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# Exploring the Mysteries of Plasma Turbulence

Max Planck Institute for Plasma Physics, Garching  
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# Acknowledgements

Turbulence in Laboratory  
& Astrophysical Plasmas



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*Four main research themes:*

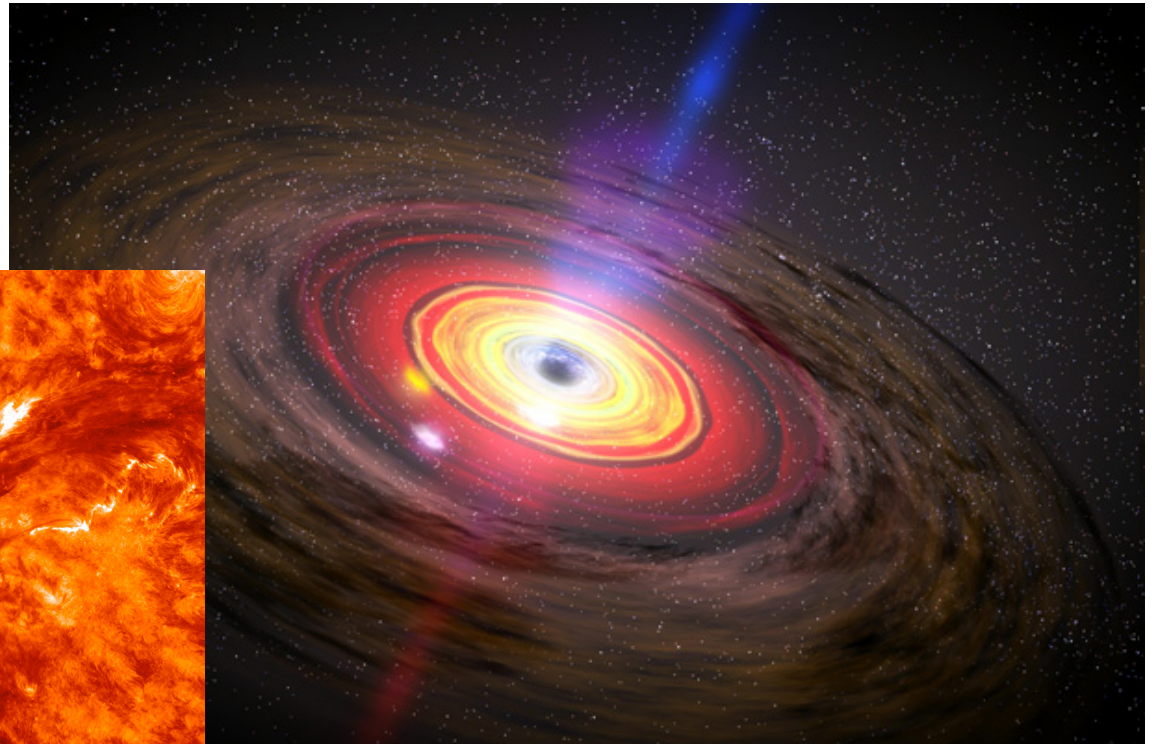
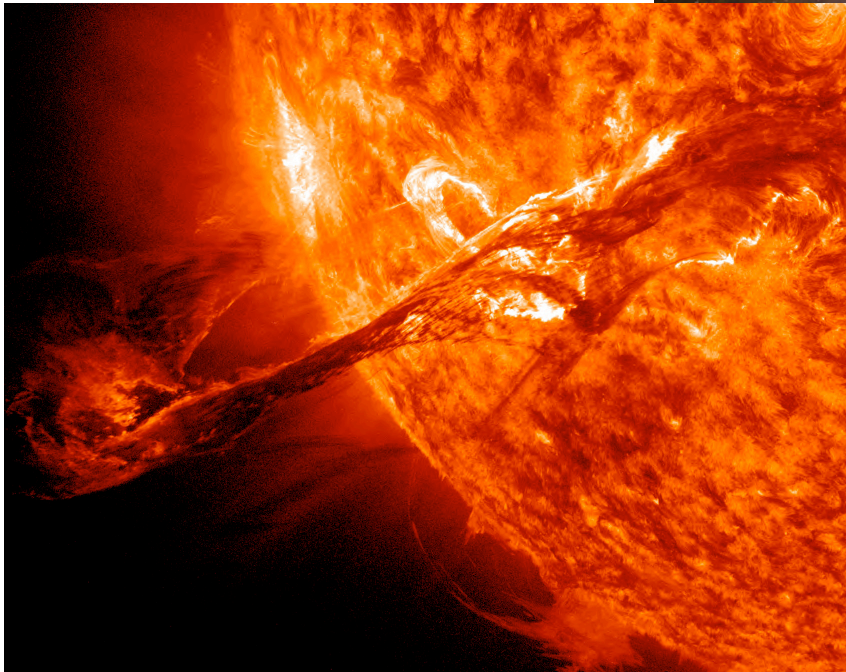
- Magnetic Reconnection
- Magneto-Rotational Instability
- Energetic particles
- **Plasma Turbulence**



[www.princeton.edu/plasmacenter](http://www.princeton.edu/plasmacenter)

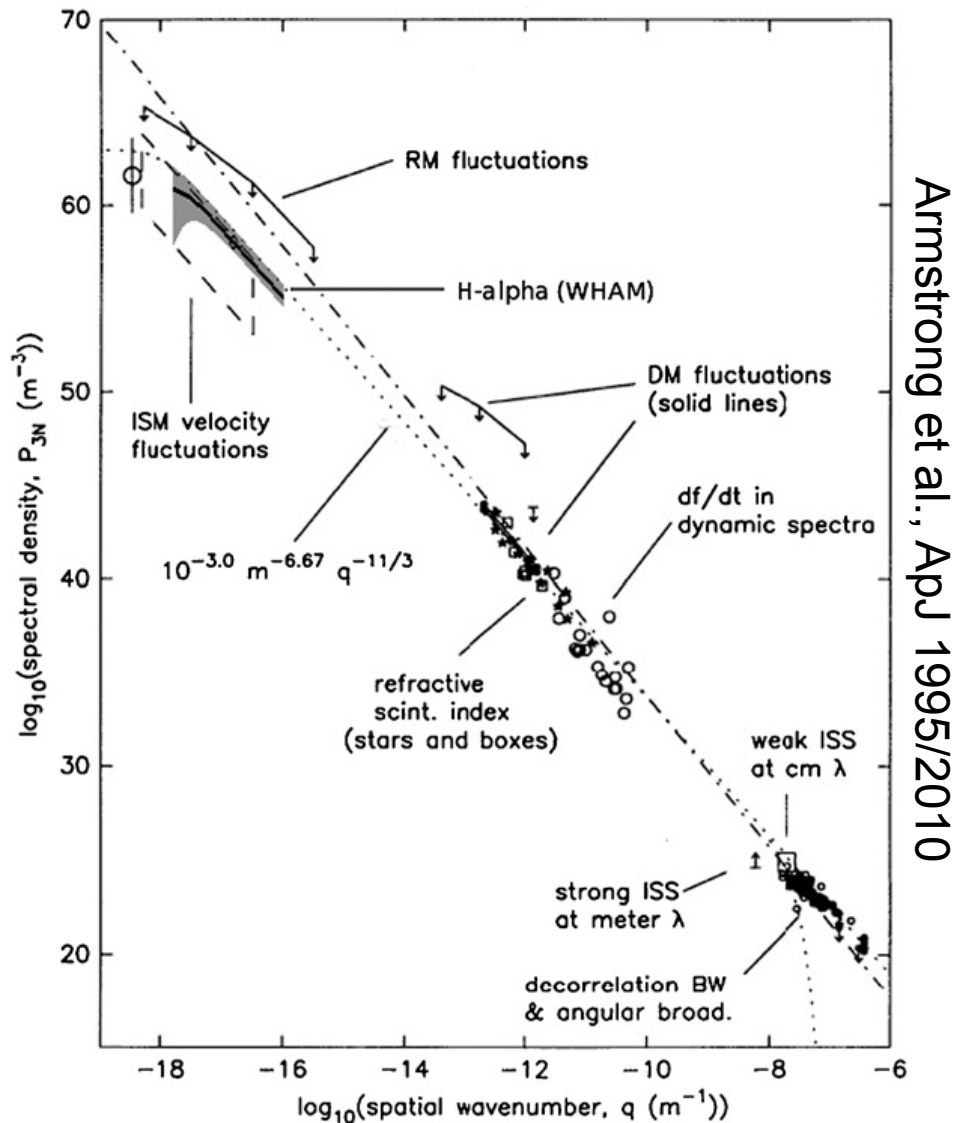
# Most of the visible universe: Turbulent magnetized plasmas

Stellar activity



Black hole accretion disks

# The “big power law” in the sky



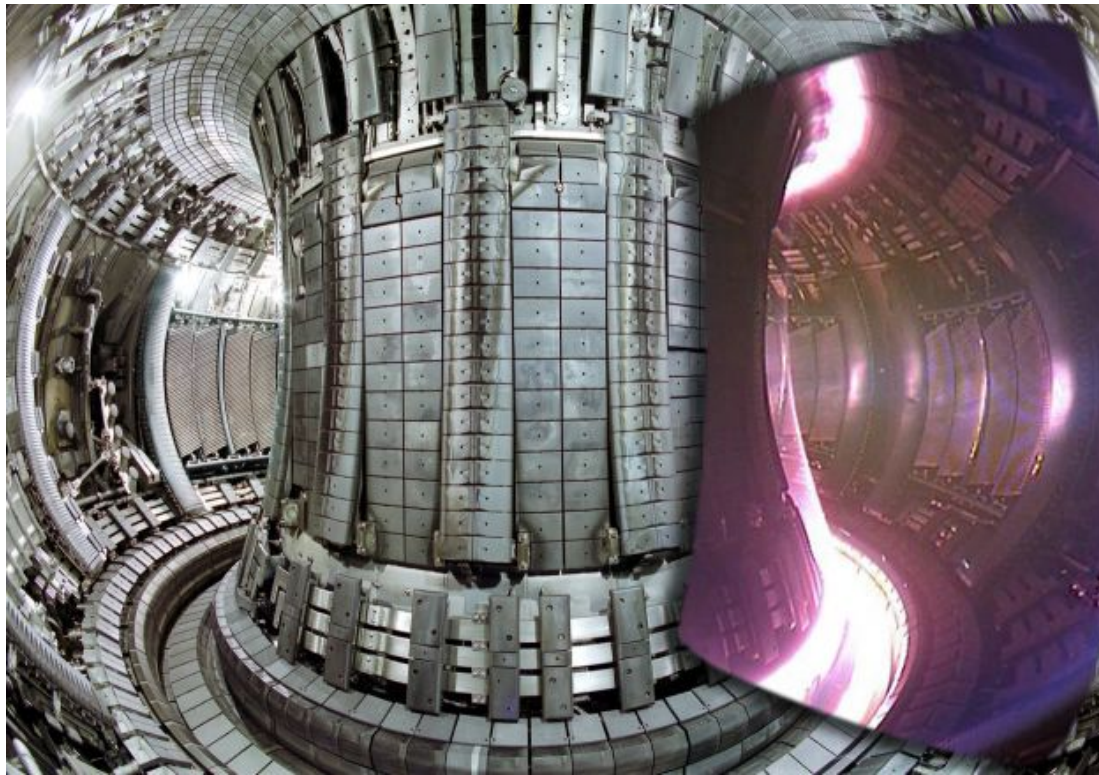
Armstrong et al., ApJ 1995/2010

Electron density fluctuations in the interstellar medium of our galaxy:

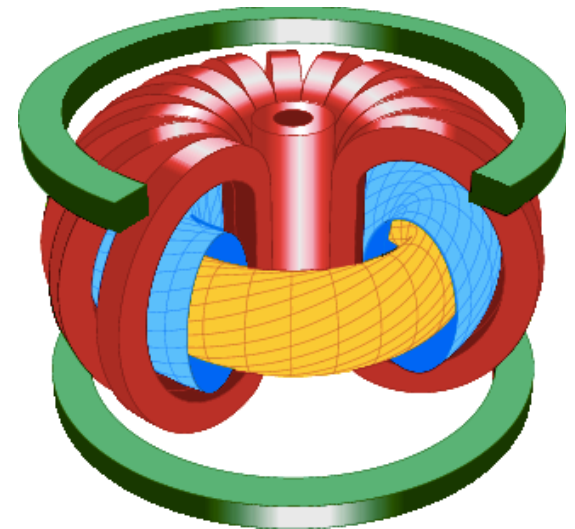
A (near) power law spanning over 10 decades

# Plasma turbulence and fusion energy

Idea: Abundant, low-carbon energy for future generations



Magnetic field lines span nested toroidal surfaces; cross-field transport due to collisions & turbulence



JET (Joint European Torus), the world's largest fusion experiment  
1997: 16 MW of fusion power from D-T reactions (world record!)

