Notes for KINGFISH on SPIRE Photometry

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ABSTRACT

Some notes on interpretation of SPIRE photometry of extended sources, and recommendations for use by the KINGFISH collaboration.

1. Introduction

I (and, I know, some others) have found the existing documentation regarding SPIRE photometry to be sometimes confusing: the distinction between beam sizes to use for point sources or extended sources, the distinction between the relative spectral response function (RSRF) for point sources and extended sources, and the correct application of "color corrections".

Here I state my current understanding of how KINGFISH should employ SPIRE photometry for the extended sources in the KINGFISH sample. I think that the information below is consistent with the current recommendations of the SPIRE team.[1][2]

It should not be assumed that the SPIRE team endorses all statements in this document. The present document is intended for circulation within the KINGFISH collaboration, to ensure that we are all "on the same page" with regard to use of SPIRE photometry.

Comments and corrections – especially by members of the SPIRE team – are welcomed.

2. Some Definitions

For each instrument there is a nominal wavelength λ_0 and nominal frequency $\nu_0 = c/\lambda_0$. The SPIRE team has chosen $\lambda_0 = 250, 350, 500\mu$m as the reference wavelengths for measurements of flux density or specific intensity in the 3 SPIRE photometric bands, with the understanding that *if* the source spectrum is a power-law with flux density $F_\nu \propto \nu^{-1}$ (or with specific intensity $I_\nu \propto \nu^{-1}$), the SPIRE flux density (or specific intensity) will be the correct value at frequency $\nu = c/\lambda_0$.

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[1]As noted below, I have benefited from helpful communications with Matt Griffin and Chris North from the SPIRE team.

[2]In a previous version (v4) of this document, I failed to recognize that the SPIRE team defines the relative spectral response function RSRF such that the overall response is proportional not to just RSRF(ν), but rather to $\eta(\nu) \times$ RSRF(ν), where $\eta(\nu)$ is the aperture efficiency. This was corrected in v5. Inclusion of $\eta(\nu)$ resulted in numerical changes in the correction factors $C(\alpha)$ and $C(\beta, T)$. v6 differs from v5, with updated coefficients for the conversion from flux/beam from the point source pipeline to extended source surface brightness, and a slight streamlining of the presentation. The present v8 has been shortened by removal of comparison to numbers in the SPIRE Data Reduction Guide v2.1, as it was not entirely clear whether the comparison was appropriate, or whether additional correction factors (of order unity) needed to be applied to the numbers in the SPIRE DRG before comparison to what is here denoted Ω_E.

We are primarily interested in extended sources. Therefore, the discussion below will focus on specific intensity I_ν (power per unit area per unit frequency per unit solid angle).

The design and performance of the SPIRE instrument are discussed by Griffin et al. (2010). Each bolometer views the sky through an optical system that includes the telescope optics, a filter, and a conical feedhorn (Chattopadhyay et al. 2003) for each bolometer. Each SPIRE detector produces a response V (e.g., a voltage change) in response to the absorbed power. In the linear regime,

$$V = \int d\nu \; I_\nu \; R_E(\nu) \quad , \tag{1}$$

where $R_E(\nu)$ characterizes the SPIRE detector response for an extended (uniform surface brightness) source. $R_E(\nu)$ includes *all* wavelength-dependent factors that come between the "sky" and the detector response V, including filter transmission, telescope efficiencies, and variations in the solid angle seen by each detector.

Let $\eta(\nu)$ be the "aperture efficiency", the fraction of the power from an on-axis source that is coupled to one detector. The SPIRE team defines a "Relative Spectral Response Function" $\mathrm{RSRF}_E(\nu)$ that does *not* include the variation with frequency of $\eta(\nu)$. Thus, with the SPIRE team's definition of RSRF_E, the overall response

$$R_E(\nu) = \eta(\nu) \times \mathrm{RSRF}_E(\nu) \quad . \tag{2}$$

The frequency dependence of RSRF_E and R_E will be discussed below, in §3.

