Collapse of Low-Mass Protostellar Cores: Part I

Isothermal Unmagnetized Solutions and Observational Diagnostics

Andrea Kulier

AST 541

October 9, 2012

Outline

- Models of Isothermal Unmagnetized Collapse
  - Early Solutions:
    - Larson-Penston Solution (1969)
    - Shu solution (1977)
  - Recent Work: Gong and Ostriker (2009)
- Core Collapse Measurements from Asymmetric Spectral Lines

Larson-Penston Solution

- prior to this time, it was often assumed that the collapse of a protostar was roughly homologous

- hydrodynamic equations numerically solved and limiting solution given in Larson 1969

- dynamical assumptions:
  - spherical symmetry
  - no rotation
  - no magnetic fields
  - no internal turbulent motions

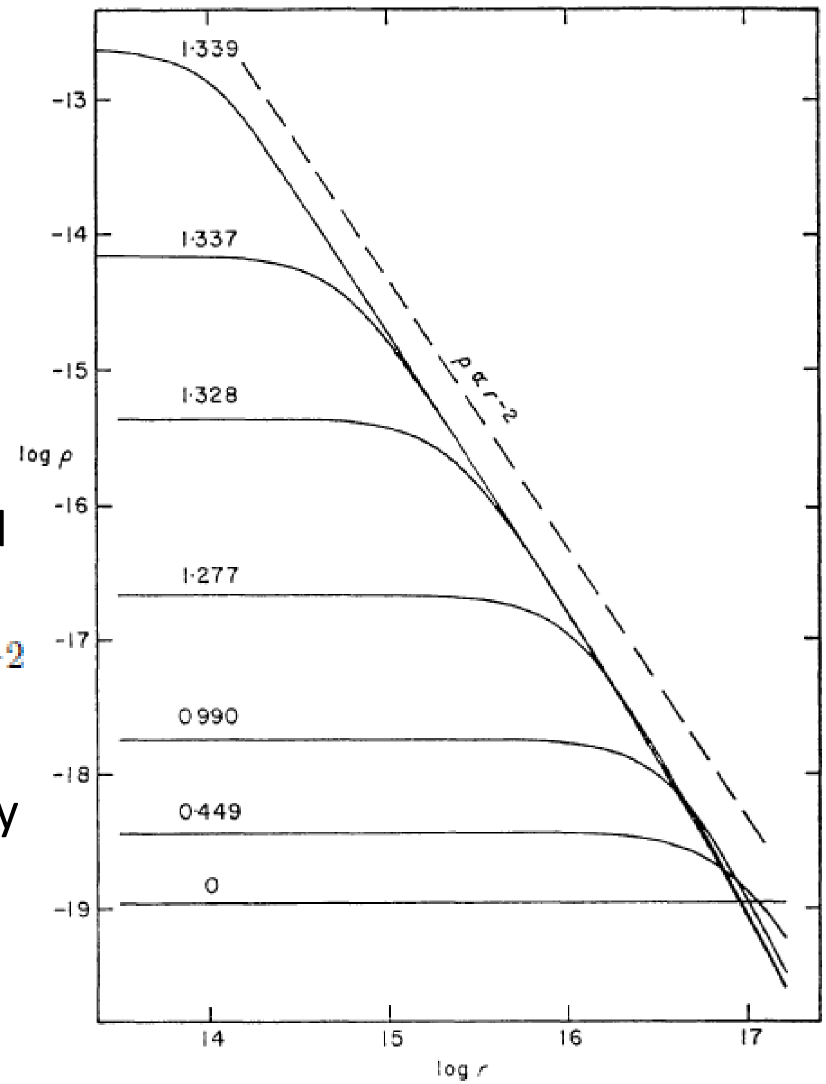
  - extremely efficient radiative cooling due to dust grains $\Longrightarrow T \approx \text{const.} \approx 10\,K$ for $10^{-19}\,g/cm^3 \lesssim \rho \lesssim 10^{-13}\,g/cm^3$

  - boundary condition: outer boundary of the cloud remains fixed in space at constant radius

  - collapse begins from rest with a uniform density distribution

Larson-Penston Solution

- at first, the whole cloud begins to collapse in free fall
- because the density in the interior of the cloud rises while that near the boundary drops, a pressure gradient is created
- $t_{ff} \propto \rho^{-1/2}$, so the collapse is fastest near the center where the density is highest
- changes in the density distribution occur closer and closer to the center of the cloud and on smaller and smaller timescales
- the density distribution approaches $\rho \propto r^{-2}$
  - in fact, this collapse pattern tends to occur regardless of initial and boundary conditions