Energy confinement time is determined by plasma turbulence

# Turbulent mixing in magnetoplasmas

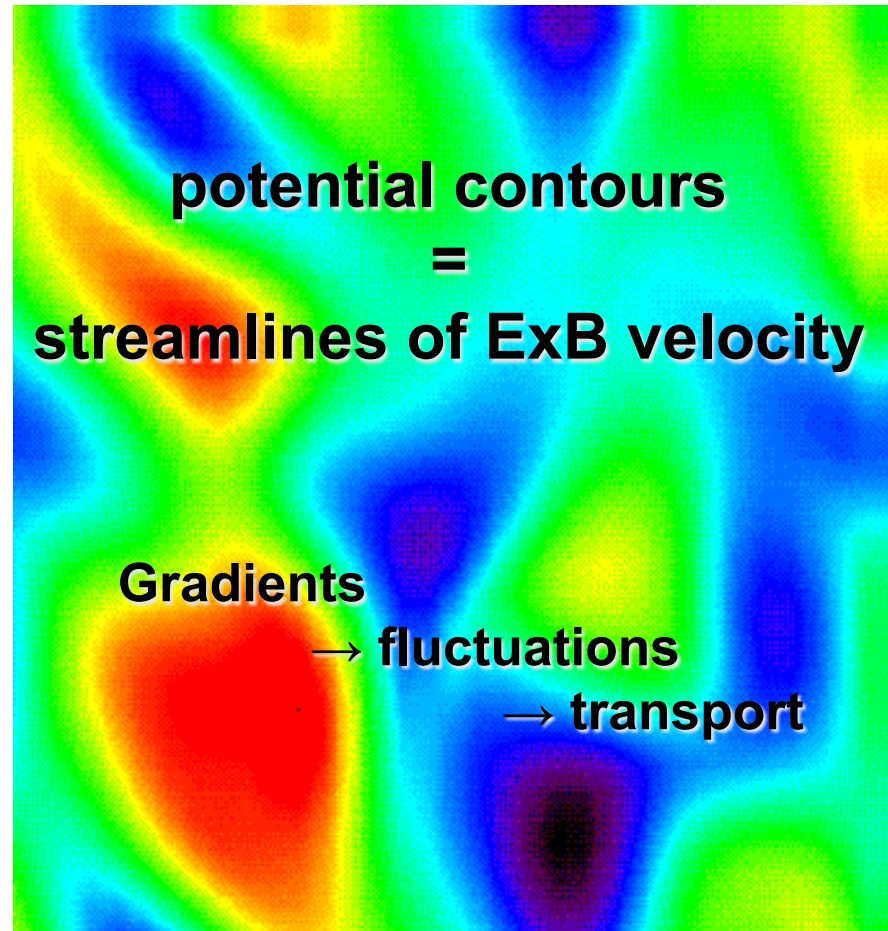
ExB drift velocity

$$\tilde{\mathbf{v}}_E = \frac{c}{B^2} \mathbf{B} \times \nabla \tilde{\phi}$$

$$Q \equiv \frac{3}{2} \langle \tilde{p} \tilde{\mathbf{v}}_E \rangle = -n\chi \nabla T$$

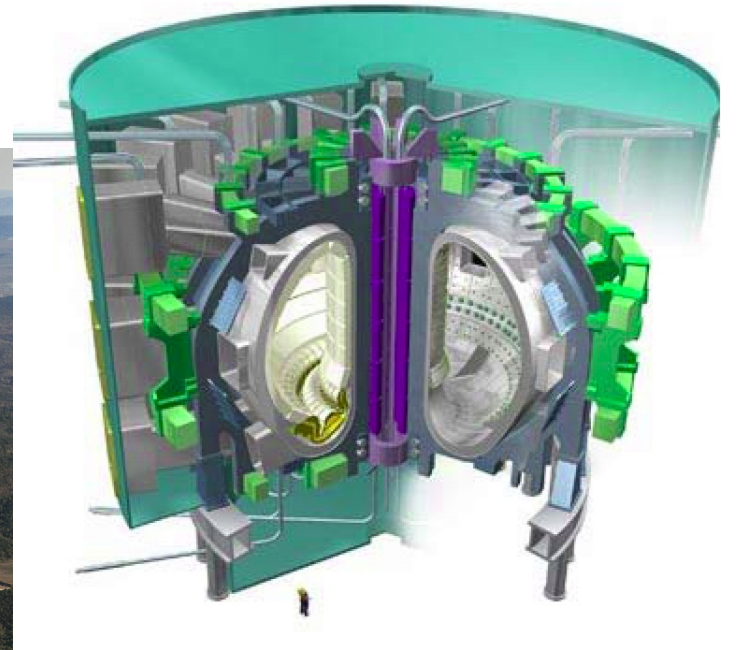
$$\chi \sim \frac{(\delta x)^2}{\delta t} \sim \frac{\rho^2 v_t}{L_T}$$

(random walk/mixing length estimates)



Typical heat and particle diffusivities are of the order of 1 m<sup>2</sup>/s

# The next step: ITER



[www.iter.org](http://www.iter.org)

Key challenge: Understand  
and control plasma turbulence

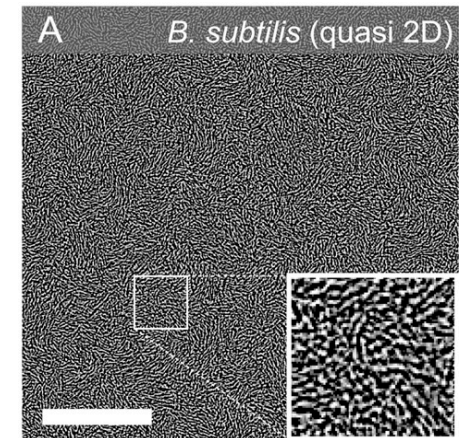
# The turbulence challenge

A general view of turbulence:

*Self-organized non-equilibrium state mediated by the nonlinear coupling of many degrees of freedom in an open system*

Examples:

- Simple & complex fluids
- Chemical & biological systems
- *Astrophysical & laboratory plasmas*



Bacterial suspensions

Challenge: Complex interplay between order and disorder; largely defies an *ab initio* analytical treatment up to now

*„There might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts...“*

John von Neumann



# Plasma turbulence: Basic questions

What are the fundamental nonlinear equations?

What are the drive, damping, and nonlinear redistribution processes of the fluctuation (free) energy?

Plasmas tend to sustain many different types of linear waves; what is their role and nature in a turbulent environment?

How to characterize and the quasi-stationary turbulent state?

Which dimensional reduction techniques may be applied?

**This talk: A snapshot!**



# Fundamental nonlinear equations

# From magneto-hydrodynamics to kinetics

Charged plasma particles undergo mostly small-angle (distant) Coulomb collisions.

Hot and/or dilute plasmas are **almost collisionless**.

Here, **MHD is not applicable; one must use a kinetic description!**

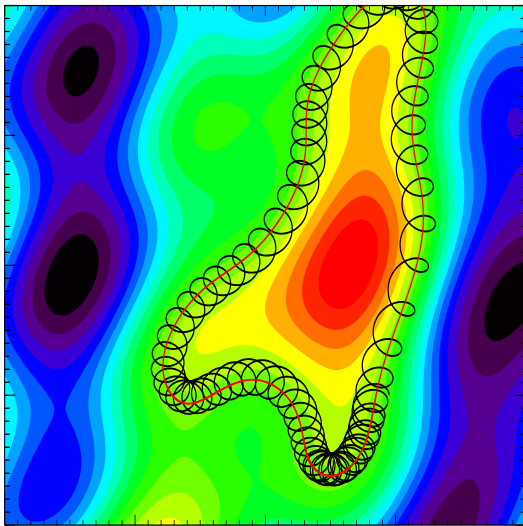
Vlasov (collisionless Boltzmann) equations ( $\alpha$ =species label)

$$\frac{\partial f_{\alpha}}{\partial t} + \mathbf{v} \cdot \nabla f_{\alpha} + \frac{q_{\alpha}}{m_{\alpha}} \left[ \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right] \cdot \nabla_v f_{\alpha} = 0$$

$$f_{\alpha} = f_{\alpha}(\mathbf{x}, \mathbf{v}, t) \quad \dots \text{from Liouville equation via BBGKY hierarchy}$$

...plus Maxwell's equations (w/o displacement current)

# From kinetics to gyrokinetics



**Strong magnetic field** (points into the plane)

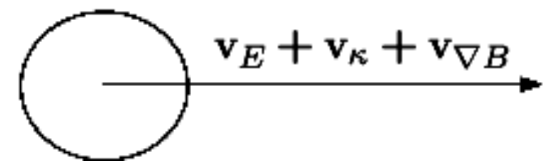
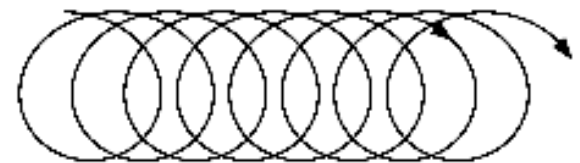
Electrostatic potential fluctuations (color-coded)

Particle orbit = fast gyromotion + slow (ExB) drift

## Basic idea of gyrokinetics:

Remove the fast gyromotion  $\omega \ll \Omega$

Introduce charged rings as quasiparticles;  
go from particle to **gyrocenter coordinates**



# A Lagrangian approach

Phase space trajectories (characteristics)

$f = \text{const}$  along:

$$\begin{aligned}\dot{\vec{x}} &= \vec{v} \\ \dot{\vec{v}} &= \frac{e}{m} \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right)\end{aligned}$$

Can be derived from Lagrangian

$$\begin{aligned}L &= \left( \frac{e}{c} \vec{A}(\vec{x}, t) + m\vec{v} \right) \cdot \dot{\vec{x}} - H(\vec{x}, \vec{v}) \\ H &= \frac{m}{2} v^2 + e\phi(\vec{x}, t)\end{aligned}$$

Add low-frequency, anisotropic, small-amplitude fluctuations

$$\omega/\Omega_i \sim \epsilon_g \ll 1 \qquad k_{\parallel}/k_{\perp} \sim \epsilon_g \ll 1 \qquad e\phi/T_e \sim \epsilon_g \ll 1$$

# Transition to gyrocenter coordinates

## New Lagrangian (using Lie transforms)

$$\Gamma = \left( m v_{\parallel} \mathbf{b}_0 + \frac{e}{c} \bar{A}_{1\parallel} \mathbf{b}_0 + \frac{e}{c} \mathbf{A}_0 \right) \cdot d\mathbf{X} + \frac{mc}{e} \mu d\theta - \left( \frac{m}{2} v_{\parallel}^2 + \mu B_0 + \mu \bar{B}_{1\parallel} + e \bar{\phi}_1 \right) dt$$

## Euler-Lagrange equations

$$\dot{\mathbf{X}} = v_{\parallel} \mathbf{b} + \frac{B}{B_{\parallel}^*} \left( \frac{v_{\parallel}}{B} \bar{\mathbf{B}}_{1\perp} + \frac{c}{B^2} \bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega} \mathbf{b} \times \nabla (B + \bar{B}_{1\parallel}) + \frac{v_{\parallel}^2}{\Omega} (\nabla \times \mathbf{b})_{\perp} \right)$$

$$\dot{v}_{\parallel} = \frac{\dot{\mathbf{X}}}{m v_{\parallel}} \cdot (e \bar{\mathbf{E}}_1 - \mu \nabla (B + \bar{B}_{1\parallel})) \quad \dot{\mu} = 0 \quad \begin{array}{l} \text{magnetic moment} \\ \text{(adiabatic invariant)} \end{array}$$

Remark: The overbar denotes a gyroaveraging operation

# The nonlinear gyrokinetic equations

$$f = f(\mathbf{X}, v_{\parallel}, \mu; t)$$

Appropriate field equations (from Maxwell)

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_{\parallel} \frac{\partial f}{\partial v_{\parallel}} = 0$$

$$\frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - (1 - \|I_0^2\|) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\parallel}}{B}$$

$$\nabla_{\perp}^2 A_{1\parallel} = -\frac{4\pi}{c} \sum \bar{J}_{1\parallel}$$

$\mathbf{X}$  = gyrocenter position

$v_{\parallel}$  = parallel velocity

$\mu$  = magnetic moment

$$\frac{B_{1\parallel}}{B} = -\sum \epsilon_{\beta} \left( \frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\parallel}}{B} \right)$$

Nonlinear 5D equations; removal of irrelevant space-time scales

Saves a factor of more than  $10^{10}$  for ITER computations...

# Charney-Hasegawa-Mima equation

Hasegawa & Mima, PRL 1977


In a certain limiting case (in particular: cold ions), gyrokinetics leads to the CHM equation which is closely related to the 2D NS equation; used in geophysics already since 1948...

$$\frac{d}{dt}(\phi - \nabla^2 \phi - x) = 0$$
$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y}$$

One-field model (for the  
electrostatic potential);  
no linear drive/damping



J. G. Charney



# Nonlinear gyrokinetics on large supercomputers: Some (surprising) findings

# Gyrokinetic simulation & GENE code

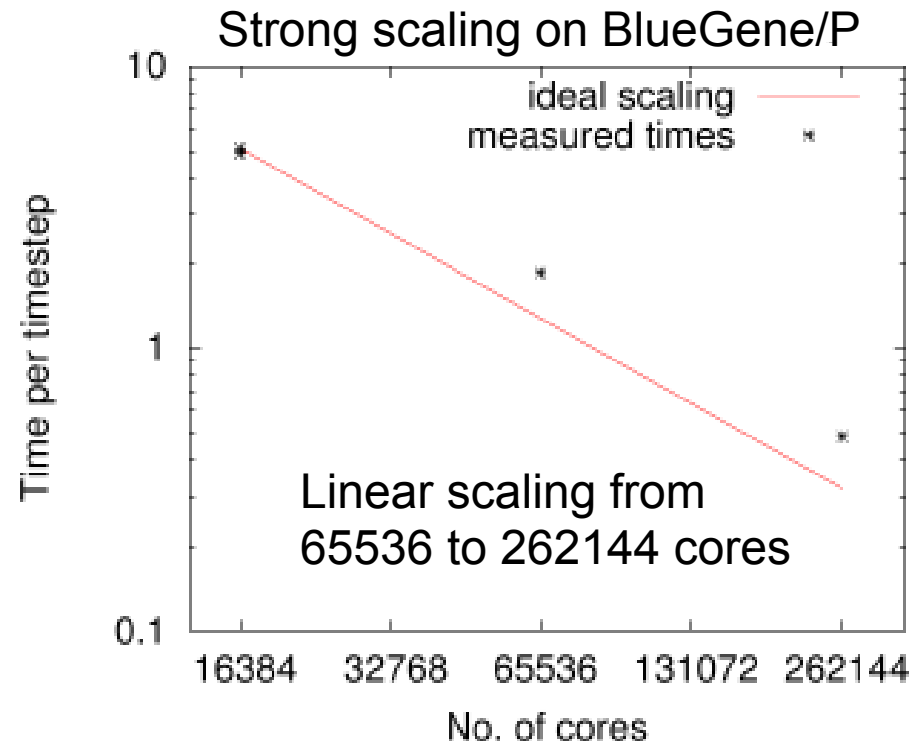
## Nonlinear gyrokinetic simulations

- 1983: First particle-in-cell („PIC“) codes
- 1999: First grid-based („Vlasov“) codes [GS2, GENE]
- 2013: Numerous applications to laboratory & natural plasmas