Let $V_{\rm cal}$ be the response when viewing a source with intensity given by

$$I_\nu = I_{\rm cal} \times (\nu/\nu_0)^{-1} \quad . \tag{3}$$

Thus

$$V_{\rm cal} = I_{\rm cal} \int d\nu \, (\nu/\nu_0)^{-1} \, R_E(\nu) \quad . \tag{4}$$

Then, for a general spectrum I_ν, the "reported" SPIRE intensity is $I_\nu^{(S)} = (I_{\rm cal}/V_{\rm cal}) \times V$:

$$I_\nu^{(S)} = (I_{\rm cal}/V_{\rm cal}) \times \int d\nu \, I_\nu \, R_E(\nu) \tag{5}$$

$$= \frac{\int d\nu \, I_\nu \, R_E(\nu)}{\int d\nu \, (\nu/\nu_0)^{-1} \, R_E(\nu)} \quad . \tag{6}$$

Note that $R_E(\nu)$ appears in both numerator and denominator, hence the absolute normalization used for R_E is unimportant – all that matters is the *relative* response as a function of wavelength. The calibration process is responsible for measuring $V_{\rm cal}/I_{\rm cal}$ for the nominal ν^{-1} spectrum, so that the measured response V can be converted to SPIRE specific intensities $I_\nu^{(S)}$.

3. Response Functions $R_E(\nu)$

Fig. 5.5 of the SPIRE Observer's Manual v2.4 (June 2011) gives the response function $\mathrm{RSRF}_E(\nu)$ for each of the SPIRE bands. For SPIRE, the extended source $\mathrm{RSRF}_E(\nu)$ differs from the point source $\mathrm{RSRF}_P(\nu)$ because each bolometer is coupled through a feedhorn whose beam width (i.e., acceptance angle) is an increasing function of wavelength λ. In the SPIRE Observer's Manual v2.4, $\mathrm{RSRF}_E(\nu) = (\nu_0/\nu)^2 \times$

