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Dynamics of viscous slugs fall in dry capillaries

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The dynamics of viscous slug fall in vertical dry capillaries is investigated by extending a published model (perfect wetting case) accounting for the film left behind the slug as it falls, and Laplace pressures at both ends of the slug in the momentum balance. The present investigation provides and uses an advancing contact-angle correlation determined based on a published theoretical work. The results are found in excellent agreement with published experimental values for falling slugs. The present model does not require any fitting parameter in the perfect wetting case, and is extended to include the non-perfect wetting case along with the unsteady-state dynamics using the quasi-steady-state approximation.

Keywords: slug; fall; Poiseuille law; dynamic contact angle; correlation; dry tube; dynamics; capillary

1. Introduction

The investigation of Bico and Quéré [1] provides a model and experimental data for the dynamics of viscous slugs fall in capillary tubes in the case of prewetted and dry (perfect wetting case) tubes. The model in [1] was found in excellent agreement with the experimental data using a value of Γ equal to 15.7 for $\ln(R/a)$, where a is a cut-off length of molecular size in the case of dry capillary tube [1] and Γ is estimated of the order 13 for a tube of millimetric size radius.[1] Jensen's theoretical model for the prewetted capillary tube case [2] agrees favorably with the prewetted-tube experimental data [1] and the agreement is excellent for sufficiently long slugs.

In the present investigation, an advancing contact-angle relation is determined based on the theoretical results of Chebbi (perfect wetting case).[3] Following Bico and Quéré [1], the results of Bretherton [4] are used in order to characterize the film left behind the slug and Laplace pressures are included in the momentum balance. The dynamics of slugs fall in dry capillary tubes is obtained using the determined advancing contact-angle relation. The results of the present model are compared with the published experimental data in Bico and Quéré.[1] The model is extended to include the non-perfect wetting case using Jiang et al.'s correlation [5] based on Hoffman's data [6] for advancing contact angle in capillary tubes as a function of capillary number and static contact angle. A model for unsteady-state dynamics, using the quasi steady-state approximation, is proposed in the last part.

While the model of Bico and Quéré [1] addresses the perfect wetting case only using an adjustable value of Γ obtained from the experimental data for the dynamics of

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slugs fall,[1] the present investigation uses a dynamic contact angle correlation obtained in this work from the theoretical work (perfect wetting case) [3] using the numerical results in Figure 9.[3] Furthermore, this work considers slug dynamics in the non-wetting case using Jiang et al.'s correlation,[5] which is based on Hoffman's experimental data.[6] The falling slug dynamics results obtained in this work (as slug length as a function of slug velocity for both wetting and non-wetting cases) provide quasi-steady results, as the slug length decreases (strictly speaking) due to the liquid film left behind as slug fall progresses. This quasi-steady-state approach is valid if the change in the slug length is slow. The requirement for the accuracy of the quasi-steady-state approximation is analyzed using a mass balance that provides the rate of change of the slug length. Integrating the mass balance yields an integral expression providing time as a function of slug velocity.

2. Quasi-steady-state model and results

2.1. Momentum balance

Figure 1 is a schematic of a falling slug of length L. The slug is subject to gravity promoting the fall of the slug, resisted by viscous forces. The Laplace pressure force at the bottom resists the slug motion whereas the top one promotes it. The slug length L

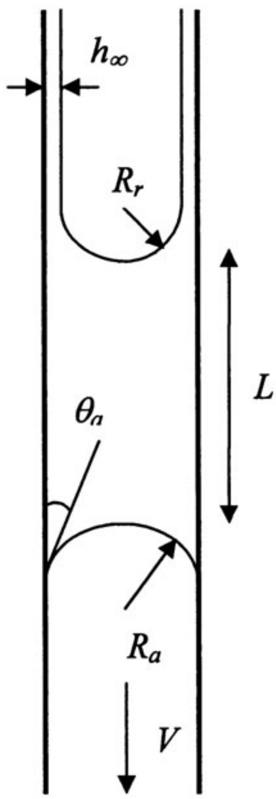

Figure 1. A schematic diagram of a falling slug.

is assumed large compared to the inner capillary radius R. Neglecting inertia terms, the momentum balance yields

$$-2\,\pi\,R\,L\,\tau + \rho\,\pi\,R^2 L\,g + \pi\,R^2(p_t - p_b) = 0 \tag{1}$$

where ρ is the liquid density, g is the acceleration of gravity, p_t and p_b are the top and bottom Laplace pressures, respectively, and τ is the shear stress given by Poiseuille equation

$$\tau = 4\,\mu\frac{V}{R} \tag{2}$$

in which μ represents the liquid viscosity.