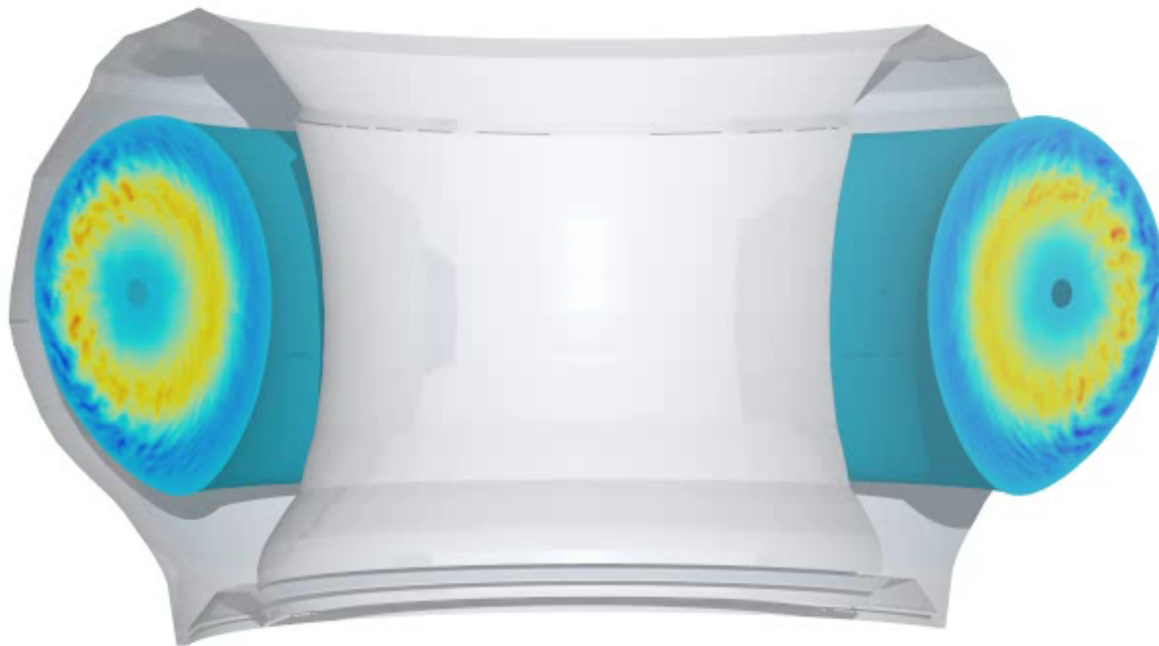
## The GENE code

- Mix of appropriate CFD-type numerical methods
- Automatic adaptation to chosen platform and grid layout

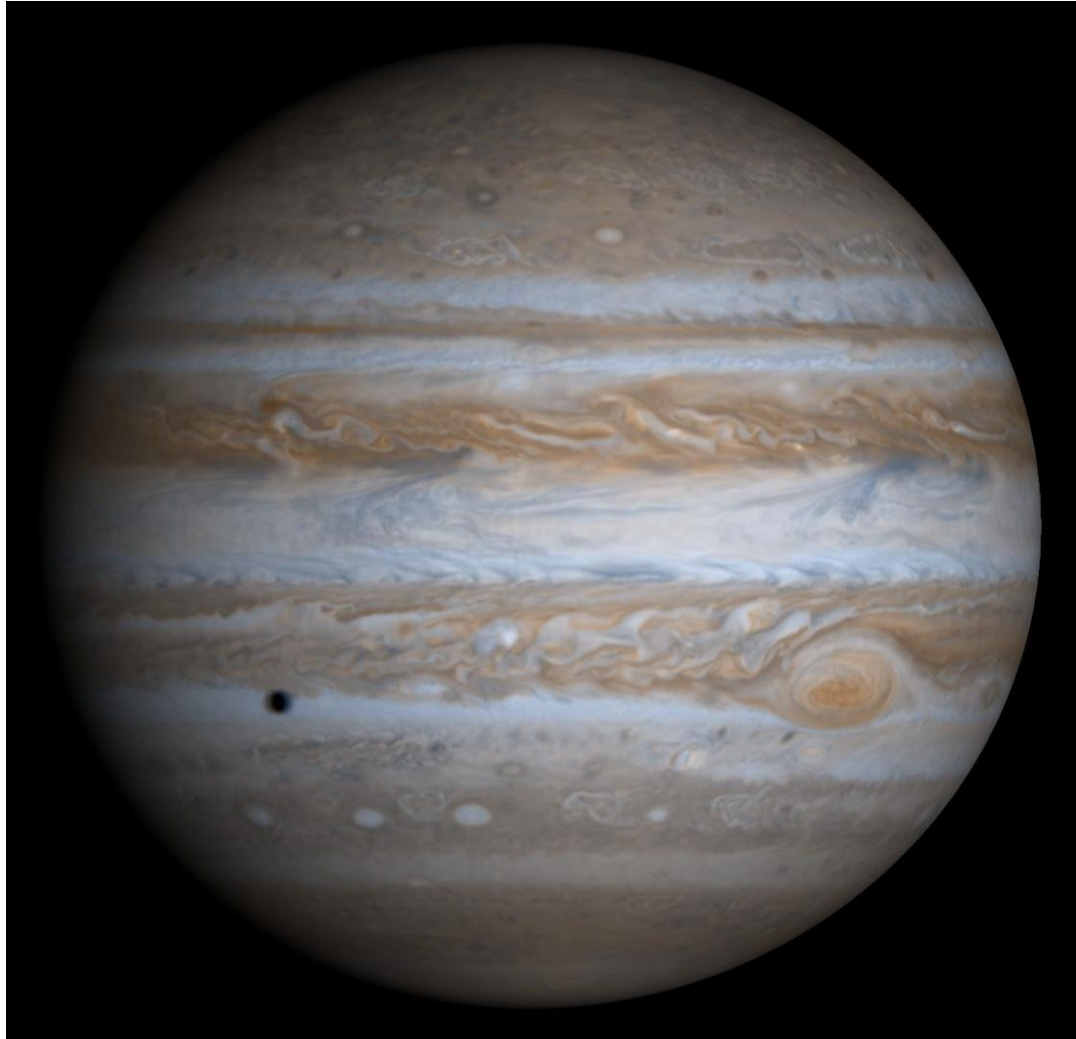
<http://gene.rzg.mpg.de>



# Self-organization: Zonal flow generation

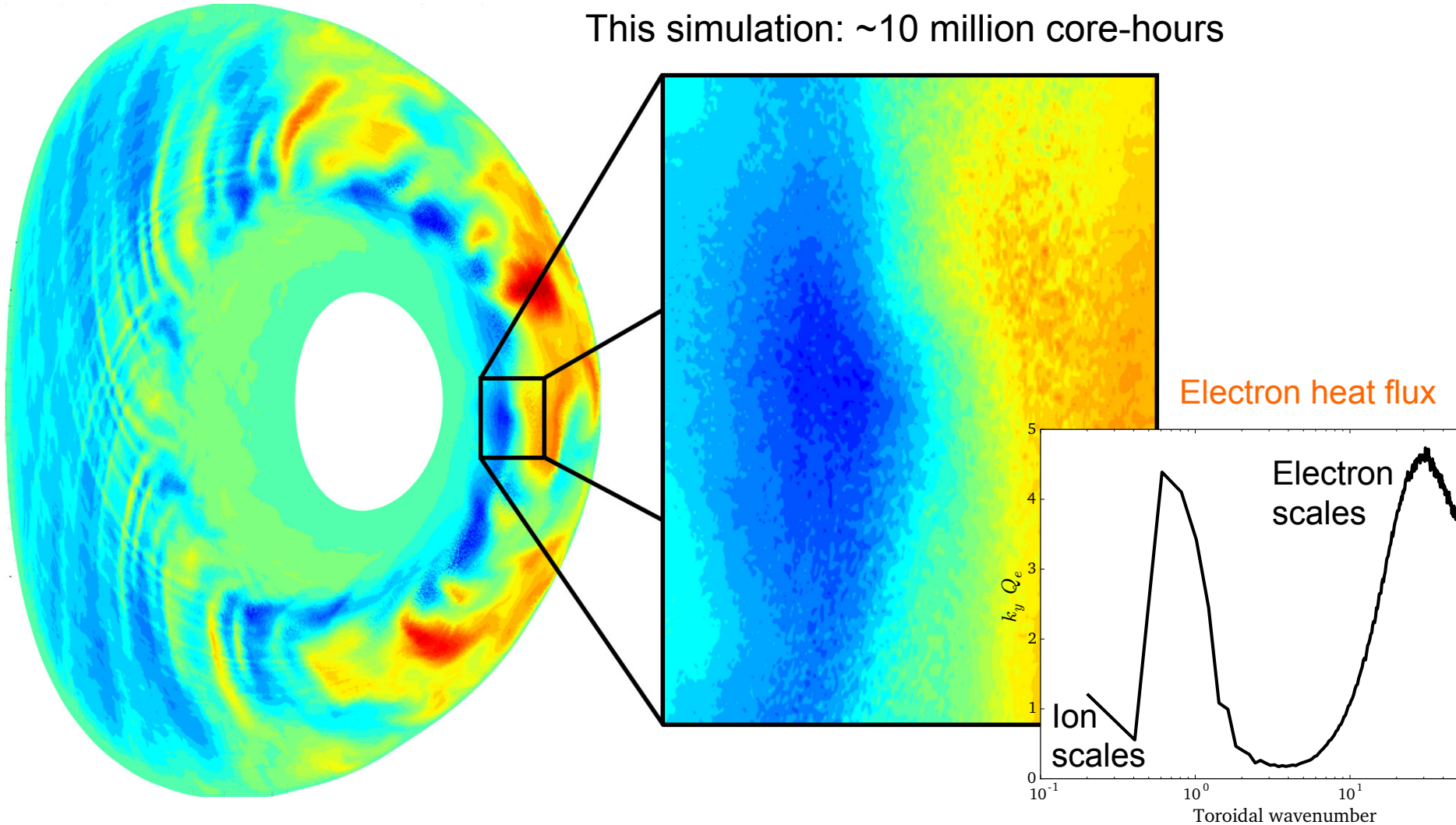


# Zonal flows in planetary atmospheres



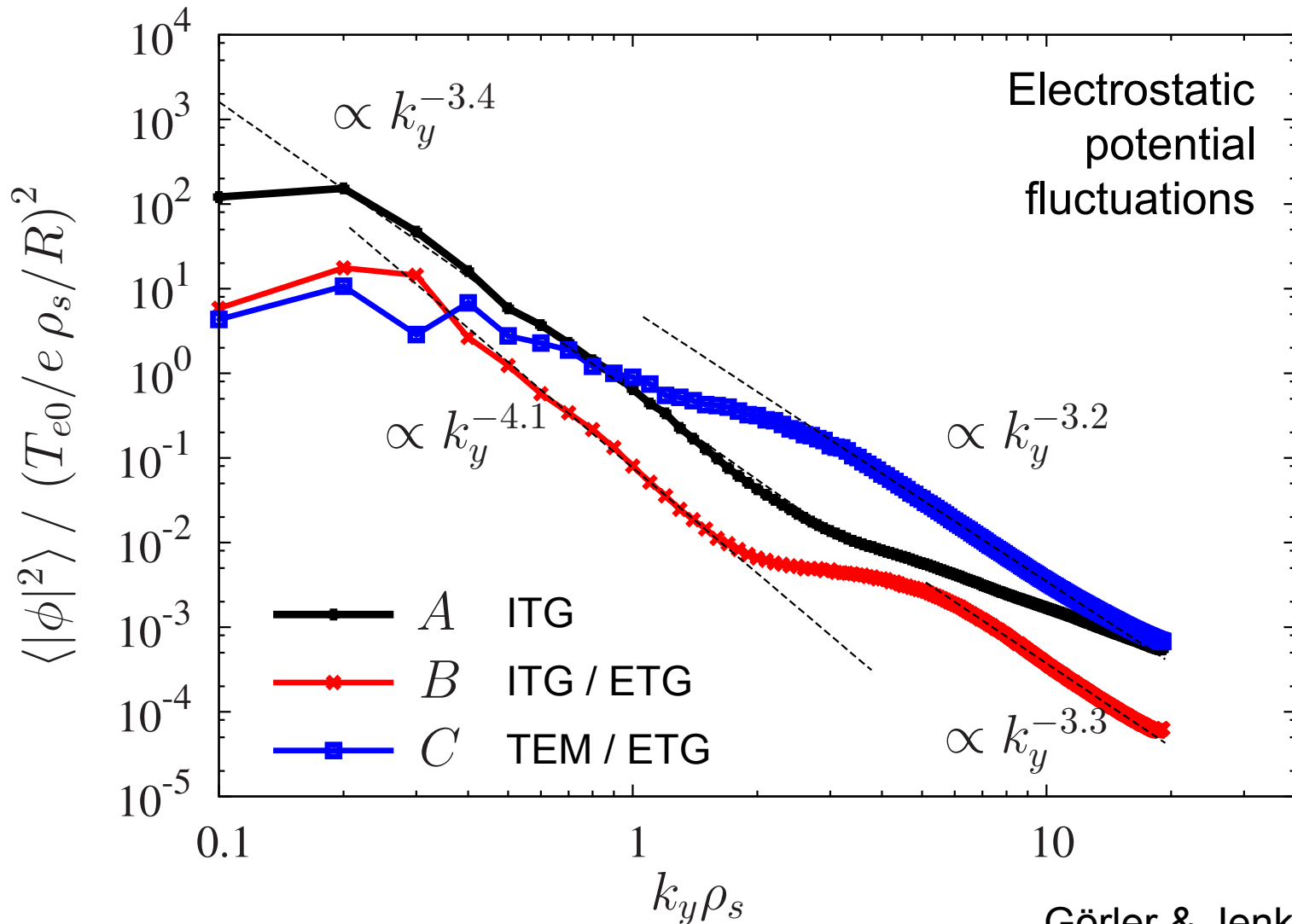
# Self-organization: Electron-scale turbulence

