Multiscale, multiphysics modeling of turbulent transport and heating in collisionless, magnetized plasmas

Michael Barnes

Plasma Science & Fusion Center

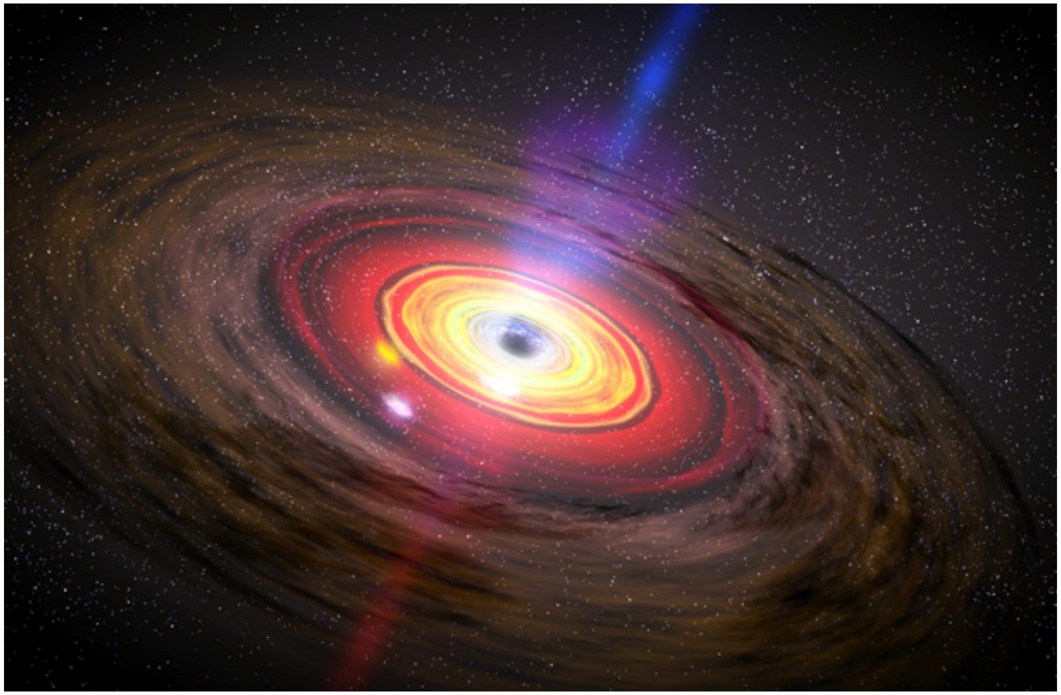
Massachusetts Institute of Technology

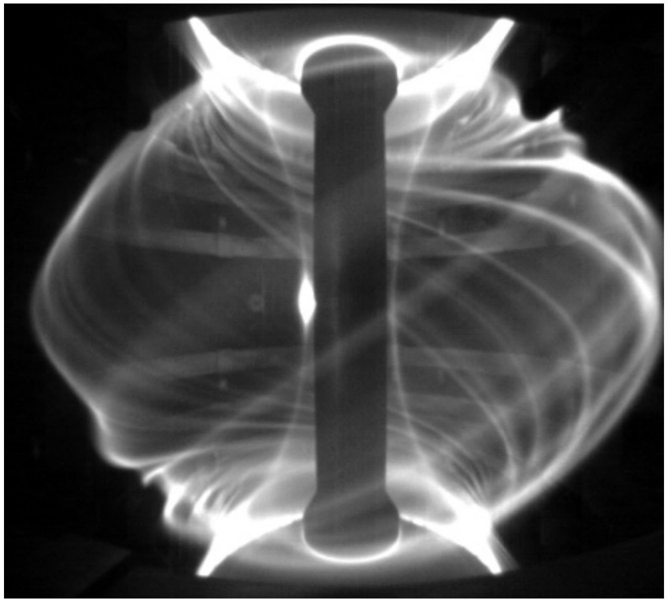
Collaborators: W. Dorland (Maryland), F. I. Parra (MIT), G. W. Hammett (PPPL), I. G. Abel, A. A. Schekochihin (Oxford), S. C. Cowley (Culham), T. Gorler (IPP-Garching), F. Jenko (IPP-Garching), …

Overview

- Collisionless plasma dynamics are hard
- Let’s make life simpler: a mean field approach for turbulence and transport/heating in tokamak plasmas
- Numerical modeling approach: multiscale, hybrid kinetic/fluid code Trinity
- Issues and opportunities

Hot, magnetized plasmas





What do we want?

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{\Gamma} = \ldots$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{P} = \ldots$$

$$\ldots$$

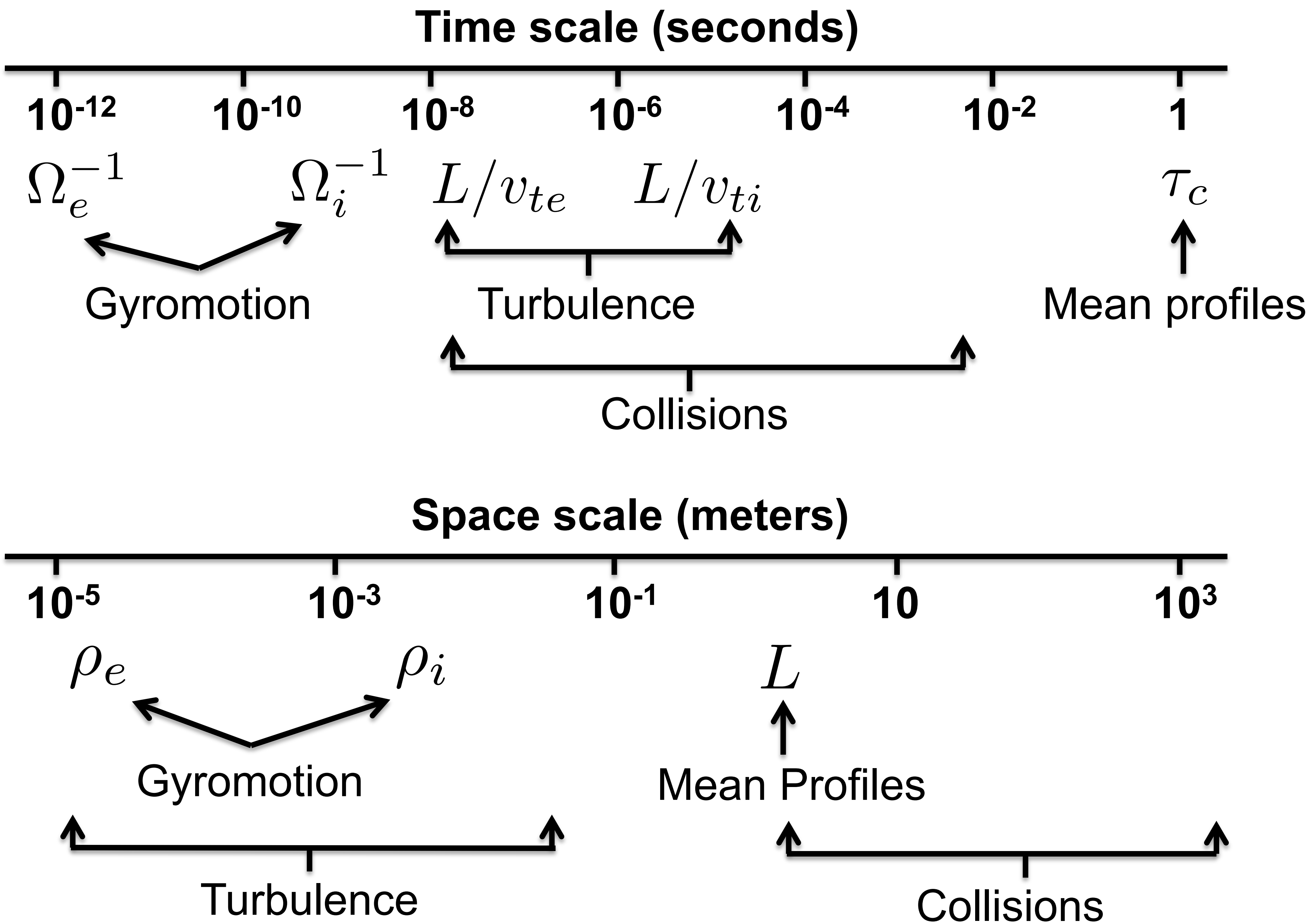
$$\frac{df}{dt} = C[f]$$

A tempting thought

“Think? Why think?! We have computers to do that for us.”

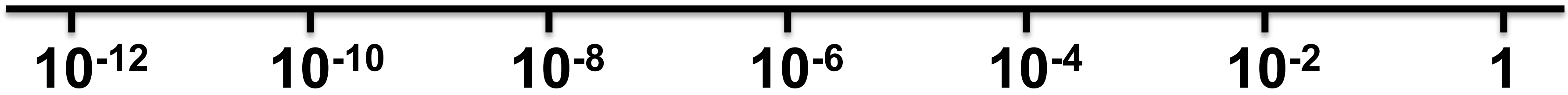
-- Jean Rostand

Range of scales



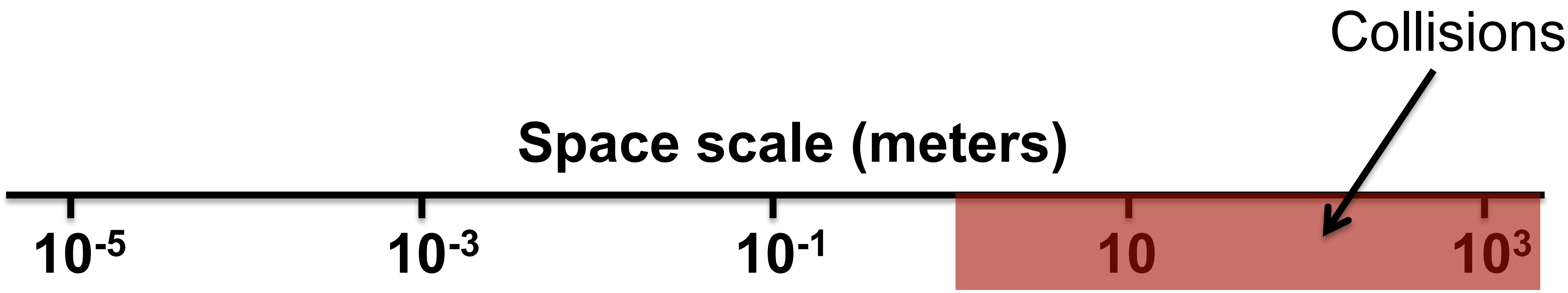
Expense (brute force)

Time scale (seconds)



Temporal grid: ~10^{13} time steps

Space scale (meters)

