Pre-Algebra Practise Exam 2 - Solutions

1. **Express as a decimal and as a percentage:**

   (a) $\frac{1}{2}$

   Answer:

   $$0.5 \qquad 50\%$$

   (b) twenty-five thousandths

   Answer:

   $$0.025 \qquad 2.5\%$$

   Order from smallest to largest:

   $\frac{1}{4}$, 0.001, 0.55

   Answer:

   $$0.001, \quad \frac{1}{4}, \quad 0.55$$

   Round to the nearest tenth and hundredth:

   (c) 5.2935

   Answer:

   $$5.3, \quad 5.29$$

   (d) 3.1234

   Answer:

   $$3.1, \quad 3.12$$

2. **Solve:**

(a) $\frac{1}{4}(x+3) = 2x - 1$

Answer:

$$\begin{aligned} x + 3 &= 4(2x - 1) = 8x - 4 \\ 4 + 3 &= 8x - x \\ 7 &= 7x \end{aligned}$$

$\boxed{x = 1}$

(b) $-2x < 10$

Answer:

$$x > \frac{10}{-2} = -5$$

$\boxed{x > -5}$

3. **Simplify, leaving no negative exponents:**

(a) $\frac{x^2y^2x}{x^2}$

Answer:

$$x^{2-2+1}y^2 = x^1y^2$$

$\boxed{xy^2}$

(b) $(2x^2y^2)^2(x^2y)^{-2}$

Answer:

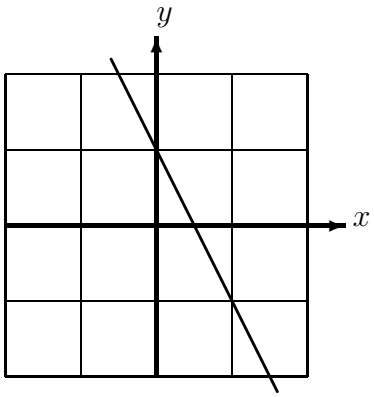
$$(2^2x^4y^4)(x^{-4}y^{-2}) = 4x^{4-4}y^{4-2} = 4x^0y^2$$

$\boxed{4y^2}$

4. **For each of the following linear equations, determine the slope and intercept. Plot the equation.**

(a) $y = -2x + 1$

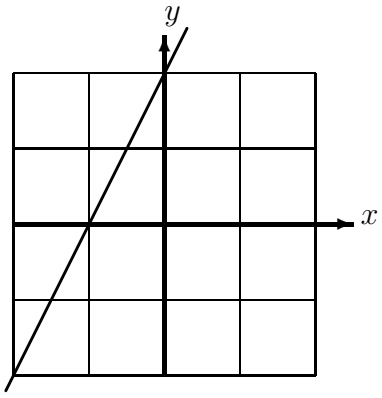
Answer:



$\boxed{\text{Slope } m = -2}$, $\boxed{\text{intercept } b = 1}$.

(b) $y - 2x = 2$

Answer:



This is equivalent to $y = 2x + 2$, hence $\boxed{\text{slope } m = 2}$, $\boxed{\text{intercept } b = 2}$.

5. I am building a ramp. The length of the ramp along the ground is 12 meters and it's 5 meters high and 5 meters wide. I want to paint it with wood preserver all over (including the base) before I install it. If the cost to paint 1 sq meter is \$1, how much is this going to cost me?

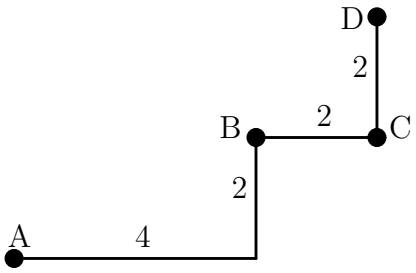
Answer: The sides of the ramp are right triangles with base 12m and altitude 5m, so each has area $\frac{1}{2} \times 12 \times 5 = 30\text{m}^2$. The base is a rectangle $12\text{m} \times 5\text{m}$, area 60m^2. The vertical back is a rectangle $5\text{m} \times 5\text{m} = 25\text{m}^2$. Finally, the length of the ramp along its slope is the hypotenuse of a right triangle of sides 12 and 5 (meters, of course), hence this length is $\sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$ meters. Thus the upper surface of the ramp is a rectangle of area $5\text{m} \times 13\text{m} = 65\text{m}^2$. The total area is therefore

$$2 \times 30 + 60 + 25 + 65 \text{ m}^2 = 210\text{m}^2 \,,$$

so the paint will cost $\boxed{\$210}$.

