Astro 403 Spring 2015: Problem Set 1

due: Wed. 2/11, 5:00 pm

Chapter 1. Introduction

1.1 Suppose that the total mass of neutral gas in the Galaxy is $5{\times}10^9\,M_\odot$. Assume that it is uniformly distributed in a disk of radius $R_{\rm disk} = 15\,{\rm kpc}$ and thickness $H = 300\,{\rm pc}$, and that it is a mixture of H and He with He/H=0.1 (by number). Assume ionized hydrogen to be negligible in this problem. [Note: even though the assumptions in this problem are very approximate, please carry out calculations to **two** significant digits.]

(a) What is the average number density of hydrogen nuclei within the disk?

(b) If 0.7% of the interstellar mass is in the form of dust in spherical particles of radius $a = 1000\,\text{Å} = 0.1\,\mu{\rm m}$ and density $2\,{\rm g\,cm^{-3}}$, what is the mean number density of dust grains in interstellar space? Assume the dust to be uniformly mixed with the gas.

(c) Let $Q_{\rm ext}$ be the ratio of the visual (V band, $\lambda = 0.55\,\mu{\rm m}$) extinction cross section to the geometric cross section πa^2. Suppose that $Q_{\rm ext} \approx 1$. Calculate the optical depth τ_V. [If α is the probability per unit pathlength of an absorption or scattering event, then the "optical depth" for path L is $\tau \equiv \alpha L$.] What would be the visual extinction A_V (in magnitudes!) between the Sun and the Galactic Center (assumed to be $8.5\,{\rm kpc}$ away).

(d) Now assume that 30% of the gas and dust mass is in spherical molecular clouds of radius 15 pc and mean density $n({\rm H_2}) = 100\,{\rm cm^{-3}}$.
What would be the mass of one such cloud?
How many such molecular clouds would there be in the Galaxy?

(e) With 30% of the gas and dust mass in molecular clouds as in (d), what is the expectation value for the visual extinction A_V to the Galactic Center?

(f) With 30% of the material in molecular clouds as in (d), what is the expectation value for the number of molecular clouds that will be intersected by the line of sight to the Galactic center?
What is the probability that zero molecular clouds will be intersected? [Hint: the number of molecular clouds in the Galaxy is large, and they occupy a small fraction of the volume, so think of this as a "Poisson process", where the presence or absence of each molecular cloud on the line-of-sight is treated as an independent event (like the number of radioactive decays in a fixed time interval).]

(g) If the line of sight to the Galactic center happens not to intersect any molecular clouds, and if the atomic hydrogen and associated dust are distributed uniformly throughout the disk volume, what will be the visual extinction to the Galactic center?

1.2 Suppose that we approximate hydrogen atoms as hard spheres with radii $a = 1.5\,\text{Å}$. In a neutral atomic hydrogen cloud with density $n_{\rm H} = 30\,{\rm cm^{-3}}$, what is the mean free path for an H atom against scattering by other H atoms (assuming the other H atoms to be at rest)?

1.3 The "very local" interstellar medium has $n_{\rm H} \approx 0.22\,{\rm cm^{-3}}$ (Lallement et al. 2004: Astr. & Astrophys. 426, 875; Slavin & Frisch 2007: Sp. Sci. Revs. 130, 409). The Sun is moving at $v_W = 26 \pm 1\,{\rm km\,s^{-1}}$ relative to this local gas (Möbius et al. 2004: Astr. & Astrophys. 426, 897).

Suppose that this gas has He/H=0.1, and contains dust particles with total mass equal to 0.5% of the mass of the gas. Suppose these particles are of radius $a = 0.015\,\mu{\rm m}$ and density $\rho = 2\,{\rm g\,cm^{-3}}$, and we wish to design a spacecraft to collect them for study.

How large a collecting area A should this spacecraft have in order to have an expected collection rate of 1 interstellar grain per day? Neglect the motion of the spacecraft relative to the Sun, and assume that the interstellar grains are unaffected by solar gravity, radiation pressure, and the solar wind (and interplanetary magnetic field).

Chapter 2. Collisional Processes

2.3 Consider a dust grain of radius a, and mass $M \gg m_{\rm H}$, where $m_{\rm H}$ is the mass of an H atom. Suppose that the grain is initially at rest in a gas of H atoms with number density $n_{\rm H}$ and temperature T. Assume the grain is large compared to the radius of an H atom. Suppose that the H atoms "stick" to the grain when they collide with it, so that all of their momentum is transferred to the grain, and that they subsequently "evaporate" from the grain with no change in the grain velocity during the evaporation.

(a) What is the mean *speed* $\langle v_{\rm H} \rangle$ of the H atoms (in terms of $m_{\rm H}$, T, and Boltzmann's constant k)?

(b) Calculate the time τ_M for the grain to be hit by its own mass M in gas atoms. Express τ_M in terms of M, a, $n_{\rm H}$, and $\langle v_{\rm H} \rangle$.

(c) Evaluate $\langle v_{\rm H} \rangle$ and τ_M for a grain of radius $a = 10^{-5}\,{\rm cm}$ and density $\rho = 3\,{\rm g\,cm^{-3}}$, in a gas with $n_{\rm H} = 30\,{\rm cm^{-3}}$ and $T = 10^2\,{\rm K}$.

(d) If the collisions are random, the grain velocity will undergo a random walk. Estimate the *initial* rate of increase $(dE/dt)_0$ of the grain kinetic energy E due to these random collisions. Express $(dE/dt)_0$ in terms of $n_{\rm H}$, $m_{\rm H}$, kT, a, and M. [Hint: think of the random walk that the grain momentum $\vec{p}$ undergoes, starting from the initial state $\vec{p} = 0$. What is the rate at which $\langle p^2 \rangle$ increases?]

(e) Eventually the grain motion will be "thermalized", with time-averaged kinetic energy $\langle E \rangle = (3/2)kT$. Calculate the timescale

$$\tau_E \equiv \frac{(3/2)kT}{(dE/dt)_0}$$

for thermalization of the grain speed. Compare to τ_M calculated in (b).

Chapter 5. Energy Levels of Molecules

5.1 Most interstellar CO is $^{12}{\rm C}^{16}{\rm O}$. The $J = 1 \rightarrow 0$ transition is at $\nu = 115.27\,{\rm GHz}$, or $\lambda = 0.261\,{\rm cm}$, and the $v = 1 \rightarrow 0$ transition is at $\lambda = 4.61\,\mu{\rm m}$.

(a) Estimate the frequencies of the $J = 1 \rightarrow 0$ transitions in $^{13}{\rm C}^{16}{\rm O}$ and $^{12}{\rm C}^{17}{\rm O}$.

(b) Estimate the wavelengths of the $v = 1 \rightarrow 0$ transitions in $^{13}{\rm C}^{16}{\rm O}$ and $^{12}{\rm C}^{17}{\rm O}$.

(c) Suppose that the $^{13}{\rm C}^{16}{\rm O}\ J=1-0$ line were mistaken for the $^{12}{\rm C}^{16}{\rm O}\ J=1-0$ line.
What would be the error in the inferred radial velocity of the emitting gas?