LECTURE 11: The Visible Universe I: The Formation of Hydrogen and Helium

AST204 March 10 2008

I. The Planck Era and the Notion of Time and Space

We have covered a lot of ground, much of it very unfamiliar. It would be well at this point to review where we have been and see where we might go from here, and fill in a couple of blanks that we hurried past. Let us begin *before* the first epoch we have considered, the epoch of inflation. As we discussed, it is not entirely clear that there *was* an era before, but some workers in the field believe that there was an era of more-or-less 'ordinary' expansion earlier than the inflation era, and that inflation is associated with breaking the symmetry which unifies the electroweak forces with the strong interactions at the GUT (Grand Unified Theory) energy scale—a kind of phase transition in the early universe. This transition certainly happened if any of the GUTs are correct, but whether it did so in a way which had anything to do with inflation is certainly not at this point clear. In any case, we know there is an epoch, associated with a density, a temperature, and a timescale 'earlier' than which it makes no sense to even discuss the notions of time and space. This is the *Planck Time*, which we have discussed already. The following is largely a repeat for the sake of continuity.

The energy of a photon or other highly relativistic ($\gamma >> 1$) particle is just $E = h\nu = hc/\lambda$. The Schwarzschild radius associated with a black hole of mass m is $r_s = 2Gm/c^2$ in ordinary units, or, since the energy E associated with a mass m is mc^2, $r_s = 2GE/c^4$. Thus an energetic particle so energetic that its own self-gravity makes a black hole in which its wavelength just fits has a wavelength which satisfies

$$\lambda = r_s = \frac{2GE}{c^4} = \frac{2Gh}{\lambda c^3},$$

so this critical wavelength, the *Planck length*, is

$$l_P = \lambda = \sqrt{\frac{2Gh}{c^3}} = 5 \times 10^{-33} cm$$

The energy, the *Planck energy*, is

$$E_P = \frac{hc}{\lambda} = \sqrt{\frac{hc^5}{2G}} = 4 \times 10^{16} erg = 2 \times 10^{19} GeV$$

And, of course, an associated temperature, the *Planck temperature*

$$T_P = E_P/k = 3 \times 10^{32} K.$$

The associated mass, the *Planck mass*, is

$$m_P = E/c^2 = 4 \times 10^{-5} g$$

There is a *density* associated with this notion, namely one Planck mass per Planck length cubed, or

$$\rho_P = m_P/l_P^3 = 3 \times 10^{92} g/cm^3$$

and a time which might be either of two expressions–the inverse of the *frequency* associated with the wavelength l_P *or* the expansion time of the universe associated with ρ_P. These two quantities are the same within a factor of two; the *Planck time* is usually called $l_p/c = 2 \times 10^{-43} sec$. The fact that the two notions agree is profound. Can you figure out why they agree?

We do not know whether the universe was ever this dense, hot, or rapidly expanding, but we do know that it does not make sense to discuss conditions even more extreme than this. In order to *study* a more extreme condition (smaller length scales, say), one would have to resolve finer detail, which would involve particles of even shorter wavelength. But Planck energy particles already cannot propagate because they are within their own event horizon. So the concepts of space and time as a classical arena for events breaks down here, or probably actually well before—*it makes no sense to talk about the spacetime continuum on scales shorter than the Planck length and time; the universe is a fundamentally quantum gravitational entity at this epoch.* The notion of even what quantum gravity *is* is rapidly evolving at the present time, and the whole field of fundamental physics is currently doing a grand experiment of an unprecedented kind today. We will see (hopefully within *your* lifetimes, but I suspect not within mine) whether this crusade will succeed. Perhaps the inflation is associated with *this* epoch, not with the GUT epoch at all, and the birthplace of this and possibly/probably an infinity of other universes in a seething timeless quantum gravitational froth, some tiny fluctuation in which spawned the whole thing.

II. The Inflation Era and Afterward–Some Thoughts on Conserved Quantities

The more 'conventional' picture is that the universe expanded more-or-less normally between this age of 10^{-43} seconds and something like the GUT epoch at 10^{-36} seconds, whereupon it began to inflate; the density was about $10^{77} g/cm^3$, the temperature about $10^{28} K$, and the thermal energies about 10^{15} GeV. At some point, the inflation ended, by some mechanism as yet ill-understood....not that the *start* is well-understood, either.