This simulation: ~10 million core-hours



**Simulation of electron internal transport barrier in TCV with GENE**

# Nonuniversal power law spectra?



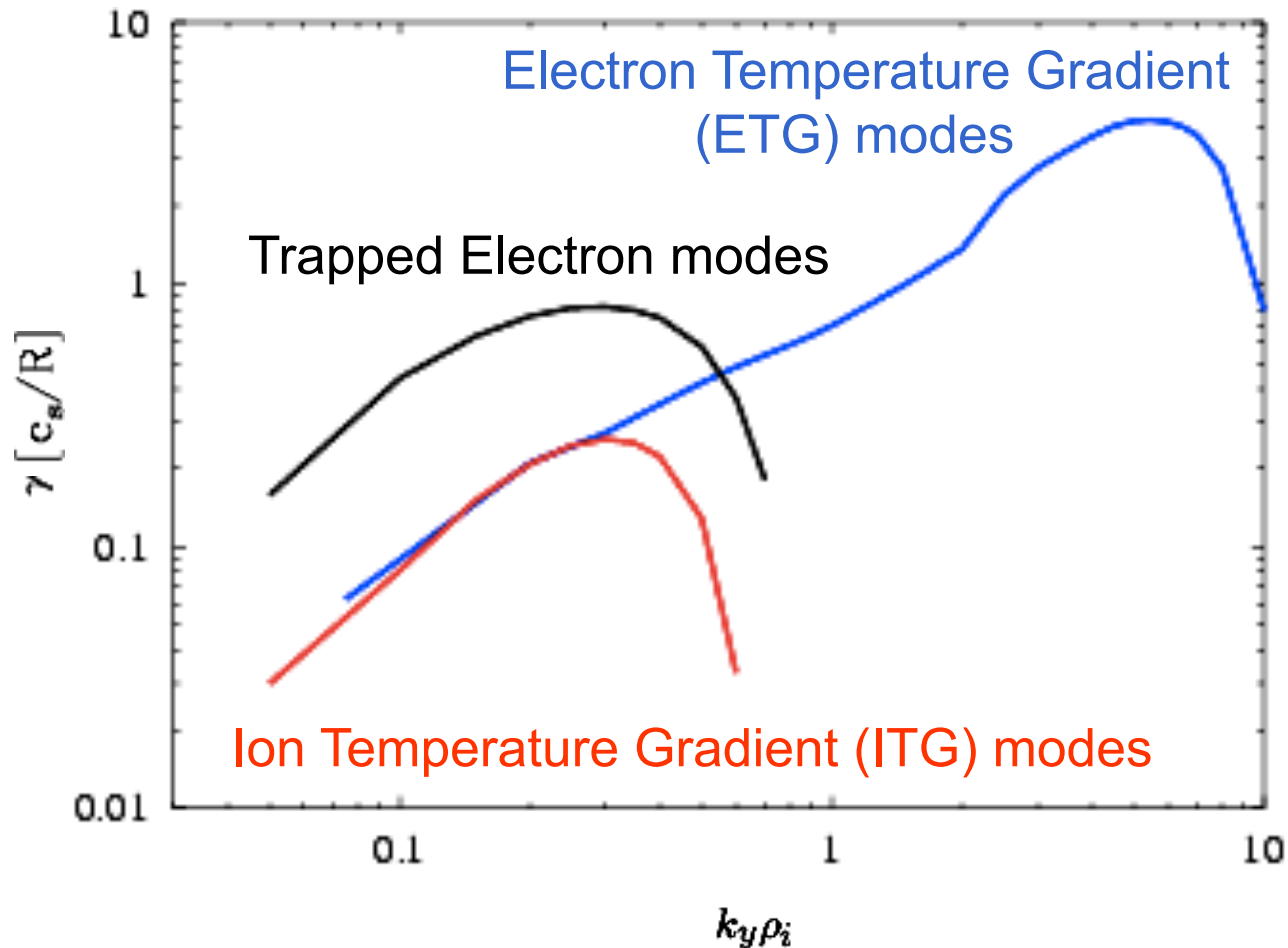
Görler & Jenko 2008

$R/L_{T_e}=6.9$  and (A)  $R/L_{T_i}=6.9$ , (B)  $R/L_{T_i}=5.5$ , and (C)  $R/L_{T_i}=0.0$



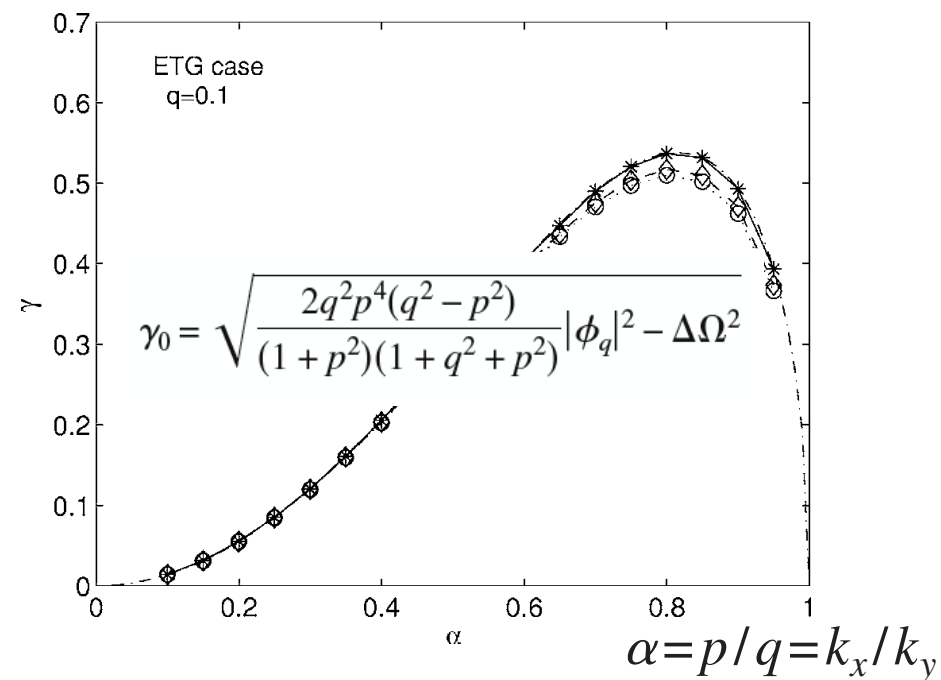
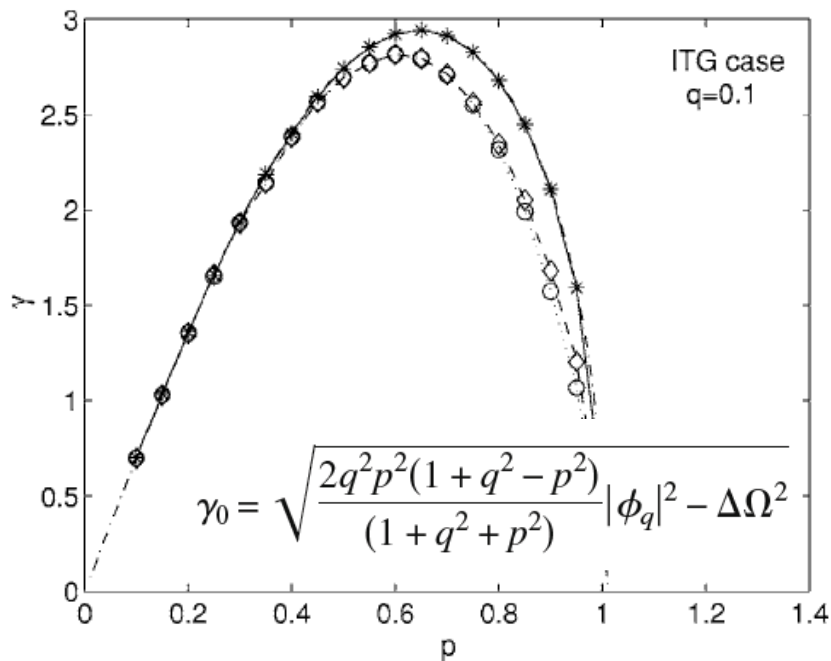
# Gyrokinetic turbulence: The cast

# Primary instabilities (key players)



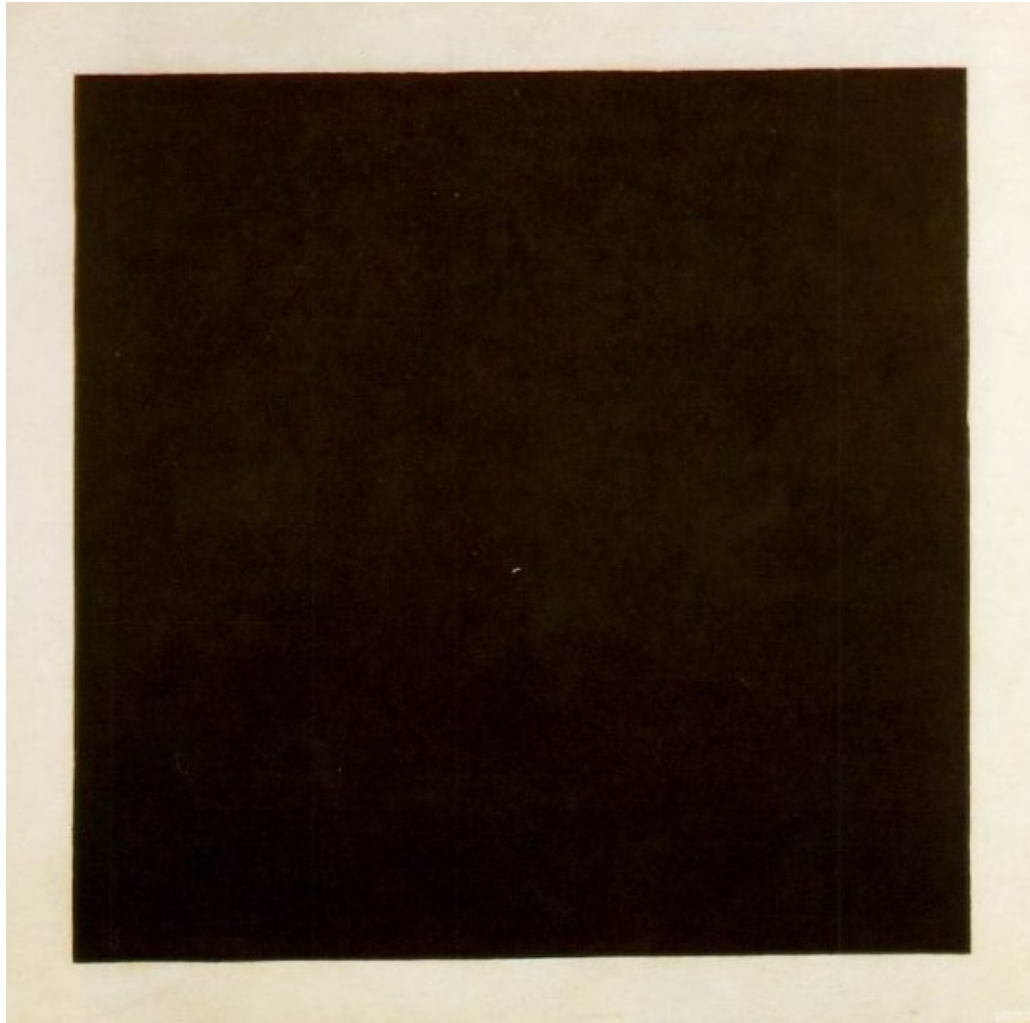
# Secondary instabilities (zonal flows!)

- Large-amplitude primaries are Kelvin-Helmholtz unstable  
[Cowley et al. 1991; Dorland & Jenko PRL 2000]
- This secondary instability contains a zonal-flow component
- Near-equivalence to 4-mode approach (here: CHM equation)



ETG modes saturate at higher amplitudes

# Damped eigenmodes




K. Malevich, Black Square (1915)

Largely unnoticed  
until fairly recently

Requires a change  
of perspective

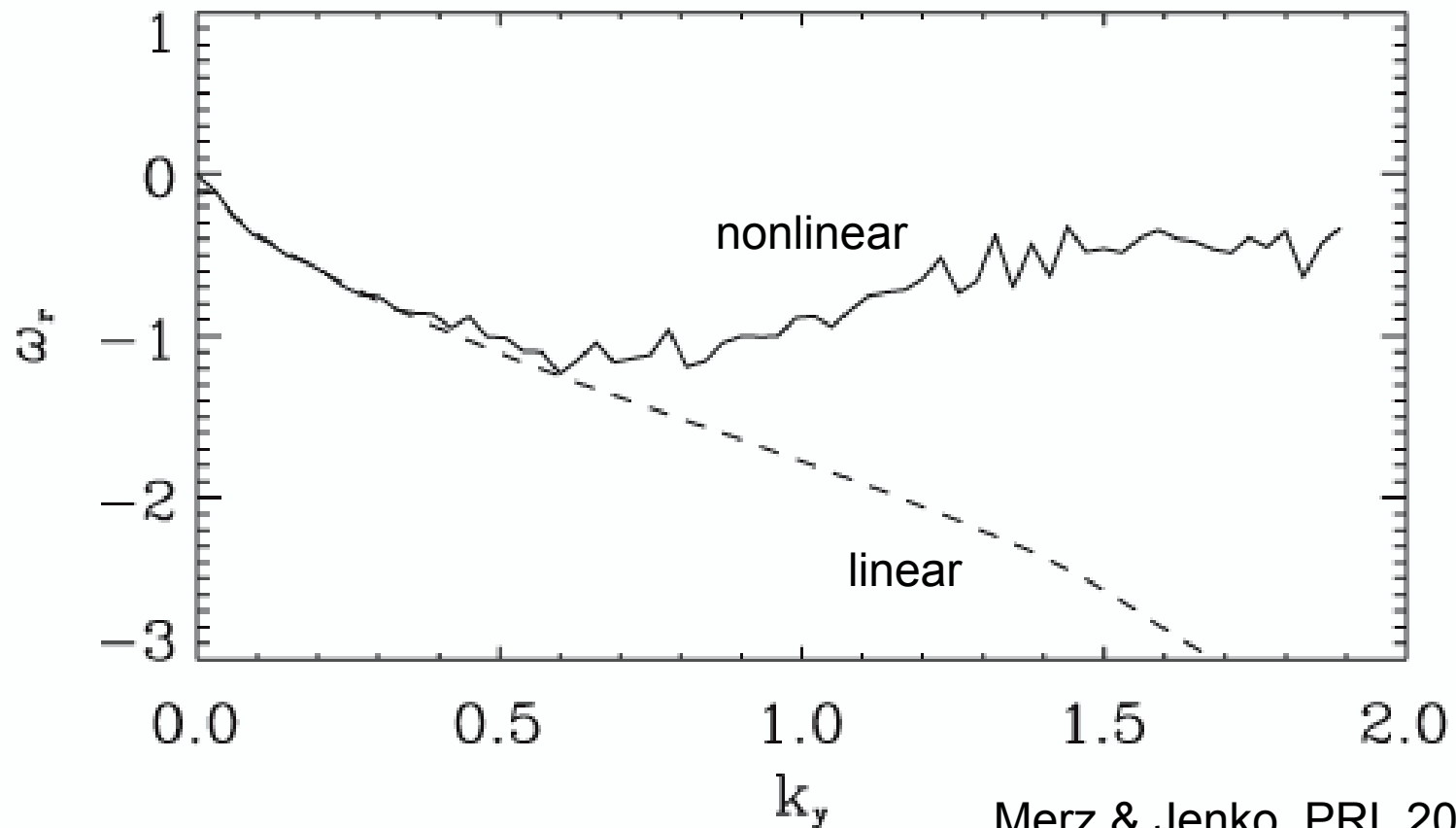
Helps explain some  
longstanding puzzles,  
physical & numerical