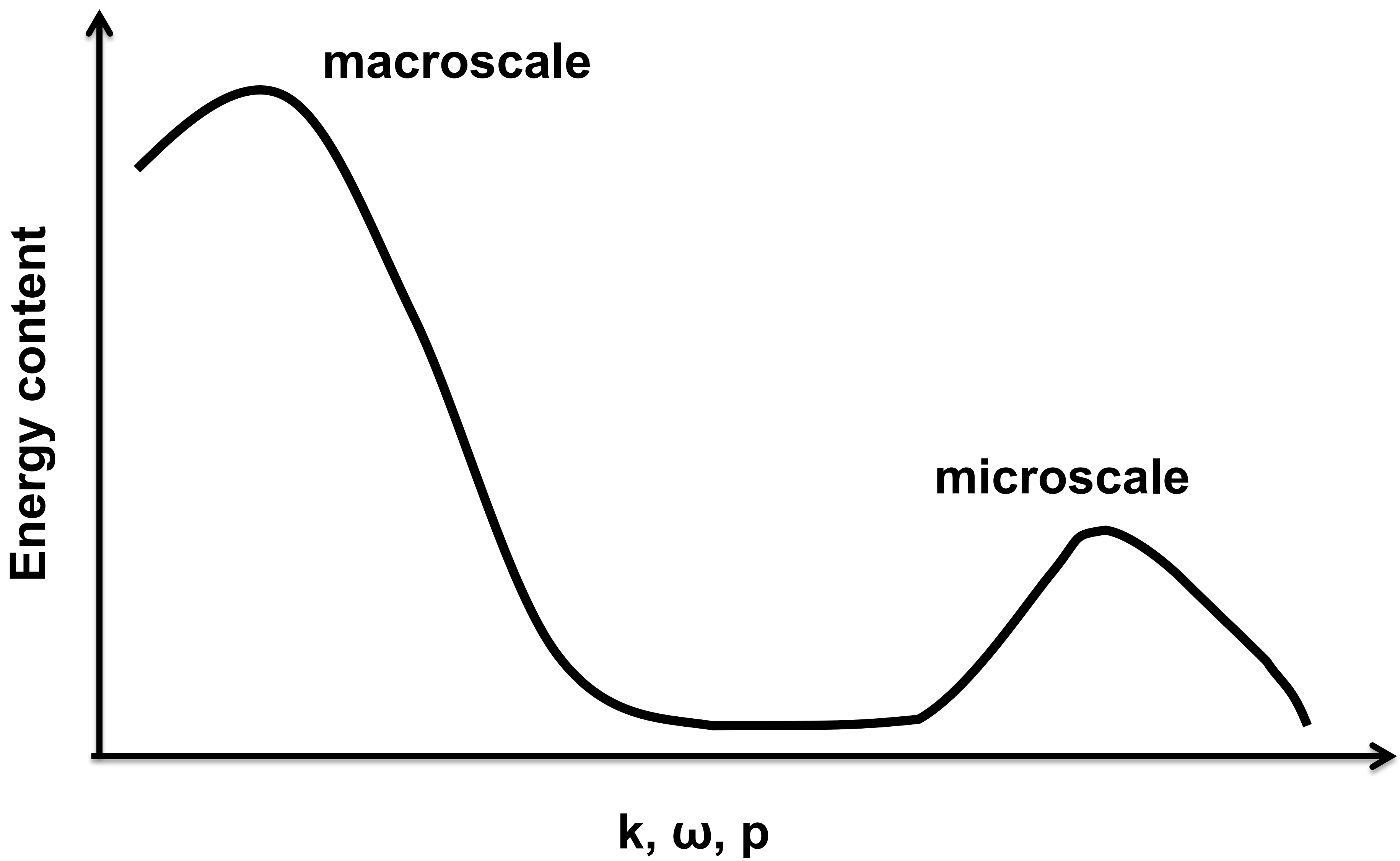


Spatial grid: ~10^{6} grid points x 3-D = 10^{18} grid points

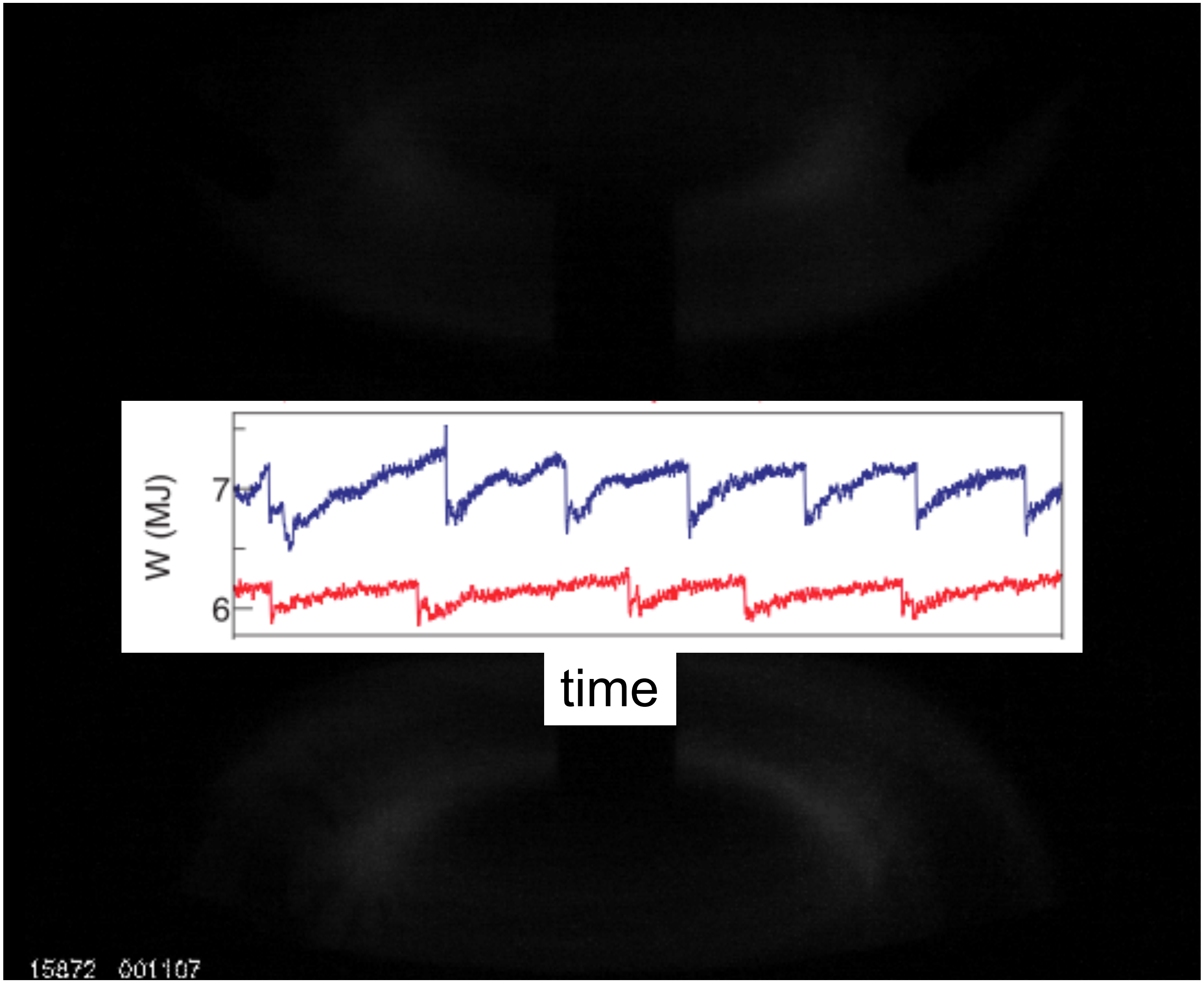
Velocity grid: ~10 grid points x 3-D = 10^{3} grid points

