A402 Interstellar Medium

Final Exam: Due 6pm, Tue May 17, 2005

The final exam will be taken *entirely* from the following, with something like 3 questions from Section I, 10 from II, 2 from III and 2 from IV. We **may** change the numbers slightly in some of the calculations, but that's all. The exam will be *closed book* but we'll give you all the constants. Calculators necessary. It's OK to work with your friends (except during the exam itself!!).

You may pick up the exam anytime after May 11.

Section I

1. The photoionization cross section of HI is about $6 \times 10^{-18}(\mathrm{E}/\mathrm{E_o})^{-3}$ cm^2, where $\mathrm{E_o}$ is the energy at the ionization threshold. What is the mean free path of a photon in an interstellar medium of density 1 cm^{-3} at the ionization threshold and at 5 keV?

2. The mean HI column density to extinction ratio in the interstellar medium is $\mathrm{N(HI)/A_V} = 2 \times 10^{21}$ atom cm^{-2} mag^{-1}. Make a rough estimate of the gas to dust ratio by mass.

3. Derive the equation of radiative transfer through a slab of optical depth τ in equilibrium at temperature T.

4. An HII region forms in a molecular cloud. How do the mass and radius of the HII region depend on its density?

5. Derive the relationships between the Einstein A and B coefficients.

6. Derive the relationship between the upward and downward collisional excitation cross sections $\sigma_{12}(\mathrm{E})$ and $\sigma_{21}(\mathrm{E})$, for the transition $2 \rightarrow 1$ where E is the center of mass energy and $\mathrm{E_{21}}$ is the energy of the transition.

7. Derive the formula for the rotational line frequencies of molecule XY using the rigid rotor approximation.

8. Derive the Langevin rate for ion-molecule reactions. Calculate its approximate value, and show that it is independent of temperature.

9. What is the optical depth of an HII region of density $\mathrm{n_e} = 10^4$ cm^{-3} and radius 2 pc due to electron scattering? What is the optical depth at 5000 Å due to dust (assuming that none of the dust has been destroyed)?

10. Consider molecule $\mathrm{XH^+}$, whose principle formation mechanism is the radiative association reaction

$$\mathrm{X^+ + H \rightarrow XH^+ + h\nu}$$

with a rate coefficient of $\alpha = 3 \times 10^{-17}$ cm^{3}s^{-1}. The principle reactions for destroying XH$^+$ are dissociative recombination

$$\mathrm{XH^+ + e^- \rightarrow X + H}$$

with a rate coefficient k $= 10^{-7}$cm^3 s^{-1}, and photodissociation

$$XH^+ + h\nu \rightarrow X^+ + H$$

due to ambient starlight, with rate $\beta = 10^{-10}$ s^{-1}. If only these chemical processes act, compute the steady-state density of XH$^+$ in a diffuse cloud with n(H) $= 10$ cm^{-3}, $n(X^+) = 5 \times 10^{-3}\ cm^{-3}$ and n(e$^-$) $= 0.01$ cm^{-3}.

11. A supernova explosion ejects 5 $M_\odot$ of material at 4000 km/s. If the supernova occurs in a molecular cloud with density $n_H \sim 10^3$ cm^{-3}, when does the free expansion stage of supernova evolution end?

12. The Be^{10}/Be^9 isotope ratio suggests a cosmic ray lifetime of 10^7 years. Given that the energy density in cosmic rays is ~ 1 eV/cm^3 and that the galactic disk has a radius of ~ 15 *kpc* and a scale height of 100 *pc*, estimate the cosmic ray luminosity of the Galaxy in ergs/year. How does this compare with the kinetic energy input rate of supernova into the ISM of the Milky Way (assume 1 SN/100 years).

13. The supernova rate in a starburst galaxy is 100 times higher than the rate in the Milky Way. Assume that the cosmic ray energy density in the starburst galaxy is also 100 times higher and that the magnetic field energy density is in equilibrium with the cosmic ray energy density in both systems. Estimate the expected ratio of synchotron emission in the starburst galaxy to synchotron emission in our own Galaxy.