# Unstable plasma waves in a turbulent environment

# Example: Trapped electron modes

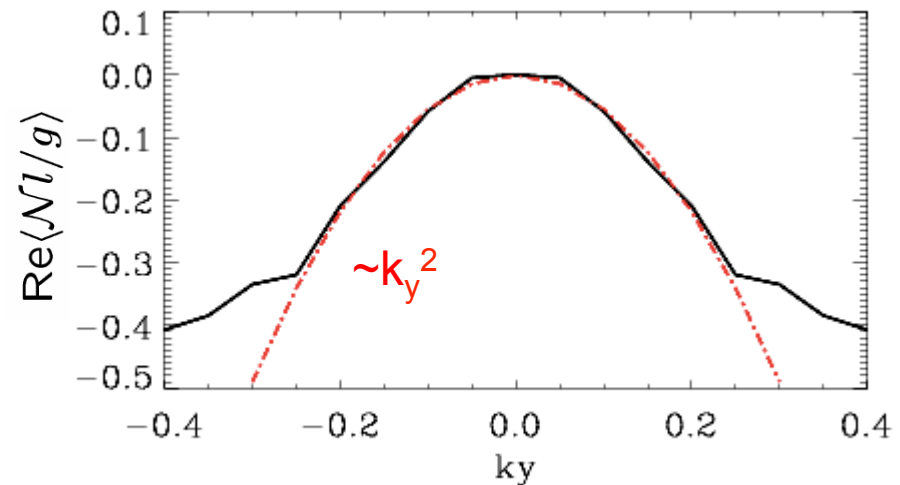
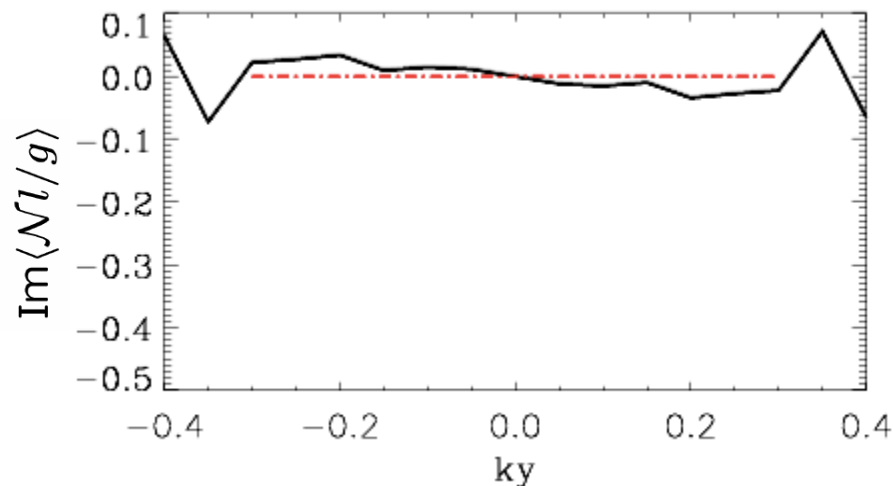
In the long-wavelength (drive) range,  
nonlinear and linear modes match closely



# Statistical analysis of the ExB nonlinearity

ExB nonlinearity in the low- $k_y$  range: large transport contributions; small random noise, while the coherent part can be written as:

$$\mathcal{N}l[g] \simeq D(-k_{\perp}^2)g = D\nabla_{\perp}^2 g$$



$$\frac{\partial g}{\partial t} = \mathcal{L}g + \mathcal{N}l[g] \simeq (i\omega_r + \gamma - D_0 \langle k_{\perp}^2 \rangle)g$$

$$D_0 \sim \frac{\gamma}{\langle k_{\perp}^2 \rangle}$$

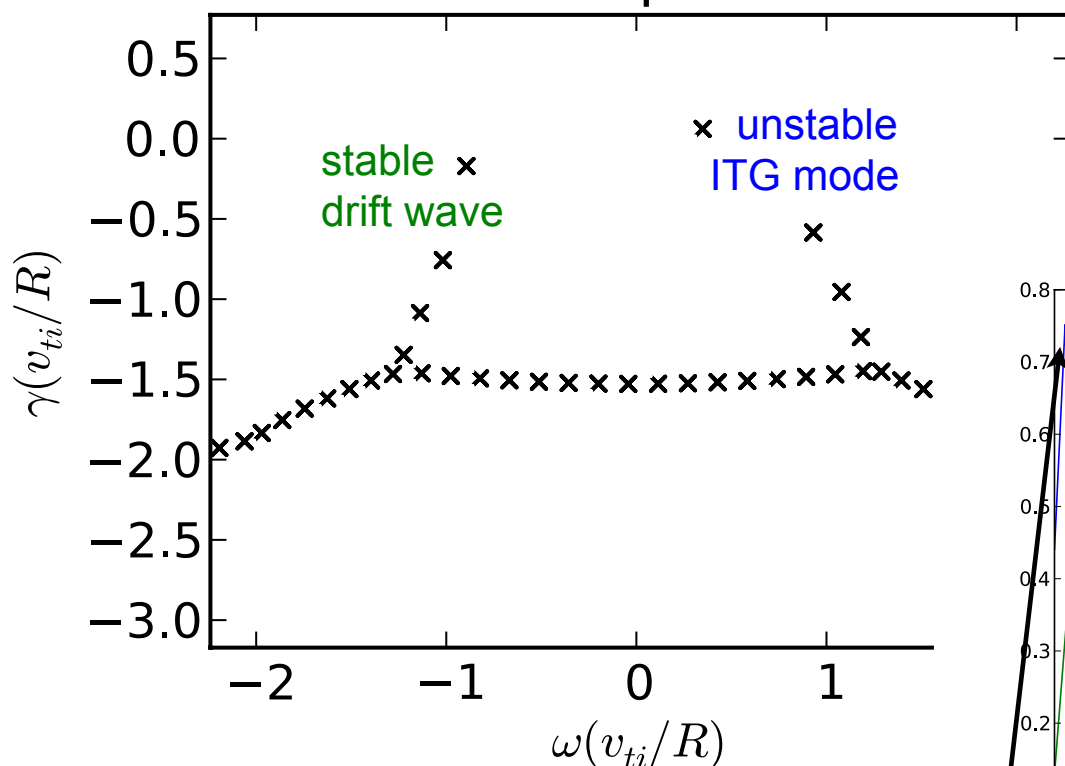


# Role of damped gyrokinetic eigenmodes

# Characteristics of eigenvalue spectra

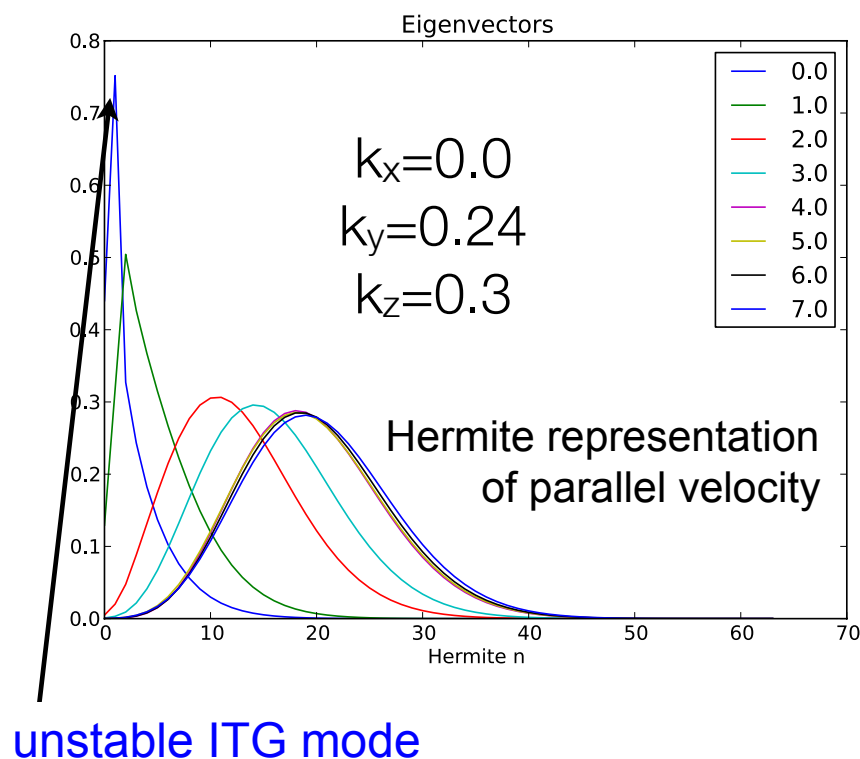
Note: fixed wavenumber!

## Linear Spectrum



Linear eigenvalue spectrum for (weakly collisional) gyrokinetic system (homogeneous B field)

Only two dominant eigenmodes (ITG mode & drift wave) have a smooth velocity space structure



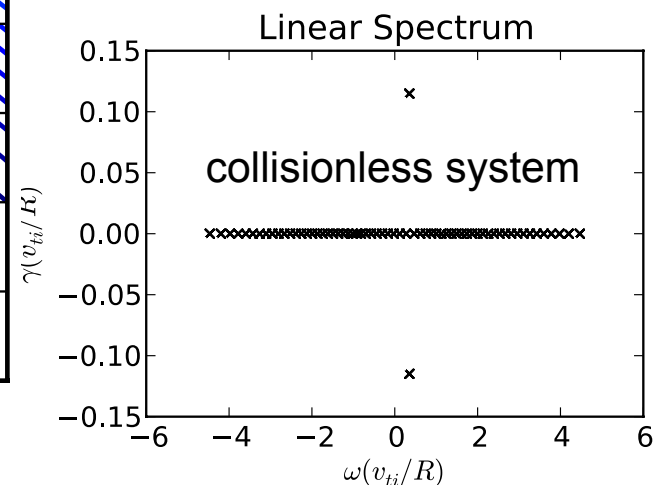
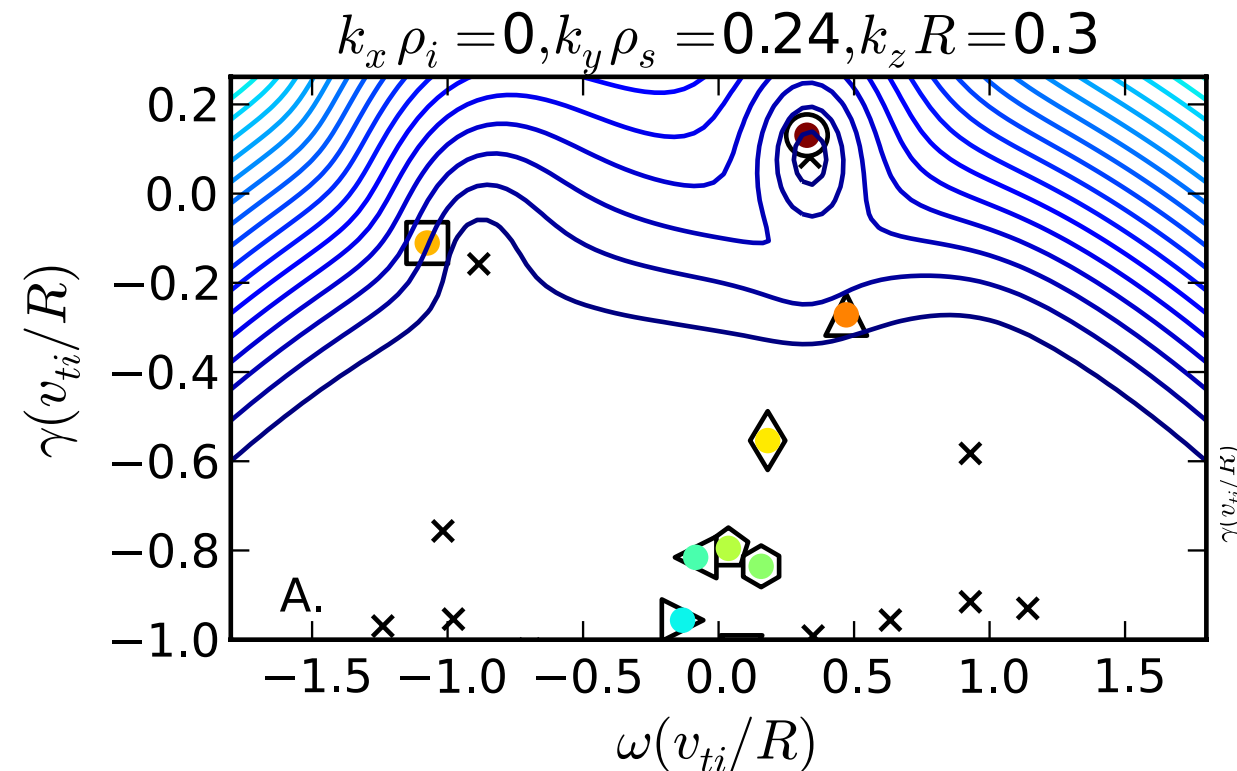
# Nonlinear versus linear spectra

Direct decomposition of nonlinear data in terms of (highly non-orthogonal) linear eigenmodes is not helpful; instead, compute **pseudospectra of POD modes** (for each  $k$ ), i.e., minimize  $\|A\hat{g}_n - z\hat{g}_n\|$

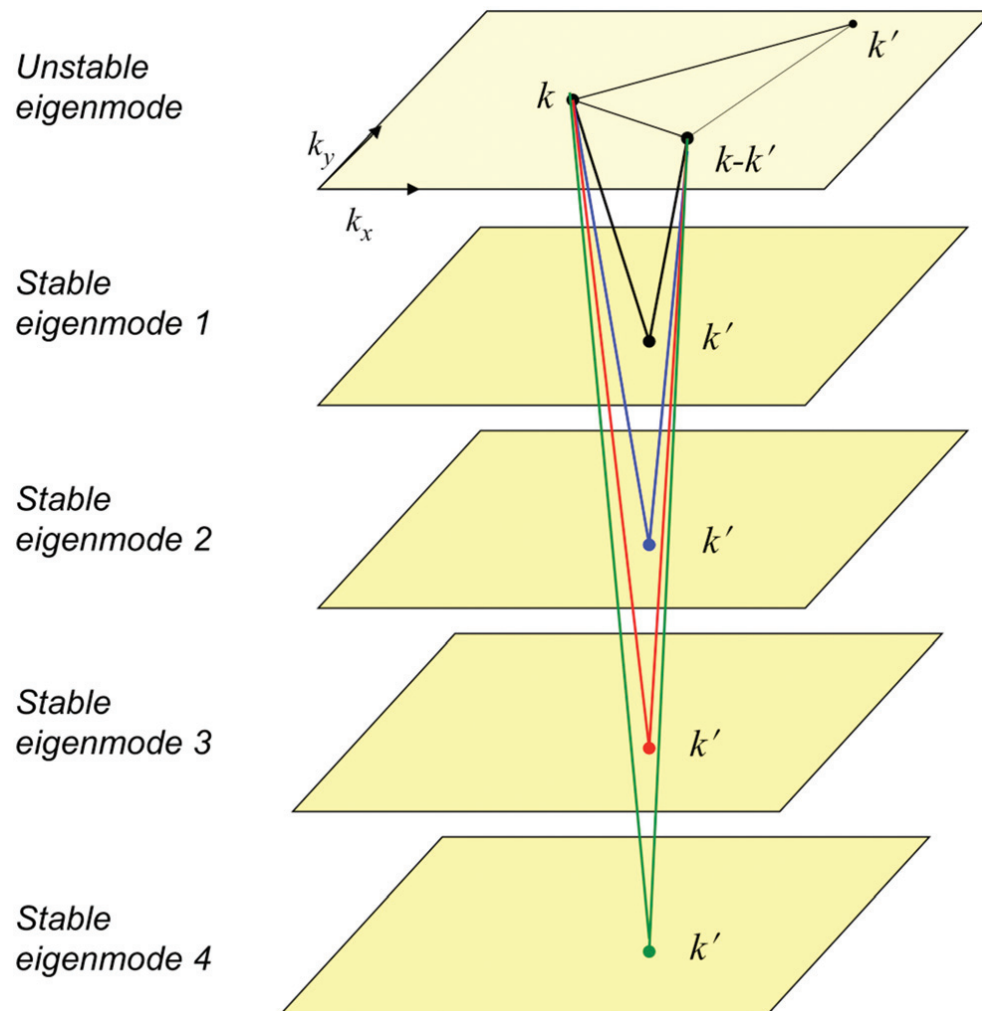
$$\Lambda_\epsilon(A) = \{z \in \mathbb{C} : \|(zI - A)^{-1}\| \geq \epsilon^{-1}\}$$

ITG mode and drift wave can be clearly identified

In addition, approximate c.c. mirror image of ITG mode (re-)appears (!!!)