Total: ~10^{34} total grid points

Scale separation

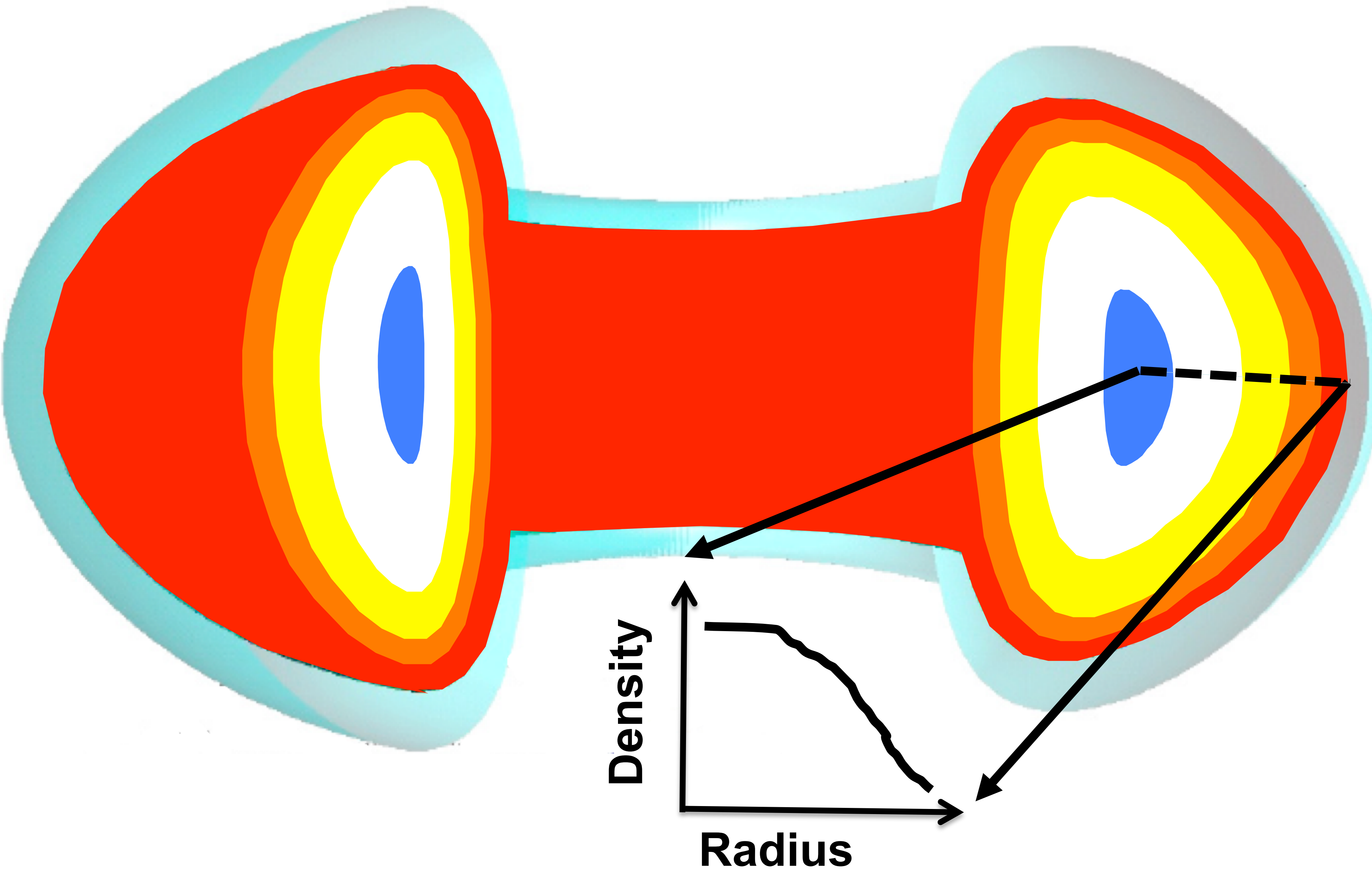


Multiscale, multiphysics

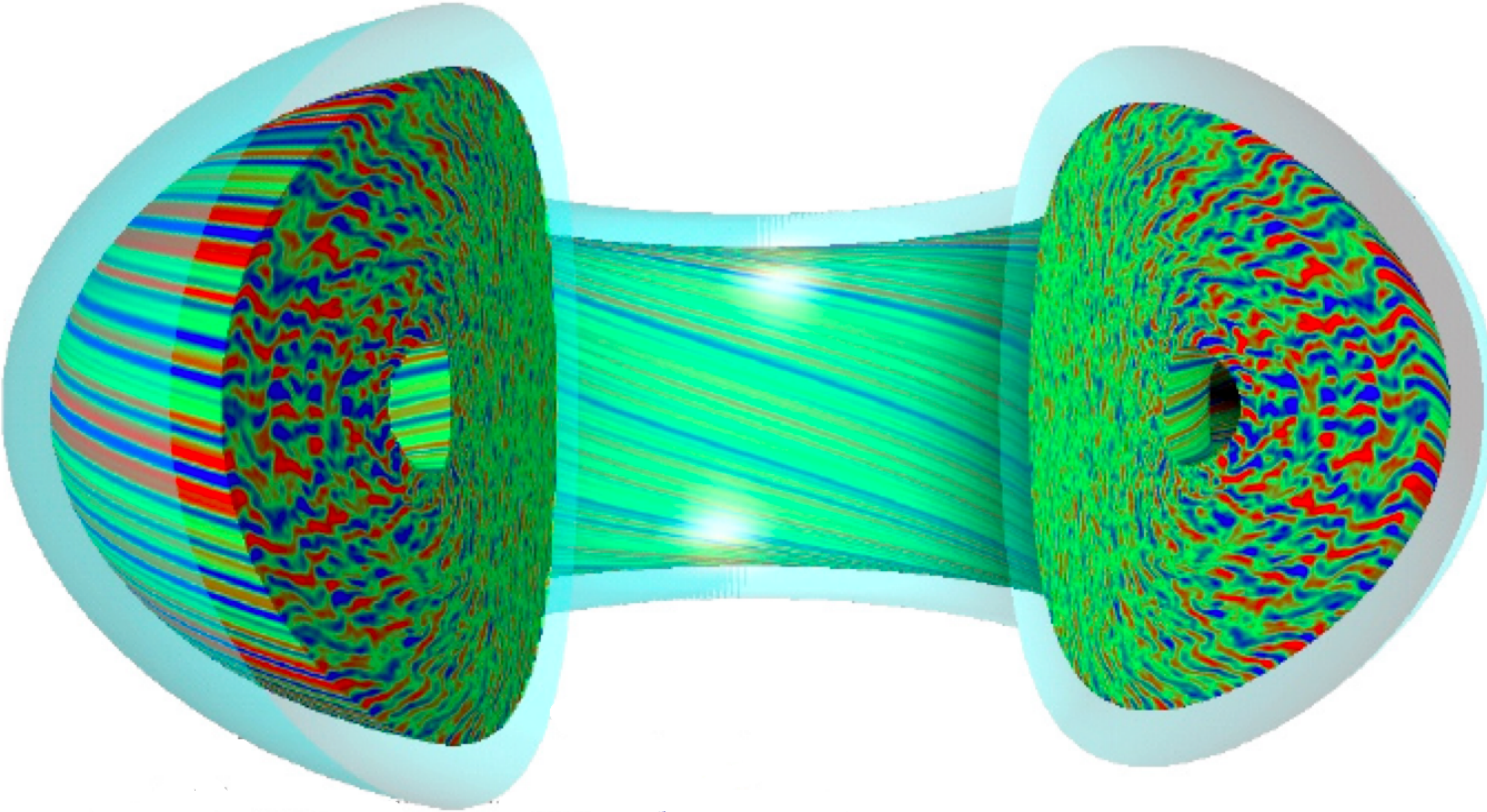


Fast camera image of MAST plasma

Equilibrium macroscale spatial profile



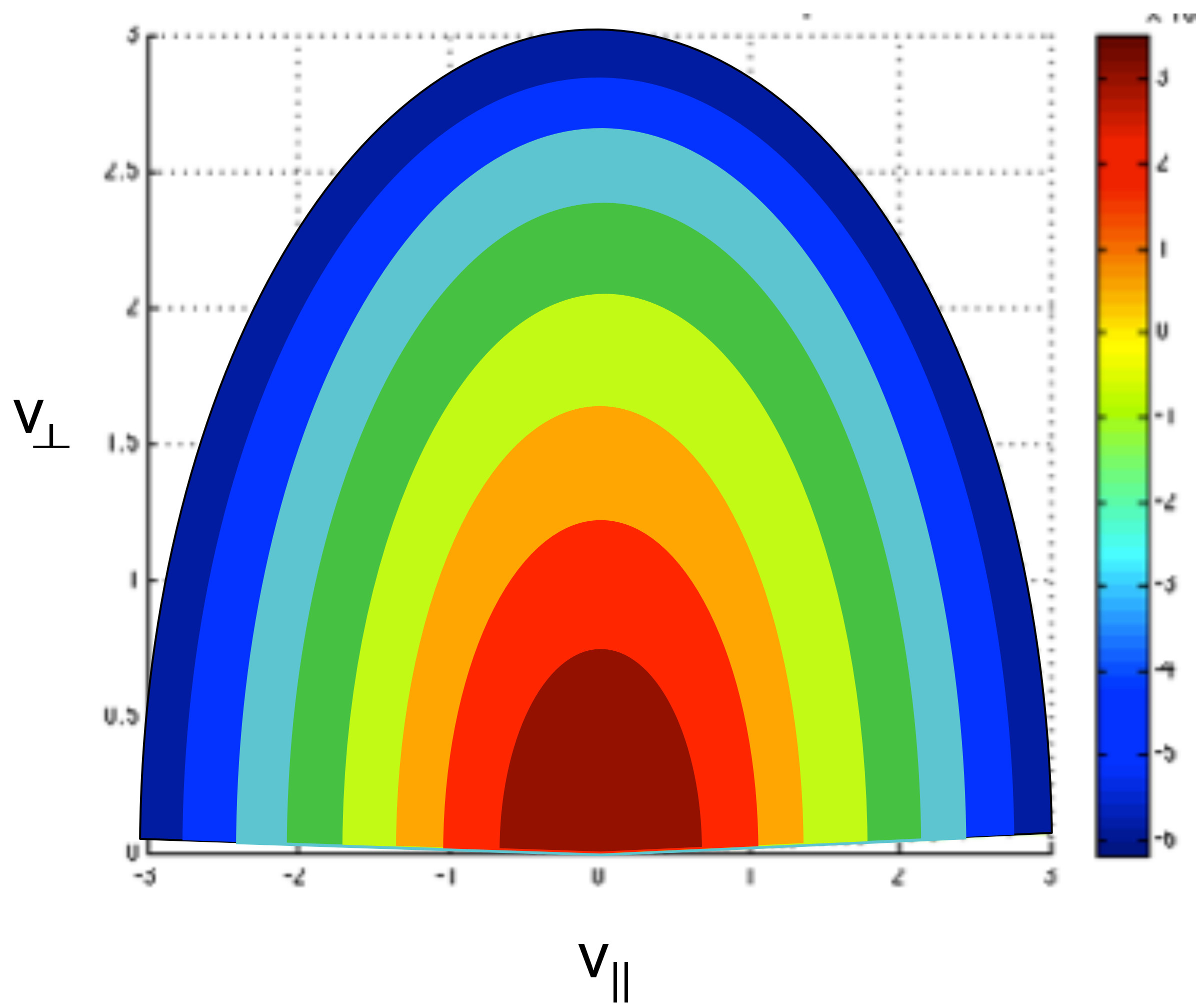
Microscale fluctuations



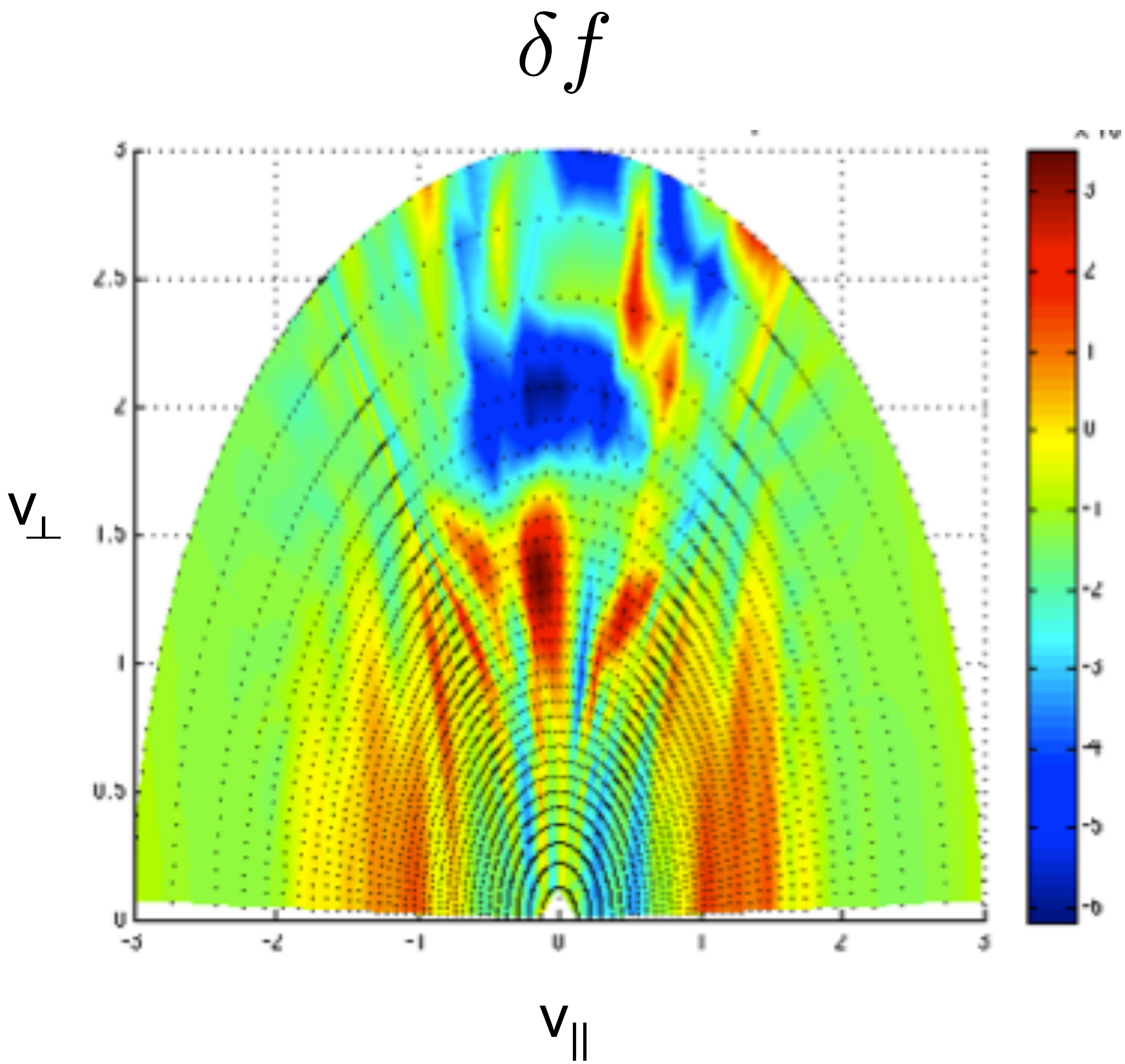
GYRO simulation
J. Candy and R. Waltz

Equilibrium macroscale speed distribution

$$f = f(\mathbf{r}, \mathbf{v}, t)$$



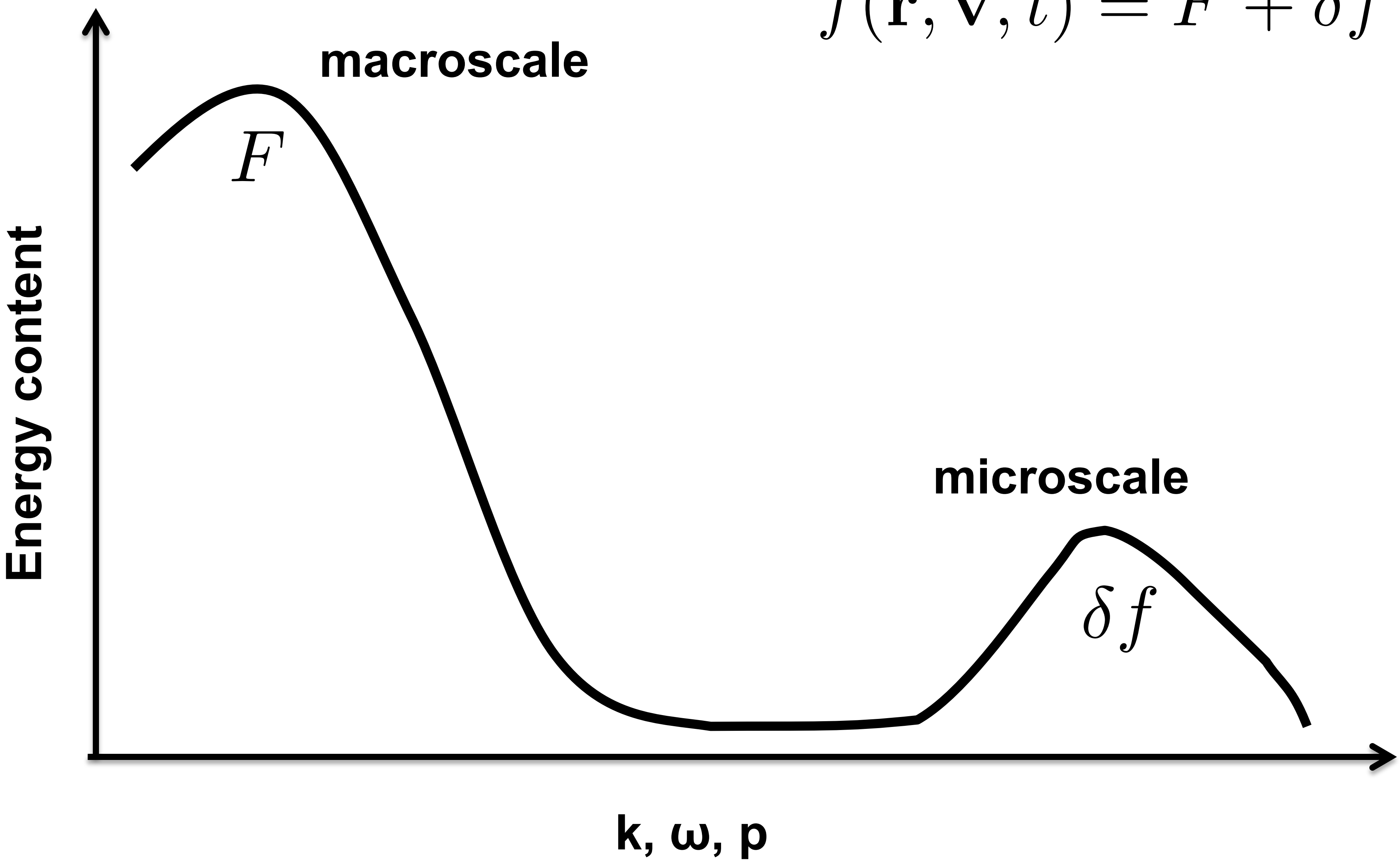
Microscale fluctuations



GS2 simulation

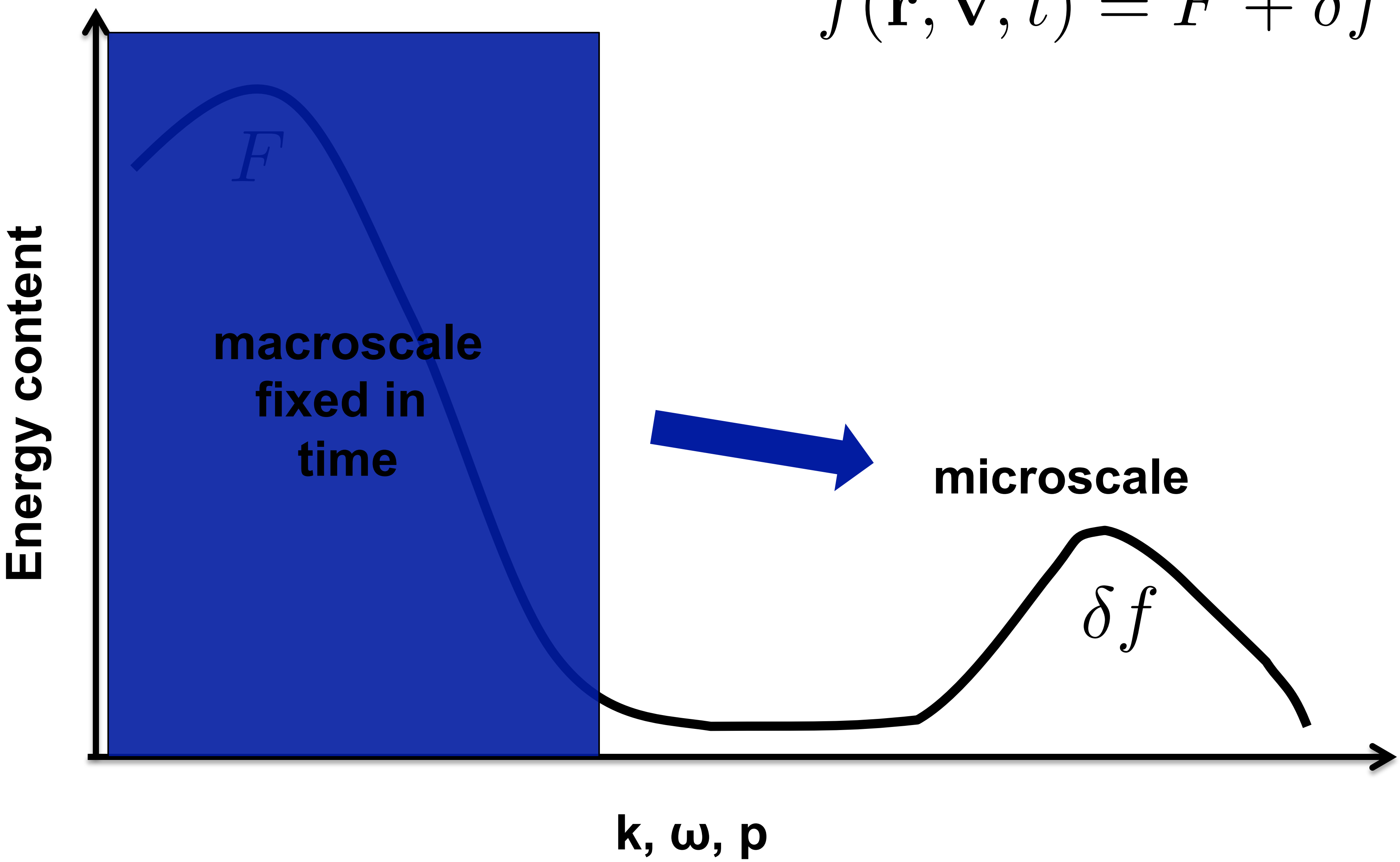
Scale separation

$$f(\mathbf{r}, \mathbf{v}, t) = F + \delta f$$



