Atoms and Starlight

- Electromagnetic radiation
- Blackbody spectra
- Spectral lines & atomic structure
- Doppler effect
- Classification of Stellar Spectra

Electromagnetic Radiation

- In 1870s, Maxwell's unified electricity and magnetism: predicted electromagnetic waves
- Oscillating electric & magnetic fields that travel through space at speed of light
  - c = 300,000 km/sec
- Most knowledge of the universe is conveyed to us by electromagnetic radiation

Electromagnetic Radiation

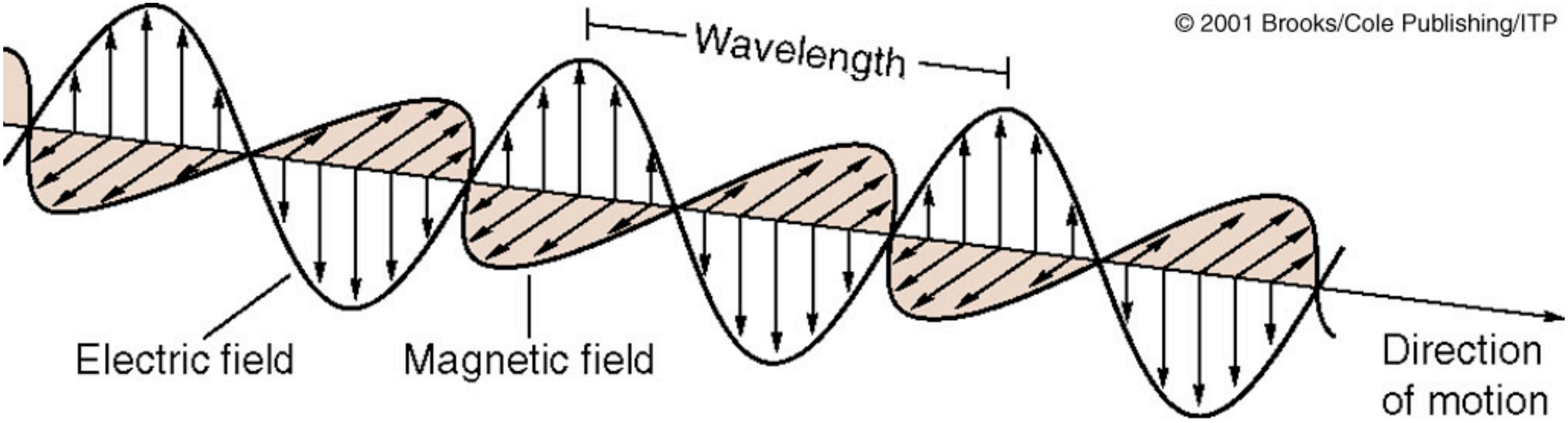
Characterized by:

wavelength λ – distance between crests

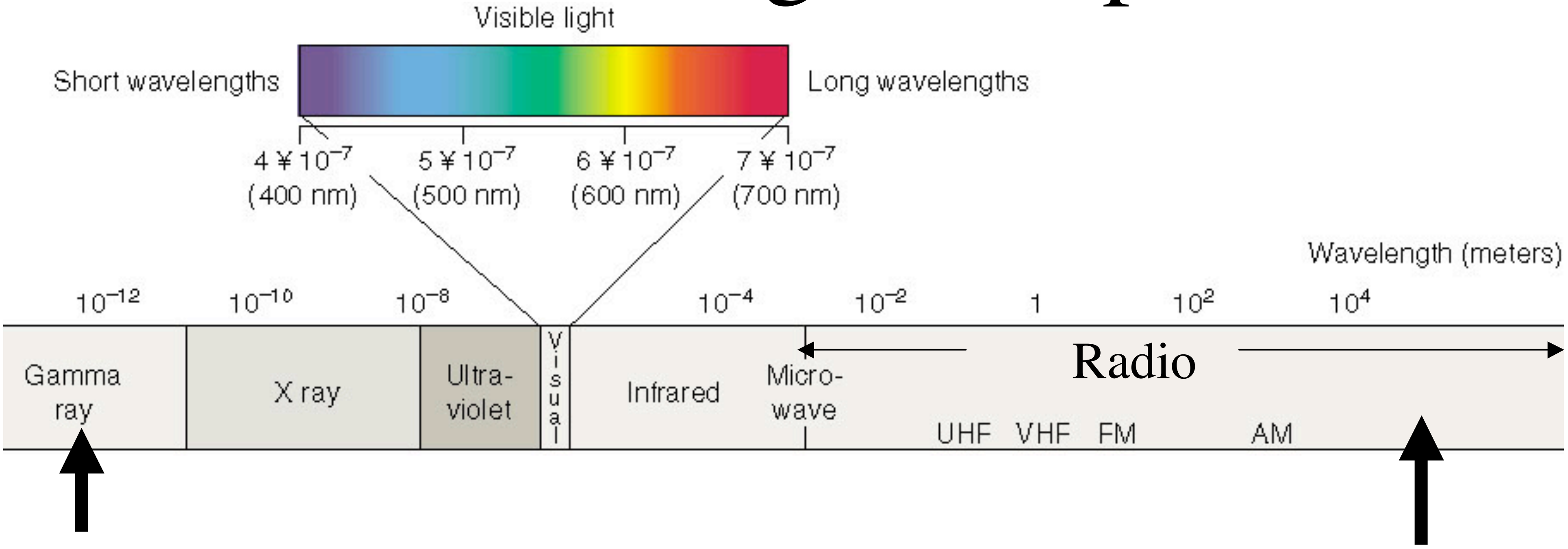
frequency ν – # of crests passing a point per unit time