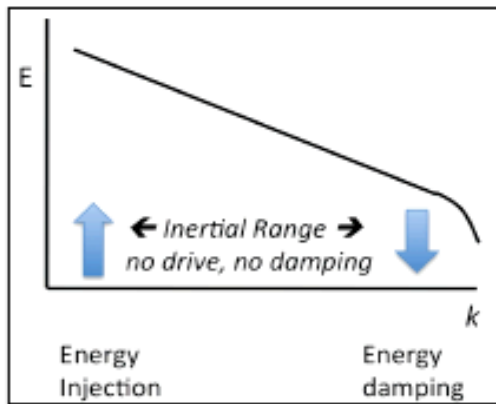
# Nonlinear excitation of stable eigenmodes



Nonlinear interaction  
and energy transfer  
between a series of  
modes at the same  
perpendicular scales

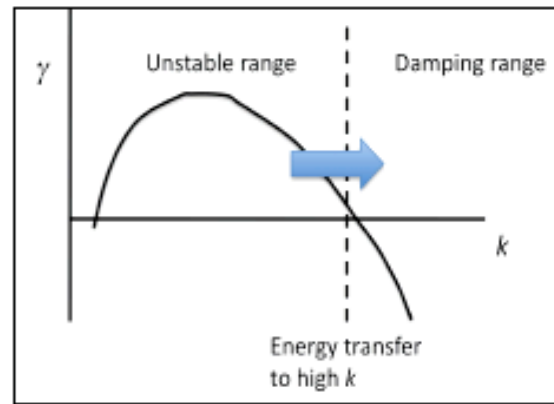
# Turbulence in fluids and plasmas – Three basic scenarios

## 1. Hydrodynamic cascade



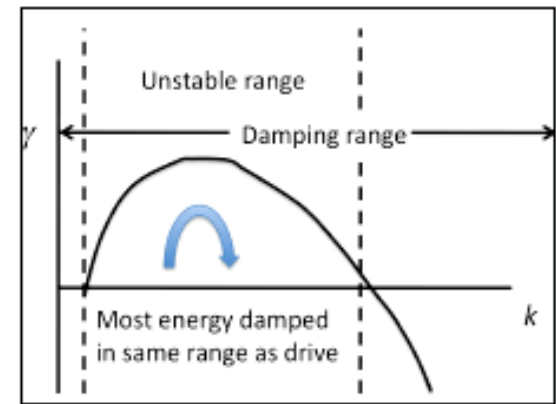
Inertial range  
→ no dissipation  
→ scale invariant dynamics  
→ power law spectrum

## 2. Conventional $\mu$ -turbulence

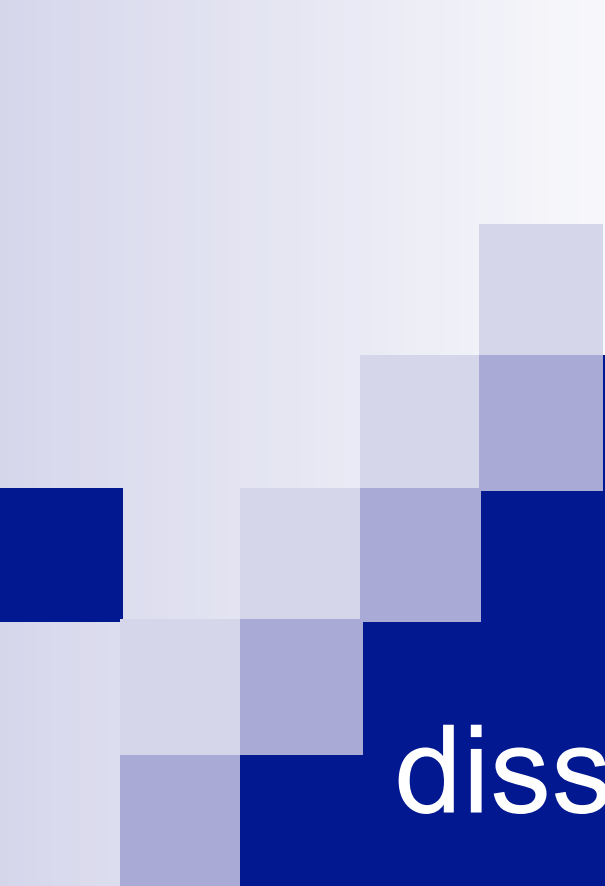


Energy transfer to high  $k$   
like hydro – no inertial range  
adjacent unstable,  
damping ranges

## 3. Saturation by damped eigenmode



Energy can go to high  $k$   
but most of it is lost at  
low  $k$  in driving range



# Turbulent free energy: Nonlinear redistribution, dissipation, and power laws

# Ideal quadratic invariants

**Kinetics:** Free energy  $\mathcal{E} = U - T_0 S = K + \mathcal{E}_E + \mathcal{E}_M - T_0 S$

$$= \sum_j T_{0j} \int d^3\mathbf{x} d^3\mathbf{v} \frac{\tilde{f}_{1j}^2}{2 F_{0j}} + \int d^3\mathbf{x} \frac{\mathbf{E}^2}{8\pi} + \int d^3\mathbf{x} \frac{\mathbf{B}^2}{8\pi}$$

up to order two in  $\tilde{f}_{1j}$

distribution function  $\tilde{f}_j =$  Maxwellian distribution function  $F_{0j}$   
 + fluctuation part  $\tilde{f}_{1j}$

**Gyrokinetics:** Free energy balance  $\partial_t (\mathcal{E}_f + \mathcal{E}_\phi) = \mathcal{G} + \mathcal{D}$   
 (Boltzmann electrons)

Entropy part (tends to dominate):  $\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2}$

$T_i$  gradient drive:  $\mathcal{G} = \omega_{Ti} Q_i$

Collisional dissipation:  $\mathcal{D}$

# (Non-)Linear transfer of free energy

Symbolic form of dynamic equation for f:

$$\frac{\partial f}{\partial t} = G[f] + L_C[f] + L_{\parallel}[f] + D[f] + N[f, f]$$

Free energy balance:  $\mathcal{E} = \sum_{\mathbf{k}} \mathcal{E}^{\mathbf{k}} \quad \partial_t \mathcal{E}^{\mathbf{k}} = \mathcal{G}^{\mathbf{k}} + \mathcal{L}^{\mathbf{k}} + \mathcal{D}^{\mathbf{k}} + \mathcal{T}^{\mathbf{k}}$

Quadratic nonlinearity:  $\mathcal{T}^{\mathbf{k}, \mathbf{p}, \mathbf{q}} = \int d\Theta \frac{T_0}{2F_0} [q_x p_y - q_y p_x] [\bar{\phi}_1^{\mathbf{q}} h^{\mathbf{p}} - \bar{\phi}_1^{\mathbf{p}} h^{\mathbf{q}}] h^{\mathbf{k}}$

$$\mathcal{T}^{\mathbf{k}, \mathbf{p}, \mathbf{q}} + \mathcal{T}^{\mathbf{p}, \mathbf{q}, \mathbf{k}} + \mathcal{T}^{\mathbf{q}, \mathbf{k}, \mathbf{p}} = 0$$

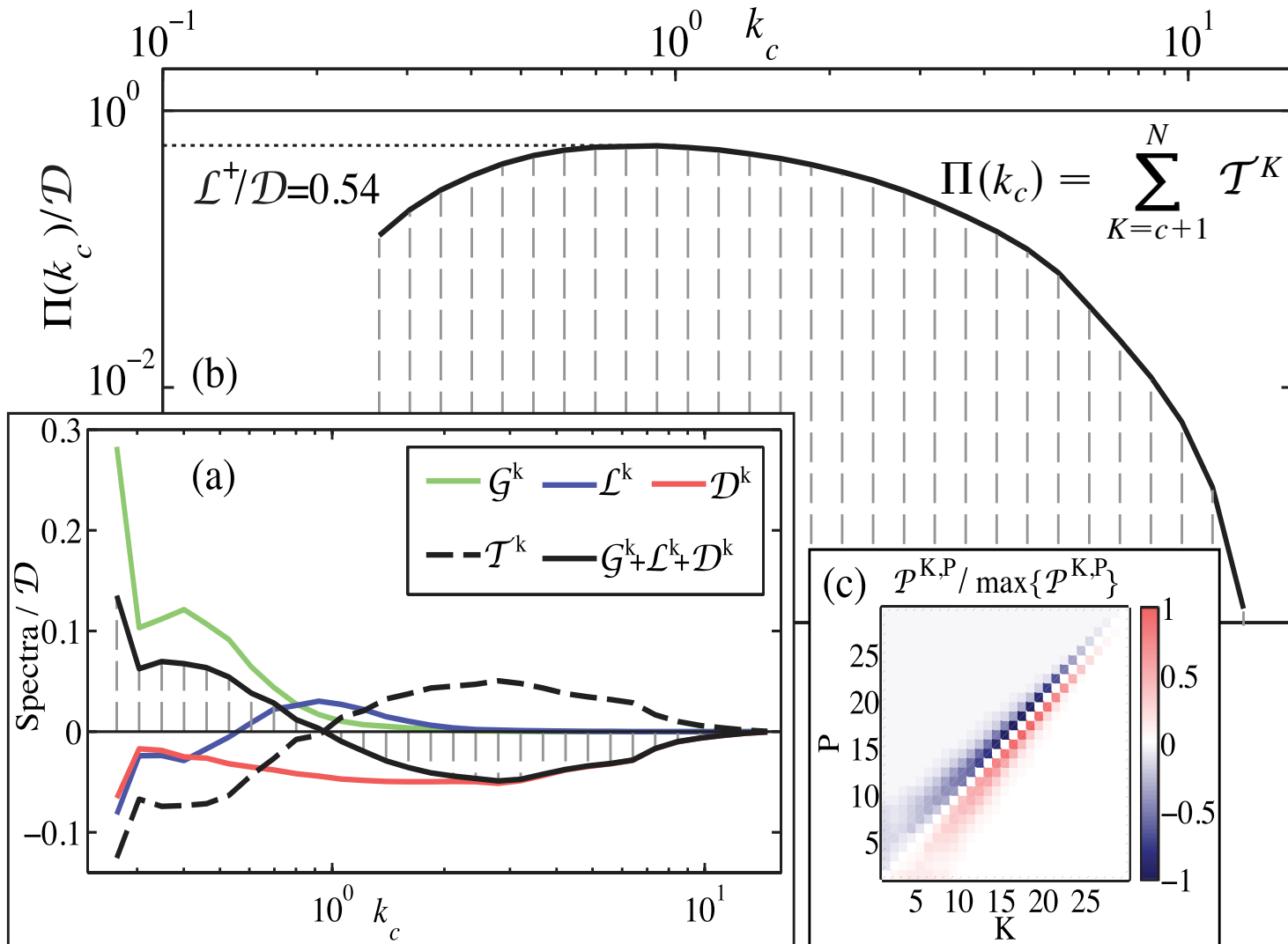
Logarithmically spaced shells  
in perp. wavenumber space:

$$\mathcal{S}^{K, P, Q} = \sum_{\mathbf{q} \in s_Q} \sum_{\mathbf{p} \in s_P} \sum_{\mathbf{k} \in s_K} \mathcal{T}^{\mathbf{k}, \mathbf{p}, \mathbf{q}} \delta_{\mathbf{k} + \mathbf{p} + \mathbf{q}}$$

$$\mathcal{T}^K = \sum_P \mathcal{P}^{K, P} = \sum_P \sum_Q \mathcal{S}^{K, P, Q}$$

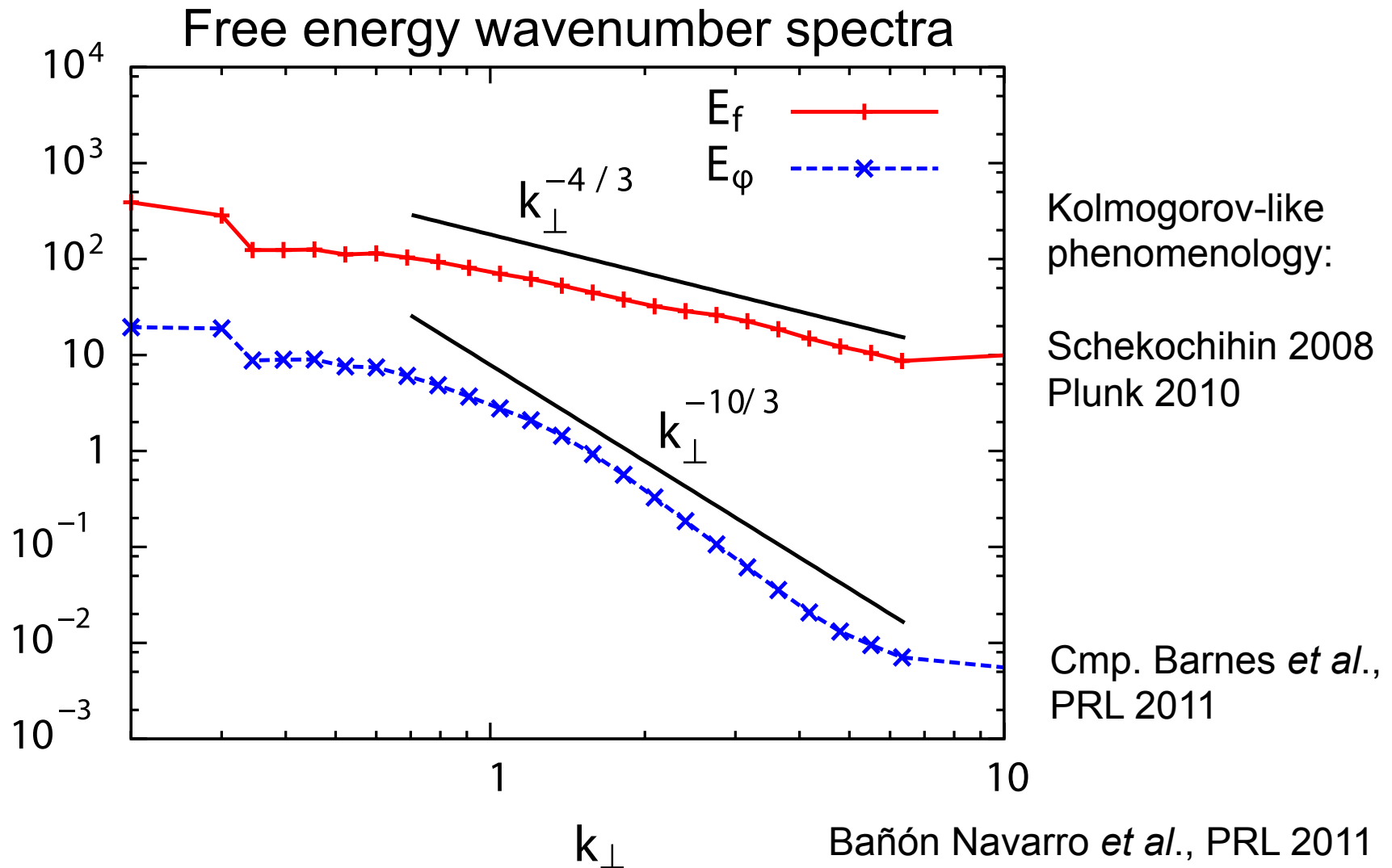
# Multiscale dissipative processes

ITG turbulence (Boltzmann electrons)



