The Evolution of Self-Gravitating, Magnetized, Turbulent Clouds: Numerical Experiments†

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Interstellar giant molecular clouds and dark clouds are observed to have comparable kinetic and gravitational energies, and low enough temperatures that their internal turbulent velocity amplitudes are highly supersonic. It has been believed for some time that the presence of magnetic fields can have important consequences for the properties of turbulence in these clouds, and for clouds' gravitational stability. In this paper, I outline how the physical parameters of clouds can be translated to dimensionless ratios (the Mach number, the Jeans number, and the plasma β), describe a series of numerical experiments underway to evaluate how the character of the turbulence depends on these parameters, and present selections from our results to date.

1. Introduction

The turbulence in Galactic molecular clouds has a rather different character from the other forms of interstellar turbulence discussed at this meeting. Strong molecular cooling brings the ambient temperatures to the range $T = 10 - 30$ K, which renders turbulence with velocities of a few km s^{-1} not just nonlinear, but hypersonic. Most observational evidence on magnetic field strengths suggest that Alfvén speeds are of order the turbulent speeds or a few times larger, and in any case unlikely to be smaller than the sound speed. Thus, the turbulence in molecular clouds is highly compressible, and strongly magnetic. In addition, although high-latitude unbound molecular clouds exist, most of the molecular material in the Galaxy resides in much more massive, self-gravitating, giant molecular clouds and cloud complexes (GMCs). The tendency for gravitational contraction of clouds must influence, and be influenced by, the evolution of the turbulent flows inside. In addition, phenomena associated with gravity (either local instabilities, or winds arising from the star formation process – which is effectively a self-gravitational runaway) may be responsible for exciting turbulence in the first place.

Understanding the evolution of turbulent flow under these conditions is a fascinating hydrodynamics problem in itself, and is also crucial to developing the theory of star formation. The rate of star formation within a cloud, the efficiency of conversion of gas to stars in a cloud before it is destroyed, and the correlations among and individual properties of the stars formed (i.e. clustered vs. dispersed star formation, distribution of stellar masses in the IMF) must all depend on the properties of the turbulence in GMCs. To understand cloud properties and to predict their evolution, we must be able to characterize how the "background state" of a cloud – e.g. its mean temperature, density, magnetic field strength, and ionization state, as well as its overall size – affects the character of its internal turbulence (including its dissipation rate, its modes of dissipation via shocks, ambipolar diffusion, or turbulent cascades, and its distributions, power spectra, and spatial correlations of density, magnetic fields, and velocity fields, etc.). This is obviously an extremely complex problem. However, even fairly simple computational

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models (using, e.g., an isothermal equation of state, periodic boundary conditions, and arbitrary initial or forced velocity fields) can be used in the initial attack on these these dynamical questions. The quantitative results and insight gained from such experiments will help inform ideas about the yet-more-complicated process of creating a collection of stars within a turbulent cloud.

Our research group has, over the last few years, begun a campaign of using numerical experiments with the ZEUS code for MHD (Stone & Norman (1992a),Stone & Norman (1992b)) to simulate turbulent molecular clouds and to characterize how their properties depend on input parameters. In this paper, I provide background on how the relevant physical parameters can be modeled, and on expectations for self-gravitating cloud evolution based on linear theory. I also present highlights from some of our numerical surveys, addressing in particular questions of whether turbulent clouds collapse under self-gravity, and how magnetic fields affect the clumping of matter.

2. Cloud Model Parameters and Expectations from Linear Theory

2.1. *Dimensional Scales for Cloud Dynamics*

In modeling molecular cloud dynamics, we start by identifying and parameterizing physical properties that are expected to have significant effects on cloud evolution. As in other astrophysical situations, the relative importance of different effects can by characterized by corresponding time scales. Four important dynamical timescales are those associated, respectively, with the times for gravitational contraction,

$$t_{\rm g} \equiv \frac{L_J}{c_s} = \left(\frac{\pi}{G\rho}\right)^{1/2} = 0.99 \times 10^7 \ {\rm yr} \left(\frac{n_{H_2}}{10^2 \ {\rm cm}^{-3}}\right)^{-1/2}, \tag{2.1}$$