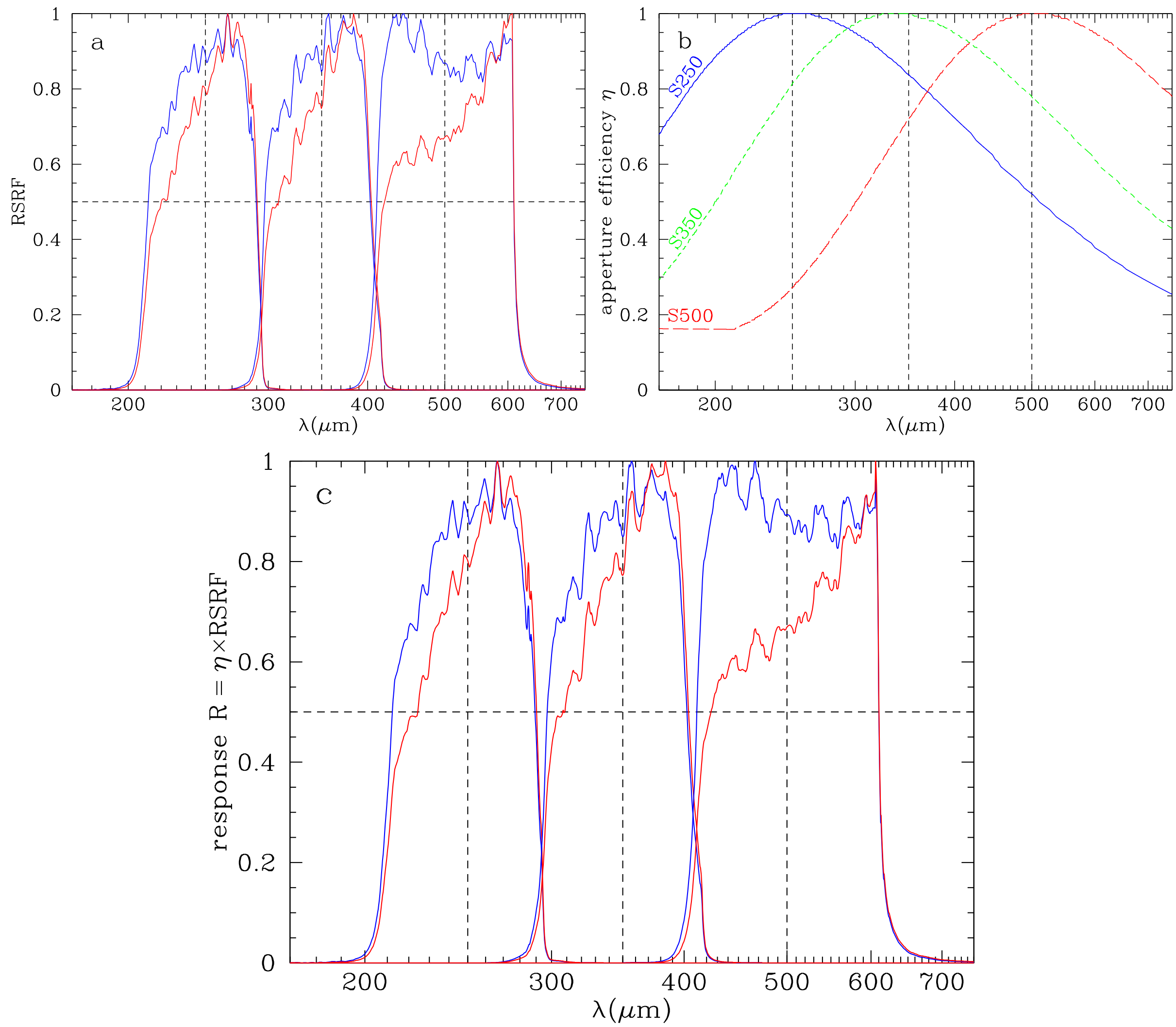

Fig. 1.— (a) Relative Spectral Response Function (RSRF) for the 3 SPIRE bands, as defined the SPIRE team. This does not include variations in beam efficiency with wavelength. Blue curves: RSRF_P for point sources. Red curves: RSRF_E for extended sources, calculated using eq. (7) with $\gamma = 0.85$. Each RSRF is normalized to its peak value. (b) Aperture efficiency factors $\eta(\nu)$ for the 3 SPIRE bands. (c) Overall response R_P and R_E to point sources and extended sources, normalized to peak values.

$\mathrm{RSRF}_P(\nu)$. However, the (more recent) SPIRE Photometer Beam Profile Analysis[3] document states that the FWHM of the beam varies as λ^γ. I take this to imply that

$$\mathrm{RSRF}_E(\nu) = \mathrm{RSRF}_P(\nu) \times (\lambda/\lambda_0)^{2\gamma} \quad . \tag{7}$$

[3]http://herschel.esac.esa.int/twiki/bin/view/Public/SpirePhotometerBeamProfileAnalysis , by Bernhard Schulz, September 2012. The document on the WWW is labelled "r2 – 19 October 2012 – Bernhard Schulz".

Griffin et al. (2013) recommend $\gamma = 0.85$ for "the power-law index of the frequency scaling of the beam profile" for all 3 bands. Fig. 1a shows RSRF_P and RSRF_E.[4] Figure 1(a) shows RSRF_P and RSRF_E for the 3 SPIRE bands.

The overall response function R_E also depends on the aperture efficiency $\eta(\nu)$:

$$R_P(\nu) = \eta(\nu) \times \mathrm{RSRF}_P(\nu) \tag{8}$$

$$R_E(\nu) = \eta(\nu) \times \mathrm{RSRF}_E(\nu) \quad . \tag{9}$$

Figure 1b shows $\eta(\nu)$ for the 3 SPIRE bands.[5] Figure 1c shows the final adopted $R_P(\nu)$ and $R_E(\nu)$ for the 3 SPIRE bands, assuming $\gamma = 0.85$.

4. Beam Solid Angle

The SPIRE point source pipeline delivers a "flux density per beam" $F_\nu^{(S)}$. For an extended source with a spectrum $I_\nu \propto \nu^{-1}$, the "reported" SPIRE flux intensity $I_\nu^{(S)}$ (what Griffin et al. (2013) refer to as "the standard SPIRE extended pipeline surface brightness") is related to the "point source pipeline" flux density per beam $F_\nu^{(S)}$ via a multiplicative constant X:[6]

$$I_\nu^{(S)} = X \times F_\nu^{(S)} \quad . \tag{10}$$

It is natural to define an effective solid angle Ω_E for extended sources[7] with $I_\nu \propto \nu^{-1}$:

$$\Omega_E \equiv \frac{1}{X} \quad , \tag{11}$$

such that the "extended pipeline" surface brightness and "point source pipeline" flux/beam are related by

$$I_\nu^{(S)} = \frac{F_\nu^{(S)}}{\Omega_E} \quad . \tag{12}$$

The SPIRE team is currently preparing a paper discussing flux calibration of broadband far infrared instruments (Griffin et al. 2013). The values of X from Griffin et al. (2013) and the related solid angles $\Omega_E \equiv X^{-1}$ are given in Table 1 for each of the SPIRE bands.

