VIRIATO: a Fourier-Hermite spectral code for strongly magnetised, fluid-kinetic plasma dynamics

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Stability, Energetics, and Turbulent Transport in Astrophysical, Fusion, and Solar Plasmas

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ipfn INSTITUTO DE PLASMAS E FUSÃO NUCLEAR

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, INOVAÇÃO E DO ENSINO SUPERIOR

Introduction

- Key problems (e.g., **turbulence**, **reconnection**) have seen tremendous progress over the last decade or so due to a combination of *reduced descriptions* [e.g., gyrokinetics (GK)] and *high-performance-computing*.
- GK is (numerically) simpler than full kinetics but two-species, multiscale simulations remain very challenging.
- This talk focuses on a new reduced-GK model (KREHM) + new code (VIRIATO). Only 4D (3+1). Goal is *insight*.
- Usefulness of this approach exemplified with detailed analysis of the energetics of reconnection in weakly collisional, strongly magnetised plasmas.

The KREHM model

[Zocco & Schekochihin, PoP **18**, 102309 (2011)]

- Minimal set of fluid-kinetic equations; captures electron Landau damping in *low beta, strongly magnetised* plasmas.
- Rigorous limit of the GK equation when $\beta \sim m_e/m_i$; fully GK ions.

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Vorticity equation

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Vorticity equation

$$\frac{d}{dt}\left(A_\parallel - d_e^2\nabla_\perp^2 A_\parallel\right) = \eta\nabla_\perp^2 A_\parallel - c\frac{\partial\varphi}{\partial z} - \frac{cT_{0e}}{e}\hat{\mathbf{b}}\cdot\nabla\left[\frac{Z}{\tau}(1-\hat{\Gamma}_0)\frac{e\varphi}{T_{0e}} - \frac{\delta T_{\parallel e}}{T_{0e}}\right]$$

Ohm's law

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Ohm's law

$$\boxed{\frac{dg_e}{dt} + v_\parallel\hat{\mathbf{b}}\cdot\nabla\left(g_e - \frac{\delta T_{\parallel e}}{T_{0e}}F_{0e}\right) = C[g_e] + \left(1-\frac{2v_\parallel^2}{v_{the}^2}\right)F_{0e}\hat{\mathbf{b}}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel,}$$

$$\frac{\delta T_{\parallel e}}{T_{0e}} = \frac{1}{n_{0e}}\int d^3\mathbf{v}\,\frac{2v_\parallel^2}{v_{the}^2}g_e$$

Electron drift-kinetic equation

Hermite expansion

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Note no explicit dependence on $v_\perp$
If such a dependence is not introduced by the coll. op., $v_\perp$ can be integrated out => 4D description!

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It is convenient to introduce the Hermite expansion of g_{e}:

$$g_e(x,y,t,v_\parallel) = \sum_{m=0}^{\infty} H_m(v_\parallel/v_{\mathrm{the}}) g_m(x,y,t) \frac{F_{0e}(v_\parallel)}{\sqrt{2^m m!}}$$

Hermite expansion

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$$\frac{d\hat{g}_m}{dt} + v_{\rm the} \hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{m+1}{2}} \hat{g}_{m+1} + \sqrt{\frac{m}{2}} \hat{g}_{m-1} - \delta_{m,1} \hat{g}_2 \right)$$
$$= -\sqrt{2} \delta_{m,2} \hat{\mathbf{b}} \cdot \nabla \frac{e}{cm_e} d_e^2 \nabla_\perp^2 A_\| - \nu_{ei} \left(m \hat{g}_m - 2\delta_{m,2} \hat{g}_2 \right).$$

Coupled set of fluid-like equations for the Hermite coeficients

Closure

- In a simulation, must chose how many m's to keep. What to do about g_{M+1}?
- Lots of work done on this in the past (Hammett, Dorland, Beer, Smith, etc.)
- Adding collisions/hyper-collisions may provide a satisfactory way to close the system, regardless of the actual closure, though convergence may require many m's.

Closure

Eq. for g_m couples to g_{m+1}. How to close the system at some m=M?

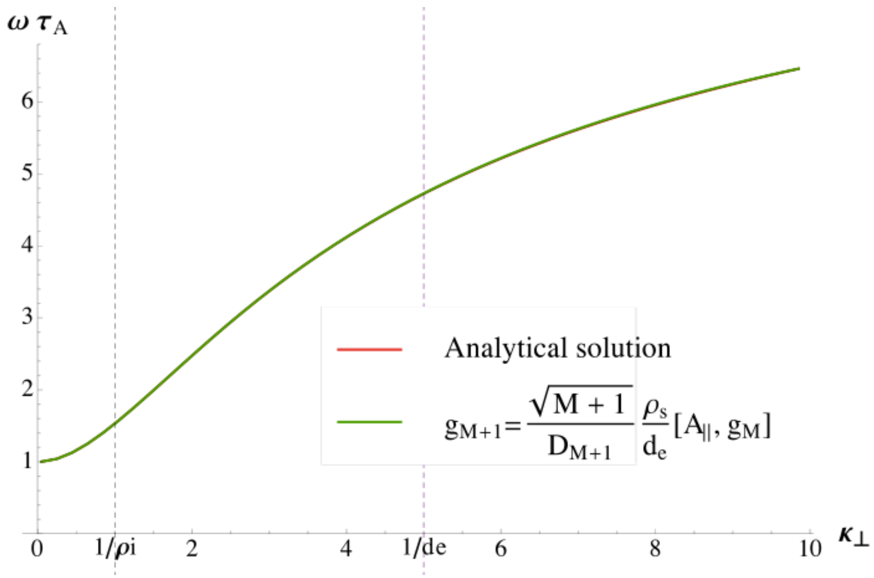
$$\frac{dg_{M+1}}{dt} + v_{\text{the}}\hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{M+2}{2}} g_{M+2} + \sqrt{\frac{M+1}{2}} g_M \right) = D_{M+1} g_{M+1}$$

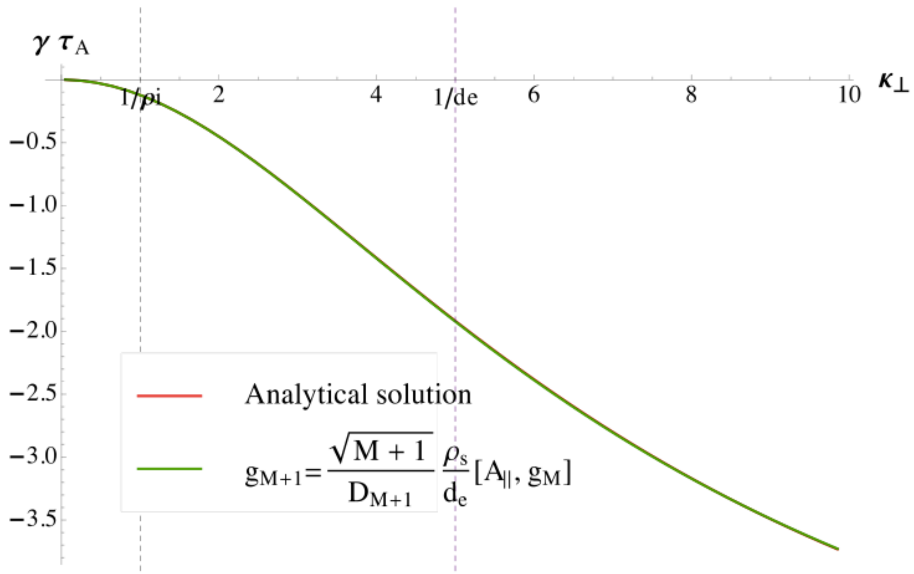
For sufficiently large m=M, must have $D_M \gg \omega \Rightarrow \frac{g_{M+1}}{g_M} \ll 1$

Nonlinear closure:

$$\boxed{g_{M+1} = \frac{v_{\text{the}}}{D_{M+1}} \hat{\mathbf{b}} \cdot \nabla \sqrt{\frac{M+1}{2}} g_M}$$

Closure: linear tests





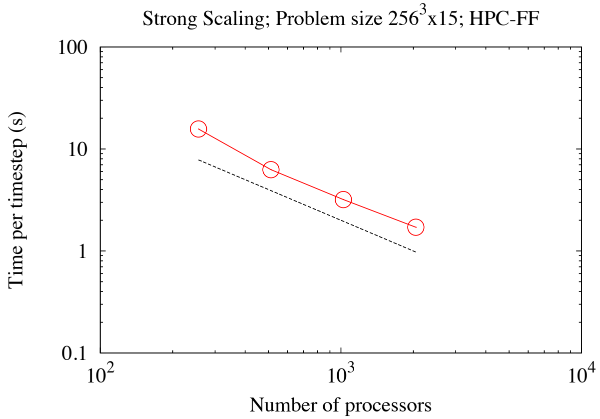
KAW dispersion relation reproduced exactly with M=15 using hypercollisions $D_m = \nu_H (m/M)^4$

(with regular collisions, for $\nu_{ei}\tau_A = 0.1$ need a few hundred m's)