Teaca *et al.*,  
PRL 2012

# Approximate power law spectra





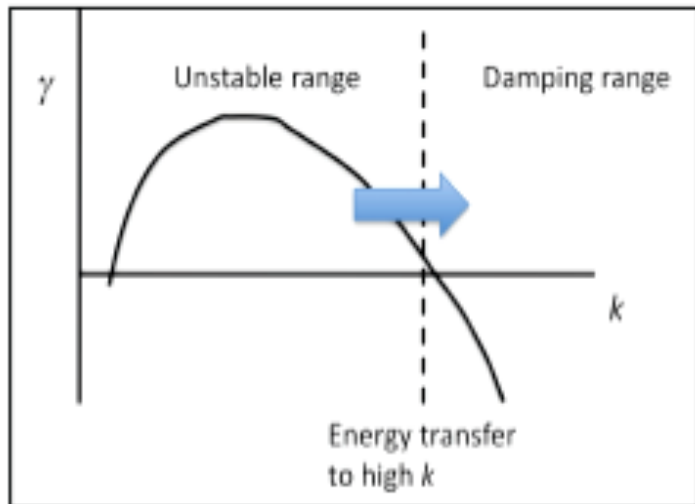
Nonuniversal  
power laws?

# Turbulence w/ multiscale drive/damping

Simple 1D model: **Kuramoto-Sivashinsky equation** (linked to CGLE)

$$\frac{\partial u(x, t)}{\partial t} = -u(x, t) \frac{\partial u(x, t)}{\partial x} - \mu \frac{\partial^2 u(x, t)}{\partial x^2} - \nu \frac{\partial^4 u(x, t)}{\partial x^4}$$

$$\frac{\partial \hat{u}(k_n, t)}{\partial t} = -\frac{1}{2} i k_n \sum_{m \in \mathbb{Z}} \hat{u}(k_n - k_m, t) \hat{u}(k_m, t) + (\mu k_n^2 - \nu k_n^4) \hat{u}(k_n, t)$$



**Modification: Constant damping rate at high  $k$**

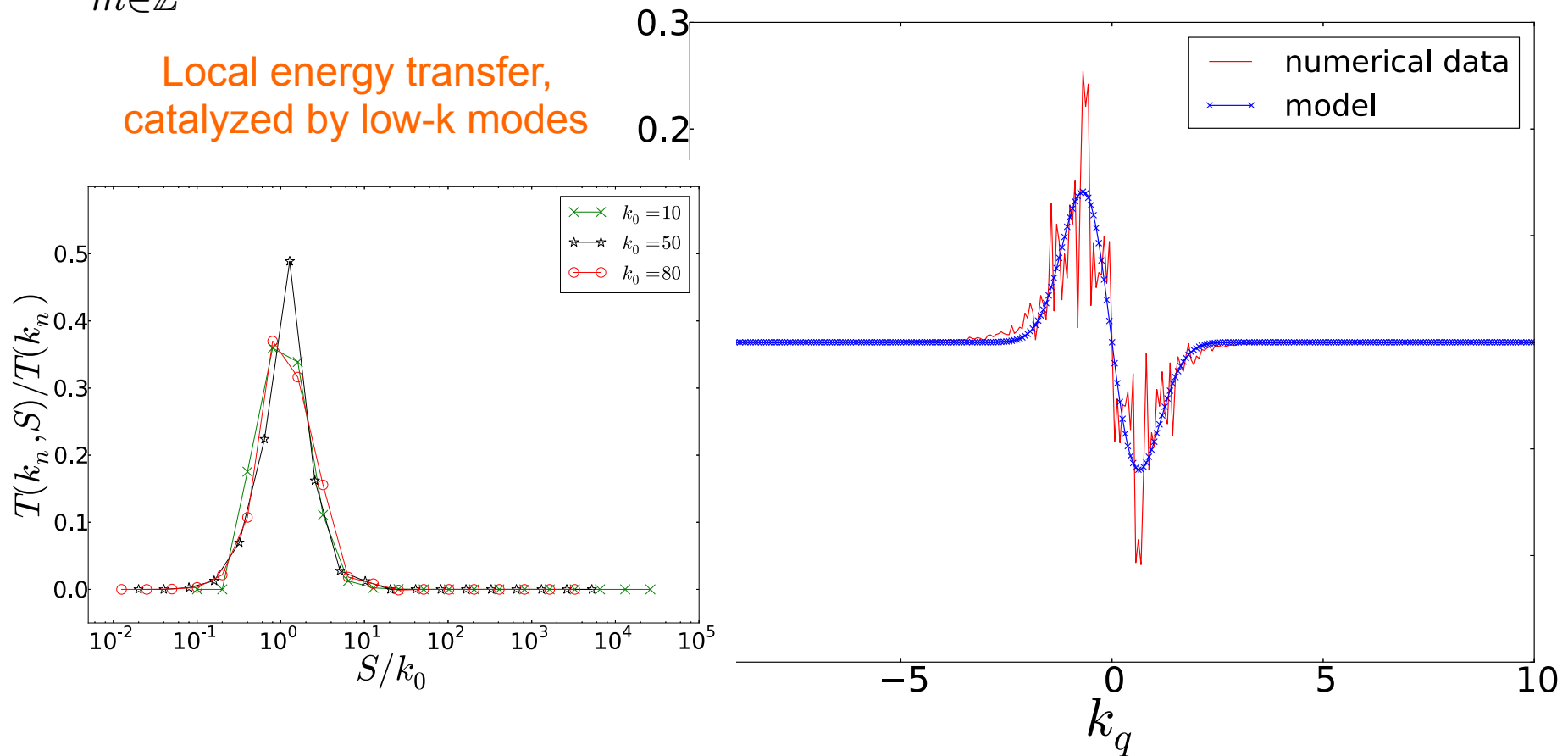
$$\mu k_n^2 - \nu k_n^4 \rightarrow (\mu k_n^2 - \nu k_n^4) / (a + b k_n^4)$$

# Nonlinear energy transfer

Energy balance (in quasi-stationary turbulent state)

$$k_n \sum_{m \in \mathbb{Z}} \Im \left( \overline{\langle \hat{u}(k_n, t) \hat{u}(k_n - k_m, t) \hat{u}(k_m, t) \rangle_\tau} \right) + 2 \frac{\mu k_n^2 - \nu k_n^4}{1 + b k_n^4} E(k_n) = 0$$

Local energy transfer,  
catalyzed by low-k modes



# Nonuniversal power laws

An analytical closure yields...

$$-2 \frac{a_1 a_2}{\Delta k} \sqrt{2\pi a_3} k \frac{dE}{dk} + 2 \frac{\mu k^2 - \nu k^4}{1 + b k^4} E(k) = 0$$

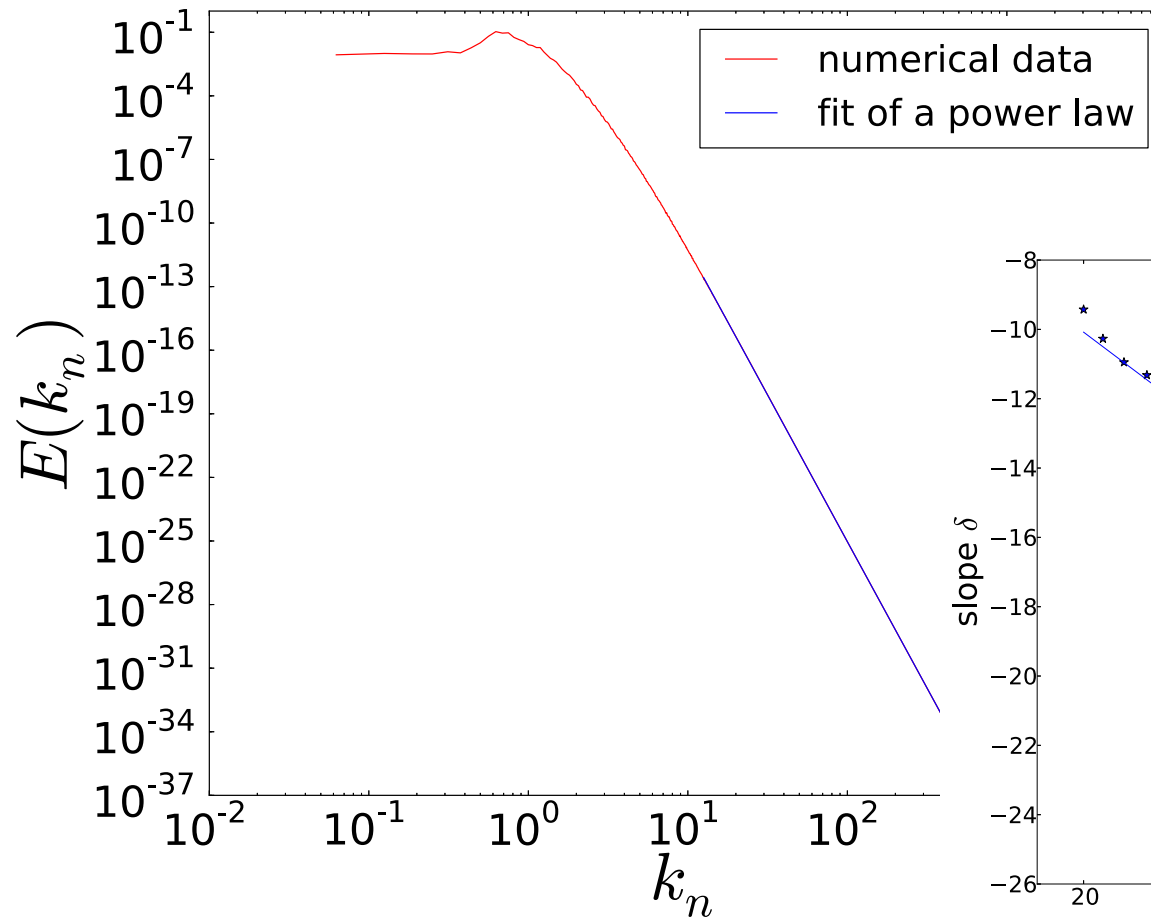
The exact solution of this equation reads...

$$E(k) = \tilde{E}_0 \exp \left( \frac{\lambda \mu}{\sqrt{b}} \arctan(\sqrt{b} k^2) - \frac{\lambda \nu}{2b} \ln(1 + b k^4) \right)$$
$$\lambda = \Delta k / (2a_1 a_2 \sqrt{2\pi a_3})$$

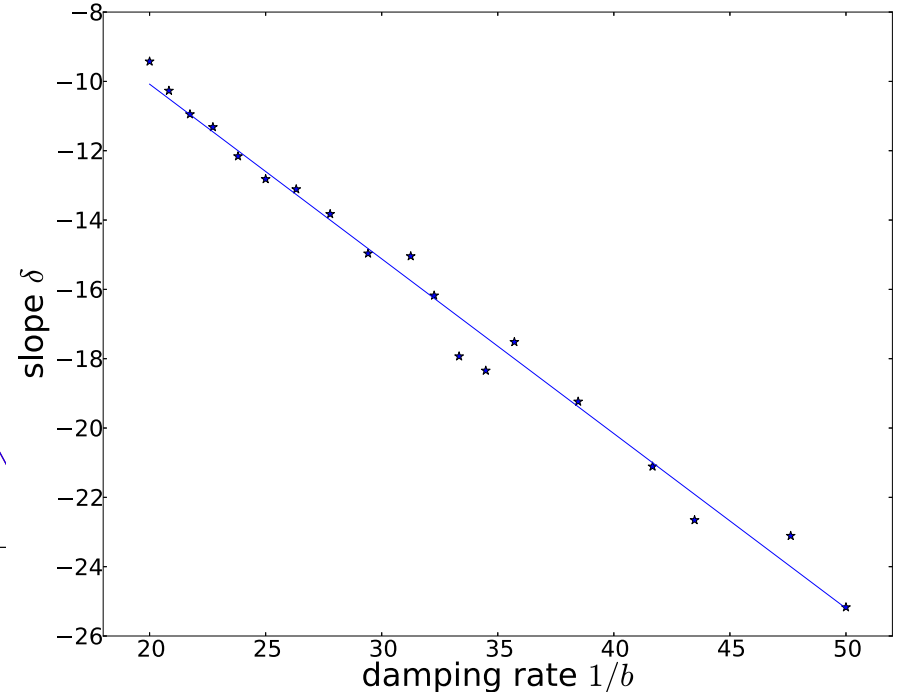
High-k limit:  $E(k) = E_0 k^{-2\lambda\nu/b}$

Spectral exponent is proportional to high-k damping rate!

