Pre-Algebra
Class 9 - Graphing

Contents

In this lecture we are going to learn about the rectangular coordinate system and how to use graphs to pictorially represent equations and trends.

1 Rectangular Coordinate System

The most commonly used plotting system is the *r*ectangular coordinate system. Here two perpendicular lines, called *a*xes cross at a point called the *o*rigin. Generally, the horizontal axes is called the x-axis while the vertical axes is called the y-axis. Any location on this coordinate system can be represented by an ordered pair of numbers called *c*oordinates. For example the point P is represented by the coordinates (2,4). The first value represents the distance along the x-axis while the second represents the distance along the y-axis.

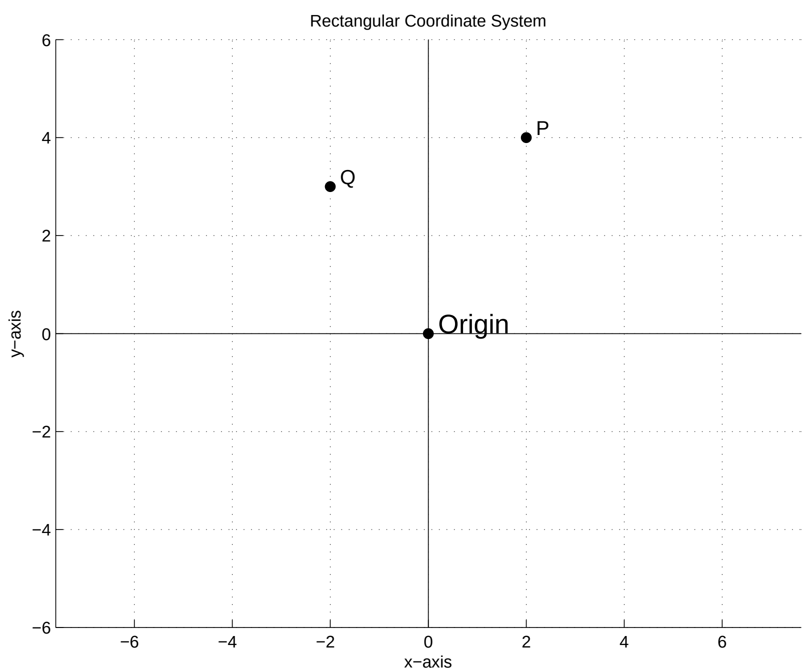

Figure 1: Rectangular Coordinate System with points P (2,4) and Q (-2,3)

Example (Refer to Figure ??)

1) Plot the point P (2,4) on the graph.

2) Where would the point Q (-2,3) go on this same graph?

3) Locate a point that is 2 units in the x-direction away from point Q. What are its coordinates? **Answer: (0,4) or (-4,4)**

4) If both coordinates are negative, where would you find the point? If both points are positive where should you look for that point?

1.1 Giving meaning to your axes. Units and Labeling

Since you will be creating a lot of graphs, let's take a moment to describe those features that make a graph truly useful to you and others. The x and y axes of the graph are a representation of a real quantity. For instance, x could represent kilometers East while y represents kilometers North from some point of origin (like the town's post office).

Example

1) Let's say you started at the post office and traveled 1 kilometer north to the bowling alley. What coordinate is the post office? What coordinate is the bowling alley?

2) Now you travel West 4 kilometers from the bowling alley to get some ice cream. What are the coordinates of the ice-cream shop?

If you were to put these locations on a graph, what information would you need to make your graph, or map, understandable by others? You would need to label the directions; x and y don't mean anything but North and East do. You would also need to label the units, traveling 1 meter is very different than traveling 1 kilometer. Without this information your graph would be useless.

1.2 Other uses for graphs.

As we shall see, graphs are used to represent trends other than distances. You can put information about how fast a car is going as it travels down the street (speed vs. distance). Or, the amount of tips a waitress earns each night (money vs time). Or, how much gas you use for each mile you drive to work (volume vs. distance). The options are truly limitless. If one value depends on another, you put it on a graph.

2 Graphing Linear Equations

A linear equation is an equation that can be written in the form

$$y = ax + b$$

x and y are both variables (the values we might want to plot on our x and y axes) while a and b are constants.

For example, let's consider the linear equation $y = 5x + 10$. There are an infinite number of values for x and y that satisfy this equation. For instance, if $x = 0$ then $y = 10$, so the ordered pair (0,10) is a solution to this linear equation. **Have each student find one more point that is a solution to this linear equation and plot it on the board.**

If you put all of our points on a graph we see that all our solutions fall on straight line, by drawing the line through these points, you graph the linear equation. The line represents all the possible solutions. Now it is easy to determine if a particular ordered pair is a solution to our linear equation. **Which point (-10,-40) or (-10,40) is a solution of our linear equation?**

Example

1) Is the equation $2y - 6 = 4x$ a linear equation? Hint: Solve the equation for y.
Answer: yes. $y = 2x + 3$ **a=2 and b=3**.

2) Find 2 solutions to this equation and plot them. Draw the line representing the equation.

3) For this equation, if y represents the number of glasses of lemonade I have sold and x represents time in hours. How many glasses of lemonade did I sell at hour 2? **Answer: 7 glasses.**

3 Functions and Graphs

Graphs that show a relationship between two quantities are not always a straight line. Graphs are an easy pictorial representation of the information and much can be inferred from the graph, even if no mathematical formula is available.

For instance, imagine you are going to plot the number of cars on I-95, passing through Philadelphia between 4am and 11am. What do you expect this graph to look like?
Have students describe the trend and then sketch a plot on the board. You would probably expect more cars around 8-9am (rush hour) than 4am or 10am.

