mode space

Turbulent fluctuations decompose

into many damped modes

- Linear eigenmodes
- Proper orthogonal
 - decomposition





Temperature gradient free energy source Q_k peaks at low k

Collisional damping C_k peaks at low k

C_k damping resides in damped modes

Energy transfer for damped modes at low k saturates instability

Explains why accurate modeling of ITG turbulence does not require huge k space



Damped modes can arise for any plasma instability in inhomogeneous medium

Also arise in homogeneous systems with instability and multiple fields Systems examined:

Magnetorotational instability

Rayleigh Taylor instability

Resistive interchange instability

Drift Waves: ITG, ETG, TEM,

Hasegawa-Wakatani Microtearing modes Ionization driven drift waves Thermally driven drift waves



Damped modes contribute to saturation provided their damping rate is not significantly larger than the instability growth rate

Three-wave coupling model (quadratic nonlinearity)

 $\dot{x}_1 = \gamma_1 x_1 + C_1 x_1^2 + D_1 x_1 x_2,$

growing mode

damped mode

 $\dot{x}_2 = -\gamma_2 x_2 + C_2 x_1^2 + \dots$

Damped mode is as important for saturation transfer to high k when:

$$P_t \equiv \frac{D_1 C_2}{C_1^2 (2 + \gamma_2 / \gamma_1)} \approx 1$$

Strong drive relative to dissipation (Re >>1) yields conjugate damped mode => Damped modes play big role in saturation



Given importance of damped modes for saturation What governs the distribution of energy across mode space?

Related questions:

- •What is most advantageous damped-mode decomposition?
- •How do you determine damping?
- •Is transfer to damped modes serial, like wavenumber cascade?
- •What damped-mode indexing is consistent with transfer properties?
- •What is energy distribution in mode space? Can it be represented by scaling properties?

Damped mode decomposition: linear eigenmodes, proper orthogonal decomposition, orthogonal polynomials

POD

- Optimal for truncations, nonlinear structure
- Unfamiliar from stability analysis, which identifies unstable mode
- Orthogonal
- Can become unphysical
- Linear eigenmodes
 - Non orthogonal in unstable systems
 - Misleadingly high energy in
 - saturation
 - Less susceptible to becoming nonphysical but more needed to capture dominant structures



POD modes develop unresolved structure as mode number increases



To compare across mode number look at



Linear modes remain "physical" to higher mode number than POD



Linear eigenmodes give better chance of observing scaled energy distribution

Non orthogonality of linear modes can yield large amplitudes

Individual linearly damped modes can have higher energy than unstable mode

Energy cancels in combination with other non orthogonal damped modes keeping total damped mode energy on par with unstable mode energy Larger hyper resistivity gives energy distributions that are comparable to orthogonalized samples

=> Non orthogonality problem related to unresolved small scale structure



Damped modes are nonlinearly driven => dissipation rate found from damping rate *and* nonlinear saturated amplitude

Simulation

Project steady state onto mode decomposition to find amplitude Evaluate magnitude of dissipative terms of gyrokinetic equation Fluctuation energy: $E = \int dz dv_{||} d\mu B_0 \pi \frac{n_0 T_0}{F_0} |g|^2 + \int dz D(k_{\perp}) |\phi|^2$ Dissipation: $\frac{\partial E_k}{\partial t}|_{N.C.} = Q_k + C_k$

Gradient Drive:

 $Q = \int dz dv_{||} d\mu \pi n_0 T_0 B_0 (v_{||}^2 + \mu B_0) \omega_T gik_y \bar{\phi}$

 C_k : collisional dissipation – largest at low k, damping rate asymptotes to small negative value => large scale damped modes dominate collisional dissipation

This is not what would be predicted from linear analysis *Predictive theory*?



In hydrodynamic turbulence, energy transfer is essentially local in k For transfer across a broad inertial range, energy must cascade serially from larger scales to smaller scales k_2 k_0 k_6 (injection) Stable modes In mode space, three-wave coupling suggests a **Parallel coupling** *n*=2 Could energy *n*=3 Unstable eiaenmode transfer be k-k parallel, so that Stable eiaenmode n=4high mode *n*=1 Stable numbers get k eiaenmode 2 similar amount Unstable Stable eigenmode 3 of energy as low mode **N-1** Stable mode number? eiaenmode

To test if transfer is serial or parallel, measure response to perturbation across mode space

- Perturb unstable mode by increasing its amplitude at one time step
- Observe response in other modes
- Plot time to first peak in responses versus mode number
- Rise time is flat across mode space Suggests transfer is parallel





Quadratic coupling in original equations => quadratic coupling in mode space

$$\frac{\partial \beta_j(k)}{\partial t} = -\gamma_j \beta_j + \sum_{l,m} \sum_{k'} C_{k,k'}^{(j)} \beta_l(k') \beta_m(k-k')$$

ITG:

Initially l = 1, m = 1 (parametric instability drive)

C is independent of *j*, so all β_j are driven equally => parallel transfer Later l = 1, m = zonal flow (k_y = 0)

This again gives same drive for all unstable modes



Can damped modes be labeled consistently with parallel transfer?

Hydrodynamic cascade:

- Modes labeled by k
- k corresponds to a scales
- Scale invariance leads to power law spectrum in k space
- Damped mode space of ITG: three very different coordinates: z, $v_{||}$, μ
- All scales are damped (no inertial range)
- Non uniformity of damping wrt scale

Seen in different way mode number varies with *z* and $v_{||}$, γ varies with mode number



Parallel energy transfer suggests using damping rate as mode index

Mode coupling model with zonal flow and unstable mode drive

$$\frac{\partial \beta_j(k)}{\partial t} = -\gamma_j \beta_j + \sum_{k'} C_{k,k'}^{(j)} ZF(k') \beta_1(k-k')$$

suggests nonlinearity is independent of *j*

Only γ_i depends on *j*

Label modes with integer indices according to magnitude of the their damping, with lowest mode number corresponding to weakest damping

Following slides show types of mode space distributions with damping rate indexing

Parallel transfer => energy space distributions might be governed by equipartition of some quantity

Transfer to all damped modes is the same, so transferred quantity should be equipartitioned

POD: suggests that amplitude attenuation rate is equipartitioned

Amplitude attenuation rate = damping rate x amplitude

However, region of equipartition corresponds to "unphysical" modes (with unresolved structure)

Amplitude attenuation rate appears to be artifact

Use better resolved linear modes



In linear eigenmode space (indexed by relative damping rate) amplitude attenuation rate is not constant, but does not vary strongly

Amplitude attenuation rate vs. mode number

Log scale (Absolute value of amplitude attenuation rate)



There is some rough notion of amplitude attenuation rate equipartition

Log log plots suggests that there may be scaling with respect to mode number indexed to damping rate



Supports notion that indexed mode number is a scaling variable for this type of dissipation range with scaling evident in various quantities

Conclusions

Excitation of damped modes in unstable plasma turbulence introduces finiteamplitude-dependent damping at largest scales of system

Analyzed with

- Mode decompositions
- Energetics
- Tests for physicality
- Indexed damping rate
- Energy distribution is amenable to analysis