At this point let us pause and ask a question of some depth. The universe, as far as we know, has a lot of *matter* in it, and that matter is made of protons, neutrons, and electrons. Before the GUT era, the particles and forces were entirely different, and there were but two forces in the universe, the GUT force and gravity. But in the standard model, there are a number of troublesome conserved quantities which should perhaps give us pause. Baryons are conserved, electrons and muons and taus are conserved. The muons and taus are no problem because they can and do decay, conserving their family lepton numbers. For example, a muon normally decays into an electron, an electron *antineutrino*, and a *muon neutrino*. Since the electron carries electron number 1, the electron antineutrino electron number -1, and the muon neutrino muon number 1, all is OK. There are no free muons or taus in nature because they decay through these channels very rapidly, but the electron is stable. The situation is similar for baryons. There are many unstable baryons, but only one stable one, the proton, and an unstable one very nearby in mass which can exist stably in nuclei, the neutron. (Why is the neutron stable in nuclei? Because it is bound more strongly than the energy which would be gained by decaying to the slightly less massive combination of a proton and an electron and an electron antineutrino, which is the normal decay route) We know almost beyond a shadow of a doubt that there is net baryon number in the universe–that is, that the universe is NOT an equal mixture of matter and antimatter, unless they are somehow segregated on very large scales. The gamma-ray background which would come about from their continual annihiliation would be much larger than observed.

There are lots of protons, and, evidence would have it, almost no antiprotons. Where did they come from? This would appear to be a real asymmetry in the world, and would mean that stuff before inflation had to have a *LOT* of baryon number to keep the number of baryons from inflating away...if you do the math carefully, this appears to be a fatal flaw. Nothing conserved during inflation can survive in any interesting quantity afterward.

So either inflation did not happen or baryons are not conserved. Most of the plethora of GUT theories handily predict the latter, as well as predicting that the various families of leptons are not conserved separately and for some that leptons are not conserved at all. At ordinary energies this nonconservation is very, very weak. This has two consequences for cosmology—one, that net baryon number can be produced in the GUT era or perhaps later by the decay of massive particles created during the GUT era and inflation. There are analogues of this kind of phenomenon at lower energy involving the non-conservation of parity. This process is called *baryogenesis*; when it happens depends on the mass of the particle which decays or interacts to make baryons. It is worth remembering that while we do have to make some in order to have some today in an inflated universe, the process was of necessity *very* inefficient. To see this, consider the number density of photons.

The energy density of photons is aT^4, which today (2.71K) is $4\times10^{-13} erg/cm^3$. The energy of a typical photon is $kT = 4 \times 10^{-16}$ erg, so the number density is of order $10^3 cm^{-3}$. The number density of baryons (protons), if the current mass density is, say, $10^{-30} g/cm^3$, is of order $10^{-6} cm^{-3}$, so the ratio is 10^{-9} (and most of the rest mass is probably *not* be baryons, so the number is probably an order of magnitude smaller). Now at some epoch in which the thermal energy kT is much larger than the rest mass m of some particle, the *number density* of such particles (and their antiparticles) is roughly the same as that for photons–there is one particle for each available state up to an energy of order kT. To estimate this number, consider a cubic centimeter box; the *wavelength* of a particle, if its energy is large compared to its rest mass, is $\lambda = hc/E \approx hc/kT$. You will see in quantum mechanics that this wavelength in a very real sense represents a 'size' for the particle; the number of states in the continuum for a free particle per unit volume is roughly $1/\lambda^3$, so the number of thermal particles per unit volume is approximately $(kT/hc)^3$. There is a sizeable dimensionless multiplier in front which is different for fermions and bosons and is proportional to the number of spin states; for photons it is $16\pi\zeta(3) \approx 60.4$; the number of photons is thus $60.4(kT/hc)^3 \approx 400 cm^{-3}$.

So whatever decayed or interacted to make the baryons, only about one in a billion needed to successfully make a baryon, or, to put it another way, the baryon-antibaryon asymmetry needed only to be about a part in 10^9. *Baryons today, even though they represent a big piece of the rest mass energy in the universe, are very much a trace constituent as regards number.* We have no idea how many leptons there are, but the best guess is that there might be roughly as many as baryons, which might seem *a priori* a little unlikely—it would seem more likely that there are precisely zero–there may well be an electron antineutrino for every electron. But we do know that there are almost certainly roughly as many neutrinos and antineutrinos, probably of each family, as there are photons, because when the temperature was high enough that electron-positron pairs were being made, lots of neutrinos were also, and even earlier they were in thermal equilibrium.