thermal pressure (sound) wave crossing,

$$t_{\rm s} \equiv \frac{L}{c_s} = 5.3 \times 10^7 \ {\rm yr} \left(\frac{L}{10 \ {\rm pc}}\right) \left(\frac{T}{10 \ {\rm K}}\right)^{-1/2}, \tag{2.2}$$

magnetic pressure/tension (magnetosonic/Alfvén) wave crossing,

$$t_{\rm A} \equiv \frac{L}{v_A} = \frac{L}{B/\sqrt{4\pi\rho}}$$
$$= 0.76 \times 10^7 \ {\rm yr} \left(\frac{L}{10 \ {\rm pc}}\right) \left(\frac{n_{H_2}}{10^2 \ {\rm cm}^{-3}}\right)^{1/2} \left(\frac{B}{10\mu{\rm G}}\right)^{-1}, \tag{2.3}$$

and fluid flow crossing†,

$$t_{\rm f} \equiv \frac{L}{\sigma_v} = 0.98 \times 10^7 \ {\rm yr} \left(\frac{L}{10 \ {\rm pc}}\right) \left(\frac{\sigma_v}{1 \ {\rm km \ s}^{-1}}\right)^{-1}; \tag{2.4}$$

all crossing times are defined over a global scale within a cloud. Here, ρ is the total mass density (n_{H_2} is the number density), L is the characteristic length of the cloud, T is the cloud temperature ($c_{\rm s} = \sqrt{kT/\mu} = 0.19 \ {\rm km \ s}^{-1}(T/10 \ {\rm K})^{1/2}$ is the sound speed), B is the mean magnetic field strength, and σ_v is the (3D) velocity dispersion. Alternative definitions, with factors of order unity difference, are given by other authors; the basic scalings with density, temperature, cloud size, magnetic field, and fluid velocity, are standard.

The simulations described herein take as their fundamental units the mean cloud density ρ, cloud edge size $L \equiv n_J L_J$ (we use periodic boundary conditions with equal lengths

† $t_{\rm f}$ is often referred to as an "eddy turnover time" in incompressible turbulence; for compressible turbulence, the present terminology is preferred.

in each dimension), and the thermal sound speed c_s. Here, n_J is the number of Jeans wavelengths

$$L_J = c_{\rm s}\left(\frac{\pi}{G\rho}\right)^{1/2} = 1.9\ {\rm pc}\left(\frac{n_{H_2}}{10^2\ {\rm cm}^{-3}}\right)^{-1/2}\left(\frac{T}{10\ {\rm K}}\right)^{1/2} \tag{2.5}$$

across the box, so that the total mass in the simulated region is $M = n_J^3 M_J$ in terms of the Jeans mass

$$M_J \equiv \rho L_J^3 = 48\ M_\odot\left(\frac{n_{H_2}}{10^2\ {\rm cm}^{-3}}\right)^{-1/2}\left(\frac{T}{10\ {\rm K}}\right)^{3/2}. \tag{2.6}$$

Time is described either in units of $t_{\rm s}$ or $t_{\rm g}$; the corresponding physical times depend on the mean cloud properties via equations (2.1) and (2.5).

In MHD models the magnetic field strength is conveniently described by the ratio $\beta = c_{\rm s}^2/v_{\rm A}^2$.† The corresponding physical field strength is given in terms of the cloud temperature and density as

$$B = 1.4\mu{\rm G}\ \beta^{-1/2}\left(\frac{T}{10\ {\rm K}}\right)^{1/2}\left(\frac{n_{H_2}}{10^2\ {\rm cm}^{-3}}\right)^{1/2}. \tag{2.7}$$