VIRIATO

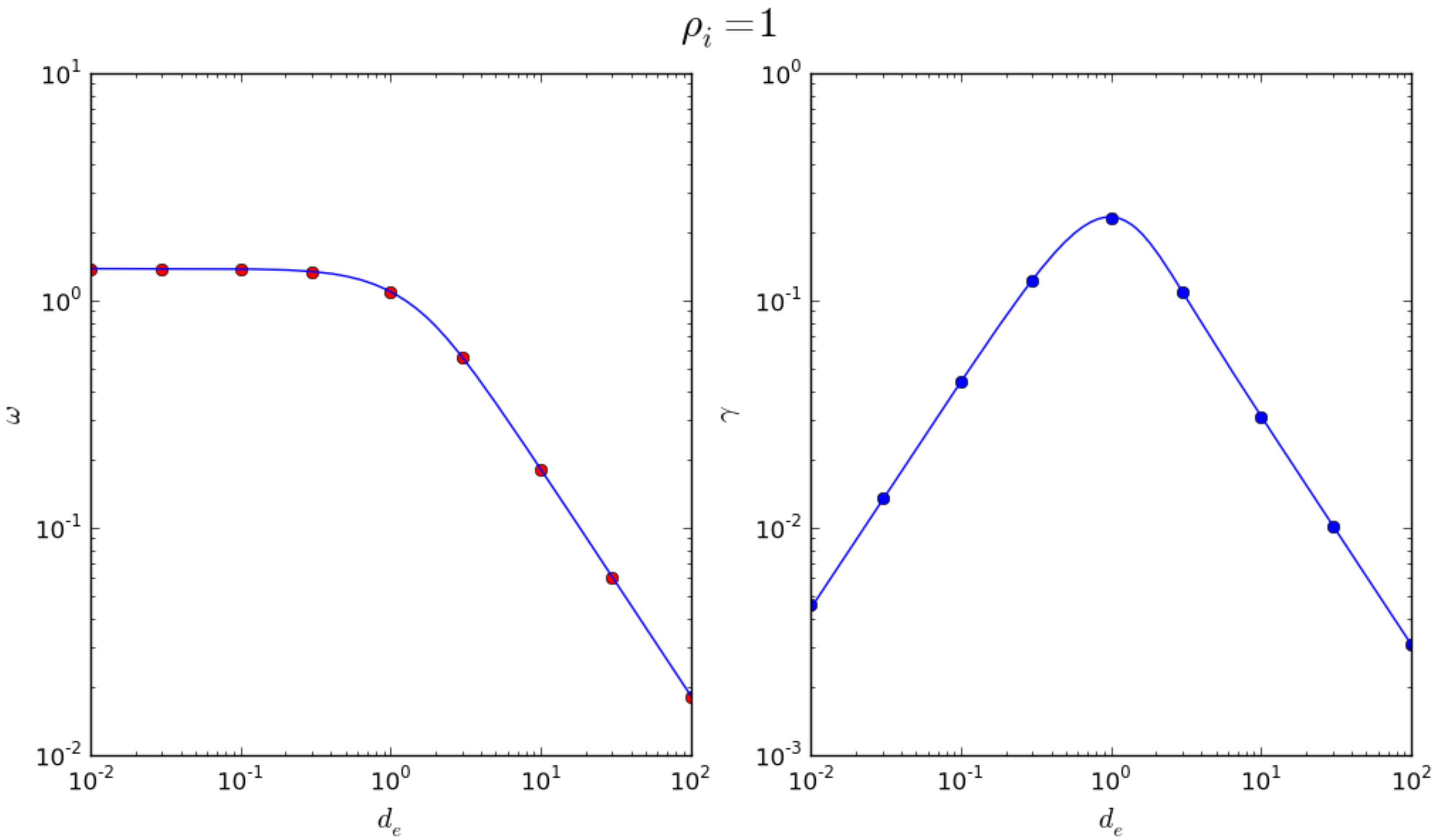
Versatile code to solve three different sets of eqs.:

- Zocco/Schekochihin reduced GK eqs. [Phys. Plasmas **18**, 102309 (2011)]
- Kinetic long wavelength slow mode eqs. [Schekochihin *et al.*, ApJ **182**:310 (2009)]
- RMHD eqs. [Strauss, PoF **19**, 134 (1976)]



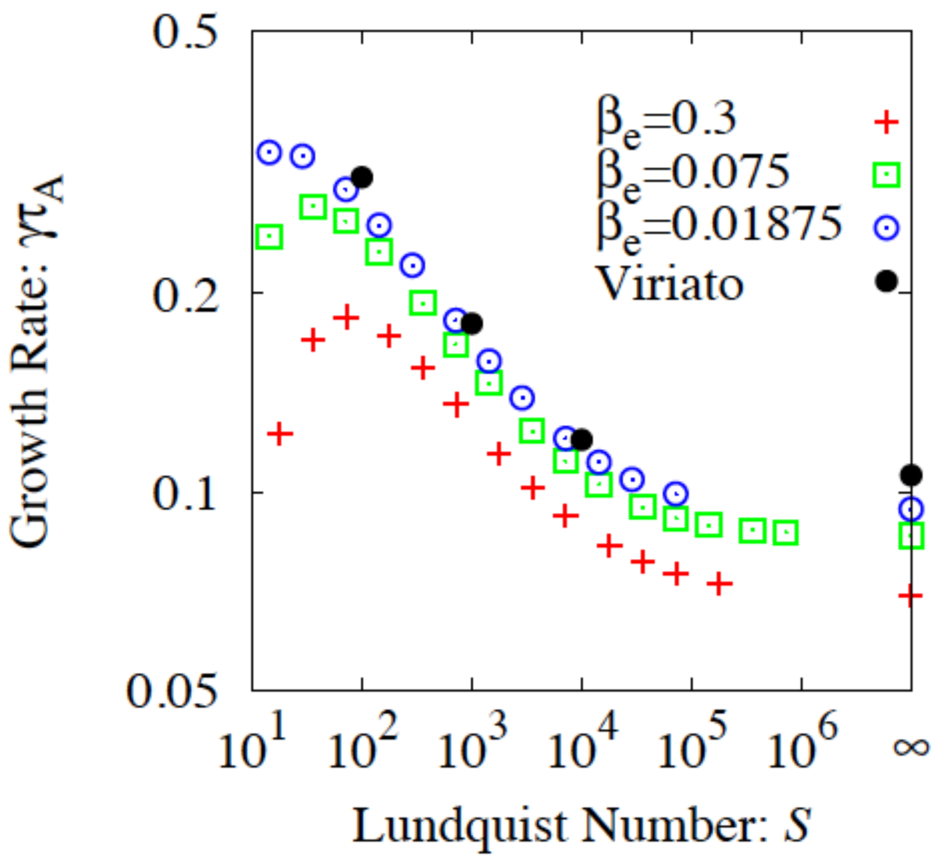
Spectral in x-y (Fourier) and $v_{||}$ (Hermite). Grid in z (MacCormack scheme). Parallel in x-y and z. Strang-split to handle separately perpendicular and parallel terms.
Good scalability (weak and strong) to few thousand processors.

VIRIATO reproduces KAW correctly



Alfred Mallet

VIRIATO agrees with AstroGK



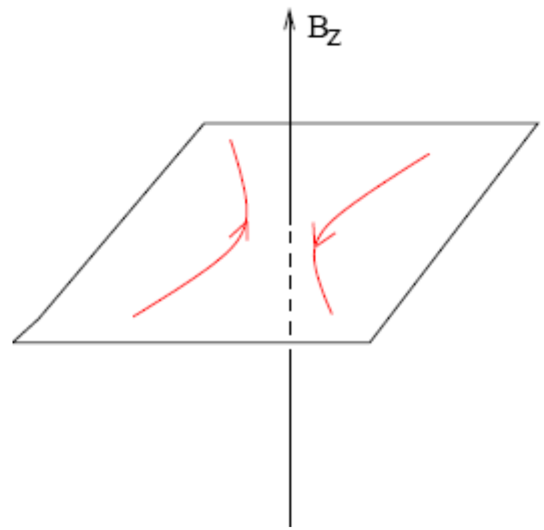
Linear tearing mode benchmark

(see Numata *et al.*, Phys. Plasmas 2011)

Simulations

- 2D doubly-periodic tearing mode setup. Finite amount of energy available.
- Initial magnetic field:

$$B_{eq} = B_z \hat{e}_z + B_y \hat{e}_y$$



- Spatial hyper-diffusion in all eqs. $\sim (k_\perp / k_\perp^{max})^6$
- Hyper-collisions in the g_m eqs. $\sim (m/M)^4$

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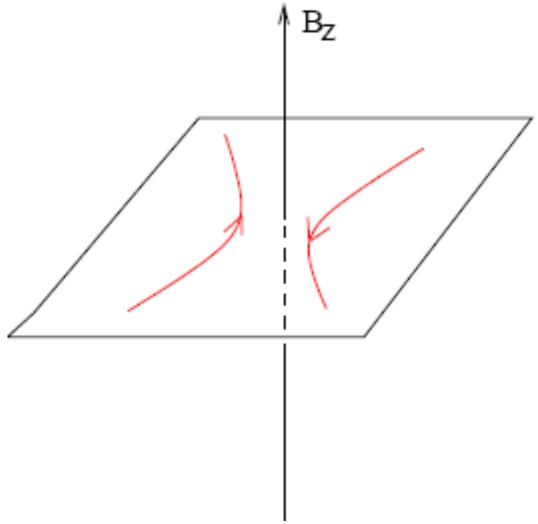

- Spatial hyper-diffusion in all eqs. $\sim (k_\perp / k_\perp^{max})^6$
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Total number of Hermite moments kept. M is proxy for collisions: higher M, less colls.

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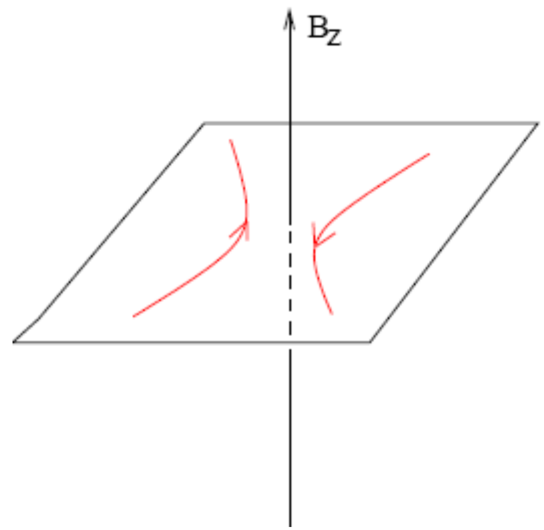

