Pre-Algebra Exam 2 Solutions

1. **Express as a decimal:**

   (a) $\frac{1}{4}$

   Ans: 0.25

   (b) three hundredths

   Ans: 0.03

   ## Order from smallest to largest:

   (c) $\frac{1}{2}$, 0.01, 0.51

   Ans: $0.01 < \frac{1}{2} < 0.51$

   (d) -1.11,-0.11,$\frac{2}{5}$,0.45

   Ans: $-1.11 < -0.11 < \frac{2}{5} < 0.45$

   ## Round to the nearest tenth and hundredth:

   (e) 1.126

   Ans:
   Nearest tenth: 1.1
   Nearest hundredth: 1.13

   (f) 1.529

   Ans:
   Nearest tenth: 1.5
   Nearest hundredth: 1.53

2. **Solve:**

(a) $\frac{1}{5}(x+4) = x - 1$
Answer:

$$\begin{aligned} x + 4 &= 5x - 5 \\ 4 + 5 &= 5x - x \\ 9 &= 4x \\ x &= \frac{9}{4} \end{aligned}$$

$\boxed{x = \frac{9}{4}}$

(b) $-3x + 2 < 20$
Answer:

$$\begin{aligned} -3x &< 18 \\ 3x &> -18 \end{aligned}$$

$\boxed{x > -6}$

3. **Simplify, leaving no negative exponents:**

(a) $\frac{x^3y^6x}{x^2}$
Answer:

$$\frac{x^4y^6}{x^2}$$

$$x^{4-2}y^6 = x^2y^6$$

$\boxed{x^2y^6}$

(b) $\frac{a^{-2}b}{a^3b^2}$
Answer:

$$\frac{b}{a^2a^3b^2}$$

$$\frac{1}{a^{2+3}b^{2-1}}$$

$\boxed{\frac{1}{a^5b}}$

(c) $(4xz^3)^2(2x^{-2}z)^4$

$$4^2 2^4 x^2 z^6 x^{-8} z^4$$

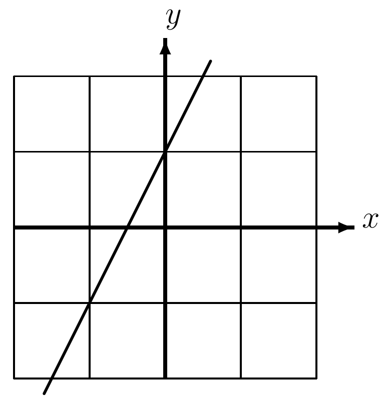
$$256\frac{z^{10}}{x^6}$$

$\boxed{256\frac{z^{10}}{x^6}}$

4. **For each of the following linear equations, determine the slope and intercept and plot:**

(a) $y = 2x + 1$

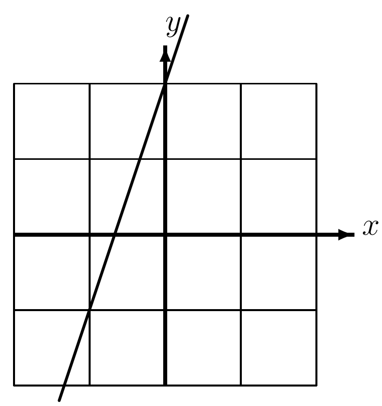
Answer:



$\boxed{\text{Slope } m = 2}$, $\boxed{\text{intercept } b = 1}$.

(b) $y - 3x = 2$

Answer:



This is equivalent to $y = 3x + 2$, hence $\boxed{\text{slope } m = 3}$, $\boxed{\text{intercept } b = 2}$.

5. **Solve:**

A woman is building a fence for her pumpkin patch. She buys 20 m of fence, and she builds a patch that is two meters longer than it is wide. (*Remember that the perimeter is* $P = 2l + 2w$.)