Larson-Penston Solution

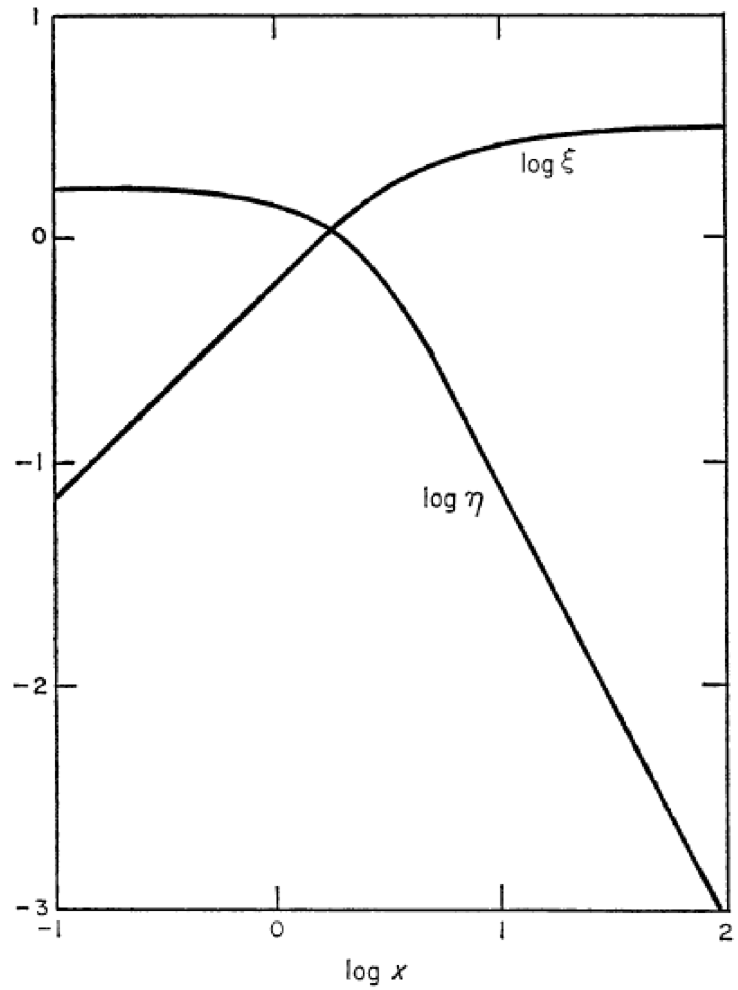
- this collapse pattern is an asymptotic similarity solution for collapse of an isothermal sphere
- the equations for isothermal collapse are:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{Rm}{r^2} + RT\frac{d\ln\rho}{dr} = 0$$
$$\frac{\partial m}{\partial t} + 4\pi r^2 \rho v = 0$$
$$\frac{\partial m}{\partial r} - 4\pi r^2 \rho = 0$$

- look for similarity solutions
- we obtain a unique solution with some criteria
- limiting values:

$$\text{for } x \ll 1, \quad \xi = \tfrac{2}{3}x, \quad \eta = 1\cdot667;$$
$$\text{for } x \gg 1, \quad \xi = 3\cdot28, \quad \eta = 8\cdot86x^{-2}.$$

- s increases for a fixed value of r over time, so the collapse will approach the second set of values