The actual beam profile is a function of wavelength, and, therefore, the effective beam profile (averaged over the band) depends on the spectrum of the source. In the present analysis we characterize each band by a single solid angle Ω_E appropriate for a spectrum $\nu I_\nu =$const, and absorb all of the variation of beam size with wavelength into the frequency-dependent response function $R_E(\nu)$ for extended sources. This approach is correct for infinitely-extended sources.

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[4] RSRF_P for the 3 SPIRE bands is from the file SPIRE-Phot-RSRF_nu.csv created by Ivan Valtchanov on 2012.01.19, downloaded from ftp://ftp.sciops.esa.int/pub/hsc-calibration/SPIRE/PHOT/Filters/.

[5] The aperture efficiency functions $\eta(\nu)$ in Figure 1b were kindly provided by Chris North.

[6] We use the symbol X here to avoid confusion with the many different usages of the symbol K (with a menagerie of subscripts) in the SPIRE documentation.

[7] Griffin et al. (2013) define quantities K_{MonP}, K_{Uniform}, and Ω_{eff}. Simone Bianchi notes that the present definition of Ω_E can be written $\Omega_E = K_{\mathrm{MonP}}(-1,\nu_0)/K_{\mathrm{Uniform}}(-1,\nu_0) = K_{\mathrm{MonP}}(-1,\nu_0) \times \Omega_{\mathrm{eff}}(-1)$.

Table 1: Conversion Factor X and $\Omega_E \equiv X^{-1}$ for Extended Sources with $I_\nu \propto 1/\nu$

	$\lambda_0(\mu\mathrm{m})$			
Quantity	250	350	500	Notes
$X(10^6\,\mathrm{sr}^{-1})$	90.687	51.430	23.907	a, b
$\Omega_E(\mathrm{arcsec}^2)$	469.1	827.2	1779.6	c
$\Omega_E(10^{-8}\,\mathrm{sr})$	1.103	1.944	4.183	c

[a] See equation (10).
[b] From §6.2.2 of Griffin et al. (2013), updated by Chris North 2013.02.14.
[c] See equation (11).

The KINGFISH galaxies are, for the most part, large compared to the SPIRE beams, and I assume that the surface brightness I_ν can be estimated from the SPIRE photometry as though the sources are infinitely extended. This is clearly an approximation.

5. Effect of Spectral Shape

If SPIRE has been correctly calibrated, then, when viewing a source of spectrum $I_\nu = I_0(\nu/\nu_0)^{-1}$, the "reported" specific intensity will be equal to the actual specific intensity at $\nu = \nu_0$:

$$I_\nu^{(S)} = I_0 \quad . \tag{13}$$

For an extended source with a general spectrum

$$I_\nu = I_0 \times \frac{f(\nu)}{f(\nu_0)} \quad , \tag{14}$$

the true specific intensity I_0 at $\nu = \nu_0$ is related to the reported intensity $I_\nu^{(S)}$ by

$$I_0 = C_f \times I_\nu^{(S)} \tag{15}$$

$$C_f \equiv \frac{\int d\nu\,(\nu/\nu_0)^{-1}\,R_E(\nu)}{\int d\nu\,[f(\nu)/f(\nu_0)]\,R_E(\nu)} \quad . \tag{16}$$

The correction factors C_f depend on the shape of the spectrum [i.e., the function $f(\nu)/f(\nu_0)$] but the C_f are independent of the absolute normalization of R_E, because R_E appears in both numerator and denominator. For the specific case of power-law spectra

$$I_\nu = I_0 \times (\nu/\nu_0)^\alpha \tag{17}$$

$$C(\alpha) = \frac{\int d\nu\,(\nu/\nu_0)^{-1}\,R_E(\nu)}{\int d\nu\,(\nu/\nu_0)^{\alpha}\,R_E(\nu)} \quad . \tag{18}$$

Griffin et al. (2013) define color correction factors K_{ColE} for extended sources. The correction factors $C(\alpha)$ defined in eq. (18) should be identical to the color correction factor $K_{\mathrm{ColE}}(\alpha,\infty)$ plotted in Figure 14a of Griffin et al. (2013).