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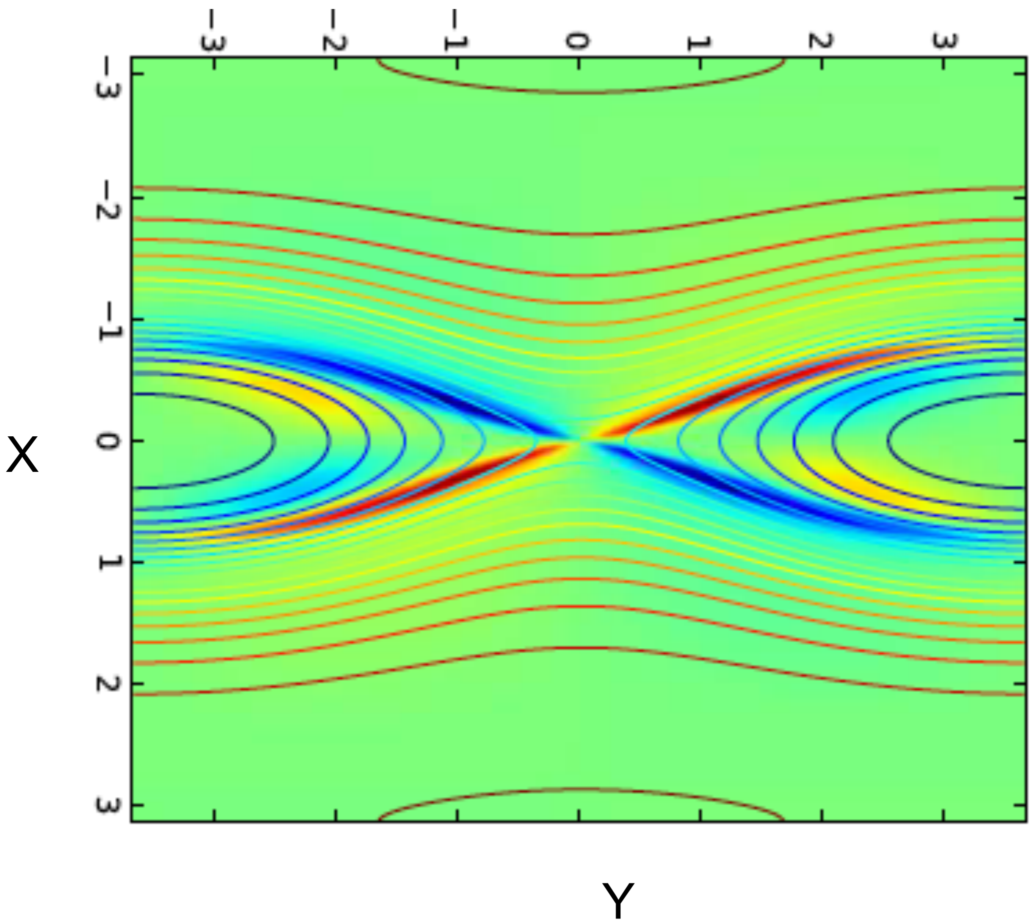
Even small amount of collisions is enough to make the system irreversible.

What happens to the converted magnetic energy?

Typical configuration



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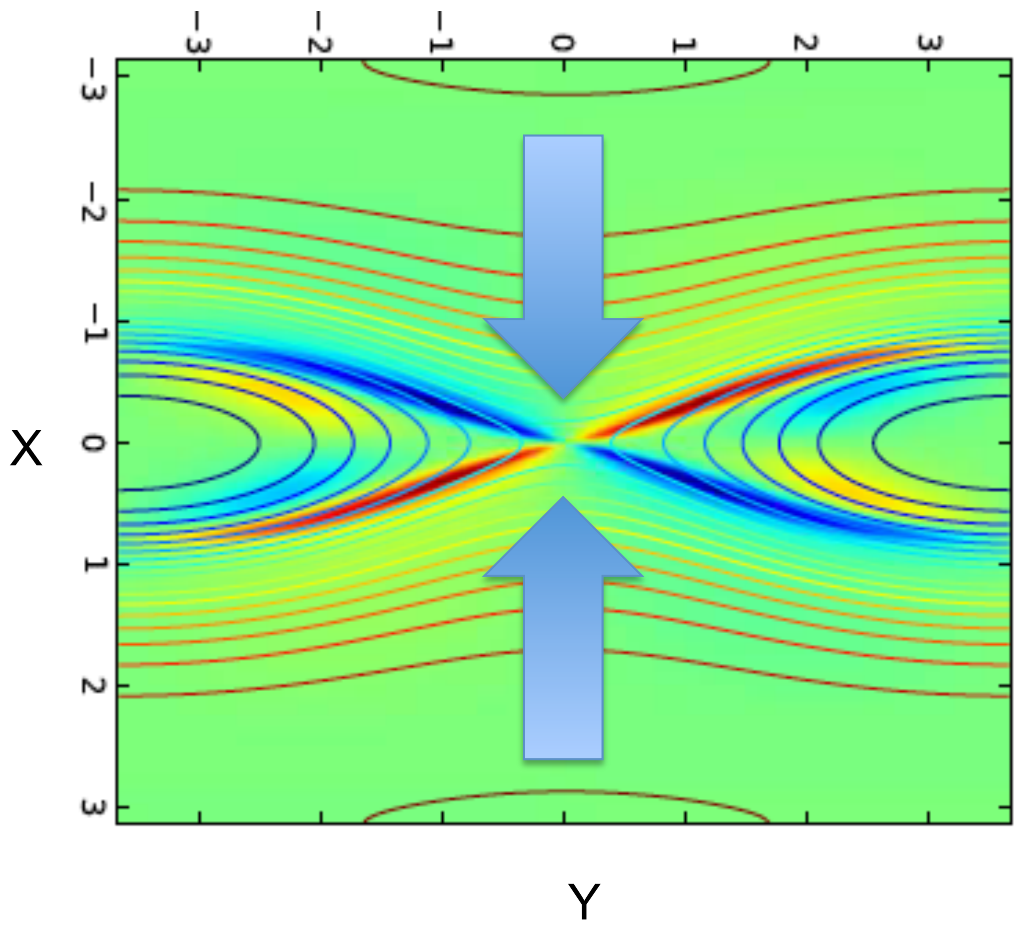

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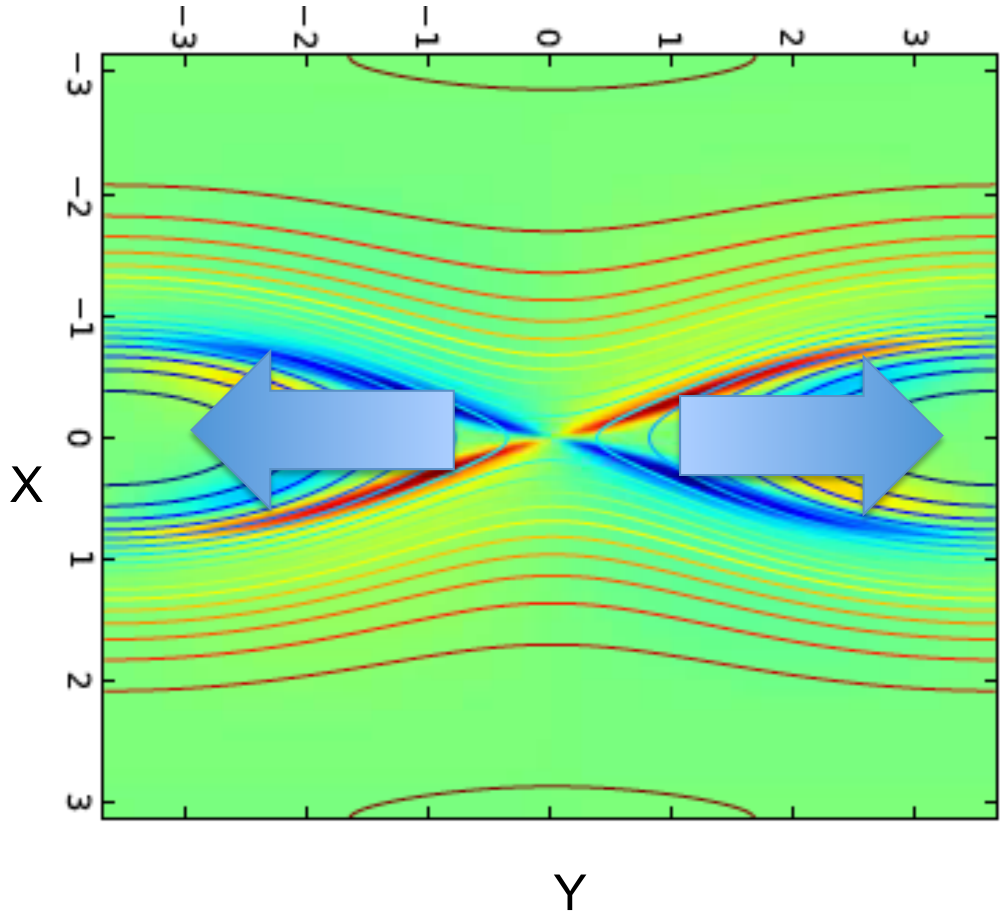

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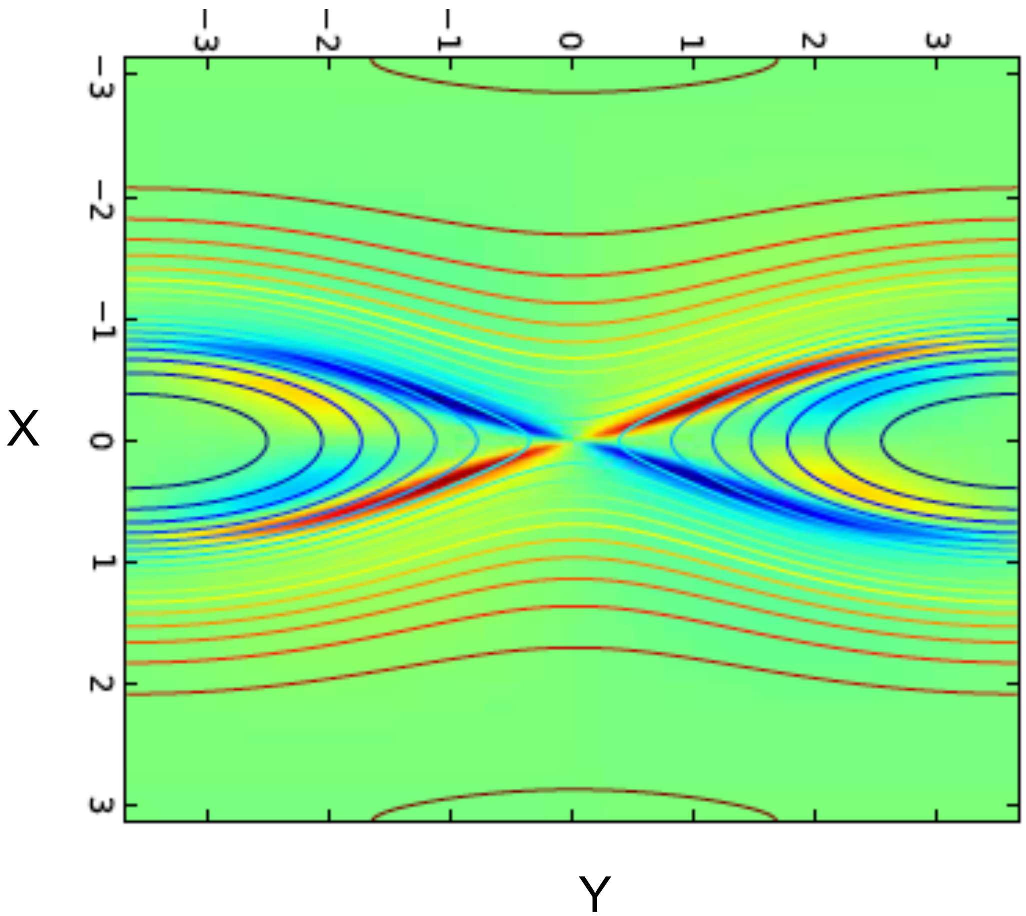