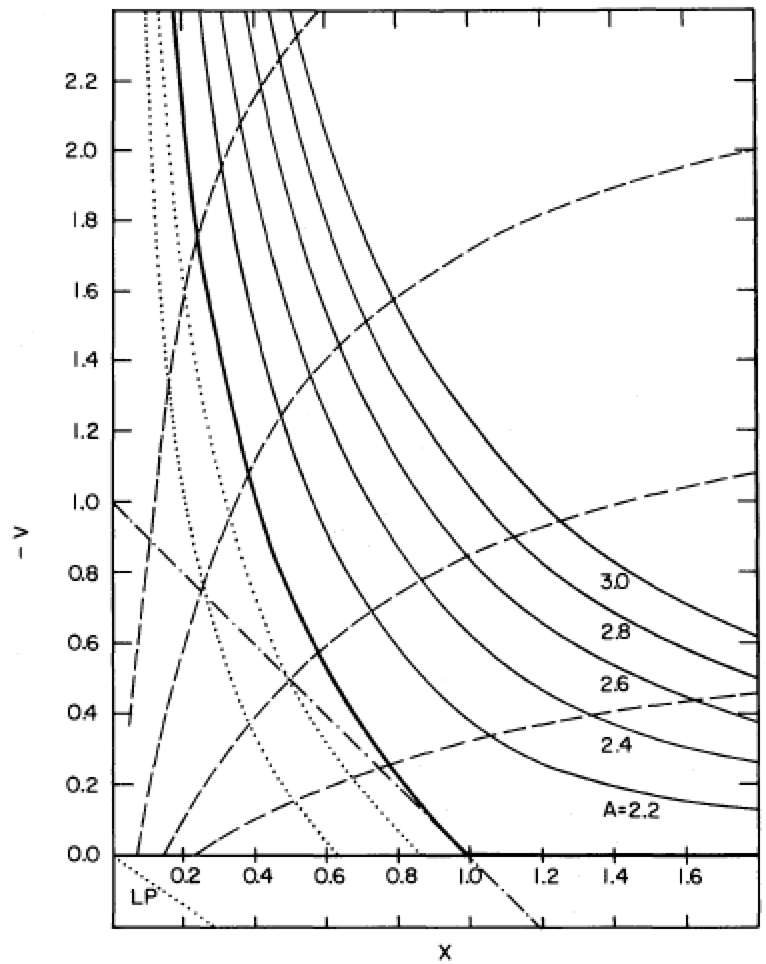


Shu Solution (Expansion Wave Collapse)

- Shu (1977) calculated similarity solutions for the collapse of an unstable isothermal sphere
- for the initial condition that v = 0 at the initial moment, one obtains solutions that have no critical points and require that at the initial time the density profile is that of a singular isothermal sphere
- near the origin, the solution tends toward

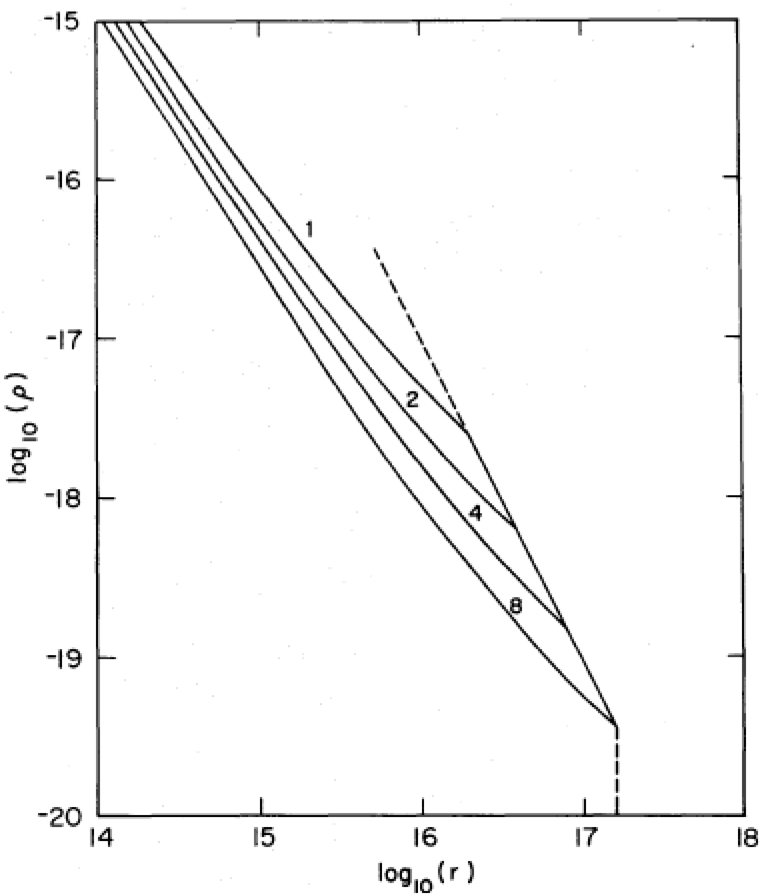
$$m \to m_0, \quad \alpha \to (m_0/2x^3)^{1/2}, \quad v \to -(2m_0/x)^{1/2} \quad \text{as } x \to 0 .$$