As a check I have calculated $C(\alpha)$ for R_E calculated with $\gamma = 0.85$ as recommmended by Griffin et al. (2013) (Table 2 columns 2–4). Also shown for comparison in Table 2 (columns 5–7) are the values of $C(\alpha)$

Table 2: Color Correction Factors $C(\alpha)$ for SPIRE Extended Source Photometry

	for $\gamma = 0.85$ [a]			from SPIRE DRG [b]		
α	250	350	500	250	350	500
-4	0.9407	0.9403	0.8689	0.9407	0.9405	0.8703
-3	0.9675	0.9672	0.9231	0.9675	0.9673	0.9239
-2	0.9876	0.9874	0.9675	0.9876	0.9874	0.9679
-1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	1.0043	1.0047	1.0190	1.0043	1.0046	1.0187
1	1.0002	1.0010	1.0235	1.0028	1.0009	1.0230
2	0.9876	0.9892	1.0135	0.9878	0.9890	1.0128
3	0.9668	0.9693	0.9897	0.9671	0.9691	0.9889
4	0.9383	0.9420	0.9535	0.9388	0.9418	0.9526

[a] R_E calculated with $\gamma = 0.85$.

[b] $C(\alpha)$ from Table 5.11 in SPIRE DRG v2.1 (2012 Dec. 13).

given in the SPIRE DRG (Photometer Mode Cookbook, Table 5.11). The two sets of numbers are in good agreement.

For convenience in comparing modified black body models $I_\nu \propto \nu^\beta B_\nu(T)$ I have also calculated tables of color correction factors

$$C(\beta, T) \equiv \frac{I_\nu(\nu_0)}{I_\nu^{(S)}} \tag{19}$$

for extended emission with a variety of opacity indices β and temperatures T – see the Appendix.

Table 3: Correction Factors for KINGFISH SPIRE v3 Images

	$\lambda_0(\mu\mathrm{m})$		
quantity	250	350	500
v3JANSCALE ($10^6\,\mathrm{sr}^{-1}$/beam)	97.700	55.000	26.000
New X ($10^6\,\mathrm{sr}^{-1}$/beam)	90.687	51.430	23.907
Correction Factor X/(v3JANSCALE)	0.9282	0.9351	0.9195

6. Prescription for KINGFISH SPIRE Photometry

Here is how I think KINGFISH should proceed with SPIRE photometry:

1. Use the flux density per beam $F_\nu^{(S)}$ delivered by HIPE or Scanamorphos.

2. Calculate
$$I_\nu^{(S)} = \frac{F_\nu^{(S)}}{\Omega_E} \tag{20}$$
where (see the second line of Table 1) $\Omega_E = (469.1, 827.2, 1779.6)\mathrm{arcsec}^2 = (1.103, 1.944, 4.183)\times 10^{-8}\,\mathrm{sr}$ from Griffin et al. (2013).

3. KINGFISH SPIRE v3 images previously delivered to the team in May 2012[8] appear to have been converted from flux density to beam (Jy) to specific intensities (MJy/sr) using "JANSCALE" conversion factors of (97.700, 55.000, 26.000).[9]

   Therefore, KINGFISH SPIRE v3 images should be multiplied by correction factors of (0.9282, 0.9351, 0.9195) – see Table 3.

4. When comparing a model to the observed $I_\nu^{(S)}$ for a SPIRE band, integrate the model $I_\nu^{(\mathrm{model})}$ over the response function using eq. (6):
$$I_\nu^{(S,\mathrm{model})} = \frac{\int d\nu\, I_\nu^{(\mathrm{model})}\, R_E(\nu)}{\int d\nu\, (\nu/\nu_0)^{-1}\, R_E(\nu)} \quad , \tag{21}$$
and compare to the observed $I_\nu^{(S)}$.