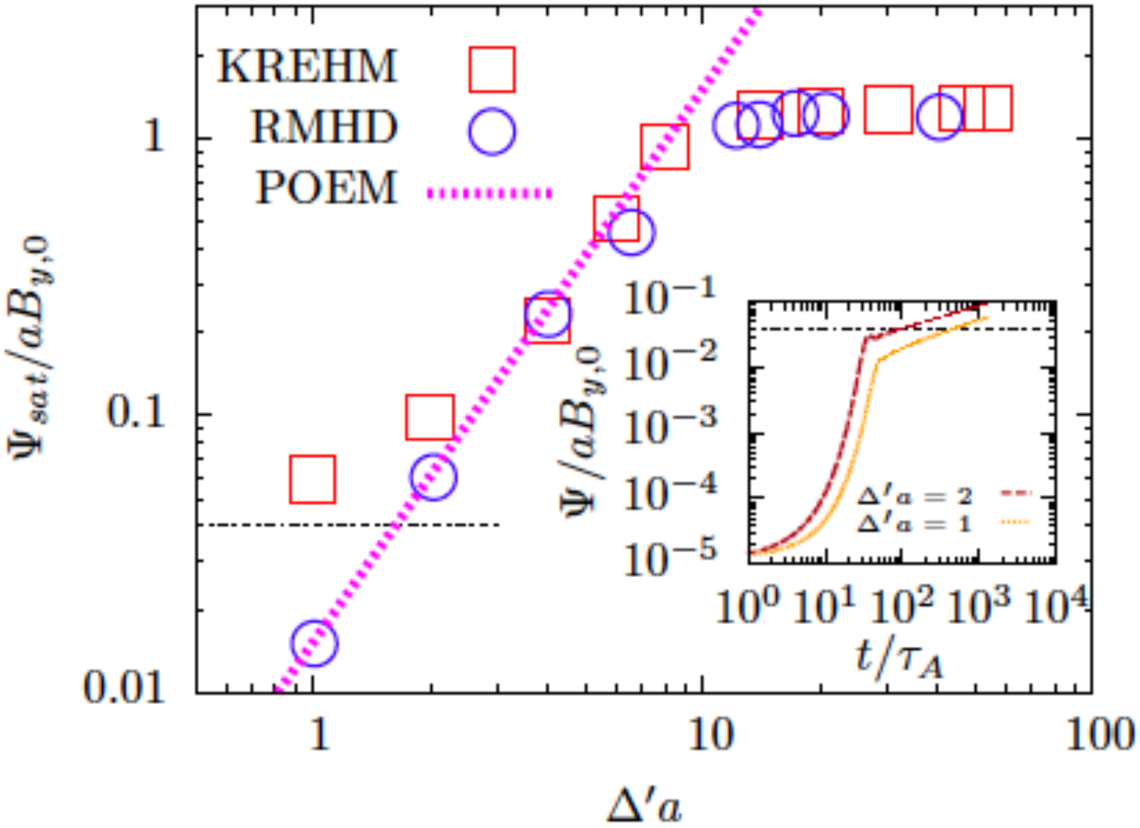
Lines are contours of magnetic flux, $A_{||}$.

Colors are temperature fluctuations – extended along separatrix, where spatial gradients are sharper

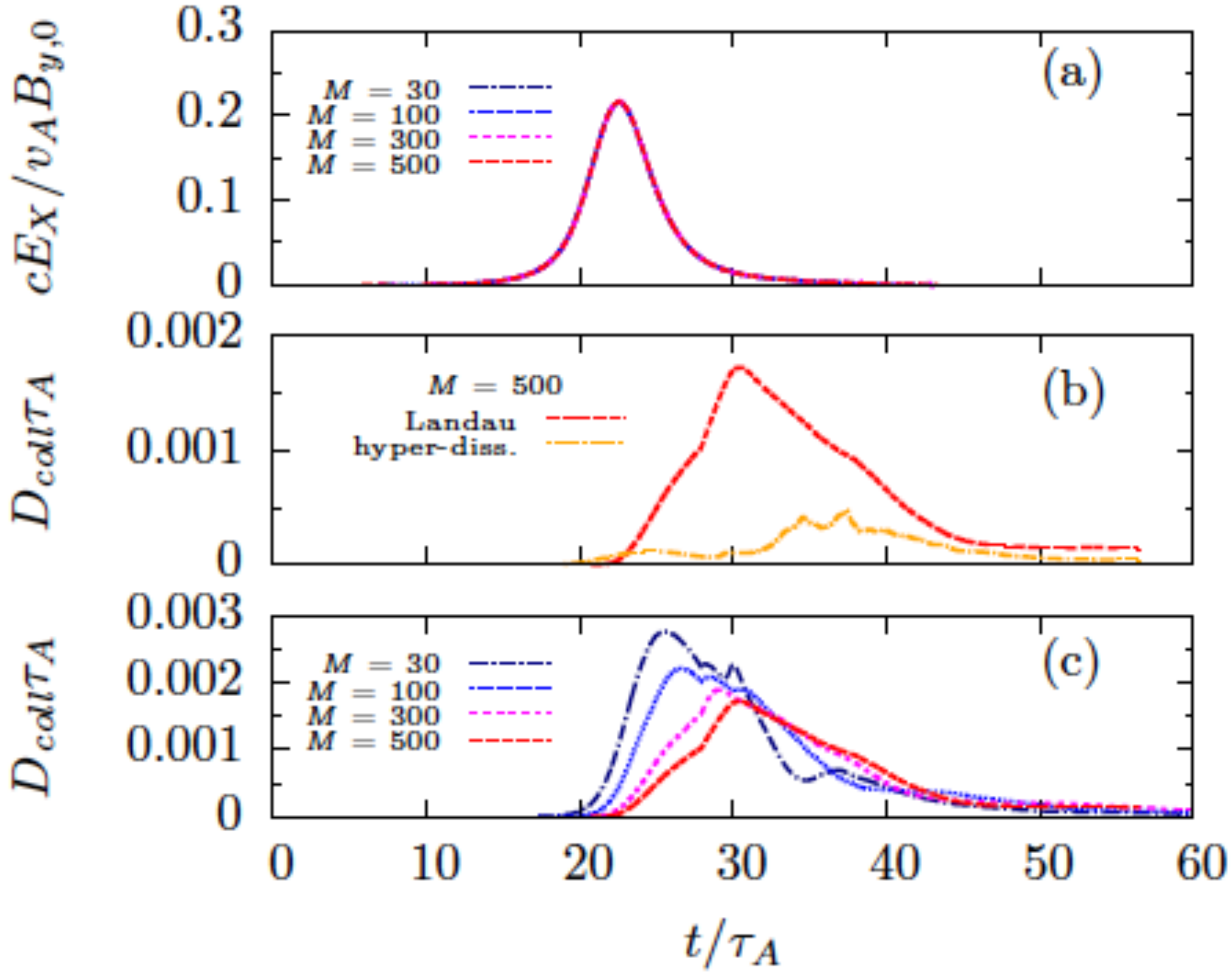
Weakly collisional tearing mode saturation

- Saturation is independent of collisions as long as W_{sat} exceeds the kinetic scales; RMHD solution recovered.

-This implies that the **same amount of magnetic energy is converted during the evolution of the tearing mode, independent of collisions. How does it dissipate in the kinetic case?**