2.2. *Cloud stability criteria*

A self-gravitating equilibrium supported by thermal pressure satisfies $c_{\rm s} \sim (GM/L)^{1/2}$, or equivalently, cloud stability requires $L < L_J$ (i.e. $t_{\rm s} < t_{\rm g}$). Molecular clouds, with their extremely low temperatures, fail to meet this (Jeans) criterion by an order of magnitude. In the absence of any other mitigating factors, a uniform, cold, quiescent cloud with $L >> L_J$ (i.e. $t_{\rm s} >> t_{\rm g}$) collapses in a time $\approx 0.3 t_g$ (the exact factor depend on the symmetry – e.g. spherical, cylindrical, slab – of the collapse). Since the Galactic star formation rate would be more than two orders of magnitude larger than observed if all Galactic GMCs collapsed and converted their material to stars in this time (assuming 100% efficiency), one of the main goals of understanding GMC internal dynamics is to identify the "mitigating factors" that prevent overall cloud collapse, and that limit the efficiency of star formation once portions of a cloud have lost their support.

A number of mitigating effects may arise from the fact that the ISM in general, and self-gravitating GMCs in particular, are magnetized. Consider first a cold, homogeneous cloud in which a uniform magnetic field is embedded. In determining stability to compressions transverse to the mean field (i.e. $\hat{k} \perp \hat{B}$), Chandrasekhar & Fermi (1953) showed that the thermal Jeans criterion is replaced by a magneto-Jeans criterion by substituting the magnetosonic speed for the sound speed in L_J; for $v_{\rm A} >> c_{\rm s}$ stability requires $t_{\rm A} < t_{\rm g}$, i.e. $n_{\rm J} \equiv L/L_J < v_{\rm A}/c_{\rm s} = \beta^{-1/2}$. The magneto-Jeans stability criterion can be translated to a maximum for the column density ρL in which transverse gravitational instabilities can be suppressed by the embedded field B: $L\rho < B/(2\sqrt{G})$, equivalent to

$$N_{H_2} < 4.1 \times 10^{21}\ {\rm cm}^{-2}\left(\frac{B}{10\mu{\rm G}}\right). \tag{2.8}$$

Field strengths in the range $7 - 24\mu$G, consistent with observations from Zeeman measurements (see the paper of Crutcher in this volume), would be adequate to make observed clouds ($N_{H_2} = 3 - 10 \times 10^{21}\ {\rm cm}^{-2}$) magneto-Jeans stable. However, since magnetic fields exert no force parallel to themselves, the stability along the field (for the uniform-field case) is subject to Jeans's original criterion; for $n_{\rm J} > 1$ all the matter in

† Our definition of β differs from the usual plasma $\beta_p \equiv P_{gas}/P_{magnetic} = 2c_s^2/v_{\rm A}^2$ by a factor of two.

a cloud would simply slide along the field to make a thin, self-gravitating pancake. The stability of a cloud pancake to perturbations with $\hat{k}$ in its plane can be described in terms of a critical ratio of column density along the field (i.e. surface density in the pancake Σ; or ρL in terms of the original mean density and length) to magnetic field strength (Nakano & Nakamura (1978), Tomisaka & Ikeuchi (1983)), with stable configurations having $\Sigma < B/(2\pi\sqrt{G})$ ‡ For a given column density, the necessary field strength for stability of a thin pancake is a factor π larger than that specified by the magneto-Jeans criterion (eq. 2.8), i.e.

$$B > 24\mu\mathrm{G}\left(\frac{L}{10\ \mathrm{pc}}\right)\left(\frac{n_{H_2}}{100\ \mathrm{cm}^{-3}}\right) = 77\mu\mathrm{G}\left(\frac{N_{H_2}}{10^{22}\ \mathrm{cm}^{-2}}\right). \tag{2.9}$$

While smaller dark clouds may be sub-critical, the strong fields required for stabilization at high column density make it likely that at least the most massive GMCs are super-critical. In addition, even when a cloud is subcritical on average, if there is variation of the mass load on field lines (i.e. the mass-to-flux ratio Σ/B) within it, then instability could arise on those field lines which exceed the critical condition.