speed of propagation $c = \lambda\nu$

polarization - direction of electric field vector



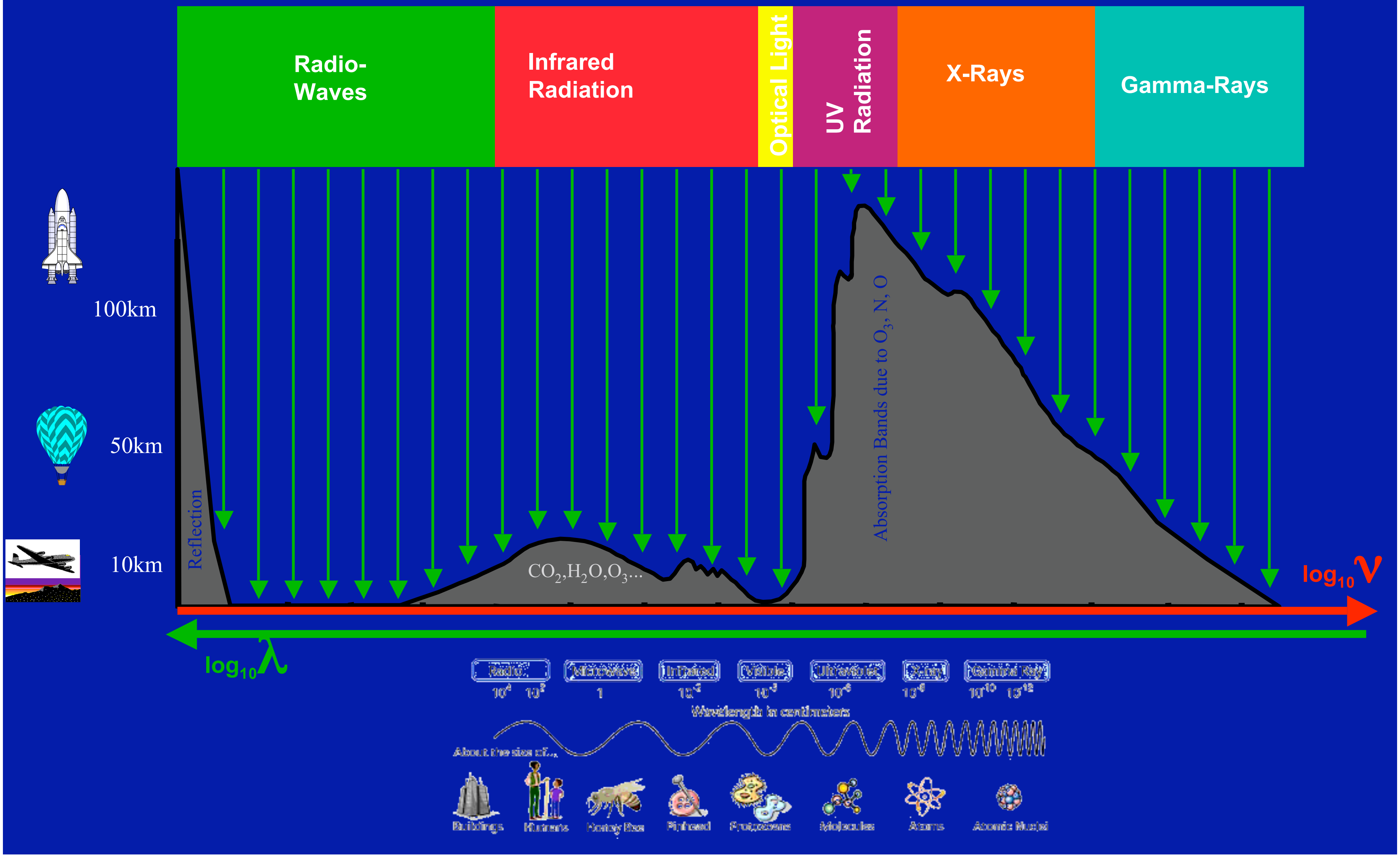
The Electromagnetic Spectrum



Shortest wavelength

Longest wavelength

Astronomical Observations throughout the e.m. Spectrum



Apparent brightness depends on distance to object

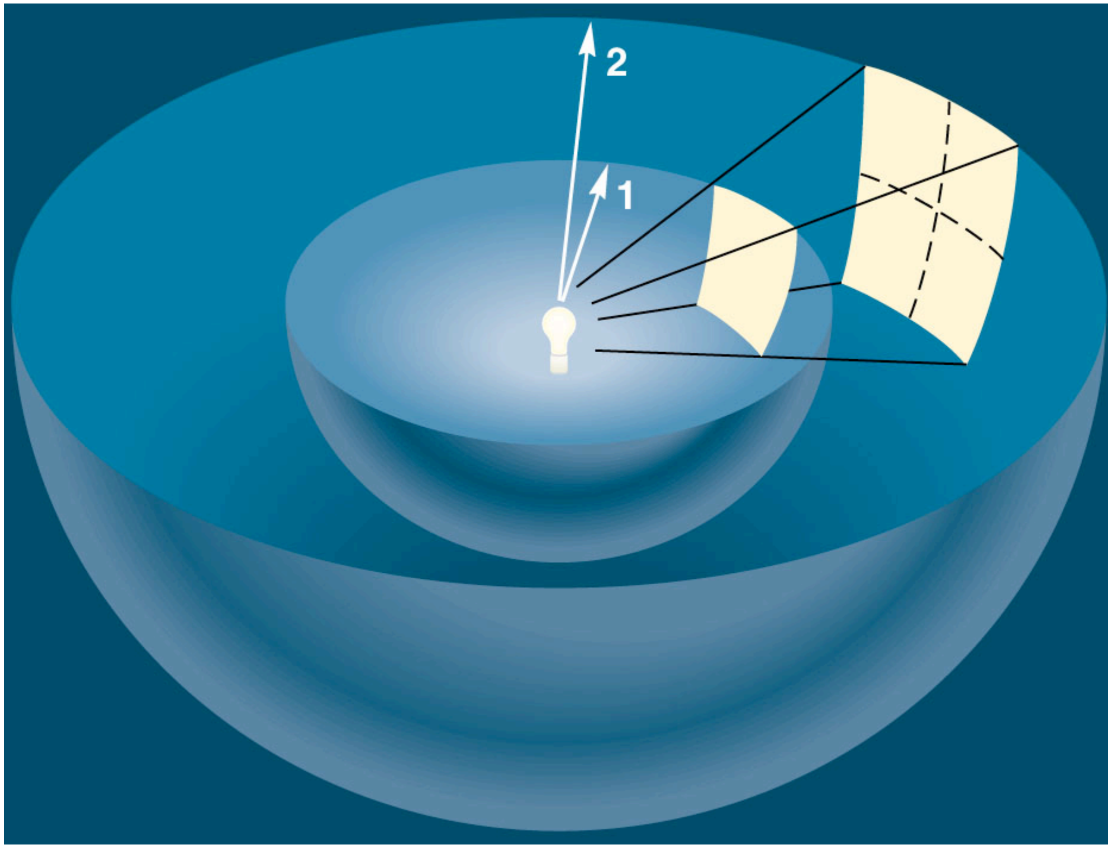
Energy is spread out over sphere of increasing area with distance from source