The Laplace pressures are functions of the radii of curvature R_a and R_r

$$p_t = p_o - \frac{2\,\sigma}{R_r};\quad p_b = p_o - \frac{2\,\sigma}{R_a} \tag{3}$$

where p_o is the gas (typically air) pressure.

Substituting for Equations (2) and (3) into Equation (1) yields

$$\frac{8\,\mu\,V\,L}{R} = \rho\,R\,g\,L - 2\,\sigma\,R\left(\frac{1}{R_r} - \frac{1}{R_a}\right) \tag{4}$$

The radius of curvature R_a is a function of the advancing contact angle, θ_a

$$\frac{1}{R_a} = \frac{\cos\,\theta_a}{R} \tag{5}$$

For the receding side, the true average curvature is taken from Bretherton's work [4]

$$\kappa_r = \frac{2}{R_r} = \frac{2}{R}[1 + 1.79 \times (3 \times\ Ca)^{2/3}] \tag{6}$$

in which $Ca = \mu\ V/\sigma$ is the capillary number.

For small contact angles, Equation (5) can be written as

$$\frac{1}{R_a} = \frac{1}{R}\left(1 - \frac{\theta_a^2}{2}\right) \tag{7}$$

In the case of sufficiently long slugs, the two terms proportional to L in Equation (4), representing viscous and gravity forces, become predominant and velocity reaches asymptotically a limiting maximum value given by the following expression as in [1]

$$V^* = \frac{\rho\,g\,R^2}{8\,\mu} \tag{8}$$

2.2. Dynamic contact-angle correlation (perfect wetting case)

The numerical results in Figure 9 [3] for liquid penetration into dry tubes (perfect wetting case) are used to find the following expression for the advancing contact angle θ_a as a function of Ca

$$\theta_a = \Omega\,Ca^{\alpha} \tag{9}$$

where

$$\Omega = 4.84 \text{ (radians)}; \quad \alpha = 0.353 \tag{10}$$

The correlation is valid for Ca in the range of about 10^{-5} to 3×10^{-4}, but extrapolation of the results in [3] is seen in Figure 9 [3] to provide an excellent match with experimental data up to a capillary number of about 10^{-2}.

2.3. *Quasi-steady-state model (perfect wetting case)*

Substituting Equations (6) and (7) into Equation (4) gives

$$\frac{8\,\mu\,V\,L}{R} = \rho\,R\,g\,L - 2\,\sigma\left[1 + 1.79 \times (3 \times\ Ca)^{2/3} - \left(1 - \frac{\theta_a^2}{2}\right)\right] \tag{11}$$

Substituting for θ_a using Equation (9), and dividing by R, leads to

$$\frac{8\,\mu\,V\,L}{R^2} = \rho\,g\,L - \frac{2\,\sigma}{R}\,\omega\,Ca^{2/3} \tag{12}$$

where ω is a function of Ca, given by

$$\omega = \frac{\Omega^2}{2}\,Ca^{2\left(\alpha - \frac{1}{3}\right)} + 3.72 \tag{13}$$

The work of Bico and Quéré [1] yields a similar Equation to (12) where β appears instead of ω; β is given by [1]

$$\beta = \frac{(6\,\Gamma)^{2/3}}{2} + 3.88 \tag{14}$$

whereas β is a constant,[1] ω depends on Ca as seen from Equation (13).

2.4. *Comparison with experimental data (perfect wetting case)*

The present model is compared with the experimental data in [1] for silicone oil slugs of density equal to 950 kg/m^3, surface tension of 20.6 mN/m, and viscosity equal to 16.7 mPa s; the dry vertical capillary tube inner radius is 127 µm. The agreement between the present model and the experimental data is excellent as seen from Figure 2.