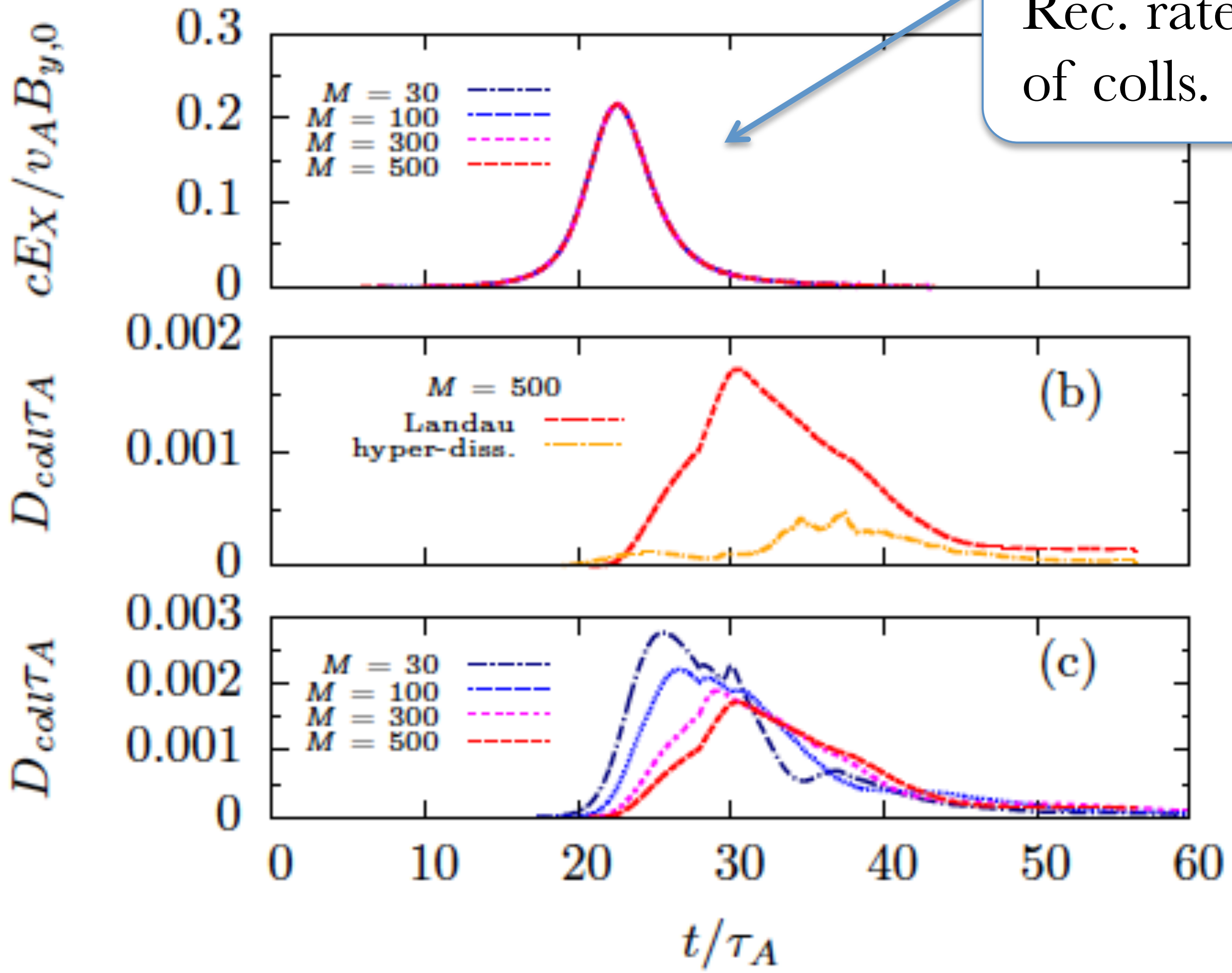


Reconnection and dissipation rates



arXiv:1301.0338

Reconnection and dissipation rates



Rec. rate independent of colls.

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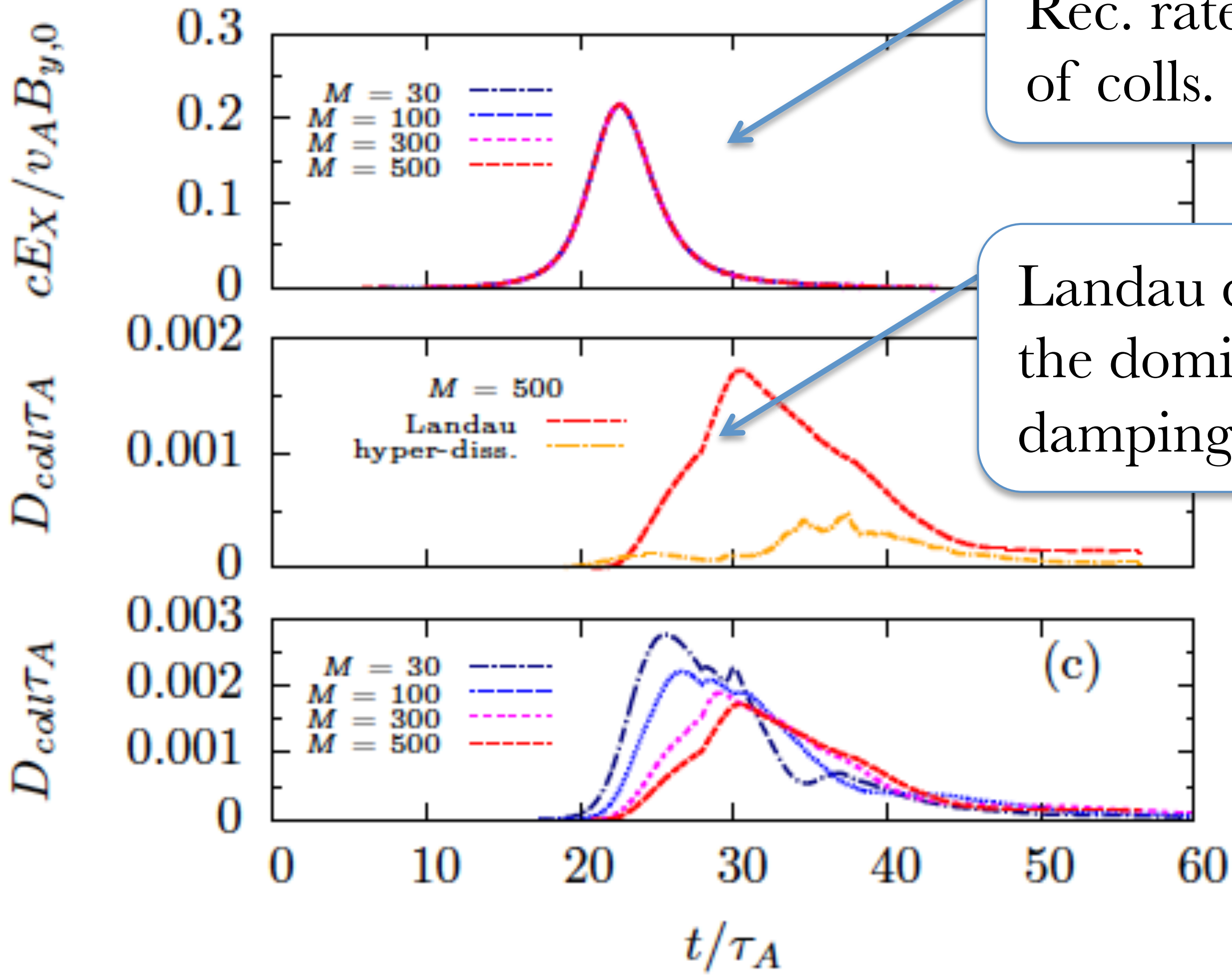

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Landau damping is the dominant damping channel

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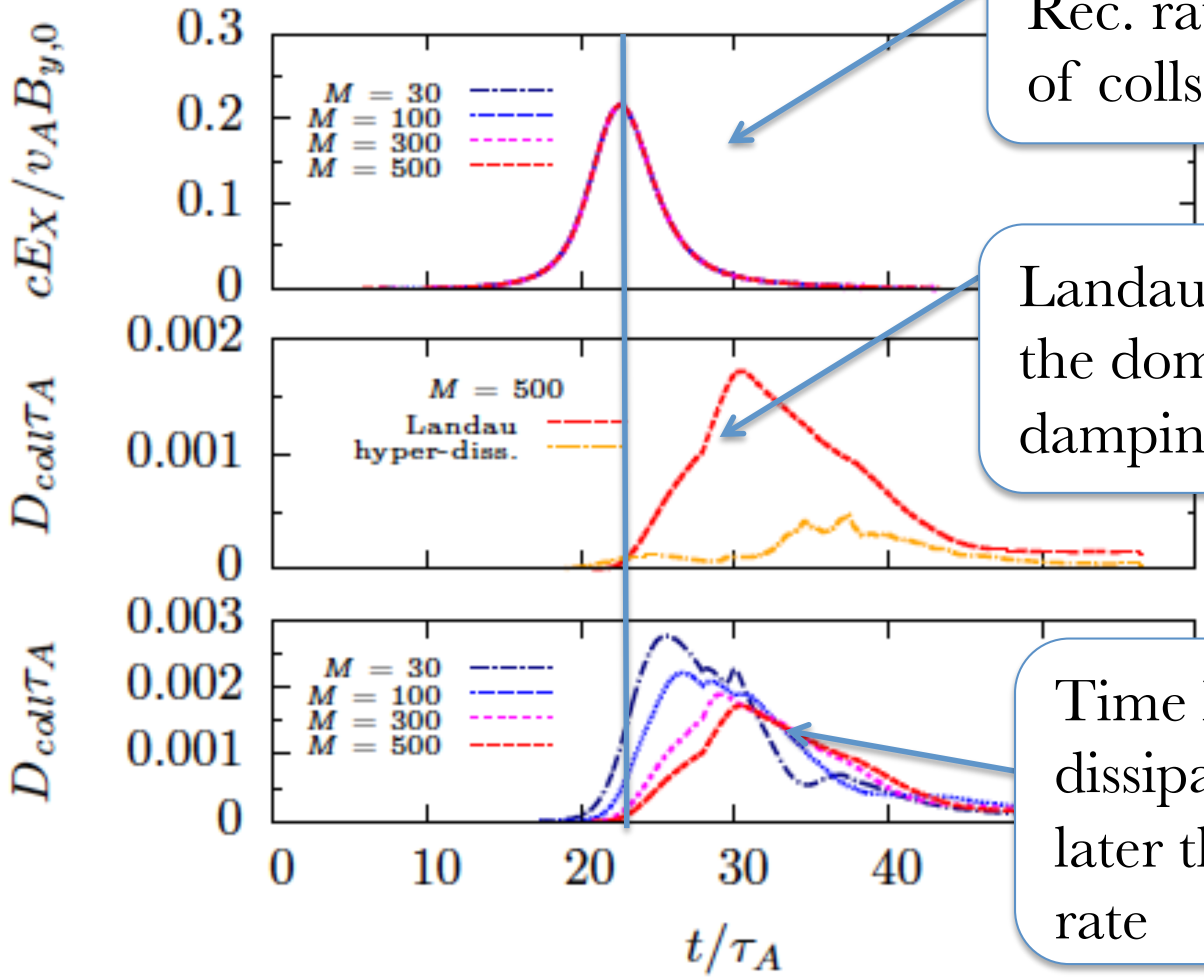