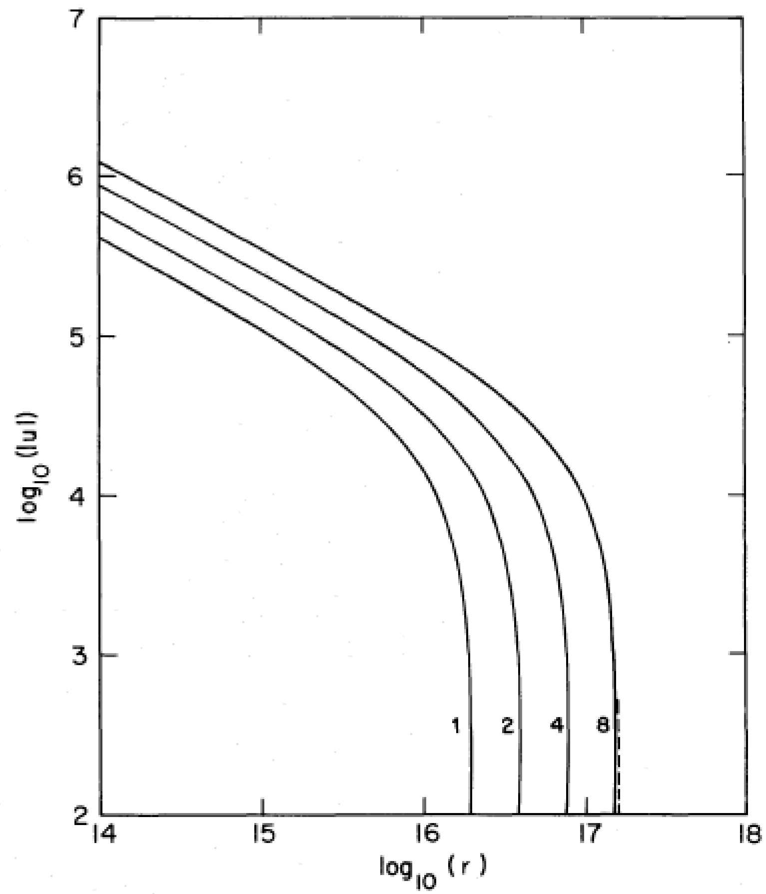
- these power-law dependencies correspond to free-fall collapse at steady mass accretion rates



Shu Solution (Expansion Wave Collapse)

- for the limit, we have a singular isothermal sphere between stability and instability
- if at t=0 a perturbation causes the central region to collapse, the infalling region will expand outward at a rate r = at