2.5. *Quasi-steady-state model (non-perfect wetting case)*

The relation by Jiang et al. [5], obtained by correlating Hoffman's data,[6] for advancing contact-angle vs. Ca is given by

$$\frac{\cos\theta_s - \cos\theta_a}{\cos\theta_s + 1} = \tanh\left(4.96\ Ca^{0.702}\right) \quad (We < 0.001,\ Bo < 0.1) \tag{15}$$

where θ_s is the static contact angle, and We and Bo are Weber and Bond numbers, respectively. Using Equation (15) leads to

$$\theta_a = \varphi(Ca) \tag{16}$$

where function φ is defined as

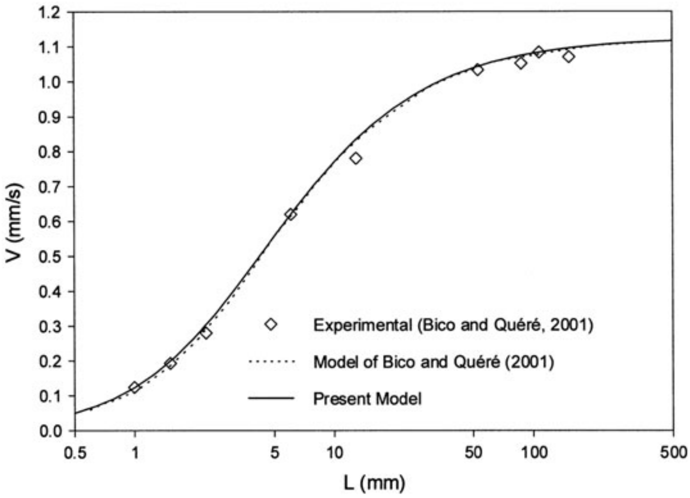

Figure 2. Comparison with the experimental data of Bico and Quéré ($R = 127$ μm, fluid: silicone oil, $\rho = 950$ kg/m^3, $\mu = 16.7$ mPa s, $\sigma = 20.6$ mN/m).[1]

$$\varphi = \arccos\left[\cos\theta_s - (1 + \cos\theta_s)\ \tanh\left(4.96\ Ca^{0.702}\right)\right] \tag{17}$$

Substituting Equations (6) and (7) into Equation (4), while using Equation (16), leads after simplification and rearrangement to a relationship between L and V given by

$$L = \frac{2\,\sigma\left\{\frac{[\varphi(\mu V/\sigma)]^2}{2} + 3.72\,(\mu V/\sigma)^{2/3}\right\}}{\rho\, g\, R - \frac{8\,\mu\, V}{R}} = \psi(V) \tag{18}$$

where function ψ is defined above to simplify equations later.

3. Unsteady-state model

As the slug moves downward, it leaves a deposited film behind it, leading to a reduction in the mass of the slug. For the case $h_\infty/R \ll 1$, a mass balance leads to

$$R\,\frac{dL}{dt} = -2\,h_\infty\,V \tag{19}$$

From reference,[4] we have

$$\frac{h_\infty}{R} = 0.643\,(3\,Ca)^{2/3} \tag{20}$$

which leads, after substitution for h_∞ into Equation (19), to

$$\frac{1}{V}\frac{dL}{dt} = -2.67\,Ca^{2/3} \tag{21}$$

For $Ca^{2/3} \ll 1$, the change in slug length is slow and the quasi-steady-state approximation can be used; Equation (18), along with the initial condition

$$V = V_0 \quad \text{at} \quad t = 0 \tag{22}$$

leads to

$$t = \frac{1}{2.67}\left(\frac{\sigma}{\mu}\right)^{2/3}\int_{V}^{V_0}\frac{\psi'}{V^{5/3}}dV \tag{23}$$

where ψ' is the derivative of ψ with respect to V.

The mass of the slug can be considered as nearly constant if

$$\frac{R\,L_0 - 2\,h_{\infty,0}\;V_0\,t}{R\,L_0} \ll 1 \tag{24}$$

where L_o is the initial slug length. If Equation (24) is satisfied, unsteady-state calculations are not required.

4. Conclusion

The present model is in excellent agreement with published data, and no fitting parameter is needed. A dynamic contact-angle correlation, used in the model, is provided based on the theoretical results for advancing gas–liquid meniscus in capillaries.[3] The exponent of the capillary number in the determined advancing contact-angle correlation, found as 0.353, is consistent with the exponent 0.351 ($\theta_s = 0$, small Ca) in Jiang et al.'s correlation [5] based on Hoffman's data;[6] both correlations deviate from the exponent value 1/3 in Tanner's equation.[7] The model is extended to include the non-perfect wetting case, and an unsteady-state model is proposed based on the quasi-steady-state approximation.

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