The criteria discussed above apply to stability about a quiescent state, although, in fact, molecular clouds are quite turbulent. The potential for time-varying magnetic fields to help stabilize self-gravitating clouds – particularly along their mean field directions – has been discussed by a number of authors (see Shu et al (1987),McKee et al (1993), and references therein). In the idealized configuration of a cloud containing a uniform mean field and Afvén waves propagating along it, a quasilinear treatment of the time-dependent terms shows that they correspond to a wave pressure $\delta B^2_{\perp,rms}/(8\pi\rho)$ (McKee & Zweibel (1995)); a Jeans-type analysis then predicts that contraction along the mean field can be stabilized as long as $\rho L < \delta B_{rms}/(4\sqrt{G})$, i.e.

$$\delta B_{rms} > 15\mu\mathrm{G}\left(\frac{L}{10\ \mathrm{pc}}\right)\left(\frac{n_{H_2}}{100\ \mathrm{cm}^{-3}}\right) = 49\mu\mathrm{G}\left(\frac{N_{H_2}}{10^{22}\ \mathrm{cm}^{-2}}\right). \tag{2.10}$$

Since Alfvén waves have $\delta v = \delta B/\sqrt{4\pi\rho}$, another way to phrase the "wave-stabilization" criterion is $\sigma_v/c_\mathrm{s} = t_\mathrm{s}/t_\mathrm{f} > 2L/L_J = 2n_\mathrm{J} = 2t_\mathrm{s}/t_\mathrm{g}$ (i.e. $t_\mathrm{g} > 2t_\mathrm{f}$). The fact that observed clouds are known to have $\sigma_v/c_\mathrm{s} = 1.7n_\mathrm{J}$ (i.e. $t_\mathrm{g} = 1.7t_\mathrm{f}$), and may well have sufficient fluctuating field strengths to meet the criterion (2.10), has led to the hypothesis that GMCs are indeed supported (in all directions) over times $\gtrsim t_\mathrm{g}$ by a combination of mean fields and fluctuating fields. Since nonlinear MHD waves are dissipative, any stabilizing effects from the time-dependent fields will decrease over time; thus, whether the quasilinear-theory expectation is realized or not must depend on the nonlinear dissipation rate of the turbulence. Over very long times, stabilization *per force* requires mechanical energy replenishment (which can be supplied in astronomical contexts by the star formation process). Direct numerical MHD simulations are required to evaluate these ideas quantitatively; some of our results are highlighted in the next section.

3. Numerical Simulations – Selected Results

Our group has performed a variety of simulations to systematically explore compressible MHD turbulence in self-gravitating cloud models in 1 2/3, 2 1/2, and 3 dimensions.†

‡ For self-gravitating magnetized cloud equilibria to exist, the ratio of the central column density to central field strength Σ/B must be smaller than the same value $1/(2\pi\sqrt{G})$ (Mouschovias & Spitzer (1976),Tomisaka, Ikeuchi, & Nakamura (1988)).

† In "1 1/2 D" and "2 2/3 D" restricted geometry, there are respectively one or two independent spatial variables, but all components of $\mathbf{v}$ and $\mathbf{B}$ evolve in time (subject to $\nabla\cdot\mathbf{B} = 0$).

These models either start with a turbulent velocity field with a spectrum $dE = (1/2)v^2(k)dk \propto k^{-2}dk$ which freely decays, or else they include stochastic impulsive forcing on wavelengths small compared to the box, such that the kinetic energy input rate is constant. In the latter case, a quasisteady state is achieved. These numerical surveys have allowed us to address a number of questions about the fundamental nature of these turbulent flows, including:

- How do the survival times for self-gravitating clouds depend on their initial kinetic energy, magnetic field strength, and size (described by the dimensionless parameters $\mathcal{M} = \sigma_v/c_{\rm s} = t_{\rm s}/t_{\rm f}$, $\beta = (t_{\rm A}/t_{\rm s})^2$, and $n_{\rm J} = t_{\rm s}/t_{\rm g}$)?
- Is it possible to support a cloud indefinitely, given sufficient mechanical energy inputs?
- How do the temporal profiles for decaying turbulence, and the dissipation rates for quasisteady turbulence, depend on $\mathcal{M}$ and β?
- How do the density, velocity, and magnetic field distributions and power spectra depend on $\mathcal{M}$, β, and $n_{\rm J}$?