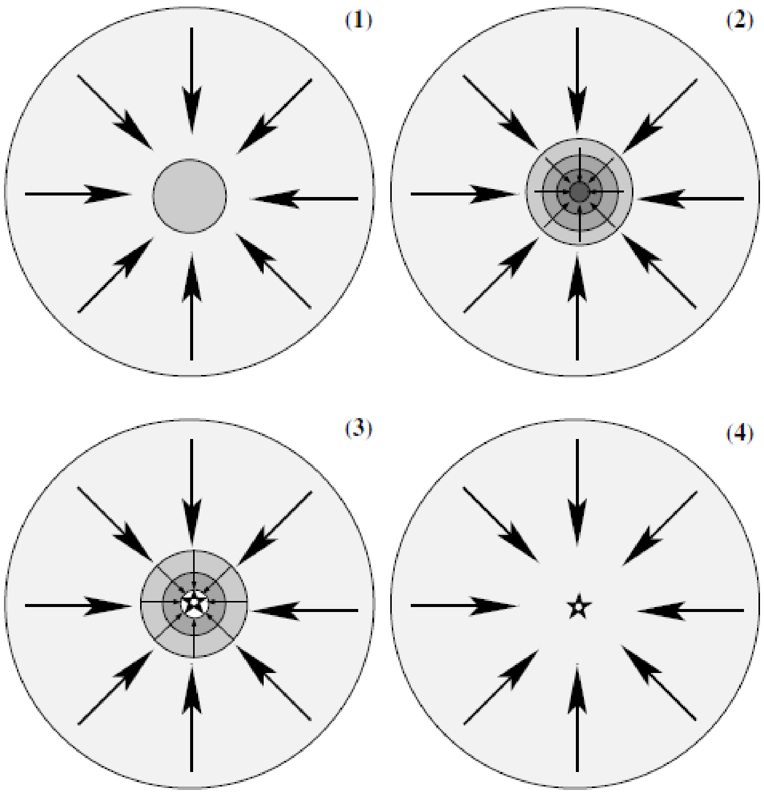


Gong and Ostriker 2009

- Gong and Ostriker simulate the formation and evolution of cores for converging spherical, supersonic flows
- boundary condition is constant density and inflow velocity at the outer radius
- vary the Mach number relative to the isothermal sound speed
- initial conditions are uniform density and uniform inflow velocity

Four Stages of Collapse

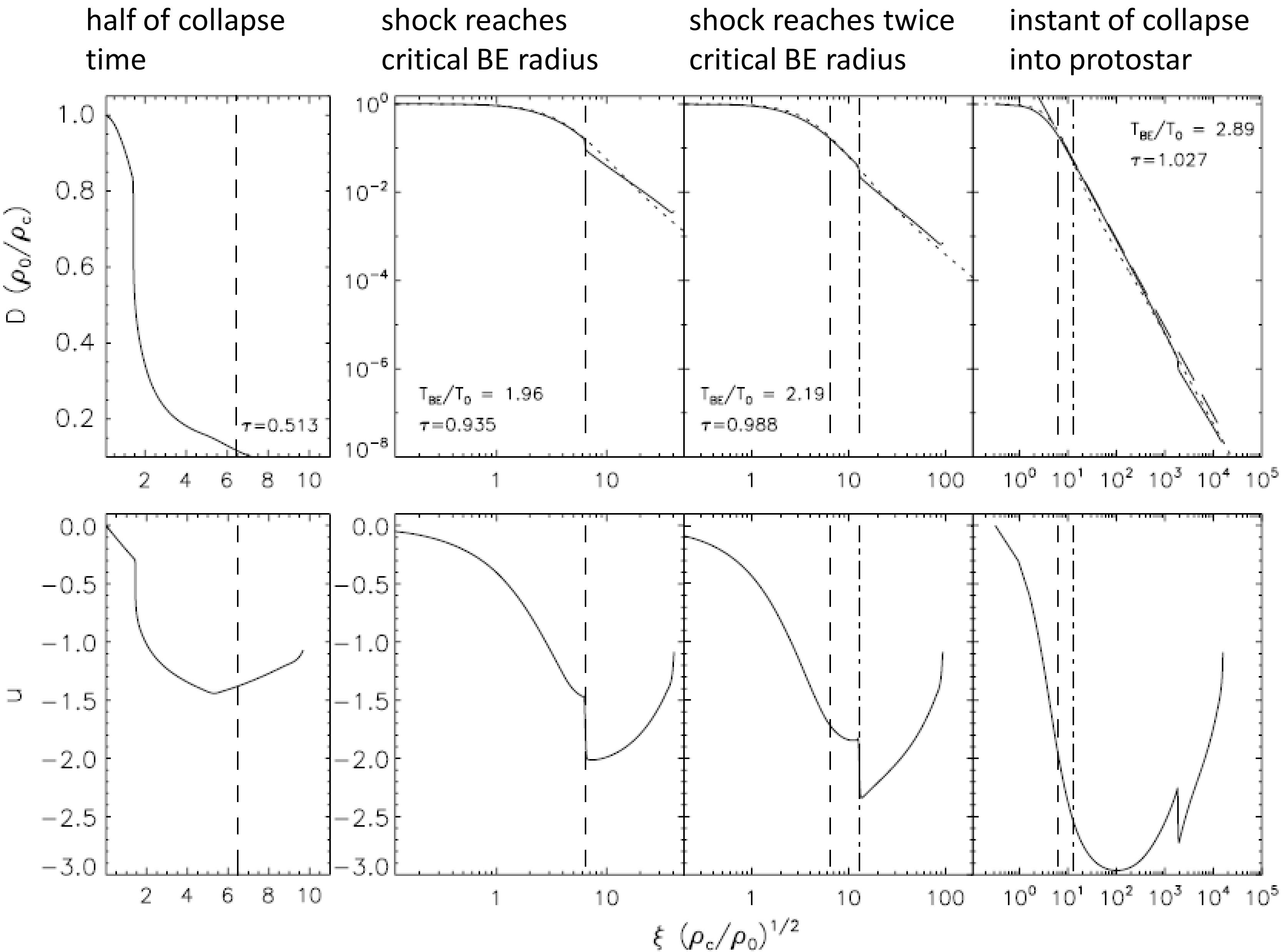
1. Supersonic inflow creates a dense subsonic core bounded by a shock.
2. The core becomes unstable and collapses supersonically, with the collapse propagating "outside-in."
3. A protostar is formed at the center of the core. A density rarefaction propagates "inside-out."
4. Late accretion of remaining gas onto the protostar.