(a) Write an equation for the width of the patch.
Answer:

The width is 2m less than the length:
$l = w + 2$

But we also know that $P = 2l + 2w$ and that $P = 20m$. Thus:

$P = 2(w + 2) + 2w$

$20 = 2(w + 2) + 2w$

$\boxed{20 = 4w + 4}$

(b) Solve for the width of the patch.

Answer:

$20 = 4w + 4$

$16 = 4w$

$\boxed{w = 4}$

(c) What is the area of the pumpkin patch?

Answer:

Area $= l \times w$

We know that $w = 4$. We can solve for l:

$w = l - 2$

$l = w + 2$

$l = 6$

$\boxed{\text{Area} = 4 \times 6 = 24 \text{ square meters}}$

6. **Solve:**

Amy is biking to the grocery store, which is 10 miles away. It takes her one hour.

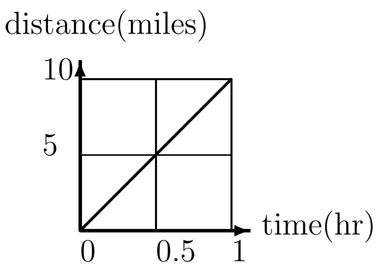
(a) What is Amy's rate (in miles per hour)?
Answer:

$rate \times time = distance$
$rate = distance/time$

$rate = 10\ miles/1\ hr$

$\boxed{\text{10 miles per hour}}$

(b) Make a plot of distance versus time for her journey, if her house is the origin (so that she starts with distance of 0 at time 0).



(c) If Joe bikes to the supermarket at a rate of 5 miles per hour, and his house is twice as far as Amy's, how long does it take him to get to the supermarket? **Answer:**

$rate \times time = distance$
$time = distance/rate$

$time = 20\ miles/5\ miles\ per\ hour$

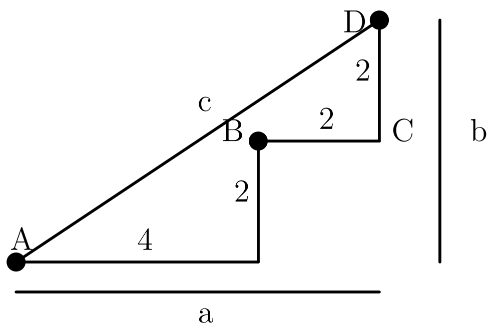
$\boxed{\text{time} = \text{4 hours}}$

7. This problem is a follow-on to problem 6 of the practice test. Here's the problem:

   Amy lives right in the center of town. Her friend Betty lives 2 miles north and 4 miles east of Amy. Each Saturday night, Amy drives to Betty's house to pick her up and they then drive 2 miles east to pick up Cathy, and then all three drive two miles north to a dance hall. Now suppose that there is an avenue which runs straight from the center of town, where Amy lives, past the dance hall. If, one Saturday, Amy's friends are out of town, so she drives directly to the dance hall without picking them up, how far does she drive?

   Answer:

   Here is the diagram of Amy's normal route:



   We want to solve for the length of c. We use the Pythagorean theorem:

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 4^2$$

$$c^2 = 36 + 16 = 52 \text{ sq miles}$$

   $\boxed{\text{c} = \sqrt{52} \text{ miles} \sim 7.2 \text{ miles.}}$

8. A peeled orange has a diameter of four inches. It has twelve equal-sized segments with no spaces between them. What is the total surface area of the segments when they have been separated? (the total area of a sphere is $4\pi r^2$. Take $\pi = 3$).

Answer:

We can think about the total surface area as the sum of the sphere (the skin of the orange) and the sides of each segment. Each side of each segment is a hemicircle. The area of a circle is πr^2, so each side has an area of $\frac{1}{2}\pi r^2$. Since there are 12 slices with two sides each, the surface area contributed by the sides of the slices totals $12 \times 2 \times \frac{1}{2}\pi r^2$ or $12\pi r^2$. In total then we have $12\pi r^2 + 4\pi r^2$ or $16\pi r^2$. For a diameter of 4 inches, $r = 2$ inches, and we have $\boxed{64\pi \approx 192 \text{ sq. in}}$.