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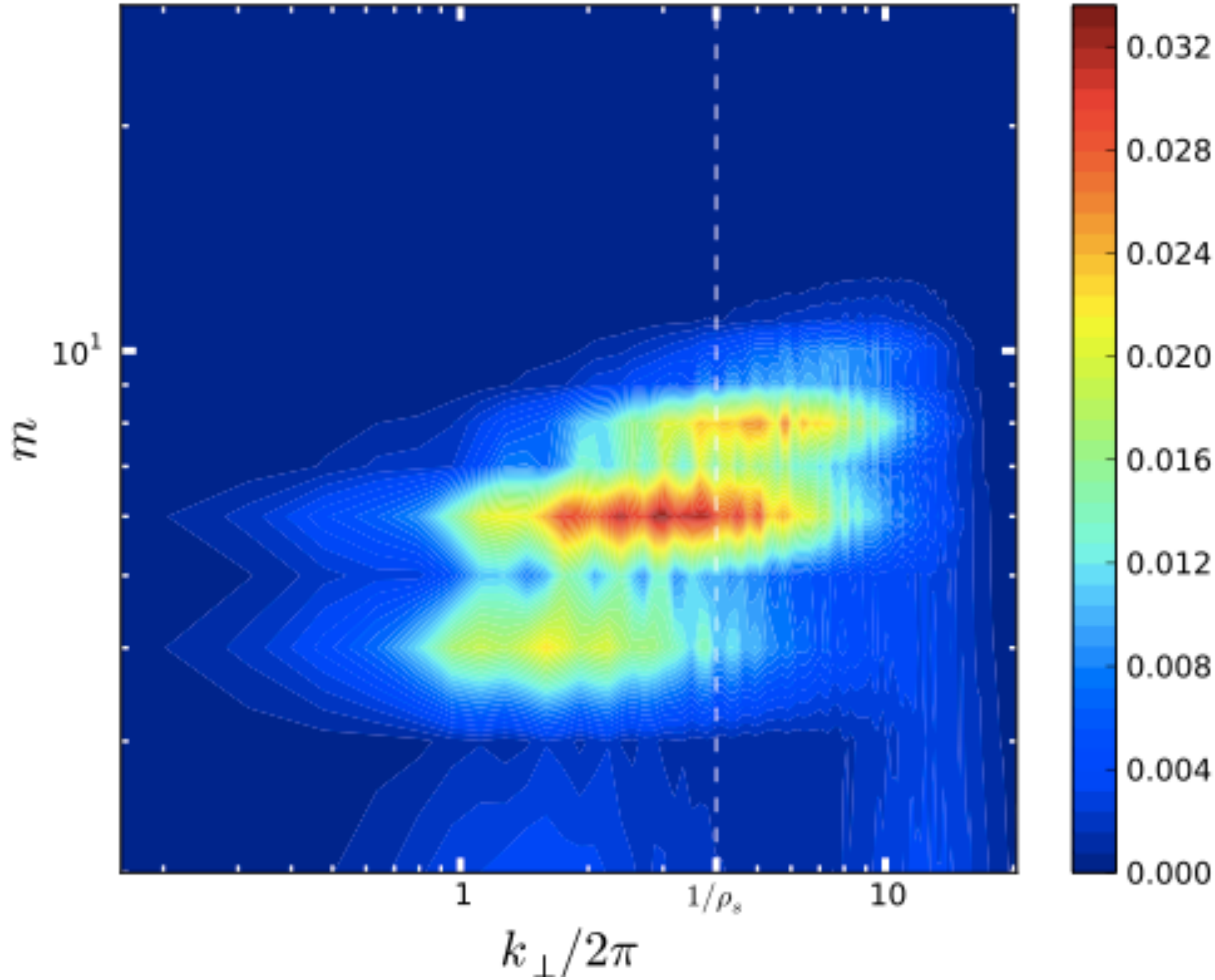
Landau damping is the dominant damping channel

Time lag: Peak dissipation occurs later than max rec. rate

arXiv:1301.0338

Dissipation spectra

M = 30



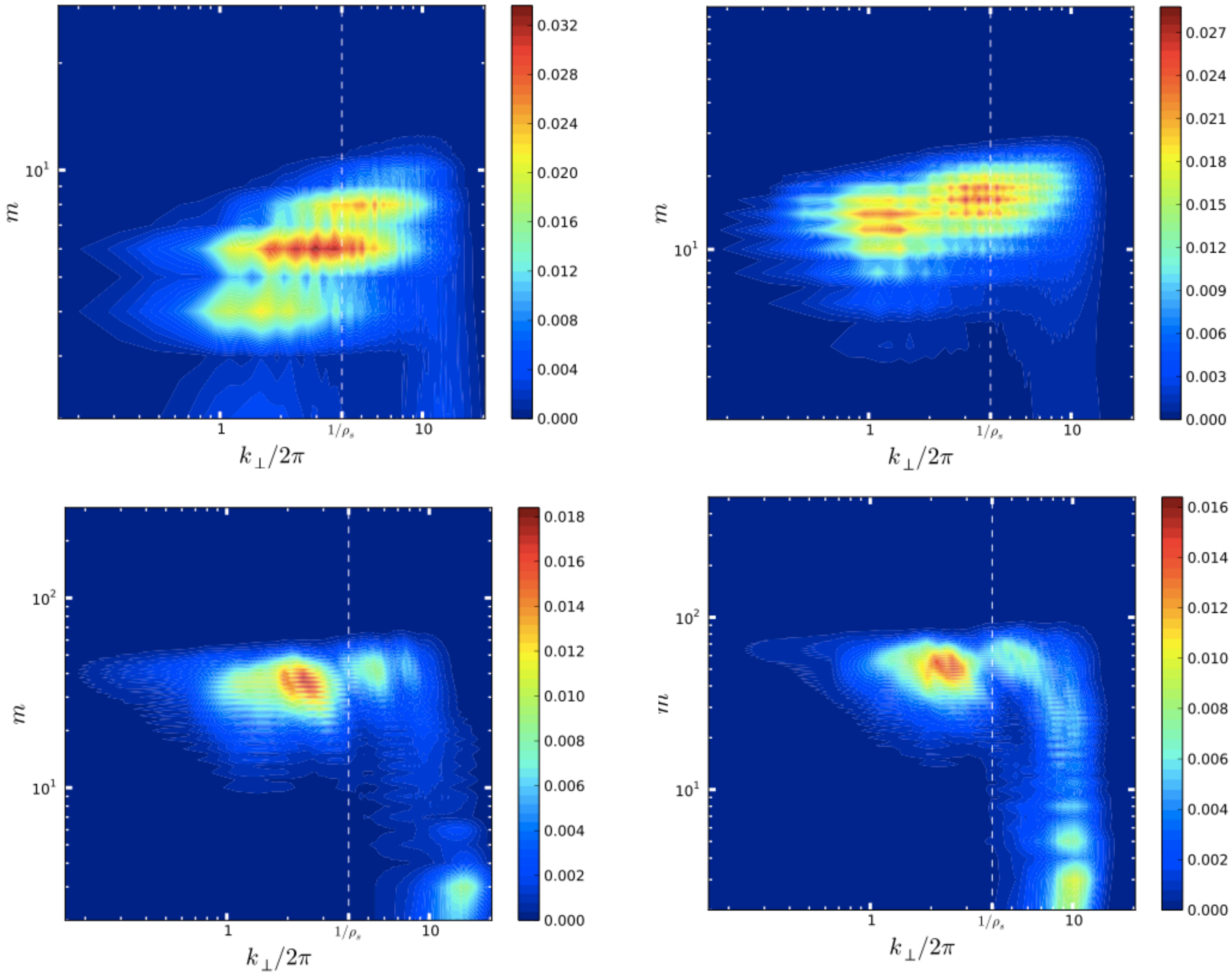
Dissipation spectra

M = 30

M = 100

M = 300

M = 500



Equation for the spectrum

$$\frac{dg_m}{dt} = \frac{v_{the}}{B_z}\left(\sqrt{\frac{m+1}{2}}\left\{A_\parallel, g_{m+1}\right\} + \sqrt{\frac{m}{2}}\left\{A_\parallel, g_{m-1}\right\}\right) - \nu_{coll} m^4 g_m$$

Define the Hermite spectrum as $E_m = |g_m|^2/2$. Linearise; for m>>1

$$\frac{\partial E_m}{\partial t} = -|k_y|\frac{B_y}{B_z} v_{\mathrm{the}} \frac{\partial}{\partial m}\sqrt{2m}E_m - 2\nu_{coll} m^4 E_m.$$

Setting $\partial E_m/\partial t = 2\gamma E_m$, we obtain

$$E_m = \frac{C(k_y)}{\sqrt{m}} \exp\left[-\left(\frac{m}{m_\gamma}\right)^{1/2} - \left(\frac{m}{m_c}\right)^{9/2}\right],$$

$$m_\gamma = \left(k_y \frac{B_y}{B_z}\frac{v_{the}}{2\sqrt{2}\gamma}\right)^2 \qquad m_c = \left(\frac{9}{2\sqrt{2}} k_y \frac{B_y}{B_z}\frac{v_{the}}{\nu_{coll}}\right)^{2/9}$$

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Does NOT depend on collisions $\Longrightarrow$ While the mode is strongly growing, there can be no collisional dissipation

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Does NOT depend on collisions $\Rightarrow$ While the mode is strongly growing, there can be no collisional dissipation

- Since $\gamma \to 0$ as the tearing mode approaches saturation, there will always be a time when $m_\gamma > m_c$

- From then onwards, cutoff is determined by m_c

- In our simulations, this happens before the time of maximum dissipation rate, independent of M, so the cutoff should be determined by m_c