$A = 4\pi r^2$

So intensity $I = L/A$

$I \propto 1/r^2$

Inverse-square law



Energy Units

- Luminosity: L (J/s)
- Flux: $F=L/4\pi r^2$ (J/s/m$^{2)}$
- Intensity: I (J/s/m^2/solid angle)
- Energy density: $u = 4\pi\langle I\rangle/c$ (J/m^3)

($\langle I\rangle$ is angle-averaged I)

- Surface-brightness, of object with radius, R: $\Sigma=L/R^{2,}$ or angular size, θ: $\mu=F/\theta^2$
- Sometimes these quantities are evaluated at specific frequencies, or, equivalently, wavelengths, e.g., F_ν, I_ν, L_λ

Continuous Spectra

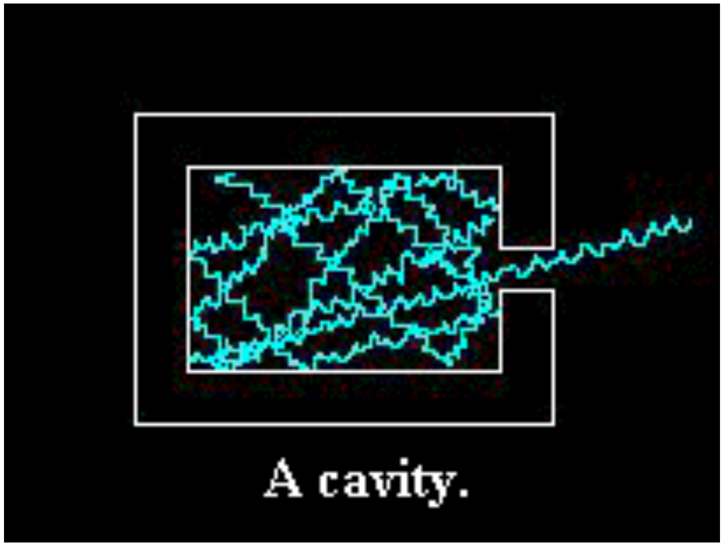
- *Continuous* spectrum has energy at all wavelengths
- Most important example: *blackbody* (thermal) spectrum
- Most astronomical objects emit spectrum that is nearly blackbody.

Blackbody Radiation

- ***Blackbody***: ideal emitter, which absorbs *all* incident radiation and reradiates this energy with a characteristic spectrum, dependent on *T*, the temperature to which object is heated
- ***Blackbody radiation***: also called "thermal" because it is emitted by matter in thermal equilibrium (characterized by temperature, *T*); distribution of blackbody photons also in thermal equilibrium

Blackbody Radiation

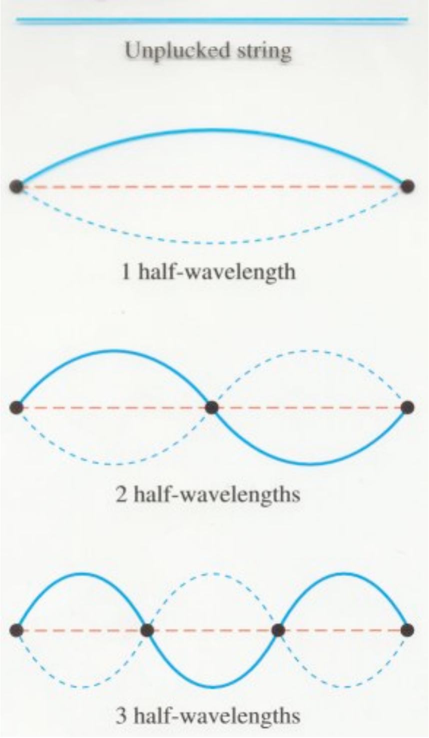
- Idealized situation: cavity maintained at temperature, *T*, in thermal equilibrium
- Cavity filled with standing waves of EM radiation. How do you calculate the spectrum?



