Pre-Algebra

Homework 10 Geometry: Solutions

Use $\pi = 3$ for your calculations

1. *I am building a ramp. The length of the ramp along the ground is 12 meters and it's 5 meters high and 5 meters wide. I want to paint it with wood preserver all over (including the base) before I install it. If the price of paint to cover 1 square meter is \$1, how much is this going to cost me?*

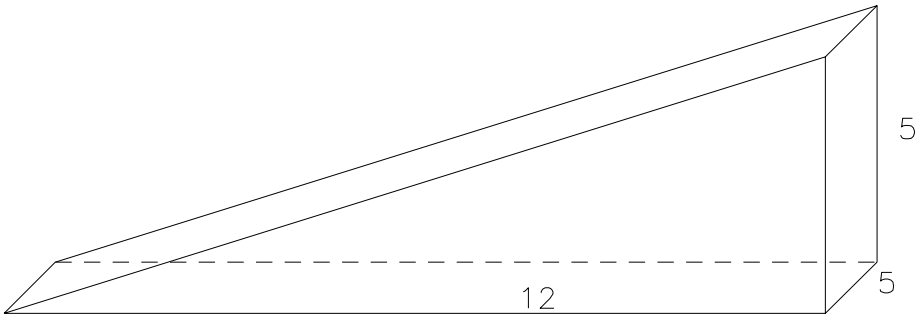

Figure 1. Ramp

See the figure. The bottom of the ramp is a rectangle measuring 12 meters by 5 meters; so it has an area of $12 \times 5 = 60$ sq m. The back is a square measuring 5×5 meters, 25 sq m. The two sides are triangles, base 12 m and height 5 m. The area of each is
$A = \frac{1}{2}$ bh $= \frac{1}{2} \times 5 \times 12 = 30$ sq m. So the sum of the areas of the two sides is 60 sq m. What about the face of the ramp itself? Its length is the *hypotenuse* of the triangle: $l^2 = 5^2 + 12^2 = 25 + 144 = 169$

So l $= \sqrt{169} = 13$ m. The ramp is then a rectangle measuring $5 \times 13 = 65$ sq m. So the total surface area of the ramp is $60 + 20 + 60 + 65 = 205$ sq m. Since it costs \$ 1 to paint 1 sq m, the total cost of wood preserver is $\boxed{\$205}$.

2. *Amy lives right in the center of town. Her friend Betty lives 2 miles north and 4 miles east of Amy. Amy drives to Betty's house to pick her up and they then drive 2 miles east to pick up Cathy, and then all three drive two miles north to a dance club. Plot these four points (call them A, B, C and D) on a map of the town and, on the assumption that all the roads in the town run north-south or east-west, draw Amy's path between her house and the dance club. If Amy can drive at 30 m.p.h., and it takes 5 minutes each to pick up Betty and Cathy, when should she leave home to get to the dance club by 7 pm?*

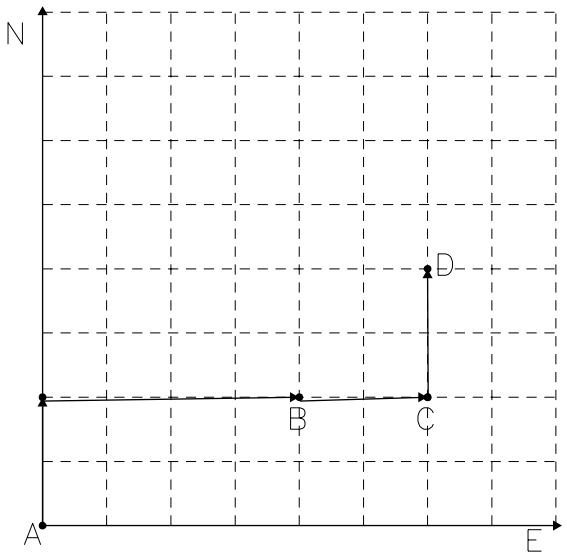

Figure 2. Amy's drive to the dance hall (D)

See the figure. It shows Amy's route through town as she picks up Betty (B), Cathy (C), and drives to the dance club (D).

Amy's total drive is $2 + 4 + 2 + 2 = 10$ miles. If she is driving at 30 mph, this takes a time $t = 10/30 = 1/3$ of an hour $= 20$ minutes. She has to wait for 5 minutes at Betty's house and 5 minutes at Cathy's house, so the total time of her trip is $20 + 5 + 5 = 30$ minutes. So she needs to leave home at $\boxed{6{:}30\text{ pm}}$ to be sure to get to the dance club at 7 pm.