Maximum dissipation

The value of m=m_{peak} at which most energy is dissipated is the solution of:

$$d(\nu_{coll} m^4 E_m)/dm = 0, \qquad m_\gamma \gg m_c$$

This yields:

$$m_{peak} = (9/7)^{2/9} m_c$$

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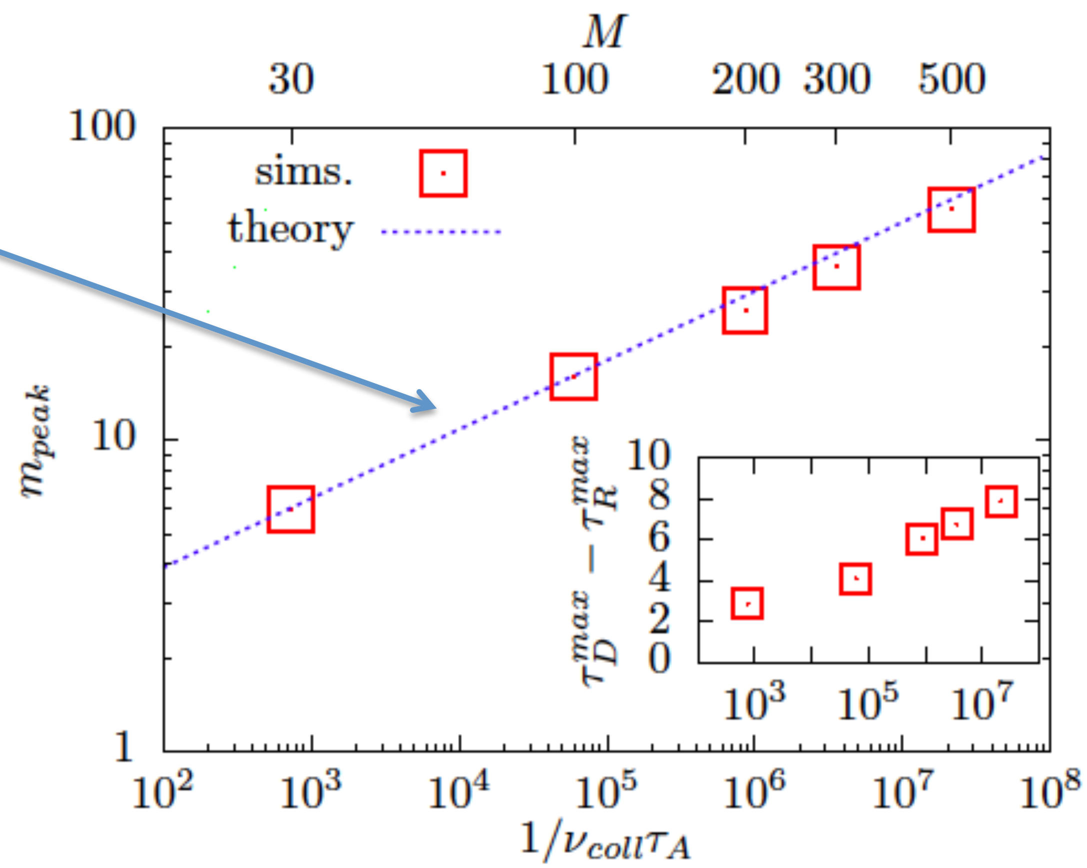
The electron heating we observe is the result of ***linear*** *phase-mixing*



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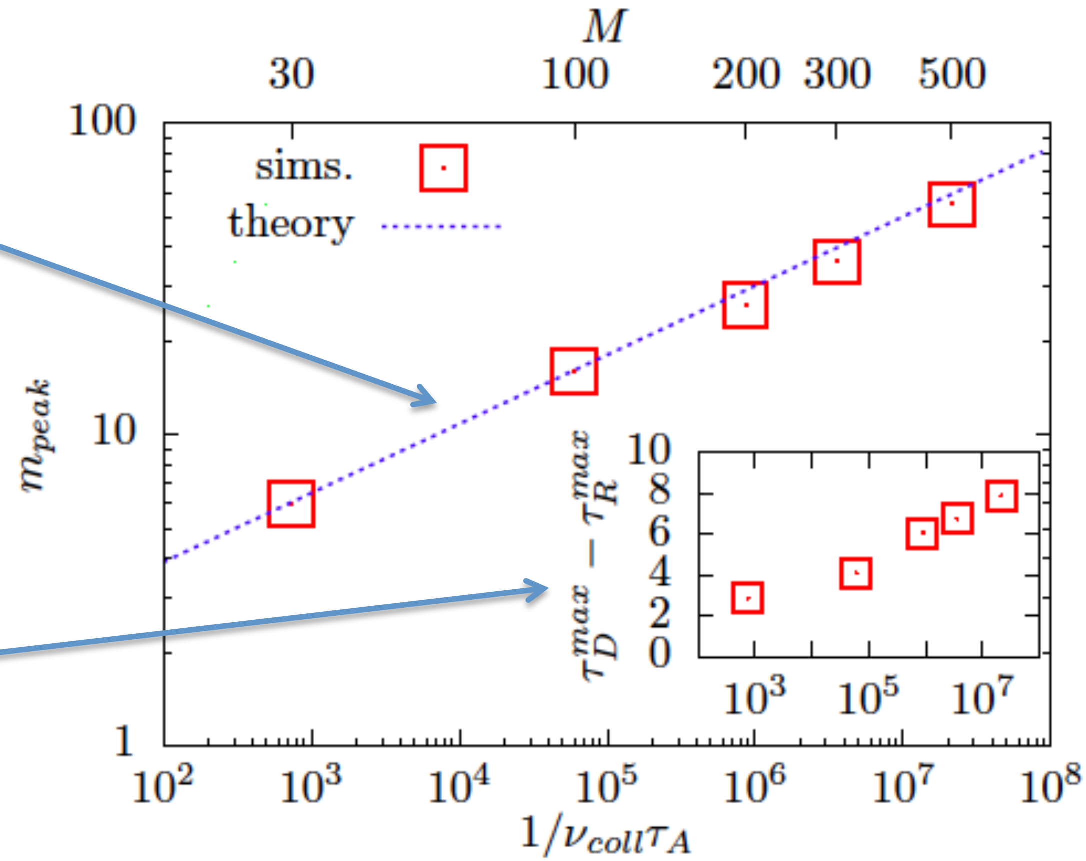
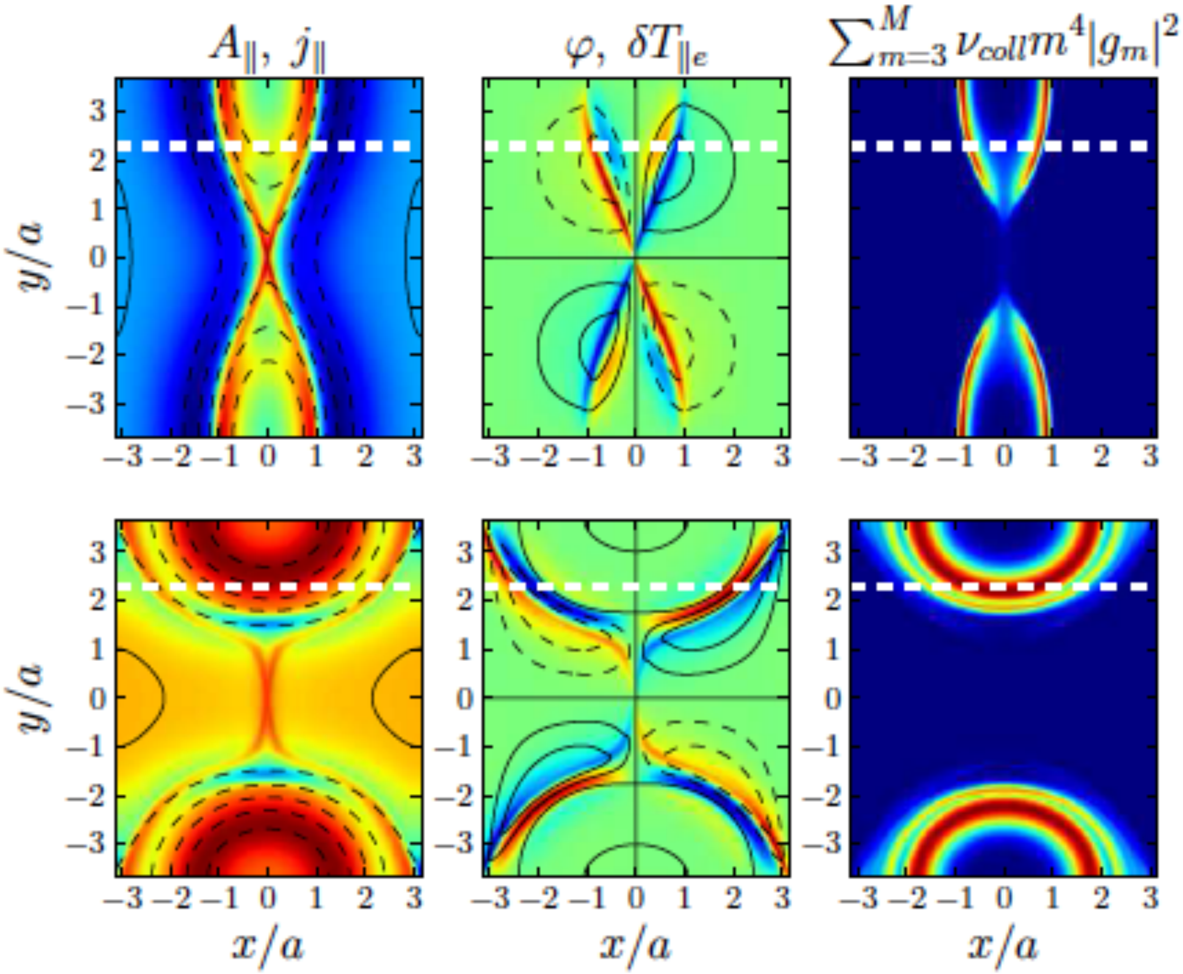