The *other* cosmological consequence of the nonconservation of baryons is that *protons are almost certainly unstable*, with a very, very long decay time, greater than 10^{29} years. This has been looked for unsuccessfully for a number of years, but would have been seen only for the very shortest times predicted–this has ruled out a few of the simplest interesting GUT theories, but many more remain.

Another question we might address while we are on the topic of inflation is how *big* the perturbations might be. We have seen that the *observations* suggest that they are tiny, even when they have had a chance to grow after they have entered the horizon. Does this even vaguely make sense? Let us estimate the number of thermal states, and therefore approximately the number of thermal particles, within the event horizon during inflation. The temperature is about $10^{28}K$, as we have seen ($kT \approx 10^{15}$ GeV), so the wavelength of thermal particles is about $hc/kT \approx 1.4 \times 10^{-28} cm$. The expansion time is, as we have seen, about $2 \times 10^{-36} sec$, and the horizon size is just this time times the velocity of light, or $6 \times 10^{-26} cm$. Thus the number of states within the horizon is of order $(6 \times 10^{-26}/1.4 \times 10^{-28})^3 \approx 8 \times 10^7$. Compare this number with that for the universe today; $(10^{28} cm)^3 \times \sim 400 \approx 4 \times 10^{86}$! The fluctuations in this number are expected to be of order the square root of the number, as for any random process, or about 10^4, and the fractional fluctuation about the inverse of this, 10^{-4}. This is about right, which says that the conditions envisaged for inflation are at least roughly consistent with what appears to have emerged from it. Notice that if inflation had occurred *later* the universe would almost certainly have been smoother, and if much earlier, closer to the Planck era, much rougher–at the Planck time, there is but *ONE* particle within the horizon!

III. The Expansion of the Universe and the Creation of the World

Recall that when everything in the Universe is in the form of radiation,

$$\rho \sim \frac{1}{t^2}$$

(where ρ is the density and t the age of the Universe. The temperature of the Universe then goes with time line

$$T \sim t^{-1/2}$$

(this is not quite true because of the creation of matter as the Universe expands, as we shall see below).

At $t = 1$ second, $T = 10^{10}$ K (remember that we can work this out by scaling to the present age of the Universe and its present temperature). Now

$$\mathrm{E} = \mathrm{mc}^2 = \mathrm{kT}$$

Thus

$$\mathrm{m} = \frac{\mathrm{kT}}{\mathrm{c}^2} = 1.5 \times 10^{-37}\mathrm{T}$$

So we have:

$$t = 10^{-6} seconds; T = 10^{13} K; m = 1.5 \times 10^{-24} gm(protons)$$

$$t = 1 second; T = 10^{10} K; m = 1.5 \times 10^{-27} gm(electrons)$$

The *Boltzmann equation*, which describes the distribution of energy among different states in thermal equilibrium at temperature T, then gives

$$\frac{N_n}{N_p} = exp\left(\frac{-(m_n - m_p)c^2}{kt}\right)$$

where N_n is the number density of neutrons, N_p the number density of protons, m_n the neutron mass, m_p the proton mass, and $m_n - m_p = 0.0014 m_p$ (2× the mass of the electron: the binding of an electron and proton to make a neutron). Thus at T = 10^{10} K, when electrons "form", the neutron/proton ratio is about 1/5.

In the early Universe, essentially all of the matter was hydrogen and helium. Thus the neutron/proton ratio is

$$\frac{N_n}{N_p} - \frac{2n(He}{2n(He) + n(H)}$$

where n(H) and n(He) are the numberdensities of hydrogen and helium. The formation of the Universe in the "Big Bang" thus predicts the helium to hydrogen ratio X = n(He)/n(H):

$$\frac{1}{5} = \frac{n(He)}{n(He) + n(N)/2} = \frac{X}{X + 0.5}$$

i.e. X = 0.1, which is the observed ratio of helium to hydrogen. Thus one of the great triumphs of the Big Bang theory of the origin of the Universe, together with observations of the microwave background, is that you can calculate abundances of the light elements which agree with the observed abundances!