5. If the user wants an estimate for the "observed" monochromatic specific intensity $I_\nu(\lambda_0)$ at the nominal wavelength λ_0:
   - Estimate the logarithmic slope $\alpha = d\ln I_\nu / d\ln\nu$ near the nominal band wavelength λ_0 (either from a model or by using the measured intensity at other bands).
   - Obtain $C(\alpha)$ from Table 2, and estimate $I_\nu \approx C(\alpha)\times I_\nu^{(S)}$.

6. If the user wants to compare modified blackbody models (single T and power-law opacities) to the observations, I will be happy to provide tables of correction factors $C(\beta, T)$ for SPIRE, MIPS, and PACS. Table 4 is an example of correction factors for extended sources observed with SPIRE 250μm band.

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[8] See the document *KINGFISH SPIRE Data, 2012 May 04* by Edward Montiel and Chad Engelbracht.

[9] The JANSCALE factor corresponds to X in the present usage.

I thank Matt Griffin and Chris North for kindly providing their draft paper (Griffin et al. 2013) in advance of submission for publication, and for helpful discussions. I am grateful to Simone Bianchi, Leslie Hunt, and Helene Roussel for helpful comments and corrections on earlier drafts of these notes. Further comments and corrections are welcomed.

REFERENCES

Chattopadhyay, G., Glenn, J., Bock, J. J., et al. 2003, IEEE Transactions on Microwave Theory Techniques, 51, 2139

Griffin, M. J., Abergel, A., Abreu, A., et al. 2010, A&A, 518, L3

Griffin, M. J., North, C. E., Schulz, B., et al. 2013, draft

A. Color Correction Factors for Modified Blackbody Spectra

It is sometimes useful to have "color correction factors"

$$C(\beta, T) \equiv \frac{I_\nu(\nu_0)}{I_\nu^{(S)}} \tag{A1}$$

for extended emission $\propto \nu^\beta B_\nu(T)$ for a variety of opacity indices β and temperatures T, allowing estimation of monochromatic intensities $I_\nu(\nu_0)$ at nominal band wavelengths ν_0 from the reported intensity $I_\nu^{(band)}$ in a photometric band.

Table 4 has $C(\beta, T)$ for the SPIRE 250μm band. Machine-readable tables of $C(\beta, T)$ for the different bands of various instruments (PACS, MIPS, SPIRE) are available from the author upon request.

Table 4: Color correction factor $C(\beta, T)$ for SPIRE 250

$C(\beta,T)$ for SPIRE 250					
	β				
T(K)	1.00	1.50	2.00	2.50	3.00
5.00	0.8472	0.8697	0.8915	0.9124	0.9323
6.00	0.9219	0.9408	0.9586	0.9749	0.9899
7.00	0.9654	0.9807	0.9944	1.0065	1.0168
8.00	0.9909	1.0029	1.0130	1.0213	1.0277
9.00	1.0058	1.0148	1.0220	1.0271	1.0303
10.00	1.0142	1.0208	1.0254	1.0279	1.0284
12.00	1.0206	1.0234	1.0241	1.0226	1.0191
14.00	1.0207	1.0206	1.0185	1.0142	1.0079
16.00	1.0182	1.0161	1.0119	1.0056	0.9972
18.00	1.0150	1.0113	1.0056	0.9977	0.9878
20.00	1.0116	1.0067	0.9998	0.9907	0.9796
22.00	1.0084	1.0026	0.9946	0.9846	0.9727
24.00	1.0055	0.9988	0.9901	0.9794	0.9667
26.00	1.0028	0.9955	0.9861	0.9748	0.9615
28.00	1.0004	0.9926	0.9827	0.9708	0.9570
30.00	0.9982	0.9899	0.9796	0.9673	0.9531
35.00	0.9938	0.9846	0.9734	0.9603	0.9454
40.00	0.9903	0.9805	0.9687	0.9551	0.9395
45.00	0.9876	0.9774	0.9651	0.9510	0.9351
50.00	0.9854	0.9748	0.9622	0.9478	0.9316