14. What is the Larmor radius of a 10 GeV cosmic ray in the Earth's magnetic field? Is this larger than the radius of the Earth?

15. Derive the ratio of the post-shock to pre-shock density for a 1D adiabatic shock.

Section II

Give one or two sentence (and/or one- formula) definitions of:

albedo
specific intensity
depletion
optical depth
HII region
reflection nebula
Compton scattering
forbidden line

flux density
diffuse bands
excitation temperature
PAH
bremstrahlung
Lyman α forest
source function
electron temperature
vibrational transition
reaction rate
two-photon emission
thermal spectrum
galactic magnetic field
Rayleigh scattering
cosmic rays
rotational transition
planetary nebula
"stationary" line
damped Lyman α lines
zone of avoidance
self-shielding
coronal gas
21 cm line
synchrotron radiation
Dispersion Measure
Rotation Measure
Alfven velocity
Faraday rotation
isothermal shock
adiabatic shock

Section III

1. Describe the physical properties of the phases of the interstellar medium, with rough numerical values for the temperature, density and volume filling factor.

2. Discuss the sources of energy input into the interstellar medium, with rough orders of magnitude (ergs cm^{-3}). Also say whether the energy is primarily mechanical or radiative, and which phase(s) of the ISM result from this energy input.

3. Discuss one or two observational probes for the different phases of the interstellar medium and how these observations are interpreted. Do you think we are missing important phases of the interstellar medium?

4. Discuss what is known about interstellar dust (size, composition, distribution, relative abundance). How do we know these properties?

5. How is the distribution of HII regions in the Galaxy found? (some details please; i.e. identification of HII regions, measurement of distance etc). What does this work tell us about star formation in the Galaxy?

6. Discuss the hypothesis of sequential star formation in a molecular cloud.

7. How would you measure the star formation rate of a galaxy (one method: state your assumptions)?

Section IV

1. The star Fomalhaut has an effective temperature of 8500 K, a luminosity of 25 $L_\odot$ and is 7 pc from the Earth. The IRAS flux densities are 26, 6, 9 and 11 Jy at 12, 25, 60, and 100 μm. Assume the star is a black body. Are these flux densities consistent with the radiation from the star, or do you need something else, e.g. a brown dwarf companion, planets, asteroids, circumstellar dust?. If so discuss what configuration these structures have with respect to the star. Which is the most likely (actually least unlikely)? In your calculations involving dust, assume that $Q_\nu \sim \nu$ and that the dust emissivity to mass ratio is $1/\kappa = 4a\rho/3Q = 0.04$ gm cm^{-2} at 100 μm, where ρ is the dust material density and a the dust grain radius in cm.

2. Suppose we have a molecule with three energy levels denoted 1, 2 and 3 such that $E_1 < E_2 < E_3$. Let g_1, g_2 and g_3 be the level degeneracies and A_{31}, A_{32} and A_{21} be the Einstein As for spontaneous emission. Suppose there is an external source of photons with $h\nu = E_3 - E_1$ and let β_{13} be the absorption probability per unit time for a molecule in level 1. (a) ignore stimulated emission and collisions, and assume that the escape probability for the photons emitted by the molecules is 1. Find an expression for n_1/n_2. (b) How large must β_{13} be to produce maser emission in the 2-1 transition? (c) Is it possible to have maser emission in the 3-2 transition? What condition(s) must be satisfied?

3. A spherical interstellar cloud has a radius of R = 20 pc and a line-of-sight velocity dispersion of 3 km/sec. If the cloud is of constant density and is in virial equilibrium, what is its mass? Suppose $n(CO)/n(H_2) = 10^{-4}$, that the CO excitation temperature is 10 K and the dust is in spheres of 2000 Å radius. If the mass absorption coefficient of the dust is $1/\kappa = 4a\rho/3Q = 0.04$ gm cm^{-2} at 100μ and the gas to dust ratio by mass is 100, what is the ratio of CO(1-0) and 100μ luminosity ($L = \nu F_\nu \times 4\pi R^2$). If the CO(1-0) radiation is the only coolant, what is the cooling time of the cloud?