3.1 Function Notation

Sometimes we refer to the y-axis as f(x). The f stands for function and the function depends on the variable x. For instance our linear equation $y = 5x + 10$ could be written as $f(x) = 5x + 10$. If this function represents the distance a car traveled over some amount of time, we might instead change the variable, x to t so we remember that we are dealing with time, $f(t) = 5t + 10$. We could even change the function name to d so we know that it is a distance. Then we have $d(t) = 5t + 10$.

The point: you can use any letter you want. We may use d(t) instead of y because it is more informative, it tells us that we are plotting a distance against time, but it is just as correct to write the variable as f(x) or simply y. Just be sure you know what your variables are representing. If you want to evaluate your function at some time, like 10 minutes, you would write d(10 min)=5(10)+10=60 meters.

4 Rate

We commonly plot distance against time, distance on the y-axis and time on the x-axis. (We might also say that we plot distance as a function of time.) For instance, consider the following example. You are traveling in a car going 25 miles/hour for 1 hour. Then you stop and do your grocery shopping for 1 hour. Then you return home at 25 miles/hour, again taking one 1 hour. What would the plot of the car's position vs. time look like for this simplistic case?

You can make a more specific graph with a bit more information. The equation

$$d = r \cdot t$$

relates distance, speed and time. Notice that the equation is a linear equation where $b = 0$. The distance simply takes the place of y, and t takes the place of x. The equation to represent your trip to the grocery store would be $d = 25t + 0$. How far have we gone after 1 hour of traveling at 25mi/hr? (25 miles) How far away was the grocery store? (25 miles)
(On the return trip you are traveling the opposite direction, you are losing distance, or going back from where you came. So your speed is negative. The rate is -25 km/s.)

If you then increased your speed to 50mi/hr, how would the equation change? $d = 50t + 0$. How long would it take to get to the grocery store. What would the plot of this linear equation look like? **Create the plot on the board. See Figure ??**

4.1 Slope, the Equation

The steepness of a line is referred to as its slope. The more inclined, the higher the slope. In this example, the rate gives us the slope of the line. The definition of slope is change in y divided by the change in x, or rise divided by run.

$$slope = \frac{rise}{run} = \frac{(y2 - y1)}{x2 - x1}$$

Let's verify that definition for when we are traveling 50 mi/hr. Find two points that satisfy the linear equation $d = 50t + 0$ and calculate the slope.

Another way to think of slope is to think of a hill. You could plot a hill where the y-axis represents the direction up, and the x-axis represents the direction East. (Draw two graphs, one with a steep hill and one with a shallow hill.) Which hill would be more difficult to climb? The steeper hill has the steeper slope. Which hill would be more fun to ski down? Again, the steeper sloped hill.

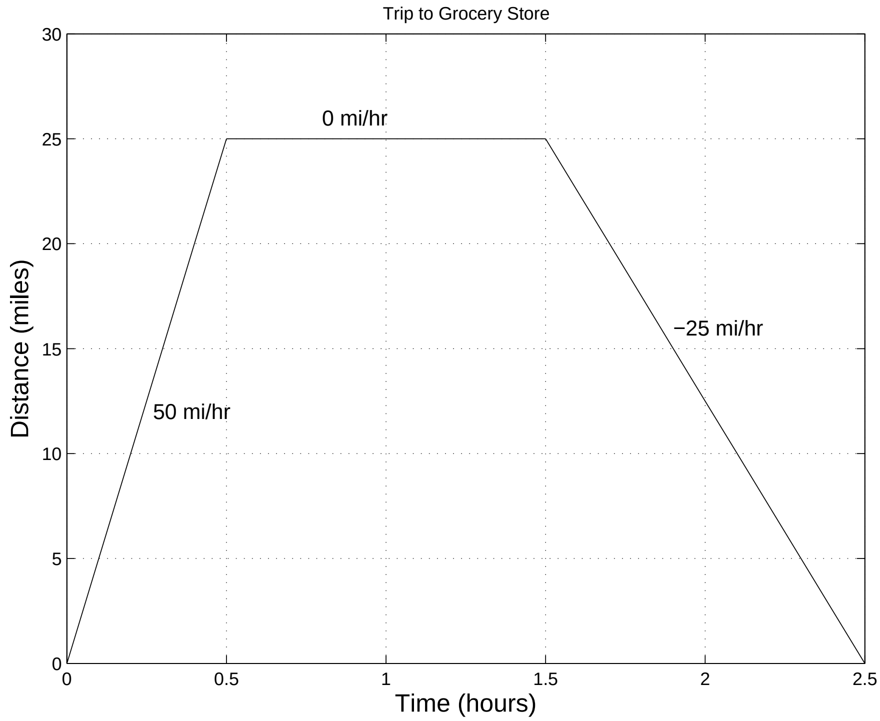

Figure 2: Plot of the location of your car versus time during the trip to the Grocery Store at 50 mi/hr, returning at 25 mi/hr.

What would a negative slope indicate? For an increase in x, there is a decrease in y. So a negative slope means a downhill line (if thinking in the positive x direction). Consider our trip to the grocery store again. For the last part of the trip you went backward and headed back the way you came. So, for an increase in time, you had a decrease in distance. This is why the plot we made shows a downhill slope.

Example

1) Consider the two equations $y = 8x + 3$ and $y = 2x + 3$. Which has a steeper slope?

2) Create a plot of each equation by finding at least 2 points that lie on each line. Were you right in question 1?

3) What does the b value in each equation $y = ax+b$ indicate? This is called the y-intercept, does that name make sense? This is the value of the function when x=0, or when the line intercepts the y-axis.

4) I start 2 miles from my house and travel at 4mi/hr toward my house. If my house is at the location 0 miles, plot this journey on a distance vs time graph. What is my rate? What is the slope of the line you drew? What is the y-intercept of this line?