Full presentations of our surveys and results to date appear in Gammie & Ostriker (1996), Ostriker (1997), and Ostriker, Gammie, & Stone (1998). Stone's contribution to this volume discusses recent results on comparative dissipation rates and power spectra with varying β in high-resolution 3D simulations. Other work in preparation analyzes the distributions of axis ratios and orientations (relative to the magnetic field) of 3D clumps and their 2D projections (Gammie, Stone, & Ostriker (1998)). Here, we will highlight recent findings in two areas: comparisons of the survival times of cloud against gravitational collapse for varying $\mathcal{M}$, β, and $n_{\rm J}$, and differences in the density structure that arise in clouds when varying the field strength.

3.1. *Simulation results on cloud support*

Our first set of surveys, in 1 2/3D restricted geometry, confirmed the quasilinear-theory predictions for cloud support along the mean field by nonlinear analogues of Alfvén waves. We found that low-turbulence model clouds (i.e. $\sigma_v/c_{\rm s} < 2n_{\rm J}$) collapsed along the mean field in times $t < t_{\rm g}$, higher-turbulence models have delayed collapse, while in the highest-turbulence cases, collapse was forced by the strong compression from nonlinear waves. Overall, survival times increased with the strength of the mean magnetic field, because the turbulent dissipation rate was lower. We also performed a survey of forced-turbulence self-gravitating experiments in 1 2/3 D, and found that quasi-steady, non-collapsed states can be attained as long as the input power (applied mainly at a scale 8 times smaller than the box) exceeded $\dot{E}_{wave} > 24\beta^{0.24}n_{\rm J}^3\bar{\rho}c_{\rm s}^3$. The corresponding energy replenishment timescale to maintain a steady state would be $E_{wave}/\dot{E} \approx 0.2\beta^{-1/4}t_{\rm g} \sim 0.3\beta^{-1/4}t_{\rm f}$.

In 2 1/2D geometry, where the fluid is free to contract in the direction transverse to the mean field (this motion is restricted in 1 2/3D), we find that turbulent motions are unable to prevent cloud collapse unless the field is strong enough to make the clouds magnetically subcritical. Unmagnetized clouds collapse at $\approx 0.5t_{\rm g}$ (corresponding to 5 Myr at typical conditions) regardless of the initial kinetic energy level; weakly-magnetized (supercritical) clouds generally last up to $0.5 - 1t_{\rm g}$ before collapsing (some small, magneto-Jeans stable clouds survive up to $1.5t_{\rm g}$before collapsing), while more-strongly magnetized (subcritical) clouds can last beyond $1.5t_{\rm g}$ with no signs of collapse. The subcritical models, which are most similar to the 1 2/3D models in that they are unable to collapse perpendicular to the mean field, confirm the conclusion that time-dependent fluctuating magnetic field can prevent gravitational collapse along the mean field for times $>> 0.3t_{\rm g}$. However, since the majority of real molecular clouds are probably supercritical, our models imply that they are unlikely to last for times greater than $5 - 10$ Myr before some parts of their interior undergo collapse and initiate star formation. Further studies in 2 1/2D geometry are now in progress to asses the continuous energy input rate, as a function of β, required to sustain clouds against collapse.

We have also performed a number of self-gravitating simulations in 3D with varying β and $n_{\rm J}$, and found that, similar to the results in 2 1/2D, supercritical clouds collapse at times $\sim 0.5t_{\rm g}$ even when their initial perturbation energies are large, whereas subcritical clouds can survive to later times. Further 3D simulations are planned to expand the parameter space, and further test these ideas.