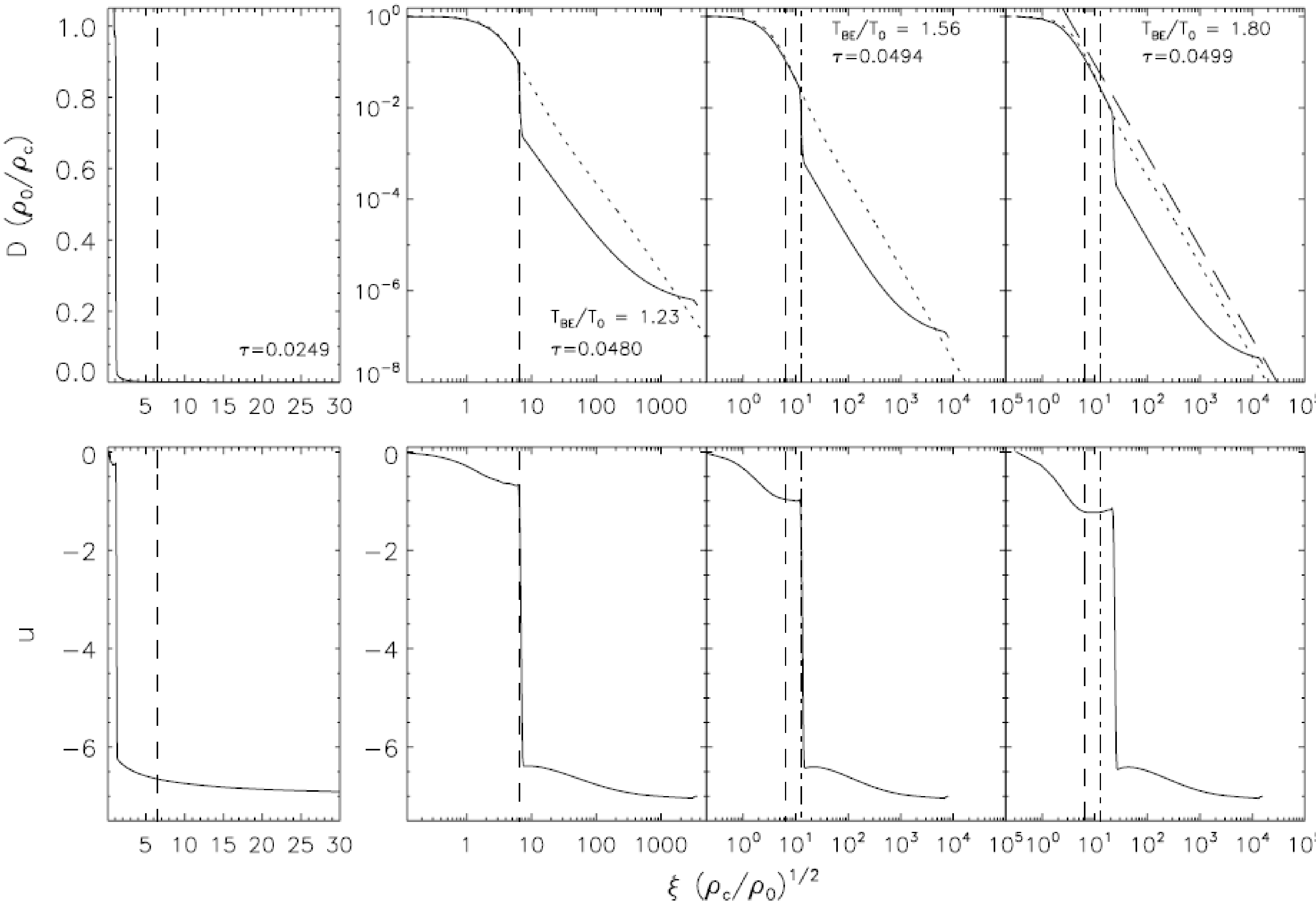
Core Creation and Collapse

- immediately at the beginning of the simulation, a shock forms at the origin and propagates outward
- this creates an inner dense subsonic region and an outer low-density region with supersonic inflow, connected by shock jump conditions
- the inner region contracts slowly into a dense core
- the density profile evolves from a uniform profile to a $\rho \propto r^{-2}$ profile consistent with supersonic radial inflow
- after a long period of time, the central dense region becomes gravitationally unstable
  - this occurs when the radius is close to that of a critical Bonnor-Ebert sphere
- the collapse now follows an “outside-in” pattern, starting from the shock front
- the inflow velocity inside the shock becomes supersonic