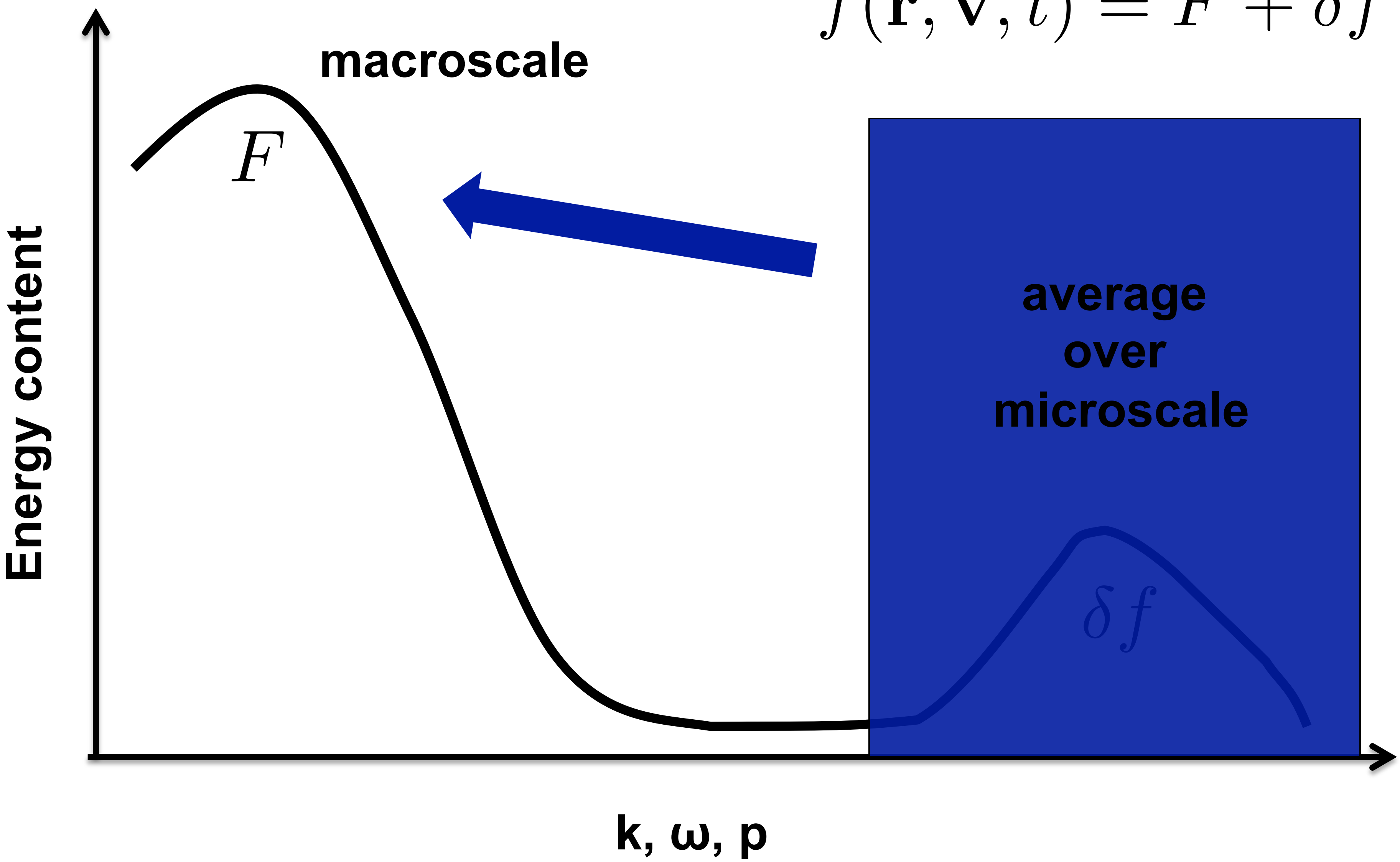
Macro drives micro

$$f(\mathbf{r}, \mathbf{v}, t) = F + \delta f$$

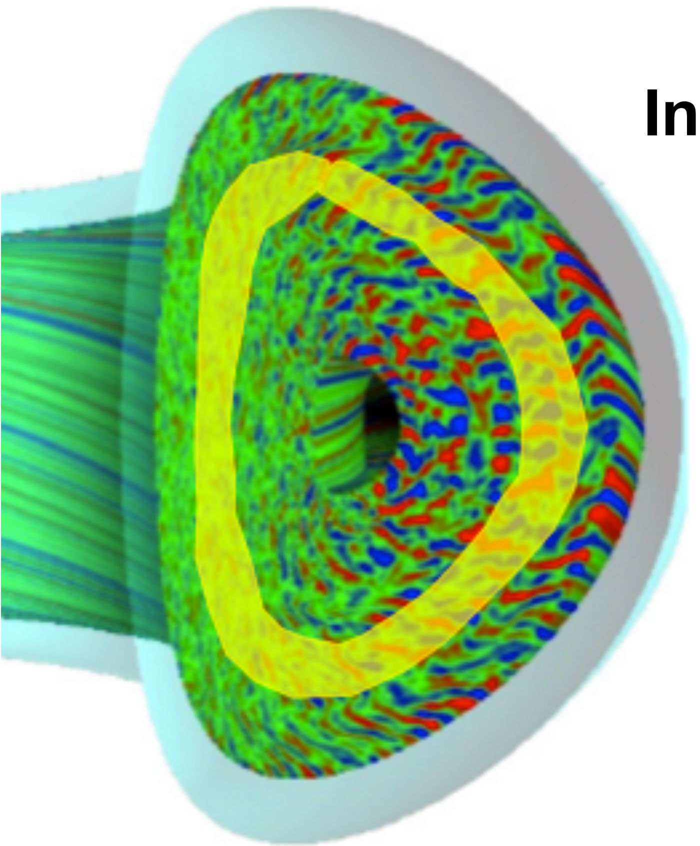


Micro feeds back into macro

$$f(\mathbf{r}, \mathbf{v}, t) = F + \delta f$$

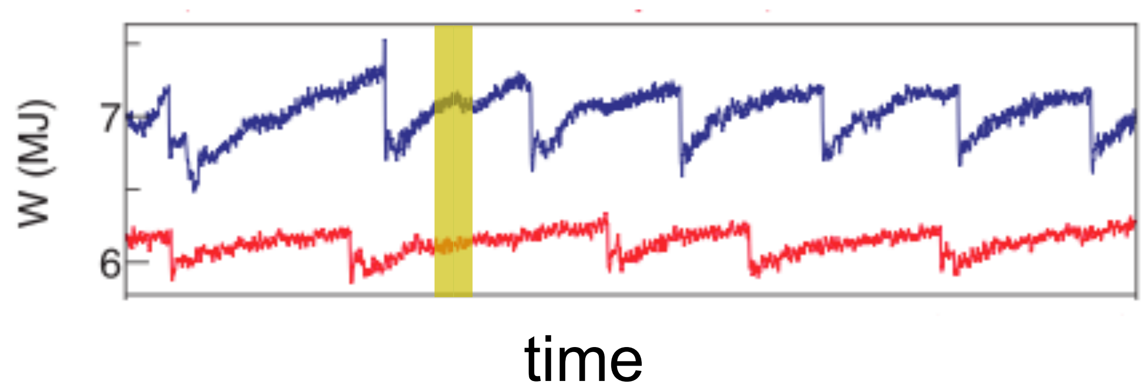


Coarse-grain average



In space…

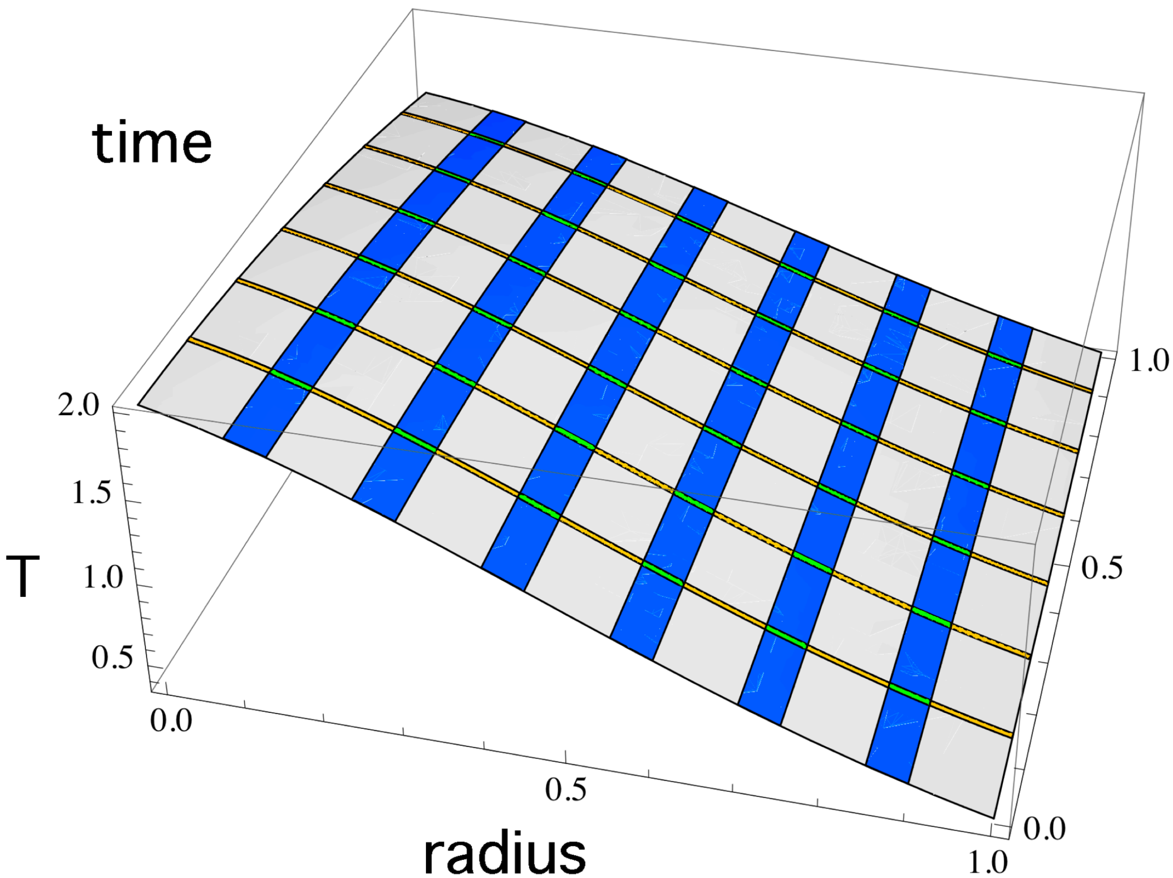
…and in time…

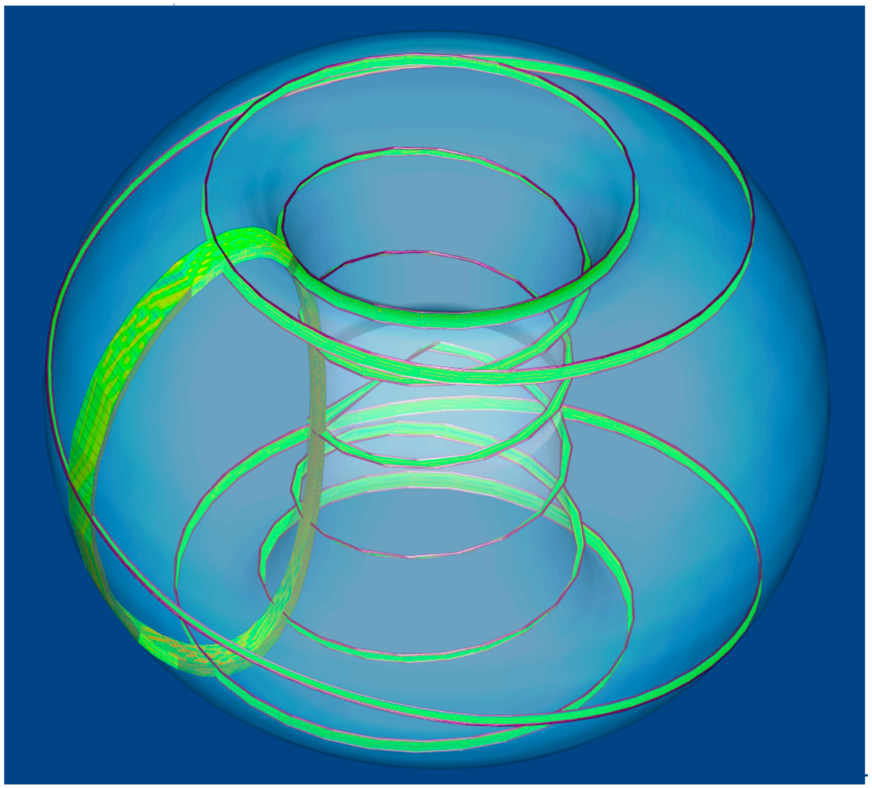


…and moment approach in velocities

Multiscale model

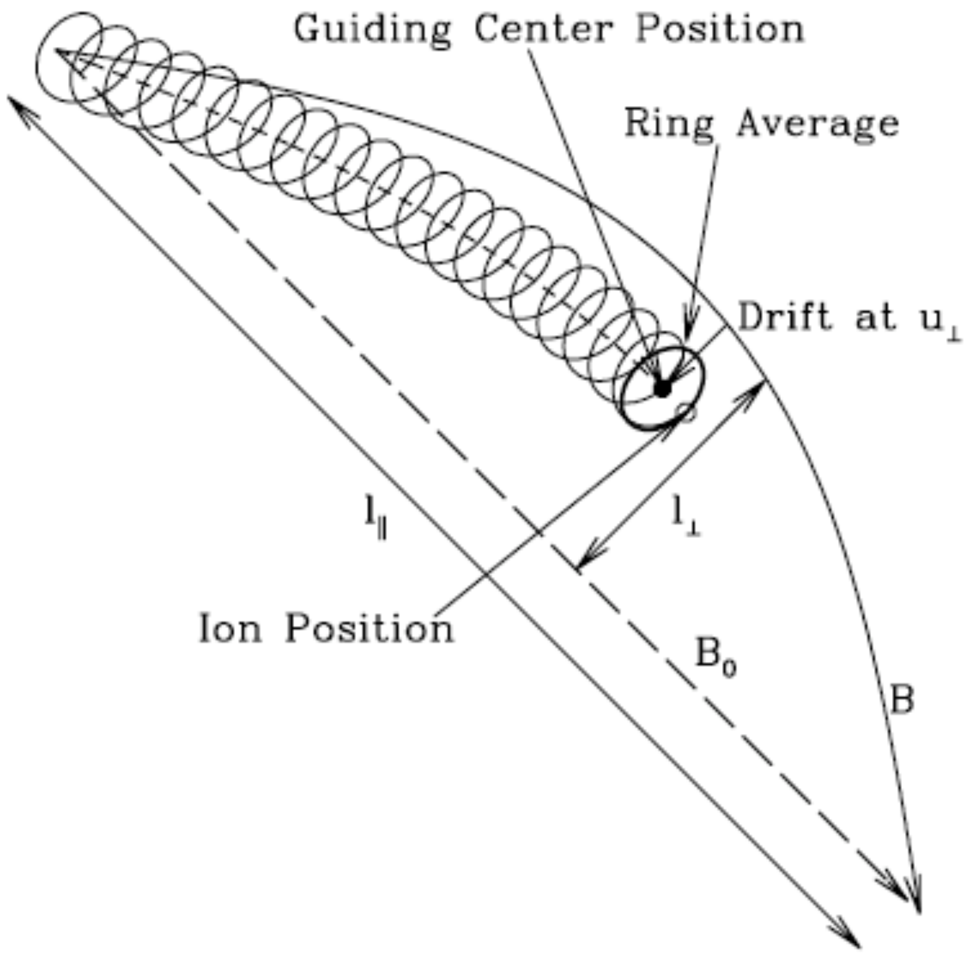
Turbulent fluctuations calculated in small regions of fine space-time grid embedded in coarse grid for mean quantities (implemented in TRINITY code)