6. Amy lives right in the center of town. Her friend Betty lives 2 miles north and 4 miles east of Amy. Amy drives to Betty's house to pick her up and they then drive 2 miles east to pick up Cathy, and then all three drive two miles north to a dance hall. Plot these four points (A, B, C and D) on a map of the town, and, on the assumption that all the roads in the town run north-south or east-west, draw Amy's path between her house and the dance hall. If Amy can drive at 30 mph, and it takes 5 minutes each to pick up Betty and Cathy, when should she leave home to get to the dance hall by 7 pm?

Answer:



Amy has $4 + 2 + 2 + 2 = 10$ miles to drive, which will take 20 minutes at 30 m.p.h., plus $5 + 5 = 10$ minutes at her friends' houses, making 30 minutes in all; so she should leave by $\boxed{\text{6:30 pm}}$.

7. I have a desk which is a rectangle measuring 5 feet by 4 feet. (a) What is the area of my desk? (b) my desk has flaps at either end which can be raised to make its surface area bigger. If each flap is a semi-circle of diameter the same as the small dimension of my desk (4 feet) what is the area of my desk with the flaps raised? (c) I buy a flat-screen TV for my living room. The salesman tells me it's a $17''$ screen, but when I get home and measure it I find it's a rectangle measuring $8'' \times 15''$. Was he lying to me? Explain.

Answer: **(a)** 20 square feet. **(b)** If combined, the two flaps would make a full circle of diameter 4 feet and radius 2 feet; the area is $\pi(2\text{ft})^2 = 4\pi\,\text{ft}^2$, so the total area is $20 + 4\pi$ square feet. **(c)** Screen areas are usually measured on the diagonal; in this case, the diagonal is $\sqrt{8^2 + 15^2} = \sqrt{289} = 17$ inches, so the salesman was truthful.

8. The Earth's diameter is roughly 8,000 miles. (a) What is its circumference? (for this problem you can take $\pi = 3$). (b) Suppose the Earth is a perfect sphere and completely flat (no

mountains, oceans etc.). I want to lay a fiber–optic cable all the way round the equator, flat on the ground. What length of cable do I need to do this? (c) Instead, I decide to hang the cable from the tops of pylons 100 feet high. If I can make the cable taut, so that it's a circle, how much more cable do I need?

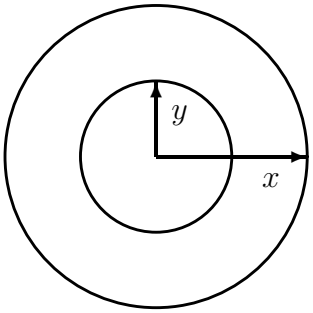
Answer: **(a)** $\pi \times 8000 \approx 24,000$ miles if $\pi \approx 3$ ($25,000$ is closer). **(b)** As above, $\pi \times 8000$ miles. **(c)** Putting the cable on pylons increases the diameter by 200 feet, which is about 0.04 miles, so $\pi \times (8000\text{mi} + 200\text{ft}) \approx \pi \times 8000.04$ miles. The difference is then $\pi \times 0.04 \approx 0.12$ miles or $\pi \times 200 = 600$ feet.

9. A grain silo is built out of sheet aluminum; it is cylindrical with a radius r and height h. The silo has a floor and a hemispherical roof of radius r. Write the formula for the total surface area of the silo, and write it in the simplest form.

Answer: The area of the floor is πr^2. The area of the sides is the circumference times the height, or $2\pi rh$. The area of the roof is $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$, where the factor of one half occurs because it is only half a sphere.[1] The area of the floor is πr^2. Hence the total area is $2\pi rh + 2\pi r^2 + \pi r^2 = \pi r(2h + 2r + r)$ i.e. the total area is $\boxed{\pi r(2h + 3r)}$.

10. A washer (a flat disk of metal with a large concentric hole) has an outer radius (the radius of the washer) of x and an inner radius (the radius of the hole) of y. Draw the washer, and write in simplest form the area of the washer.

Answer:



The area of the washer is the difference between the areas of the two circles, which is $\boxed{\pi(x^2 - y^2)}$.

[1] Note: *hemi-*, *semi-*, and *demi-* are all prefixes meaning "half." In music, the British call an eighth note a *quaver*, a sixteenth note a *semiquaver*, a $\frac{1}{32}$ note a *demisemiquaver*, and a $\frac{1}{64}$ note a *hemidemisemiquaver* (or so I'm told).