# Confirmation by numerical simulation



Direct numerical simulations  
confirm analytical prediction

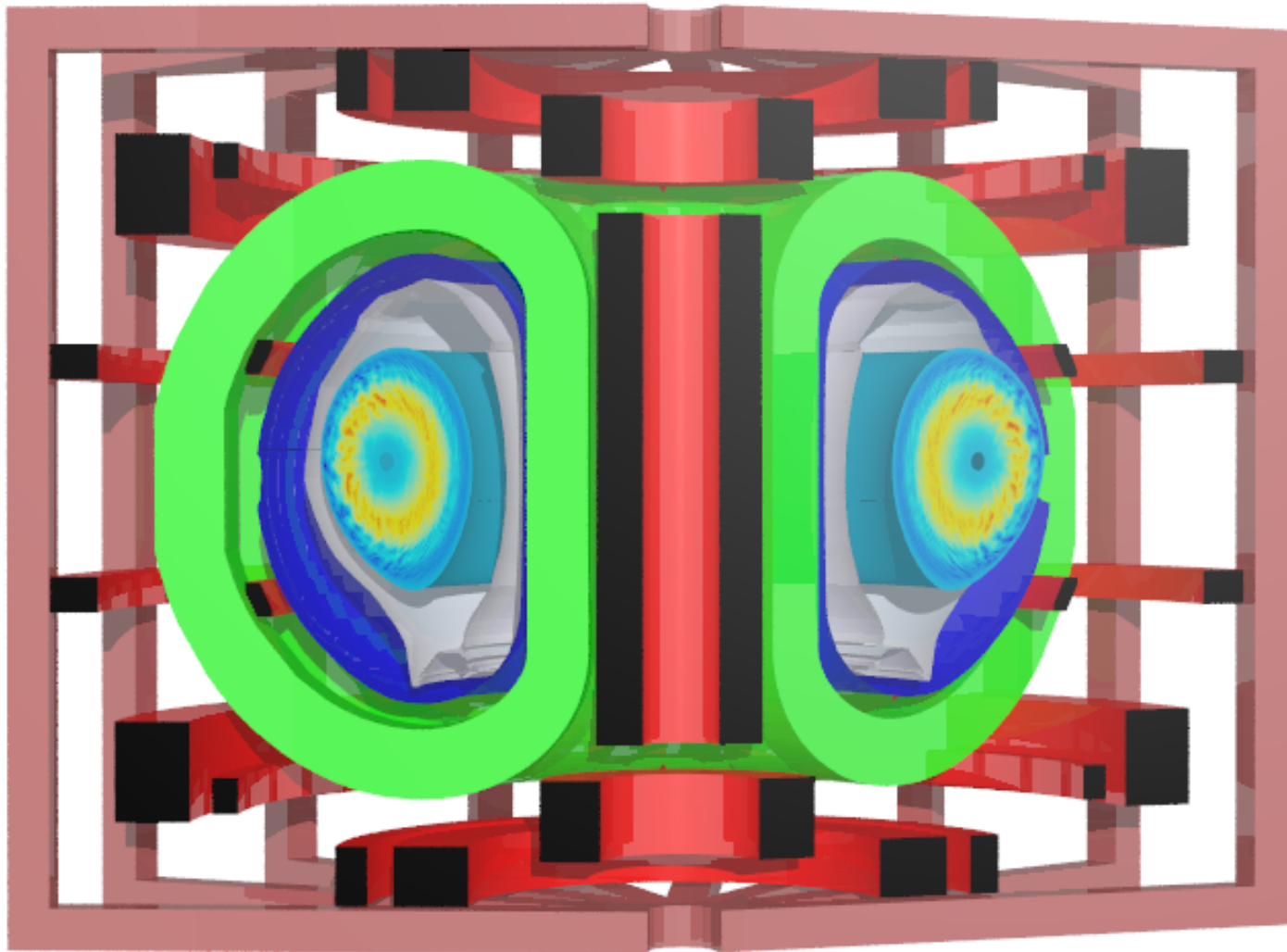


Bottom line: *Nonuniversal power laws* in a certain spectral range if the ratio of nonlinear and linear (damping) time scales is (roughly) scale-independent.



# Future challenges

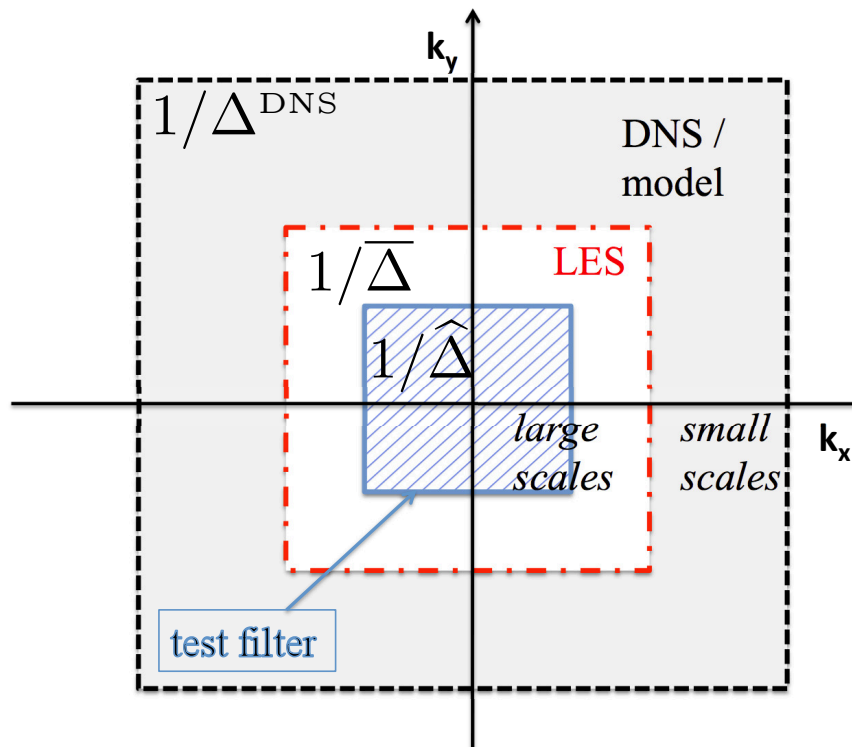
# Gyrokinetics for laboratory plasmas



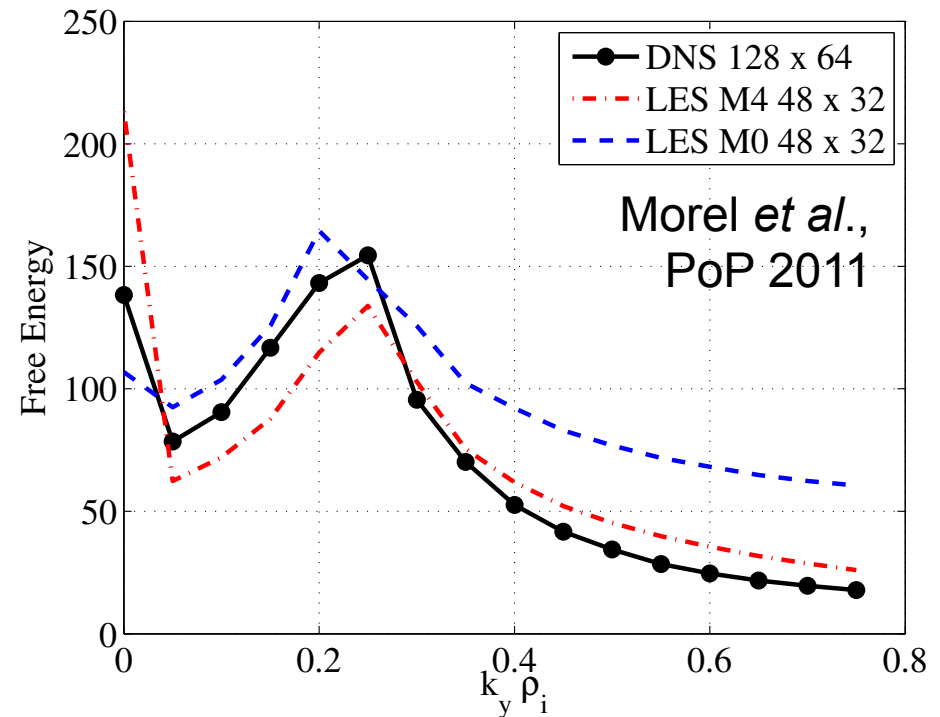
Simulation of ASDEX Upgrade with GENE (<http://gene.rzg.mpg.de>)

# Dimensional reduction in gyrokinetics

Schematic of „dynamic procedure“



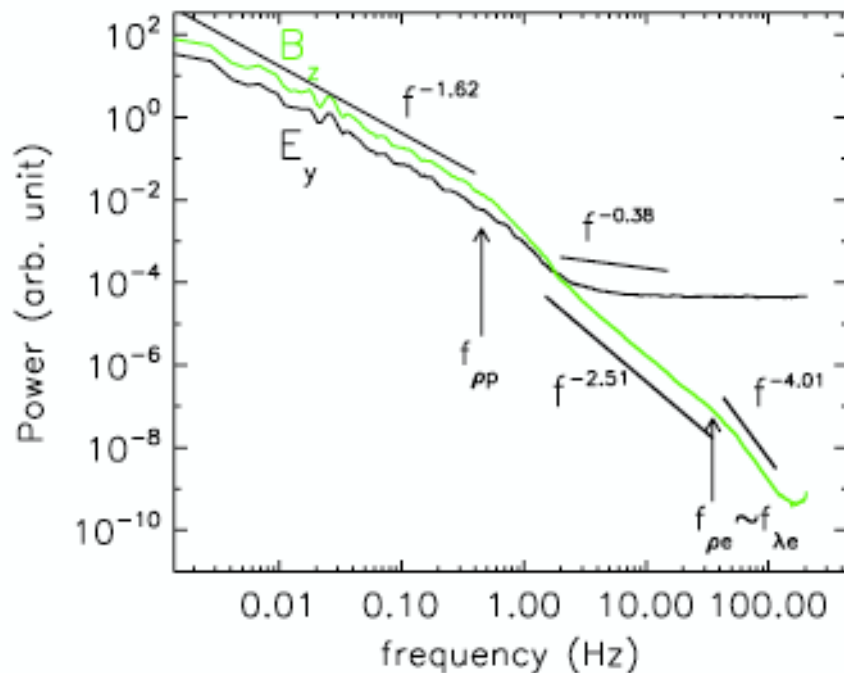
Free energy spectra  
(w/ and w/o model)



LES techniques are likely to reduce the simulation effort substantially without introducing many free parameters. This offers interesting perspectives...

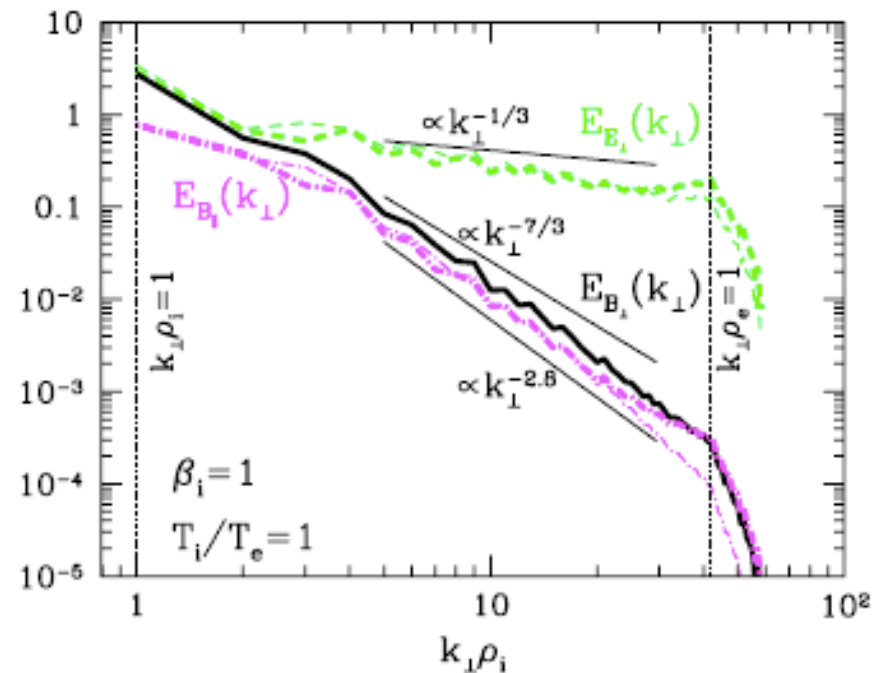
# (Gyro-)Kinetics for natural plasmas: The solar wind dissipation range

Sahraoui *et al.*, PRL 2009



Cluster spacecraft measurements

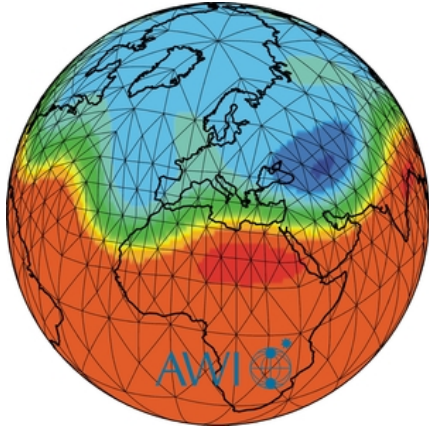
Howes *et al.*, PRL 2011



Gyrokinetic simulations below  $\rho_i$

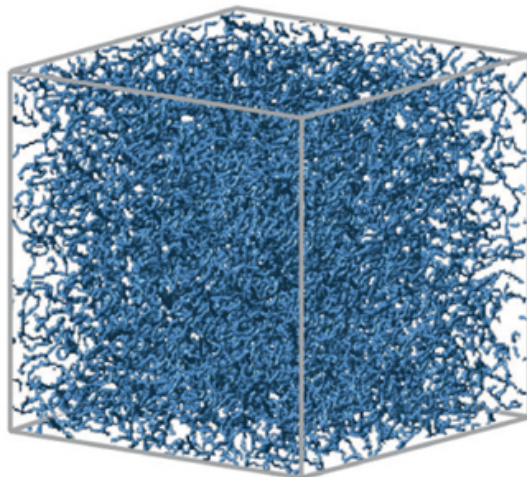
Role of linear waves in a turbulent environment?

# Some other turbulence-wave systems



Turbulence in planetary atmospheres: Rossby waves

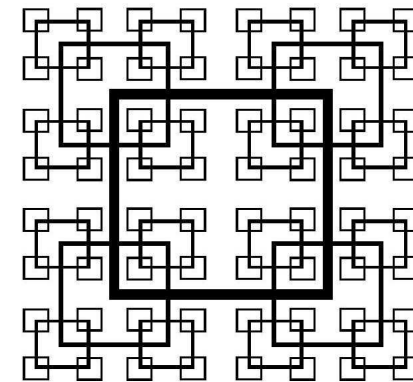
Turbulence in oceans:  
Water surface waves



Turbulence in quantum fluids:  
Kelvin waves on vortex filaments

# Beyond Richardson and Kolmogorov: Multi-scale driven/damped turbulence

- Turbulence behind space-filling square fractal grids
- Turbulence in biological systems
- Instability-driven turbulence in laboratory and astrophysical plasmas



Turbulence...

...a fascinating and challenging  
example of nonlinear dynamics  
in non-equilibrium systems

...our view keeps evolving