Gyrokinetic description of dynamics

$$\frac{\partial f_s}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{d\mathbf{v}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = C[f_s]$$



- Average over fast gyro-motion and follow ‘guiding center’ position
- Eliminates fast time scale and gyro-angle variable (6-D --> 5-D)

Multiscale gyrokinetics

Decompose f into mean and fluctuating components:

$$f = F + \delta f$$

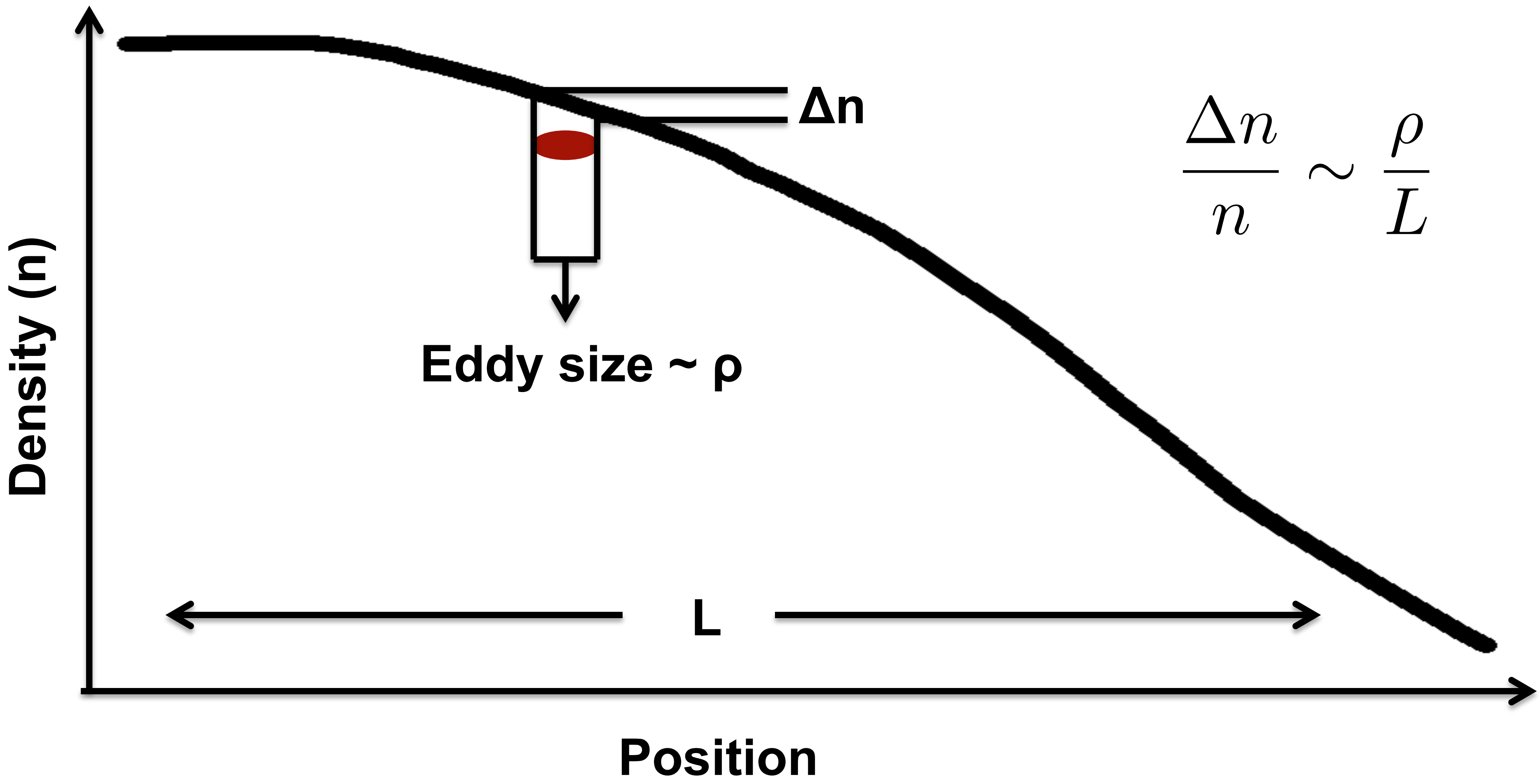
Mean varies perpendicular to mean field on system size while fluctuations vary on scale of gyro-radius:

$$\nabla_\perp \ln F \sim L^{-1} \qquad \nabla_\perp \ln \delta f \sim \rho^{-1}$$