- Measure emergent spectrum from small hole, don’t disturb equilibrium

Planck spectrum

- Hot cavity filled with EM waves
- Only standing waves with $\lambda = 2L/n$ (n=1,2,3,…) can fit in cavity, with electric field=0 on cavity walls



- Analogy with 1D plucked string
- In 3D, there are more ways you can fit a standing EM wave into the cavity, as ν increases (λ decreases)

Planck spectrum

- Planck showed that if energy in waves is quantized, E=hν = hc/λ (photons), then spectrum of waves in cavity follows

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

h=Planck’s constant

k=Boltzmann const.

c=speed of light

T=temperature in K

- One of the first successes of quantum theory, matched observed BB spectra in laboratory

Planck spectrum

- How exactly is this spectrum derived?
- Blackbody spectrum is product of ***density of states*** and ***average energy per state***, where "state" is standing wave of frequency ν
- It turns out that the ***density of states*** of frequency ν is $2\nu^2/c^3$ (# states/solid angle/volume/frequency), which includes two polarizations for each standing wave, and the fact that there are more 3D degrees of freedom for how waves fit in cavity at shorter wavelength

Planck spectrum

- How exactly is this spectrum derived?
- Blackbody spectrum is product of ***density of states*** and ***average energy per state***, where "state" is standing wave of frequency ν
- The ***average energy per state*** arises from understanding radiation field in terms of photons and the probability distribution that, at temperature, *T*, there are <n> photons of energy $h\nu$

$$\langle E \rangle = \langle n \rangle \times h\nu = \frac{h\nu}{\exp(h\nu/kT)-1}$$

You will show this for real in Stat mech.

Planck spectrum

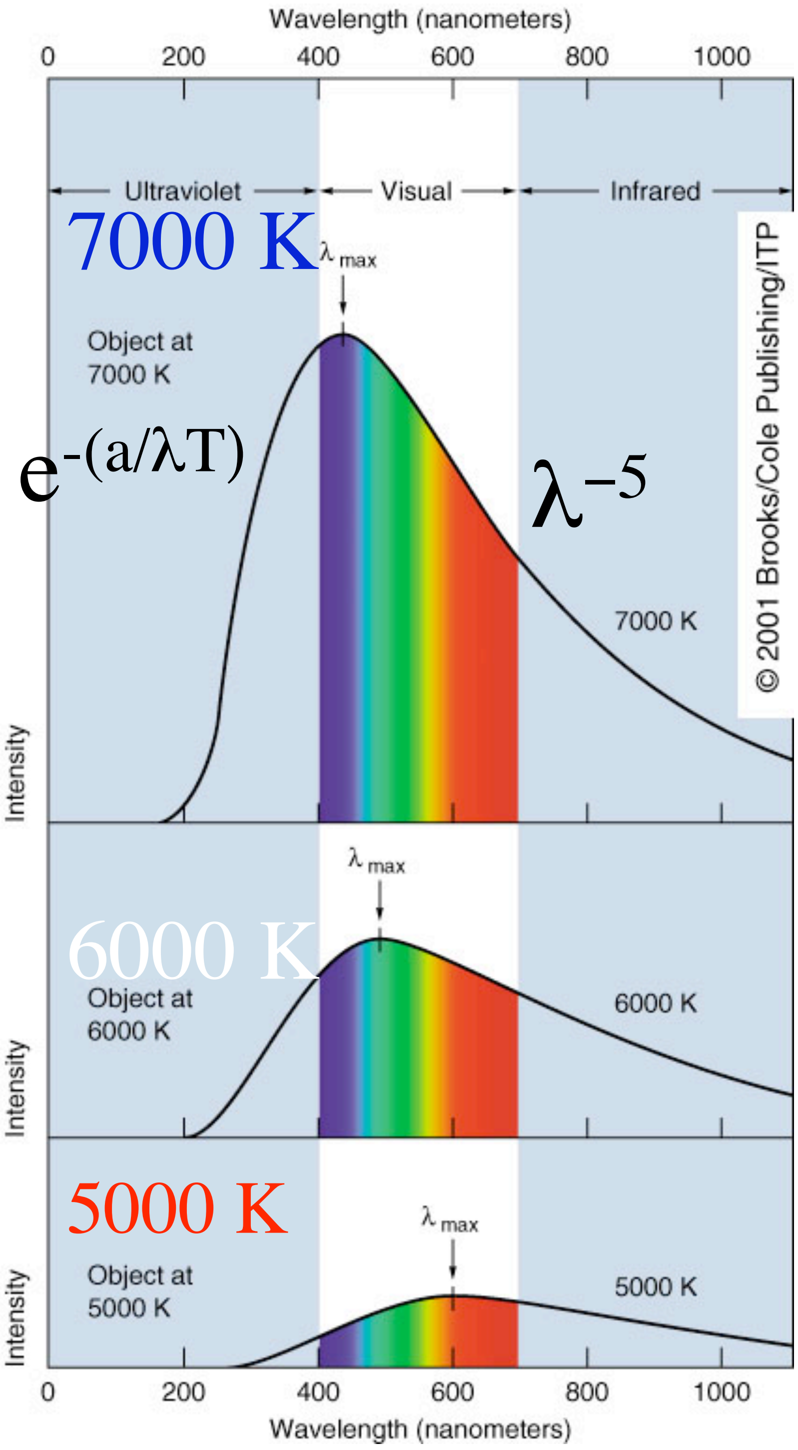
$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

