AST 204 Feb 18 2008: Due Feb 25 2008

PROBLEM SET 2

There is a famous paradox in relativity theory, the 'twin paradox', in which one twin goes on a journey on a spaceship which reaches very nearly the speed of light to some distant star and then returns similarly rapidly. Time dilation implies that the travelling twin ages less (because less time passes on the fast spacecraft–never mind the rigors of the voyage) than the stay-at home. It is a paradox is because the travelling twin can say she is at rest and her sister is moving, and reaches the opposite conclusion. People argue about acceleration, etc., and obfuscate the problem in various ways. We can state it in a very simple, though somewhat wordy manner:

1. Consider the following experiment: Three identical atomic clocks are made. One of them is sent by spaceship to far beyond Alpha Centauri, which is 4.3 light-years away; the captain is instructed then to turn around, and to accelerate to 0.95c toward home, reach this velocity before he reaches Alpha Centauri, and maintain it until he reaches Earth. He is to meet a spaceship carrying the *second* clock, which has been sent in the direction opposite and instructed to turn around and accelerate toward Alpha Centauri, reaching 0.95c before passing Earth and maintaining that velocity until it reaches Alpha Centauri. The timing is such that the spaceships meet within the Alpha Centauri system. The third clock remains on earth. Note that BOTH clocks travel uniformly at v=0.95c between Earth and Alpha Centauri.

They are synchronized in the following way. When the second spaceship passes earth on its way to Alpha Centauri, both that clock and the one on earth are set to zero. The second clock then goes to Alpha Centauri. When the first clock and the second reach the Alpha Centauri system (simultaneously, if the planning was OK), the first clock, now heading home at 0.95c, is set to read the same as the second (outbound) clock. We assume that each of these clock settings is done at very short range by radio (say) and is done without error.

When the first clock returns home, it relays its time to the lab where the stayathome clock is, and the readings are compared.

What are they? What do you infer about time and motion from this?

2. Misleading appearances: We found that dimensions of moving objects parallel to their motions are shortened by the Lorenz contraction; a moving object is measured to have length $L_0\sqrt{1-v^2/c^2}$, where L_0 is the 'proper' length—*i.e.* the length measured by an observer at rest with respect to the object. Suppose a cube which is lighted from inside (and suppose has different colored faces) is moving by at large distance at large velocity, moving parallel to one of its edges and with one face of the cube perpendicular to the line of sight when the observation is made. What does the observer see through her telescope? This problem is *not* difficult at all, but you need to *think*.

3. You are a bug living on the surface of a balloon. You notice that all the other bugs are moving away from you. You observe distances D and recession speeds V of other bugs on the balloon, and find that they obey a law V = HD, where H is constant in space and time. The distances and speeds are measured along the surface of the balloon. Assuming that the other bugs, like you, are stationary with respect to the surface of the balloon, derive a formula for the balloon's radius as a function of time.

4. You measure the spectrum of a distant quasar at the Keck telescope and detect the 1216 Å line of hydrogen at a wavelength of 8512 Å. You want to know if the quasar contains nitrogen or carbon. You know that the laboratory wavelengths of spectral lines from these atoms are 1240 Å and 1549 Å respectively. At what wavelengths should you look for these lines in your quasar spectrum?