Fluctuations are anisotropic with respect to the mean field:

$$\nabla_\| \ln \delta f \sim L^{-1}$$

Mixing length estimates



$$\frac{\Delta n}{n} \sim \frac{\rho}{L}$$

Mixing length ~ (step size) × (# steps)$^{1/2}$

Macro time scale ~ (step time) × (# steps to mix over length L)
~ (L/ρ)^2 × (step time)

Multiscale gyrokinetics

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Fluctuations are anisotropic with respect to the mean field:

$$\nabla_\| \ln \delta f \sim L^{-1}$$

=> Turbulent fluctuations are low amplitude: $\delta f \sim \epsilon f$

=> Mean profile evolution slow compared to turbulence:

$$\frac{\partial \ln F}{\partial t} \sim \epsilon^2 \omega \sim \epsilon^3 \Omega$$

$$\epsilon \equiv \frac{\rho}{L} \ll 1$$

Multiscale gyrokinetics

Gyrokinetic equation for fluctuation dynamics:

$$\frac{\partial \langle \delta f \rangle}{\partial t} + \left\langle \frac{d\mathbf{R}}{dt} \right\rangle \cdot \frac{\partial}{\partial \mathbf{R}} \left(\langle \delta f \rangle - q \langle \delta \Phi \rangle \frac{\partial F}{\partial \varepsilon} \right) + \left(\left\langle \frac{d\mathbf{R}}{dt} \right\rangle - \overline{\left\langle \frac{d\mathbf{R}}{dt} \right\rangle} \right) \cdot \frac{\partial F}{\partial \mathbf{R}} = \langle C[\delta f] \rangle$$

Fluid conservation equations for mean dynamics:

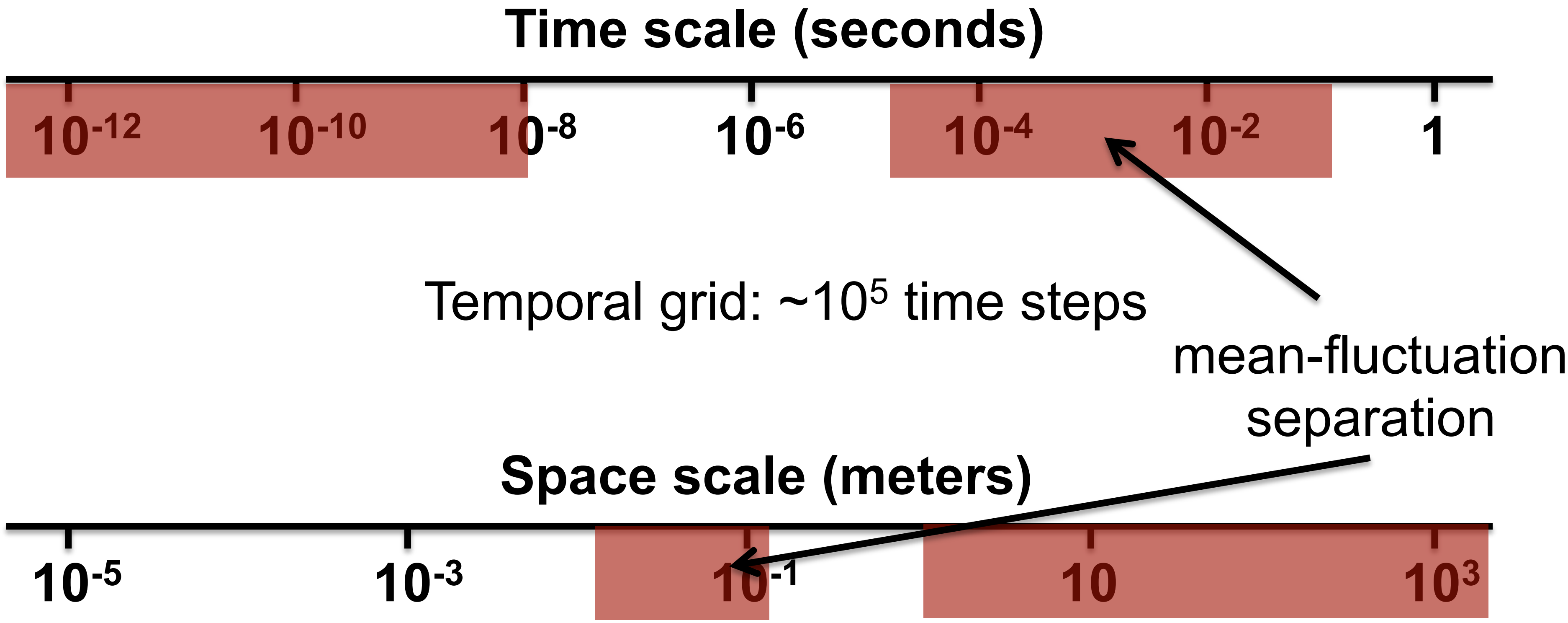
$$\frac{\partial \overline{n}}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial r} \left(V' \overline{\Gamma} \right) = \overline{S_n}$$

$$\frac{\partial \overline{L}}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial r} \left(V' \overline{\Pi} \right) = \overline{S_L}$$

$$\frac{3}{2} \frac{\partial \overline{p}}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial r} \left(V' \overline{Q} \right) = \overline{S_p}$$

Fluxes are functions of fluctuating quantities

Expense (multiscale gyrokinetics)



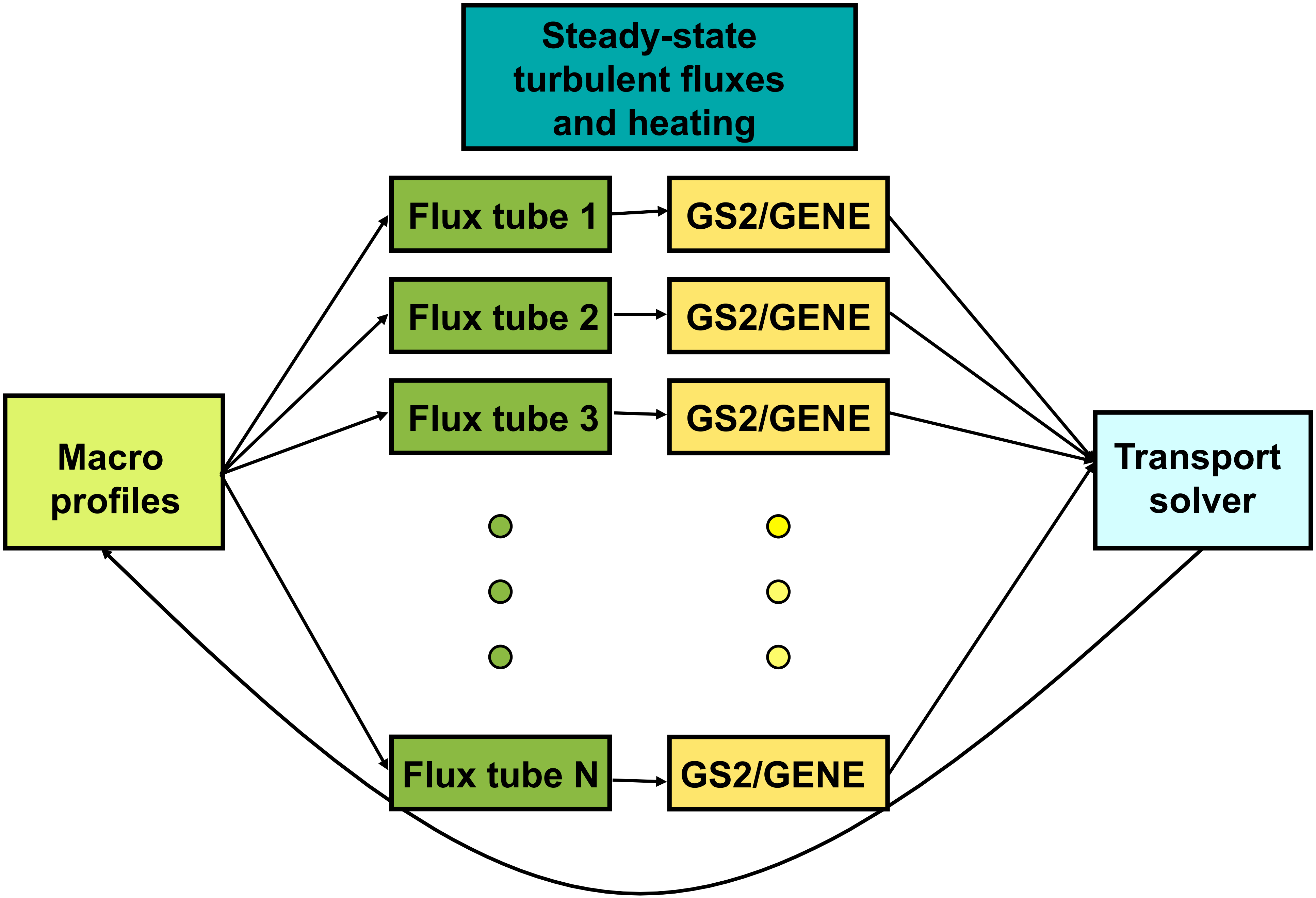
Perpendicular spatial grid: ~10^5 grid points x 2-D = 10^{10} grid points

Parallel spatial grid: ~10 grid points x 1-D = 10 grid points

Velocity grid: ~10 grid points x 2-D v-space = 10^2 grid points

Total: ~10^{18} total grid points (10^6 savings)

TRINITY schematic



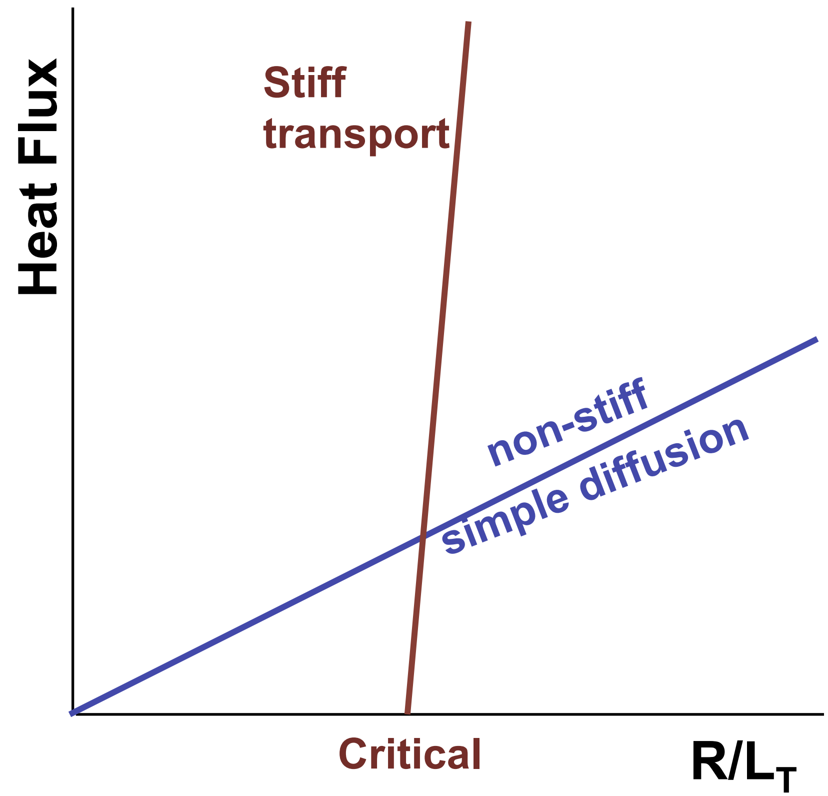
TRINITY transport solver

Transport equations are stiff, nonlinear PDEs:

$$\frac{3}{2}\frac{\partial p_s}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\left\langle \mathbf{Q}_s \cdot \nabla\psi \right\rangle\right) + \ldots$$

$$\mathbf{Q}_s = \mathbf{Q}_s\left[n(\psi,t), T(\psi,t); \psi, t\right]$$

Implicit treatment needed for stiffness



Newton solve

- Challenge: requires computation of quantities like

$$\Gamma_j^{m+1} \approx \Gamma_j^m + \left(\mathbf{y}^{m+1} - \mathbf{y}^m\right) \left.\frac{\partial \Gamma_j}{\partial \mathbf{y}}\right|_{\mathbf{y}^m}$$

$$\mathbf{y} = \left[\{n_k\}, \{p_{s_k}\}, \{L_k\}\right]^T$$

- Local approximation: $\dfrac{\partial \Gamma_j}{\partial n_k} = \dfrac{\partial \Gamma_j}{\partial n_j} + \dfrac{\partial \Gamma_j}{\partial (R/L_n)_j} \dfrac{\partial (R/L_n)_j}{\partial n_k}$