IV. Nuclear Reactions in the Early Universe

When the universe is about 1 second old, its temperature has fallen to about $10^{10} K$, and kT is about 1 MeV, the typical energy differences among various nuclear species. It is still very radiation- dominated; it is filled with photons, neutrinos and antineutrinos, electrons and positrons (the mass-energy of an electron is only 0.5 MeV, so the photons in the background are energetic enough to make positron-electron pairs), and a few baryons, roughly comparable numbers of protons and neutrons, but, as we saw above, only about 10^{-10} as many as photons. For temperatures much higher than this, the reactions

$$n \to p + e^- + \bar{\nu},$$

$$n + e^+ \to p + \bar{\nu},$$

$$p + e^- \to n + \nu$$

and some related neutrino-driven reactions are in equilibrium, and since the neutron is a bit more massive than the proton and is therefore harder to make, the neutrons are a bit less numerous. There are typically about six times as many protons as neutrons when kT drops below 1 MeV and these reactions can no longer maintain the equilibrium. Thus at this point the neutron fraction is 'frozen in'. But neutrons are unstable and decay (the first reaction in this set) with a lifetime of order 15 minutes, so their number slowly drops. There are still too many high-energy photons around for nuclei to form, because the only nucleus which can form from protons and neutrons is the deuteron, the nucleus of deuterium, which consists of one proton and one neutron, and it is fairly weakly bound, 2.2 MeV.

Well, you say, that is higher than the typical photon energy, and this is true, but there are $\approx 10^{10}$ times as many photons as protons or neutrons, and the high-energy tail still has many more photons than baryons. It is not until the temperature falls about a factor of 10, to $kT \approx 0.1 MeV$, that substantial amounts of deuterium can form by the reaction

$$p + n \to {}^2D + \gamma,$$

and not be destroyed immediately by the reverse reaction (photodissociation). As soon as a deuterium nucleus is formed, it is joined by another proton,

$$D + p \to {}^3He + \gamma,$$

to make a nucleus of the isotope Helium-3. This then almost always reacts with a deuterium nucleus,

$$D + {}^3He \to {}^4He + p.$$

so almost everything winds up as Helium-4, the most stable light nucleus, and the total effect is the reaction

$$2p + 2n \to {}^4He + photons.$$

which uses up essentially all the neutrons. When these reactions occur, there were about a seventh as many neutrons as protons (some had decayed since they froze out). Since each neutron captures a proton as it is incorporated into a helium, there are twice as many nucleons in the resulting helium nuclei as there are neutrons, and the *mass fraction* Y of Helium-4 is then

$$Y = \frac{2n}{p+n} \approx 0.25.$$

This is slightly sensitive to the abundance of baryons, since if there are more the deuterium can form earlier before the neutrons have decayed so much, and there is a larger fraction of helium formed; conversely, if there are fewer the reaction has to wait longer and there is less. It is also sensitive to how many species of massless particles there are; if there are more, the total density is higher and the expansion of the universe and the cooling more rapid; again one makes more helium.

Why does the helium not react? First, it has a high repulsive force because it has two protons, but no higher than helium-3. The real reason is that there is *no* stable nucleus with mass 5, and none with mass 8, so helium-4 can react with neither protons or other helium-4 nuclei. The world would be a *very* different place were this not so. Stars have solved this problem, as we will see, but the universe did not have time.

The measured abundance of helium is $Y = 0.238 \pm .007$ in systems in which there has been very little production of helium by stars, very close to this crude estimate. There is at the end a very small amount of deuterium and Helium-3 left (about 10^{-5} by mass) and a tiny amount of ^7Li produced (about 10^{-10} by mass, by a tiny number of reactions between helium-4 and helium-3). It is one of the great triumphs of cosmological theory that these results agree with what is observed within a cosmological model which fits all the other constraints. All of these reactions were over when the universe was about 3 minutes old and had a temperature of about 700 million degrees.

By this time all the positrons are long gone; the universe consists of photons and neutrinos (which contain nearly all of the energy), and a relatively few protons and helium nuclei, enough electrons so the soup is neutral, and another heavy component (the mysterious Dark Matter) which outweighs the ordinary matter by a factor of seven or so and whose nature we do not know, but which presumably had been there all along. The next hundred thousand years is pretty dull. Eventually the tiny fraction of baryons and dark matter dominate the total energy as the photons and neutrinos cool, and then when the universe is about two hundred thousand years old and has a temperature of about 4000K, the electrons attach themselves to the protons and helium nuclei and the baryonic matter in the universe becomes neutral hydrogen and helium gas. At this time the density of baryonic matter is about 500 particles per cubic centimeter, about $10^{-21} g/cm^3$.

