

HOMEWORK # 3, FRS 126, SPRING 2013

DUE FEBRUARY 27

1. The age of the Sun (40 points)

We saw in class that the Sun shines by burning hydrogen into helium by the process of thermonuclear fusion. We're going to explore this in some detail in this problem. First, some useful numbers and facts:

- When 4 protons fuse to make a helium nucleus, 0.7% of the original mass of the protons is converted to energy via Einstein's famous equation, $E = mc^2$ (c is of course the speed of light).
- The mass of the Sun is 2×10^{30} kg, and its luminosity is 4×10^{26} Watts. From the fact that the Earth's oceans have remained liquid for roughly 4 billion years, we are confident that the Sun's luminosity has remained close to constant that entire time. The radius of the Sun is 7×10^8 m. The Sun is 75% hydrogen by mass (the rest is mostly helium).
- When a gravitating object shrinks, it releases gravitational potential energy, just in the same way that when I drop a ball in the Earth's gravitational field, its gravitational potential energy is converted to kinetic energy (and then to heat when it hits the ground and comes to rest). The gravitational potential energy released in forming a uniform spherical body of mass M and radius R is:

$$E \simeq \frac{3}{5} \frac{GM^2}{R},$$

where G is Newton's Gravitational Constant, $G = 2/3 \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$.

a. (15 points) First, suppose you didn't know about thermonuclear fusion, and hypothesized that the Sun shone from the gravitational potential energy of its collapse. Approximating the Sun as a uniform sphere, calculate how long it could shine at its present rate from this source of energy; express your answer in years. Compare with the current lifetime of the Sun, and comment.

b. (10 points) Calculate the amount of energy that one gram of hydrogen would release if it all fused to Helium via thermonuclear fusion. Express your answer in Joules). Then calculate how much mass is converted into energy every second in the Sun. Express your answer in tons (one metric ton is 10^3 kilograms) and blue whales (a big blue whale weighs about 100 tons).

c. (10 points) Only the central 10% of the mass of the Sun is close enough to the core, i.e., hot and dense enough, for thermonuclear fusion to take place. With this in mind (and remembering that only 75% of the mass of the Sun is hydrogen) calculate how long the Sun can shine with its current luminosity. Express your answer in years. Compare with the known age of the Sun, and comment.

d. (5 points) Finally, something to ponder: it turns out in fact that the Sun has not stayed at exactly constant luminosity, but has rather increased in luminosity by something like 30% over the past four billion years, as the amount of helium in its core has gradually increased. Speculate on what effect, if any, the smaller luminosity in the past might have had on the early Earth, especially on the presence of liquid water.

2. Lifetimes of stars (25 points)

In this problem, you will calculate how the lifetimes of stars depend on their masses.

a. (10 points) The radius of a star on the main sequence is approximately proportional to its mass, while its surface temperature is approximately proportional to the square root of its mass. Assuming that the stars radiate as black bodies, calculate how the luminosities of stars depend on their mass. Your answer will have the form:

$$\text{Luminosity} \propto \text{Mass}^\alpha;$$

you have to determine α .

b. (5 points) In Problem 1, the lifetime of the Sun was determined by its mass and luminosity. Calculate how the lifetime of a star of mass M depends on its mass.

c. (10 points) Now let's plug in numbers. You know the lifetime of the Sun (you calculated it in Problem 1!). Determine the luminosity (relative to that of the Sun) and lifetime (expressed in years) for:

- an O star, $60 M_\odot$
- a B star, $8 M_\odot$
- an A star, $2 M_\odot$
- a G star, $1 M_\odot$
- an early M star, $0.5 M_\odot$
- a late M star, $0.1 M_\odot$

Given that it took complex, multi-cellular life close to 4 billion years to evolve here on Earth, comment on the possibility for complex life to have evolved on planets orbiting these different stars.