3. *My desk is a rectangle measuring 5 feet by 4 feet. (a) What is the area of my desk? (b) my desk has flaps at either end which can be raised to*

make its surface area bigger. If each flap is a semi-circle of diameter the same as that of the small dimension of my desk (4 feet), what is the area of my desk with the flaps raised? (c) I buy a flat-screen TV for my living room. The salesman tells me it's a $17''$ *screen, but when I get home and measure it I find it's a rectangle measuring* $8'' \times 15''$*. Was he lying to me? Explain.*

(a) The area of a rectangle is A = length × width = lw. l = 5 feet, w = 4 feet, so the area of the desk is A = 5 × 4 = $\boxed{\text{20 square feet}}$.

(b) Each flap is a semicircle, so both flaps together have the area of one whole circle, whose area is $A = \pi r^2$, where r is the radius of the circle. If the diameter of the circle is 4 feet, its radius is 2 feet, and its area is $A = 3 \times 2^2 = 3 \times 4 = 12$ square feet. So the total area of the desk is A = 20 + 12 = $\boxed{\text{32 square feet}}$. [What's the perimeter? It's the circumference of the circle, $2\pi r = 12$ feet, plus the two long sides, 2 × 5 feet. So the perimeter is 12 + 10 = $\boxed{\text{22 feet}}$].

(c) The flat screen has length $15''$ and width $8''$. What's its diagonal measure? Use Pythagoras' theorem:

$c^2 = a^2 + b^2$ where a is the length, b the width and c the diagonal. Your flat screen has a diagonal of $c^2 = 15^2 + 8^2 = 225 + 64 = 289$
So $c = \sqrt{289} = 17''$. So the diagonal measure of your flat screen is $17''$, and the salesman was telling the truth. Computer monitor screens and TVs are usually described by their biggest dimension - the diagonal.

4. *The Earth's diameter is roughly 8,000 miles. (a) What is its circumference? (b) suppose the Earth is a perfect sphere and completely flat (no mountains, oceans etc.). I want to lay a fiber-optic cable all the way around the Equator, flat on the ground. What length of cable do I need to do this? (c) Instead, I decide to hang the cable from the tops of pylons 100 feet high. If I can make the cable taut, so that it's a circle, how much more cable do I need? (d) I also want to lay a cable round the Earth, flat to the ground, running due north between the South Pole and the North Pole, then back down to the South Pole on the other side of the Earth. What length of cable do I need?*

(a) the circumference $C = 2\pi r = \pi D = 3 \times 8000 =$ $\boxed{\text{24,000 miles}}$, where C is the circumference, D the diameter, r the radius and we're approximating $\pi = 3$.

(b) The Earth's equator is a great circle, whose length is that of the circumference. So you need $\boxed{24{,}000}$ miles of cable.

(c) If you raise the cable 100 feet off the ground, the radius of the circle made by the cable is 4000 miles plus 100 feet (the radius of the Earth is half of its diameter, i.e. 4000 feet). The circumference of this circle is $C = 2\pi r = 6\times$ (4000 miles plus 100 feet) = 24,000 miles plus 600 feet. So the cable only needs to be $\boxed{\text{600 feet longer}}$ to lift it 100 feet off the ground.

(d) The circle which goes round the Earth through the two poles is also a great circle, so you need 24,000 miles of cable.

5. *A grain silo is built out of sheet aluminum; it is cylindrical with radius r and height h. The silo has a floor and a hemispherical roof of radius r. Write the formula for the total surface area of the silo, and write it in the simplest form.*

   The floor of the silo is a circle of radius r and so has an area of πr^2. The roof is half a sphere. The area of a sphere is $4\pi r^2$, so the area of half a sphere is $2\pi r^2$. Now take the wall of the cylinder. Imagine inrolloing it and flattening it. Its length is the circumference of a circle, $2\pi r$. Its width is the height of the cylinder, h, and its are is $2\pi rh$. The total surface area of the silo is then

   $$\boxed{A = \pi r^2 + 2\pi r^2 + 2\pi rh = \pi r(3r + 2h)}$$

6. *A washer (a flat disk of metal with a large concentric hole, used for a spacer in metal construction) has an outer radius (the radius of the washer) of x and an inner radius (the radius of the hole) of y. Draw the washer, and write in its simplest terms a formula for the area of the washer.*