V. The Universe Today: Dark Matter and Dark Energy

Ah, yes, what about the *REST* of the rest mass, the dark matter. We will see that about 85-90 percent of the mass is in this form. We do not know what it is. If it is a new kind of stable particle, we know that it interacts only very weakly and cannot be charged. We know of no stable neutral particle with mass, and even if neutrinos have mass, which many GUT theories predict, they are almost certainly too light to have much cosmological impact. The favorite extensions to physics today involve *supersymmetry*, a symmetry between bosons and fermions in which for each fundamental fermion there is a fundamental boson and vice versa, and the theory unambiguously predicts that the lightest of these supersymmetric partners is stable, but is silent so far on what particle that is–perhaps the photino or the gravitino, the fermion partners of the photon or the graviton, the hypothetical massless boson which carries gravity. Crude calculations say that it should weigh perhaps a few hundred times as much as the proton and be present in the universe in even somewhat smaller numbers than baryons. Searches for these particles, which it is supposed do interact weakly (literally, through the weak force) have been underway for a long time but have so far yielded nothing. There are other possibilities. For now we know that the stuff is there and have much fun speculating what it is, but know almost nothing about it.

So the census of the universe today includes baryons, about a billion times as many photons and roughly the same numbers of neutrinos of all three flavors. The numbers and thermal energy are vastly dominated by these zero (or nearly zero) rest-mass particles but the rest energy by the baryons and the dark matter. We do not know the mass of the dark matter particles within a range from the lightest postulated, the *axion* with a mass of about a millivolt, $10^{-36}g$, to mini black holes with masses of more than 10^14g—an uncertainty of at least a factor of 10^{50}—but one which, surprisingly, makes almost vanishingly little difference to anything going on in the universe today.

Then, of course, there is the possibility, even the probability, that the vacuum today gravitates very weakly, with an energy density which is some substantial fraction (like 0.7) of the critical density today $\rho_c = 3H^2/8\pi G$, or of order 5×10^{-30}. Note, however, that if this is the case, one must go only back a little way (in expansion, though a rather long *time*) before the force due to the vacuum is negligible. If the ratio of vacuum force to the gravitation of the matter is now 4:1 ($\Omega_m = 0.2$, $\Omega_\Lambda = 0.8$), they were equal when $(1+z) = 4^{1/3} = 1.6$, redshift 0.6, or an age of something in the neighborhood of 60 percent of the present age. Before this the universe behaved very much as if the vacuum did not gravitate, and all the development is pretty much exactly what we have done.

So the energetics of the universe today appears to be dominated by two things of which we know very little, hypothetical dark matter and hypothetical dark vacuum energy. It may give the reasonable person pause that their current densities are within a factor of 4 or 5 of each other, when the ratio could have been anything, and in any case has been changing throughout cosmic time. Indeed, as mysterious is the (constant) ratio of dark matter to baryons at 7 or 10 to 1. Probably unrelated species; why so close? The situation is not very satisfactory, and it seems very likely that there are some underlying relations of which we are unaware.

VI. A Summary Table, From the Beginning Till Now

To complete this summary, let us consider a table which summarizes the physical conditions in the universe from the Planck time to the present.