- Monochromatic specific intensity $I(\nu,T)$ (energy/time/area/solid angle/frequency), can also be written in terms of wavelength, using $I(\nu,T)d\nu=I(\lambda,T)d\lambda$
- Related to energy density, u, by $u=4\pi I/c$

Blackbody Radiation

- Has specific spectral form that only depends on the temperature, T
- Has a certain normalization
- Isotropic (the same emission in all directions)
- Completely unpolarized



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Planck spectra

- 7000 K
  - All colors brighter, but blue is brightest → object looks **blue**
- 6000 K
  - All colors roughly similar in brightness → object **white**
- 5000 K
  - All colors fainter, but red is brightest → object looks **red**

Thermal Radiation: The Rules

Rayleigh-Jeans Law: $h\nu << kT$, reduces to

$$I(\nu, T) = \frac{2kT\nu^2}{c^2}$$

Original statement of radiation law for Blackbody radiation at all frequencies, which led to "UV catastrophe," energy diverges at higher frequencies, different from what was observed

UV catastrophe came from assuming that the average energy per mode was kT, rather than $h\nu/(\exp(h\nu/kT)-1)$

Thermal Radiation: The Rules

Wien's Law: *hv>>kT*, reduces to

$$I(\nu, T) = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}}$$

Represents high-frequency tail of distribution

Thermal Radiation: The Rules

Wien Displacement Law: the wavelength of maximum intensity is *inversely* proportional to temperature

$$\lambda_{\max} = \frac{3 \times 10^6}{T} \text{ nm}$$

Derived by finding extrema: $dI(\lambda, T)/d\lambda = 0$

Stefan-Boltzman Law: flux emitted increases with fourth power of temperature

$$F = \pi \int_0^\infty I_\nu(T) d\nu = \sigma T^4$$

P = energy/(unit area)/time

Luminosity $L = \int F dA = 4\pi R^2 \sigma T^4$ (for spherical body)

Thermal Radiation: Constants

h=6.626x10^{-34} J s (Planck’s constant)

=6.626x10^{-27} erg s (cgs)

k=1.381x10^{-23} J/K (Boltzmann’s constant)

=1.381x10^{-16} erg/K (cgs)

σ=5.67x10^{-8} W/m^2/K^4 (Stefan-Boltzmann constant)

=5.67x10^{-5} erg/s/cm^2/K^4 (cgs)