- Simplifying assumption: normalized fluxes depend primarily on gradient scale lengths
- Implicit treatment allows for time steps ~0.1 seconds (vs. turbulence sim time ~0.001 seconds)

Parallelization

Calculating flux derivative approximations:

- at every radial grid point, simultaneously calculate $\Gamma_j\left[(R/L_n)_j^m\right]$ and $\Gamma_j\left[(R/L_n)_j^m+\delta\right]$ using 2 different flux tubes
- Possible because flux tubes independent (do not communicate during turbulence calculation)
- Perfect parallelization
- use 2-point finite differences for derivative:

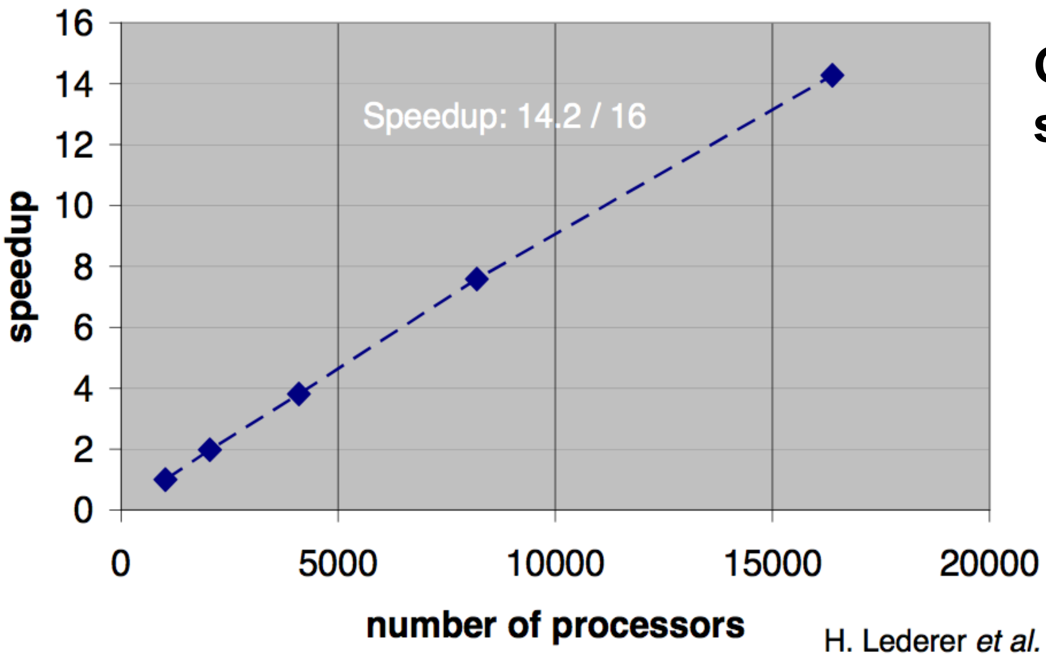
$$\frac{\partial\Gamma_j}{\partial(R/L_n)_j}\approx\frac{\Gamma_j\left[(R/L_n)_j^m\right]-\Gamma_j\left[(R/L_n)_j^m+\delta\right]}{\delta}$$

Scaling

Example calculation with 10 radial grid points:

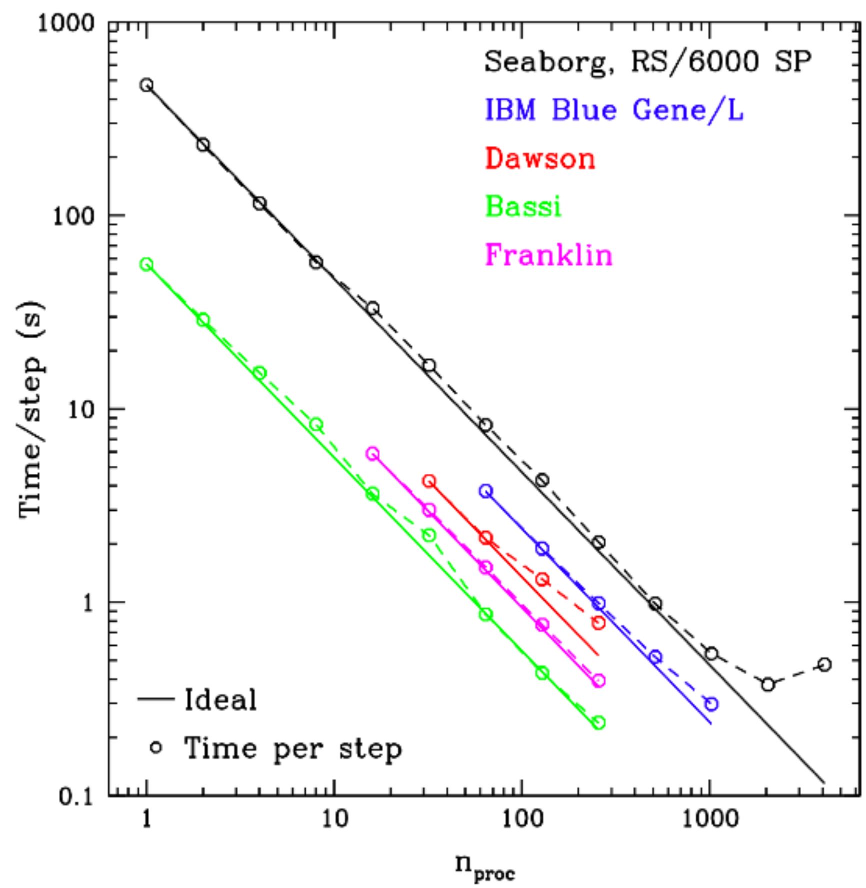
- Evolve electron density, toroidal angular momentum, and ion/electron pressure profiles
- Simultaneously calculate fluxes for equilibrium profile and for perturbed profiles (one for each time-varying gradient scale length, i.e. 4)
- Total of 50 flux tube simulations running in parallel

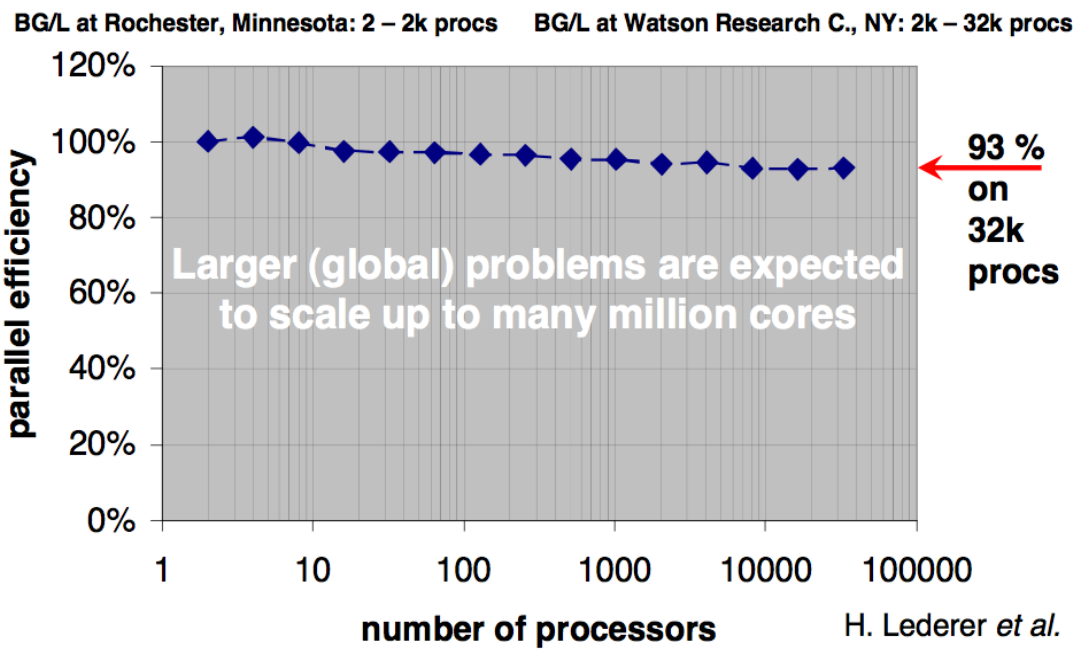
Flux tube scaling



GENE strong scaling

GS2 strong scaling





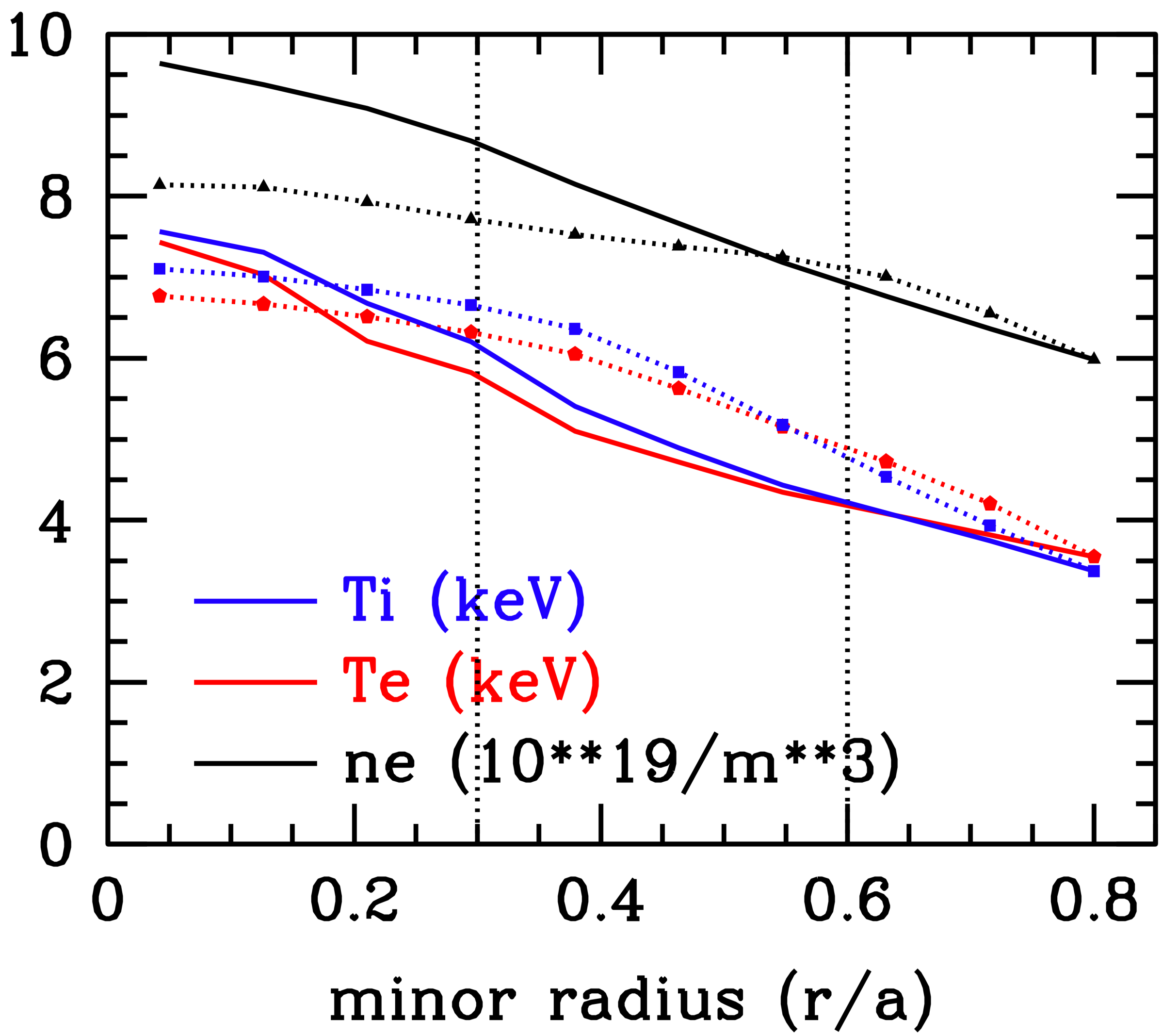
GENE weak scaling

Scaling

Example calculation with 10 radial grid points:

- Evolve electron density, toroidal angular momentum, and ion/electron pressure profiles
- Simultaneously calculate fluxes for equilibrium profile and for perturbed profiles (one for each time-varying gradient scale length, i.e. 4)
- Total of 50 flux tube simulations running in parallel
- ~2-4k cores or more per flux tube => scaling to over 100k's processors with very high efficiency

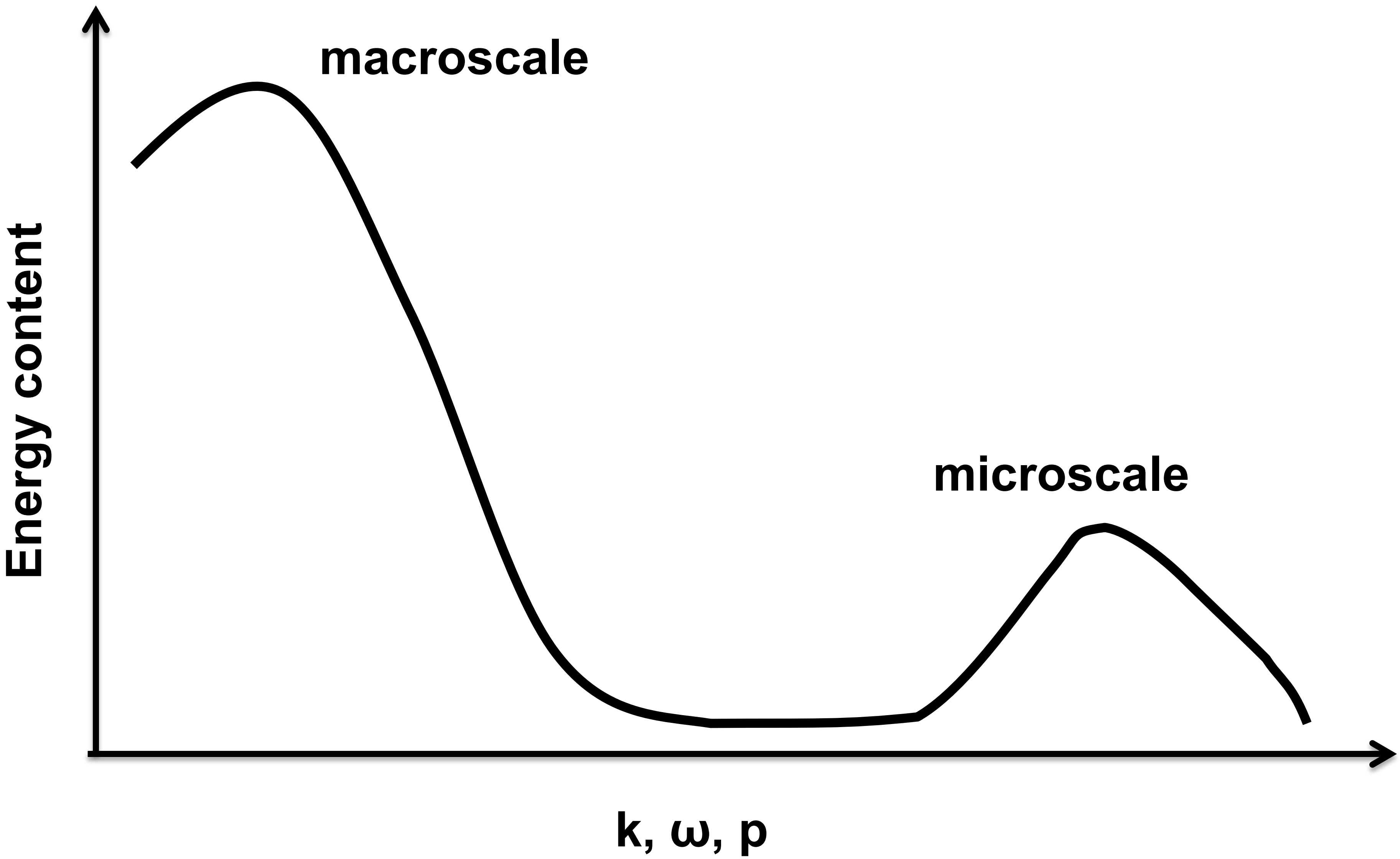
Example: JET H-mode



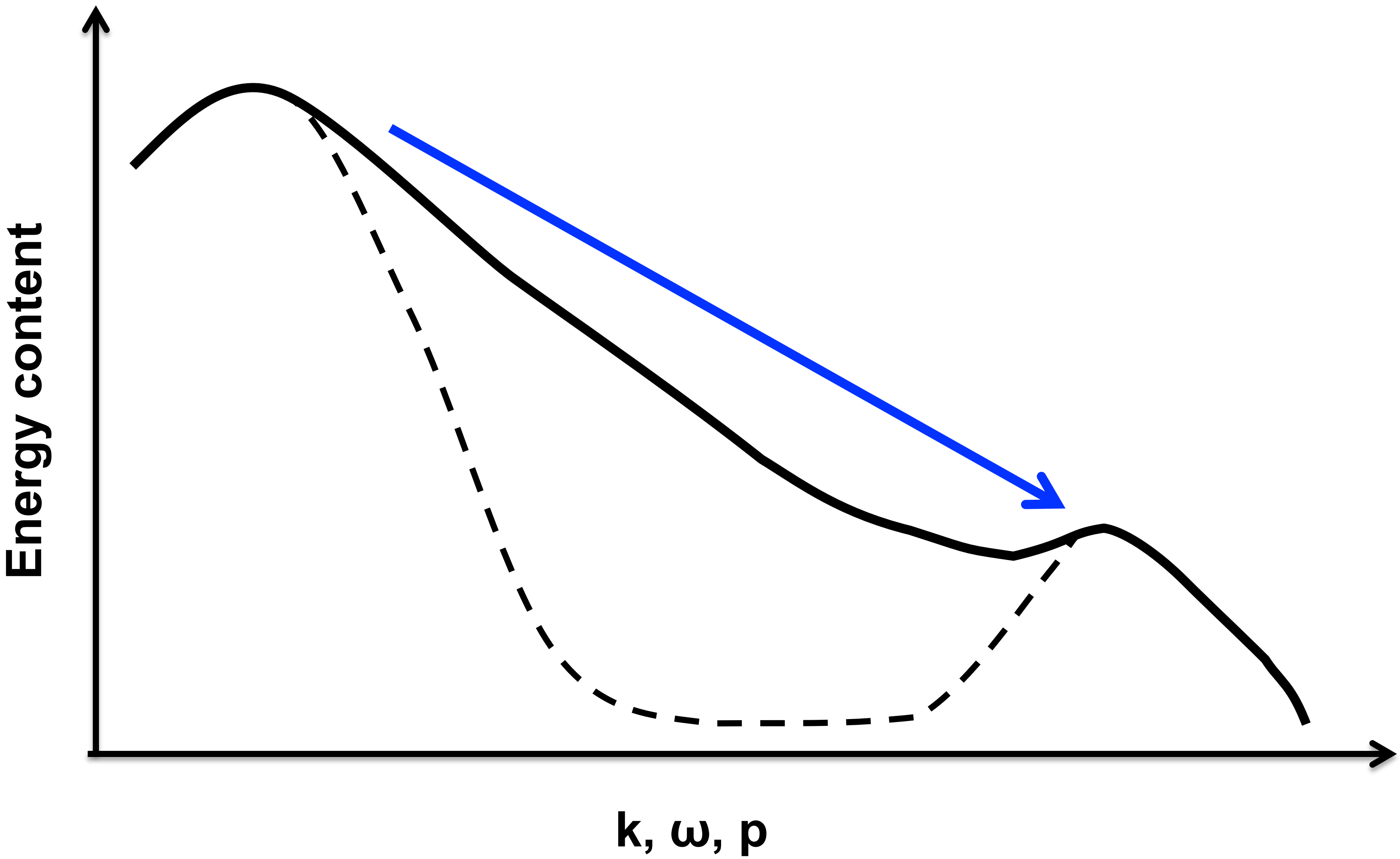
- 10 radial grid points
- Costs ~120k CPU hrs (<10 clock hrs)
- Dens and temp profiles agree within ~15% across device
- Energy off by 5%
- Incremental energy off by 15%

Issues and opportunities

Issue: Scale separation?



Issue: Scale separation?

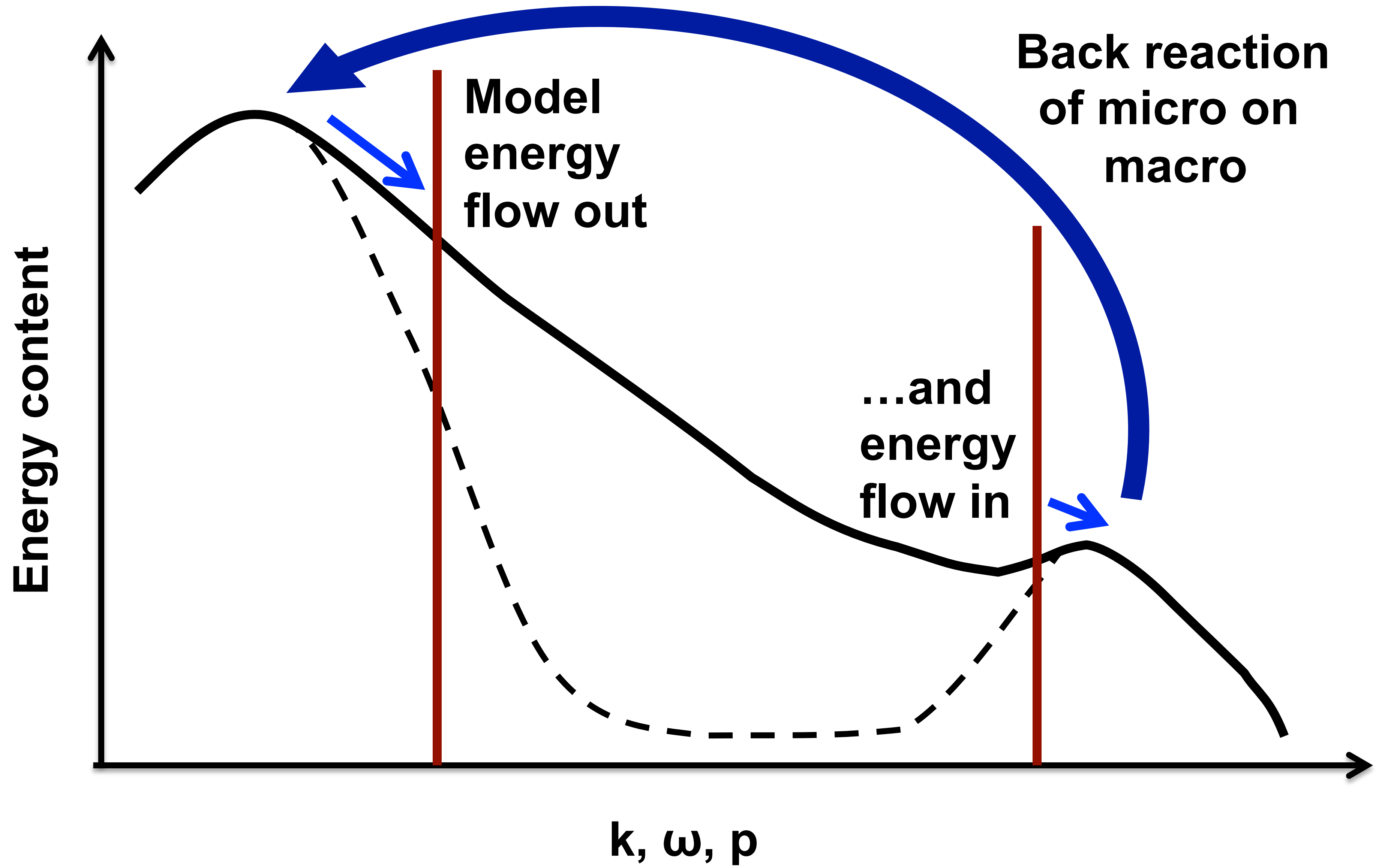


Summary

From T. S. Eliot's *Little Gidding*,

"We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time."

Opportunity



Issue: Computational expense

- Could reduce computational expense considerably by using a reduced model for the microscale physics
  - fluid model
  - proper orthogonal decomposition
  - LES for kinetics
- These approaches share weakness that they are not universal
- This weakness can be ameliorated by the multiscale approach

Equilibrium macroscale spatial profile