Lag shows weak, logarithmic dependence on collisions: *dissipation occurs in finite time even for weak collisionality.*

arXiv:1301.0338

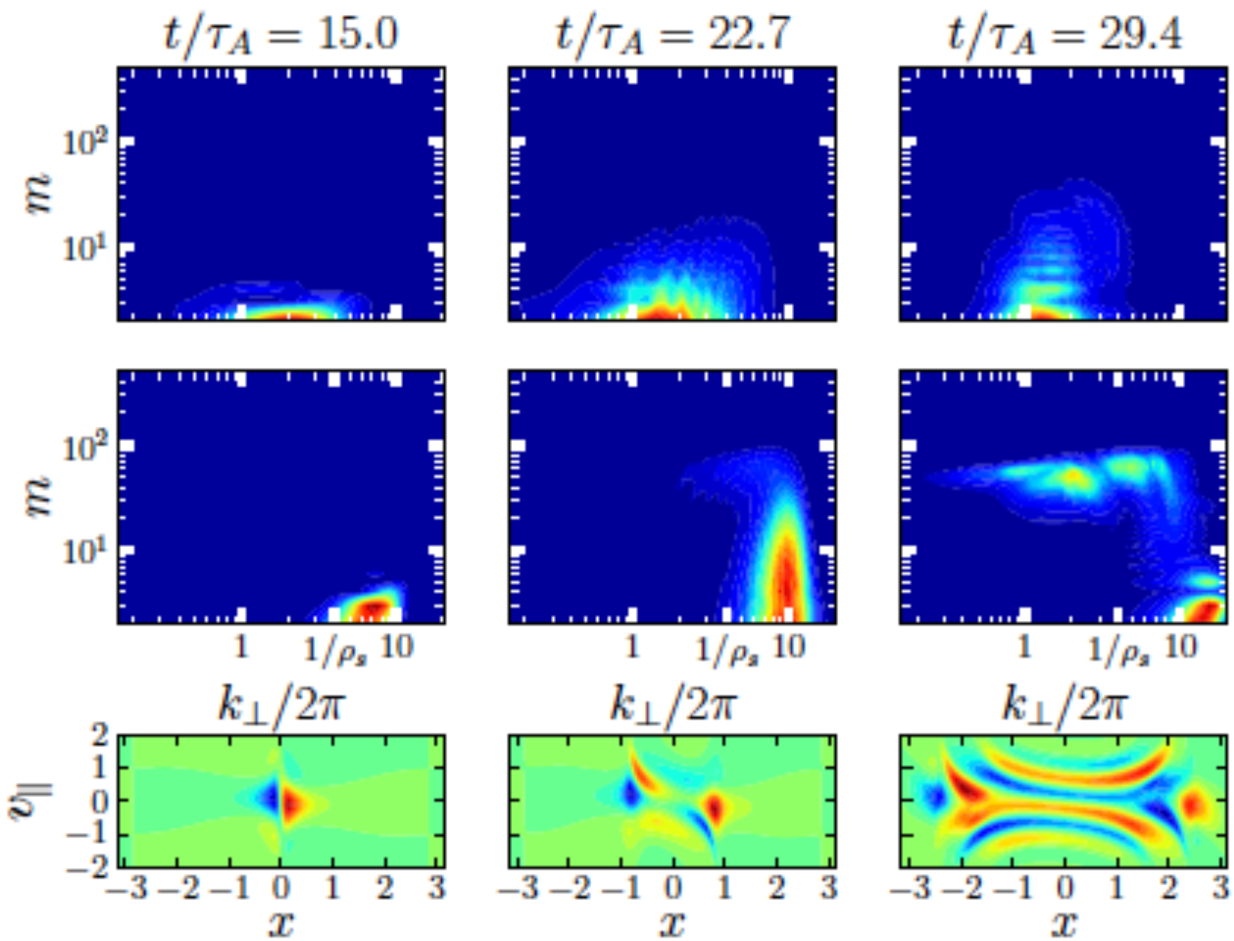
Dissipation occurs on the separatrices



System at time of max. reconnection rate

System at time of max. dissipation rate

Spectra and distribution function



Electron free energy spectra

Dissipation spectra

Reduced distribution function

arXiv:1301.0338

Weakly collisional tearing mode saturation

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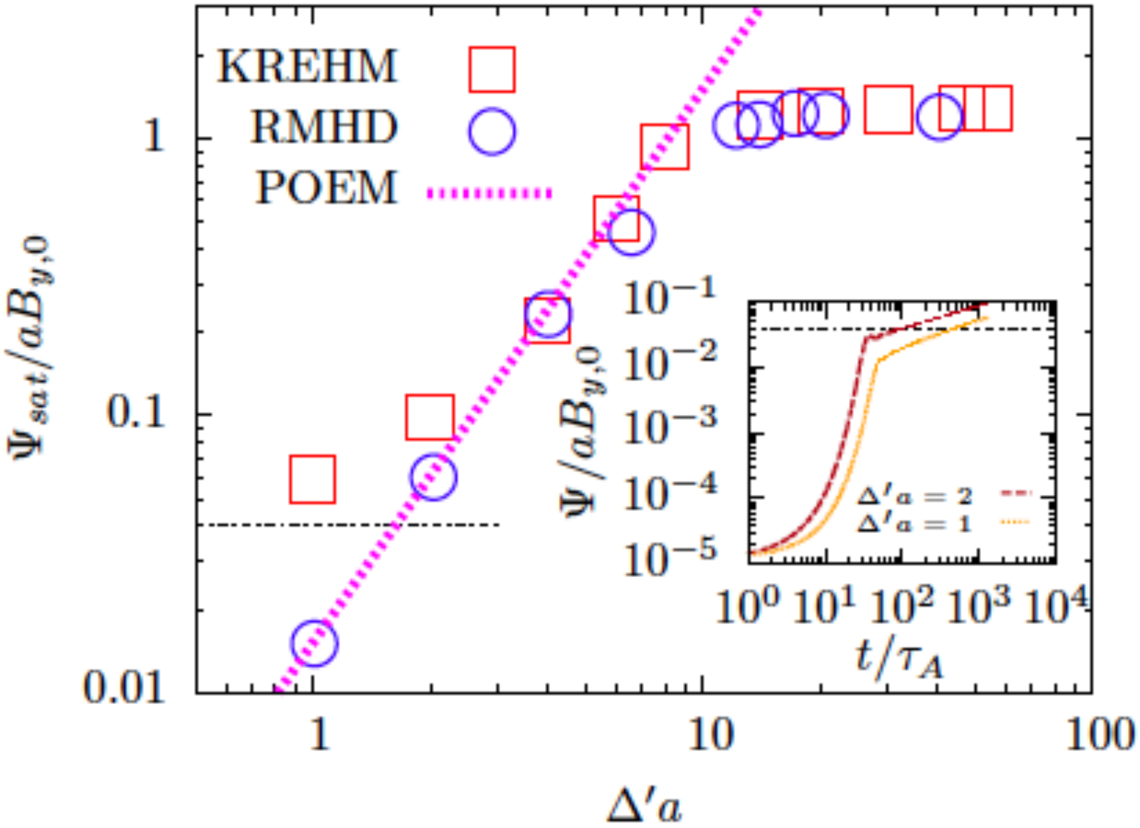

Weakly collisional tearing mode saturation

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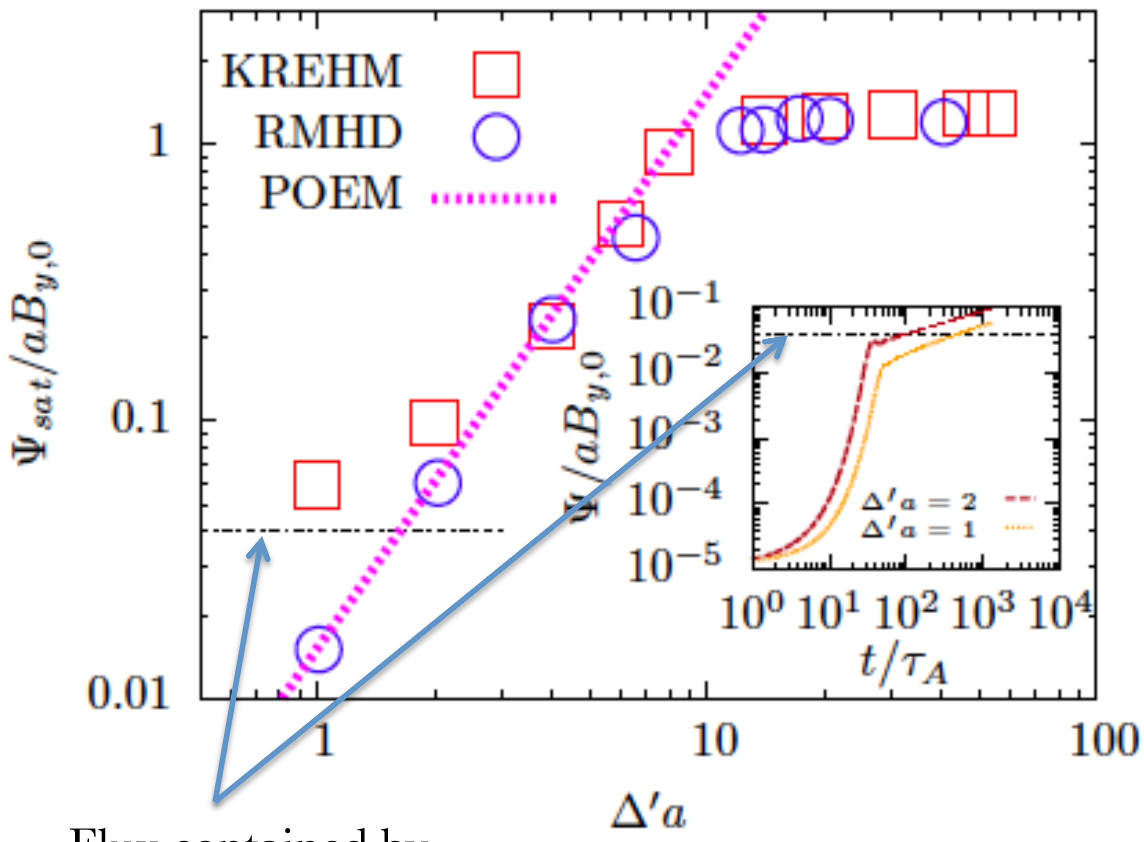
- Electron inertia scale appears to set a lower limit on the saturation size. Could have important implications for e.m turbulence (suggests existence of a minimum fluctuation amplitude)



Flux contained by island of width d_e

Conclusions

- Reduced (4D) gyrokinetic formalism of Zocco & Schekochihin [*"KREHM"*, Phys. Plasmas **18**, 102309 (2011)]: nimble tool for exploring phase-space dynamics of strongly magnetised plasmas.
- VIRIATO: new Fourier-Hermite code, reduced kinetic-fluid description (4D) --- can be used for reconnection, turbulence, slow modes (see A. Kanekar's poster), etc.
- Weakly collisional tearing mode reconnection: reconnection and heating are causally related, but spatially and temporally disconnected: heating happens after most flux has reconnected, and along the separatrices, not in the current sheet
- Heating in weakly collisional tearing mode reconnection occurs via *linear* phase-mixing / Landau damping