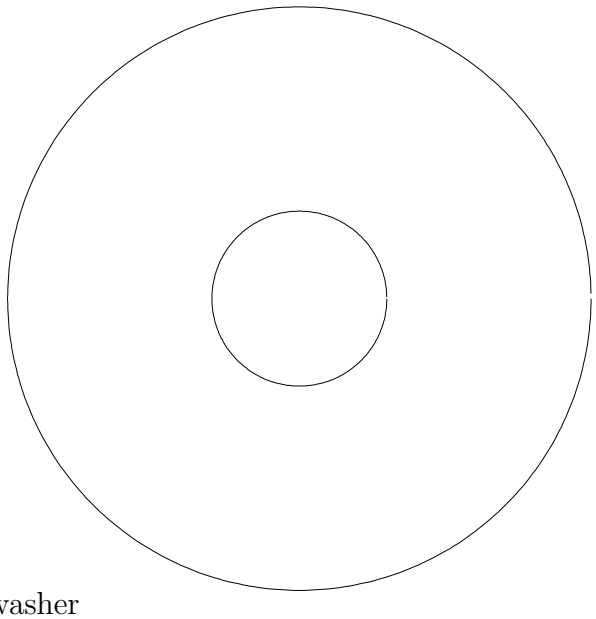

Figure 3. A washer

If the washer had no hole, its area would be $A = \pi x^2$. The area of the hole is πy^2. Thus the area of the washer is

$$A = \pi x^2 - \pi y^2 = \pi(x^2 - y^2)$$

7. *A straight line is the shortest distance between two points. What's the shortest distance between the North and the South pole of the Earth along the surface of the Earth (you can't make a tunnel).*

   The circle going through both the North and South poles on the Earth is a great circle. Thus, moving along the Earth between the North and South poles, the shortest distance is half of a great circle, or $\boxed{\text{12,000 miles}}$ (using the numbers from Problem 4).

8. *My company makes copper piping by extrusion. I'm making piping with an outer wall of 11 cm and an inner wall of 10 cm. If the density of copper is 9 gm/*cm^3 *and the price of copper is \$7 per kilogram, what is the cost of materials to make 1 meter of copper tubing?*

   Look at Figure 3. This is the cross-section (not to scale) of my copper pipe. The area of the pipe's cross-section is $\pi(x^2 - y^2$. $x = 11$ cm,

so $x^2 = 121$ sq cm. y = 10 sm, so $y^2 = 100$ sq cm, and the area of the pipe's cross-section is $\pi(121 - 100) = 63$ sq cm. The volume of a 1-meter length of pipe is then 100×63 cm^3 = 6300 cm^3, and its weight is $9 \times 6300 = 56{,}700$ gm, or 56.7 kg. The cost of materials is then \$7 $\times$ 56.7 = \$396.9 = \$396 90 cents, to make 1 meter of copper pipe.

9. *Butter has a density of about 1 gm/*cm^3*. I am making 1 kg packages of butter in (a) cubical blocks and (b) cylinders whose height is roughly 1/2 of the diameter. What is the size of the blocks of butter in both cases?*

   (a) If the density of butter is 1 gm/cm^3, 1 kg of butter has a volume of about 1000 cm^3 = 10^3 cm^3. Since
   $\sqrt[3]{10^3} = 10$
   A cubical block of butter weighing 1 kg is 10 cm on the side.

   (b) The volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the cylinder's circular base and h is the height. Here, we have h = D/2, where D = diameter, so h = r, where r, the radius, is half of the diameter, and so $V = \pi r^3 = 3r^3 = 10^3$ cm^3. The radius and height are then $10/\sqrt[3]{3} = 8.5$ cm, approximately. Thus the block of butter is 17 cm in diameter by 8.5 cm high.

10. *Time to paint the living room. The room measures 20 feet wide by 30 feet long by 12 feet high. There are two doors, 7 feet high by 3 feet wide, and four windows, each 4 feet high by 3 feet wide. I'm going to paint the walls and ceiling with two coats of paint at 25 cents per square foot. What will it cost?*

    The room is a rectangular solid in shape, with one ceiling measuring 20 x 30 feet = 600 square feet, two walls 12 x 20 feet = 240 square feet each, and two walls 12 x 30 feet = 360 sq feet each. The total area of the ceiling and 4 walls is then 600 + 600 + 600 sq feet = 1800 sq feet. Since you don't have to paint the doors and windows, you can subtract their area: $2 \times 7 \times 3 = 42$ sq feet for the doors and $4 \times 4 \times 3 =$ 48 sq feet, 90 sq feet altogether. So the total area you have to paint is 1800 - 90 = 1710 sq feet. Two coats of paint at 25c per square foot for each coat (sorry; this was not made explicit in the question) is 50 cents per square foot, or \$1710/2 = \$855 for the whole room.