We can compare our results so far to the expectations of linear and quasilinear theory for cloud support. Our numerical experiments in 2 1/2D and 3D verify that magnetically supercritical ($n_{\rm J}\beta^{1/2}\pi > 1$) clouds typically gravitationally collapse in times $\lesssim t_{\rm g}$, whereas subcritical clouds can survive much longer without collapsing. In subcritical clouds, matter eventually slides along the field and forms thin sheets which do not themselves fragment (but may oscillate with respect to one another for some time). However, for lower field strengths the turbulent dissipation is rapid enough that long-term cloud support along the mean field is not possible. That is, even if clouds are magneto-Jeans stable and have initial turbulence levels exceeding $\sigma_v/c_{\rm s} > 2n_{\rm J}$, they have sufficient dissipation that they can collapse in all directions in times shorter than $t_{\rm g}$ (contrary to what quasilinear theory would predict if dissipation is ignored).

3.2. *Simulation results on density distributions*

An important way to make comparisons between simulated clouds and real clouds, so as to discriminate among the possible values of the input model parameters, is to analyze density distributions. In all of our models, we find that the cloud density structure becomes very clumpy and filamentary, due to the combination of turbulent Reynolds (i.e. ram pressure) and magnetic stresses. Figure 1 shows examples of snapshots of cloud structure in two 2 1/2 turbulent-decay models. Both models have initial turbulent energies with $\mathcal{M}^2 = (\sigma_v/c_{\rm s})^2 = 200$. One has a stronger $\beta = 0.01$ mean field; the other has a weaker $\beta = 0.1$ mean field (corresponding to $B_0 = 14\mu$G and $B_0 = 4.4\mu$G for fiducial GMC parameters, cf. eq. 2.7). The two models' snapshots are taken when the

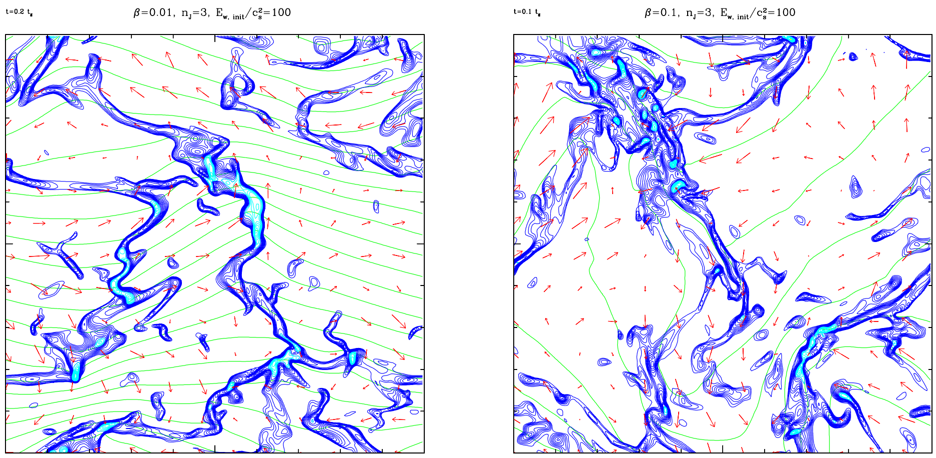

FIGURE 1. Snapshots of density (contours, starting at $\log(\rho/\bar{\rho}) = 0$ with logarithmic increments 0.1), velocity field (vectors), and magnetic field lines in $\beta = 0.01$ (left) and $\beta = 0.1$ (right) 2.5D decay simulations when $\sigma_v/c_s \approx 10$. $n_{\rm J} = 3$ for both models.

kinetic energy has declined to half of the initial value, i.e. to $\mathcal{M} = 10$ (corresponding to 3D velocity dispersion 2−3 km s^{-1} under GMC conditions); this is before self-gravity has become very important (the specific gravitational binding energy is only $-1.1c_s^2$, $-1.2c_s^2$ for the $\beta = 0.01$, 0.1 models, respectively).