- the density profile and velocity inside the shock approach that of the Larson-Penston solution, D=8.86xi, v = -3.3a for all Mach numbers
- this stage ends with the formation of a protostar at the center

Results for $M_a = 1.05$



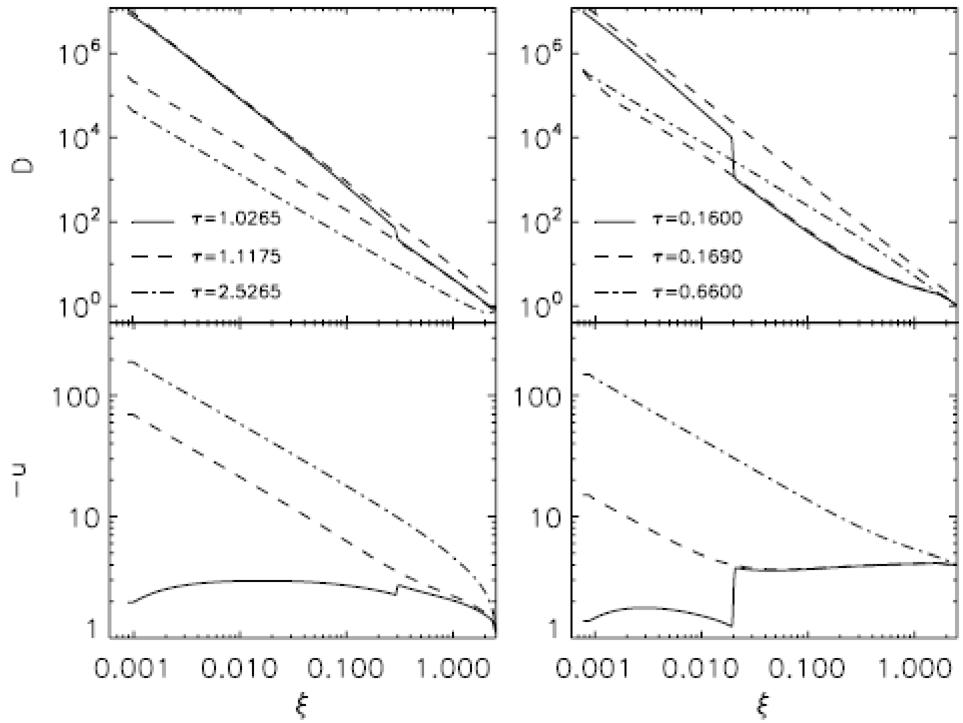
Results for $M_a = 7.0$



Infall and Accretion Stages

- after the central density becomes singular, the infall stage begins
- a region of unsupported infall begins from the center and propagates outward in an "inside-out" collapse
  - this is similar to the Shu expansion wave solution
- the matter is now in free-fall, so the density is proportional to r^(-3/2)and the velocity is proportional to r^(-1/2)
- this stage ends when the infall wave reaches the shock
- finally, the rest of the gas accretes supersonically onto the protostar
- the duration of the accretion stage depends on the how much material will fall onto the protostar from large scales