4. Suppose that the cross section for the reaction $AB + C \rightarrow A + B + C$ is $\sigma(E) = 0$ for $E < E_o$, $\sigma(E) = \sigma_o$ for $E > E_o$, where E is the center of mass

translational energy. Let the masses of AB and C be m_{AB} and m_C. Obtain the thermally averaged rate coefficient for this reaction as a function of temperature T.

5. A star is losing mass in a steady wind with an outflow speed of 20 km s^{-1} and a mass loss rate of 10^{-4} $M_\odot$ yr^{-1}. The hydrogen is lost in molecular form and the age of the envelope is 30,000 years. Let G (= 3×10^5 photons s^{-1} sr^{-1} cm^{-2}) be the number of interstellar photons of sufficient energy to excite H_2 and f = 20% be the number of excitations which lead to dissociation of an H_2 molecule. Make a very rough calculation, ignoring the effects of dust, of the number of H atoms in the envelope. At the edge of the envelope, what is the time scale for reformation of H_2 on the surfaces of dust grains if the gas to dust ratio is normal?

6. The gas in the galactic disk is in hydrostatic equilibrium.
Assume that the gas density distribution obeys

$$\rho(\mathrm{z}) = \rho(0)\exp-(\mathrm{z}/\mathrm{h})^2$$

with $\rho(0) = 3 \times 10^{-24} gm\ cm^{-3}$.

There are three terms that contribute to the gas pressure:
-turbulent motions with $< \mathrm{v_c^2} >^{1/2} = 10$ km/s
-magnetic field pressure with $B = 2 \times 10^{-6}$ G in the plane
-cosmic ray pressure with a energy density equal to the magnetic field pressure

Assume that the scale height for the magnetic field, the gas and the cosmic rays are the same, so that

$$\frac{\mathrm{dlnp_{gas}}}{\mathrm{dz}} = \frac{\mathrm{dlnp_{CR}}}{\mathrm{dz}} = \frac{\mathrm{dlnp_B}}{\mathrm{dz}}$$

and use the equation of hydrostatic equlibrium to solve for the disk scale height.

For this problem, assue that the gradient in the gravitationl field of the disk $\mathrm{dg_q/dz} = -2.2 \times 10^{-11}$ cm/s^2/pc.

7. Kelvin-Helmholtz instability. Consider two incompressible fluids streaming past each other at an interface. At $z > z_s(t = 0) = 0$, $\rho = \rho_1$, $\vec{U} = U_1\hat{x}$. At $z < z_s(t = 0) = 0$, $\rho = \rho_2$, $\vec{U} = U_2\hat{x}$ with $\rho_1 < \rho_2$. In this question, we will ignore surface tension at the boundary. (a) impose a weak perturbation to the velocity potential, $\psi = C_1 \exp[-kz + i(kx + \omega t)]$ for $z > 0$ and $\psi = C_2 \exp[kz + i(kx + \omega t)]$ for $z < 0$. Show that this perturbation is consistent

with the fluid being incompressible. (b) Relate the location of the surface to Ψ:

$$z_s^+(t=0) = z_s^-(t=0)$$

$$\frac{Dz_s}{Dt} = \left(\frac{\partial}{\partial t} + U_x \frac{\partial}{\partial X}\right) z_s = v_z(z_s)$$

(c) Use the continuity of velocity normal to the surface to show

$$(\omega + kU_1)C_2 = -(\omega + kU_2)C_1$$

(d) Show that

$$p = -\rho g z + \rho \frac{\partial \psi}{\partial t} + \rho U \frac{\partial \psi}{\partial x}$$

is consistent with the conservation of z momentum

(e) Use the continuity of pressure across the boundary, together with the results of (a) - (d) to derive the dispersion relation:

$$\omega^2 + 2\omega k(\frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2}) + k^2(\frac{\rho_1 U_1^2 + \rho_2 U_2^2}{\rho_1 + \rho_2}) + gk\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = 0$$

Note that at large k, the perturbation is always unstable.