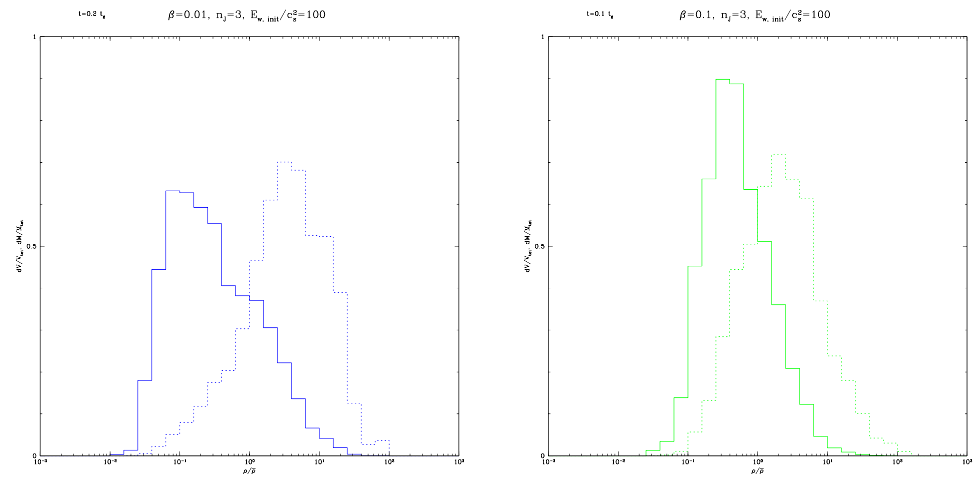

FIGURE 2. Density distributions for snapshots shown in Figure 1a,b. Solid-line histograms show fractional volume as a function of $log(\rho/\bar{\rho})$; dotted-line histograms show fractional mass as a function of $log(\rho/\bar{\rho})$;

For each of these snapshots, we have tabulated the fraction of the total volume, and the fraction of the total mass, as a function of $\log(\rho/\bar{\rho})$, the logarithmic density contrast relative to the mean ($\bar{\rho} \equiv M_{tot}/L^3$). These volume and mass distributions are shown in Figure 2. In both models, the mass distribution is centered at $\log(\rho/\bar{\rho}) > 0$, while the volume distribution is centered at $\log(\rho/\bar{\rho}) < 0$, as a consequence of matter clumping: most of the mass is in clumps which have densities higher than the mean for the whole cloud, whereas most of the volume is filled with matter at less-than-average densities. The $\beta = 0.01$ model has mass-averaged mean contrast $\langle\log(\rho/\bar{\rho})\rangle_M = 0.49$ (with median contrast 0.52), and volume-averaged mean contrast $\langle\log(\rho/\bar{\rho})\rangle_V = -0.50$ (with median contrast -0.60). The $\beta = 0.1$ model has mass-averaged mean contrast $\langle\log(\rho/\bar{\rho})\rangle_M = 0.33$ (with median contrast 0.31), and volume-averaged mean contrast $\langle\log(\rho/\bar{\rho})\rangle_V = -0.28$ (with median contrast -0.34). The distributions are very roughly log-normal, which accounts for the values of $\langle\log(\rho/\bar{\rho})\rangle_V$ and $\langle\log(\rho/\bar{\rho})\rangle_M$ being almost equal and opposite. We note that the density contrast in the $\beta = 0.01$ models is closer to the values $\sim \pm 0.4-0.6$ inferred for real clouds based on studies of clumping in ^{13}CO (e.g. Bally et al (1987), Williams, Blitz, & Stark (1995)).

The difference shown in these examples in density contrast between models with different β at the same Mach number is characteristic of all of the models examined so far: we generally find greater contrast in models with $\beta = 0.01$ than in the corresponding models with $\beta = 0.1$. We have also found that the density contrast in $\beta = 1$ models is intermediate between the $\beta = 0.1$ and $\beta = 0.01$ cases. Physically, we believe that the increase in contrast toward high β can be attributed to stronger compressions arising directly from the compressive part of the velocity field ($\nabla \cdot \mathbf{v} \neq 0$) when the magnetic pressure $B^2/8\pi$ is smaller, whereas the increase in contrast toward low β can be attributed to stronger compressions arising nonlinearly from the shear part of the velocity field – at kinks in the magnetic field lines – when B^2 is larger. Further research aimed at clarifying these issues is now underway.

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