M_a = 1.05 M_a = 4.0



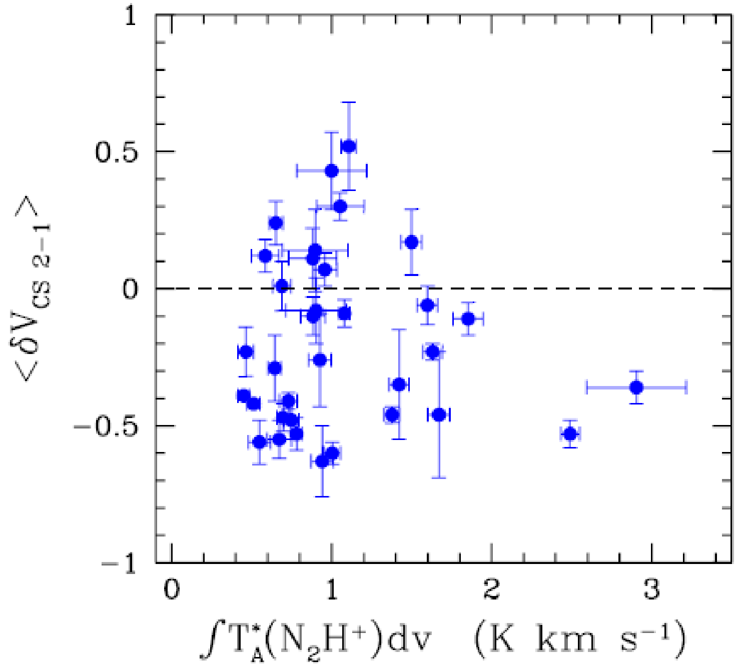
solid line – instant of core collapse
dashed line – infall wave reaches the shock
dot-dash – late accretion stage

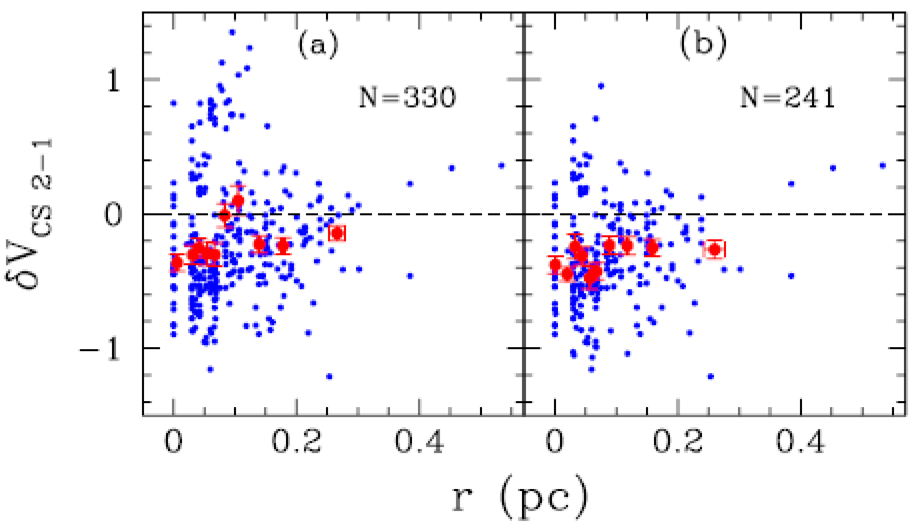
Evidence of Infall From Spectral Lines

- can detect “infall asymmetry” in optically thick spectral lines
- the degree of excitation of the line must increase with depth toward the center of the cloud
- contributions from the infall region are blueshifted for the far hemisphere and redshifted for the near hemisphere, and vice versa for expansion
- for an optically thick line, the material in the outskirts of the cloud that is nearly static will cause absorption at the rest wavelength
- the regions nearest to the center have the most excitation
- however, light from the center for the near hemisphere is self-absorbed along the line of sight
- one side of the line will be less intense

Measurements for Starless Cores

- Lee and Myers (2011) analyzed molecular line data and obtained this velocity difference for cores without stars
- found an overabundance of blue (contracting) profiles
- some cores, however, appear to be oscillating or expanding
- fraction of contracting profiles increases with cloud column density
  - above about $6x10^{21}$ cm^{-2}, all cores are contracting
- velocity difference is somewhat larger near the center of the cloud





Conclusions

- even very simplified models (spherical symmetry, no magnetic fields, no rotation) can give us some idea of the form of core collapse
- collapse occurs in stages
  - the stage before protostar formation resembles the classic Larson-Penston solution, whereas the infall stage after resembles the expansion wave collapse described by Shu
- we can learn about the nature of collapse through measurements of line asymmetries in clouds