Cosmological Conditions as Functions of Time and Redshift

τ (sec)	R(τ)	$1+z$	T (K)	kT (eV)	ρ_T g/cm^3	ρ_m g/cm^3	$R_0 u_h$ (cm)	r_h (cm)	m_h $M_\odot$	description
4(17)	—	1	2.7	2.3(−4)	9(−30)	3(−30)	4(28)	4(28)	3.8(23)	present
8(12)	$\tau^{2/3}$	1500	4000	0.3	1.2(−20)	1(−20)	1.1(27)	7(23)	(18)	combination
4(11)	—	7000	2(4)	1.9	2.5(−18)	1(−18)	1.7(26)	2.4(22)	3(16)	equal m&r
10		5(9)	1(10)	$1M$	5(5)	0.2	4(21)	(11)	130	nuclear reac
3(−7)	$\tau^{1/2}$	8(12)	1(13)	$1G$	1.6(18)	5(8)	1.4(17)	1.8(4)	5(−12)	quark-gluon
2(−11)		1(15)	1(15)	$100G$	4(26)	1(15)	1.2(5)	1.2	3(−18)	baryogenesis
2(−34)	—	1(28)	1(28)	1(15)G	3(77)	1(54)$*$	1.5(3)	1.5(−25)	7(−54)	end inflation
1(−34)	$e^{H\tau}$	2(45)	??	??	3(77)	(106)$*$	3.5(20)	1.5(−25)	0.1	mid-inflation
2(−36)	—	5(62)	1(28)	1(15)G	3(77)	2(158)$*$	8(37)	1.5(−25)	1(51)	beg. inflation
1(−43)	??	??	2(32)	2(19)G	3(92)	??	??	5(−33)	??	Planck era

The numbers in parentheses are powers of 10, so that, for example, $1(-43) = 1 \times 10^{-43}$. The columns in the table are mostly self-explanatory, but briefly are as follows: τ is the cosmic time at the epoch in question, Rlaw the relation between the scale factor and τ. $1+z$ the redshift factor, T the temperature in K, kT the typical thermal energy in eV–later MeV and GeV; ρ_T the total mass-energy density in g cm^{-3}.

The column headed ρ_m is the density of *rest mass* Actually, this is a swindle for early times, because what it is is simply the present rest mass density multiplied by $(1+z)^3$, and as such is just a 'tracer' for the mass today. Clearly early on at energies where protons and neutrons do not exist, we cannot easily calculate the rest mass density, and during inflation, if the identities of protons and neutrons were carried along, the densities would become ridiculous. These entries are noted by the presence of a (*). Note that this simple bookkeeping calculation assumes that the baryons, were they conserved, continue to carry one proton mass, which is probably wrong–in fact, they may carry no mass at all, because the mechanism which gives particles mass may do so only at relatively low energies. But this calculation illustrates graphically that baryon number cannot be conserved in inflation.

$R_0 u_h$ is the comoving size of the particle horizon computed as if *there were no inflation* referred to the present universe. At and prior to the end of the inflation era, we just follow an incoming light ray which is at that horizon radius at the end of inflation, which illustrates how immense the effective horizon becomes during inflation. Note that the *physical* size of the horizon remains at the *event horizon* during this time, but the redshift factors become so large that the comoving distances become huge. About 55 efolds are required for the inflated particle horizon to encompass the entire observable universe today, and after about 80, roughly the supposed number, the horizon is 7 orders of magnitude larger than the present observable universe. Thus any point in the universe at the end of inflation had been able to receive information and send information to a volume which is now immensely larger than the present observable universe.

The adjacent column, the physical size of the horizon at the relevant period, is just the comoving one divided by $(1 + z)$.

The column m_h is the rest mass, again referred to the present universe, within the horizon– just the present rest mass density within the comoving radius $R_0 u_h$.

We have talked about most of the phenomena identified in the table: The present, recombination, the equality of radiation and mass density, the era of nuclear reactions. Above a temperature corresponding to something like 1 GeV (though it is somewhat uncertain) baryons and mesons no longer exist and are replaced by a plasma of free quarks and gluons; at somewhat higher energies, above about 100 Gev, the electroweak symmetry is restored, and at something like these energies the processes which lead to baryogenesis probably freeze out and create the baryon number we see today.

Before this, at least according to our present hazy understanding, not very much interesting happens over a very large range in temperature and expansion until the energies of the GUT era are reached. There are only the electroweak and strong forces, only the fundamental leptons, quarks, any decaying heavy remnants of the GUT era, the gauge bosons responsible for the forces, including photons, Ws, Zs, and gluons. Then we hit the GUT era and inflation, prior to which there may (or may not) have been a more-or-less ordinary expansion era from the Planck time.

The numbers in this table are mostly, especially in the columns pertaining to the earliest times, very uncertain. The assumptions made in calculating the entries are that H_0 is about 70km/s/Mpc, that Ω_0 is unity but is made up currently of $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$; that baryogenesis is associated with the decay or interaction of weakly interacting particles of mass about 100 GeV, that inflation is associated with the GUT energy scale at about 10^{15} GeV, and that the universe undergoes about 80 efolds of inflation.