Pre-Algebra Lecture 7: Solving Equations

Throughout the class we have been solving equations. Today we will go over many of the things we have learned a bit more formally, and get some practice translating words into algebraic expressions.

Outline:

1. Properties of Multiplication
2. Solving Equations with Combined Operations
3. Problem-Solving Using Equations
4. Simplifying Equations
5. Introduction to Inequalities

1 Properties of Multiplication

- Commutative

  A change in the order in which two numbers are multiplied does not change their product.

  $$\mathbf{ab = ba}$$

  Example:
  $6 \times 2 = 12$ and $2 \times 6 = 12$

- Associative

  The product of three numbers does not depend on the grouping.

  $$\mathbf{a(bc) = (ab)c}$$

  Example: $2 \times (3 \times 1) = 2 \times 3 = 6$ whereas
  $(2 \times 3) \times 1 = 6 \times 1 = 6$
  The two are identical.

- Distributive

  The sum of two numbers multiplied by a factor is equivalent to the sum of each number multiplied by the factor.

$$\mathbf{a(b+c) = ab + ac}$$
$$\mathbf{(b+c)a = ba + ca}$$

Example: $2 \times (3+1) = 2 \times 4 = 8$ whereas
$2 \times 3 + 2 \times 1 = 6 + 2 = 8$
The two are identical.

The distributive property in particular will arise all the time in solving equations. Often times you may forget to multiply through all the terms in a sum, and this will lead to much suffering. Here is an example that is commonly confused:

$$-(5+x) = -5 - x$$

You can think of that negative sign as -1 and you must be very careful to multiply -1 by **each term** in the parenthesis.

More examples:

1. $\frac{1}{3}(x+9)$

2. $\frac{x}{2}(3+y)$

3. $-y(x+7)$

2 Solving Equations with Combined Operations

As we have discussed before, equations express an equality between expressions on either side of the equal sign. In order to solve them, it is crucial that whatever is done to one side of the equation also be done to the other. The goal in manipulating the equation is to isolate the variable. In other words, the variable should be by itself on one side of the equation while everything else is on the other side. Here are some examples:

1. $3n + 10 = 16$

2. $\frac{x}{2} + 4 = 3$

3. $4(x+1) = 7$

4. $\frac{y-7}{3} = 8$

We can write down rules for solving equations involving multiple operations.

1. Determine the order in which the operations are performed on the variable (e.g., use PEMDAS).

2. Undo these operations in reverse order, by applying the inverse operations **to both sides**.

3 Problem-Solving Using Equations

One of the hardest skills to learn in algebra is how to translate words into equations. This is the essential skill that allows us to use algebra in life. The only way to learn is to practice. First, let's practice turning phrases into algebraic expressions.

1. Julie's weekly salary, \$300, is \$150 more than twice Henry's weekly salary.

2. Mike can bike 10 mph more than half of Adele's speed, which is 20 mph.

3. The turnout for the presidential election of one million was ten percent more than twice the turnout for the senate race the year before.

Now, we can take our skills at translating words into equations, and combine them with our talents for solving equations, to solve word problems.
Here is an example problem:

Jonelle paid \$60 for a couch. She paid \$20 up front and then \$5 every month. How many months did it take her to completely pay for the couch?
Step 1: Choose a variable and decide what it stands for. In this case it is natural to use m to represent the number of months that it takes Jonelle to pay for her couch.
Step 2: Write an equation using the variable based on the information in the problem.
Step 3: Solve the equation.
Step 4: Check the answer.
Here is some practice.

1. Three-quarters of a basketball team arrived for breakfast on the morning of a game. At lunch nine more players arrive, for a total of 84 players. How many players are on the team?

2. Joan had two thirds of a crate of oranges and then gave 12 oranges away. She made a fruit salad with the remaining ten oranges. How many oranges does a crate hold?

4 Simplifying Equations

In this section we will review how to use the distributive property to combine like terms in an equation. Simplifying equations this way is necessary for solving them.
We can write the distributive property in reverse as ax + bx = x (a + b). Therefore, if two terms in an equation contain the same variable, the distributive property tells us that we can combine them. For instance:

$$2x + 4x = 30$$
$$(2 + 4)x = 30$$
$$6x = 30$$
$$x = 5$$

More examples:

1. $24z - 8z = 64$

2. $\frac{7}{8}n - \frac{3}{4}n = -6$

3. $18r - 9r = -108$

4. Cindy sold the same number of student tickets ($2 each) as adult tickets ($3 each) for the science fair. How many tickets did she sell if she collects $100?

5. $5x + yx = 10$

We can make things slightly more complicated:
$9x + 5(x + 7) = -49$
First, use the distributive property:
$9x + 5x + 35 = -49$
Then combine like terms:
$14x + 35 = -49$
Solve:
$14x = -84$
$x = -6$
Try these:

$3(x+4)+5(x-2)=66$
$-4(3+z)+6z-12=-36$
One last complication arises if the variable appears on both sides of the equation. In this case, you want to first get all the terms with the variable to one side, and then proceed as above.

For example:

$6x-2=4x+3$

5 Introduction to Inequalities

Inequalities represent relationships between numbers and variables just as equations do. For instance $x>2$ tells us that the variable x can take any value larger than 2. Remember that $x>2$ means x can be any number **larger** than 2, while $x \geq [2]$ means that x can be equal to or larger than 2. Likewise, $x<2$ means x is not equal to but only less than 2, while $x \leq [2]$ means that x can equal or be less than 2.

We can solve inequalities in the same way as we solve equalities, by doing the same thing to both sides. However, there is one complication, which is best illustrated by an example.
We know that $3<4$. However, if we multiply both sides of this inequality by -1, then we have -3 and -4. Now which side is larger?
When multiplying an inequality by a negative number, always remember to change the direction of the inequality.
This special property of inequalities is summarized here:
For all numbers a, b, c **where** $c>0$**:**

$$\textbf{If } a>b \textbf{ then } ac>bc \textbf{ and } \tfrac{a}{c}>\tfrac{b}{c}$$
$$\textbf{If } a<b \textbf{ then } ac<bc \textbf{ and } \tfrac{a}{c}<\tfrac{b}{c}$$

For all numbers a, b, c **where** $c<0$**:**

$$\textbf{If } a>b \textbf{ then } ac<bc \textbf{ and } \tfrac{a}{c}<\tfrac{b}{c}$$
$$\textbf{If } a<b \textbf{ then } ac>bc \textbf{ and } \tfrac{a}{c}>\tfrac{b}{c}$$

Practice:

1. $x + 9 < 13$

2. $-3n > 12$

3. $15 < y + 7$

4. $-2(x + 6) > 20$