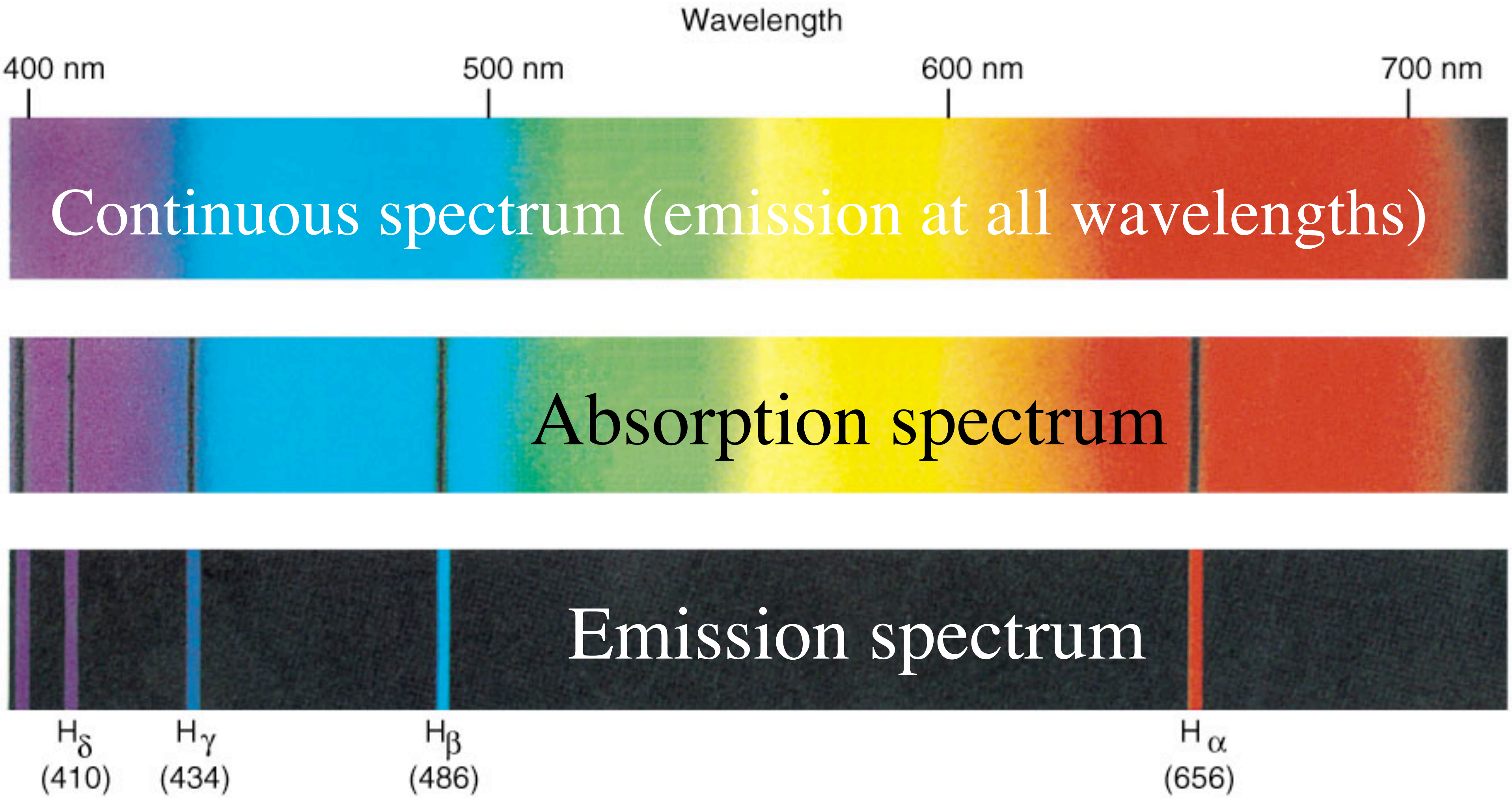
Thermal Radiation: Other References

Shu, Ch. 4, pp. 75-80

Rybicki & Lightman, “Radiative Processes in Astrophysics”, Ch. 1

Kittel & Kroemer, “Thermal Physics”, Ch. 4

Continuous, Absorption, Emission Spectra



Continuous spectrum (emission at all wavelengths)

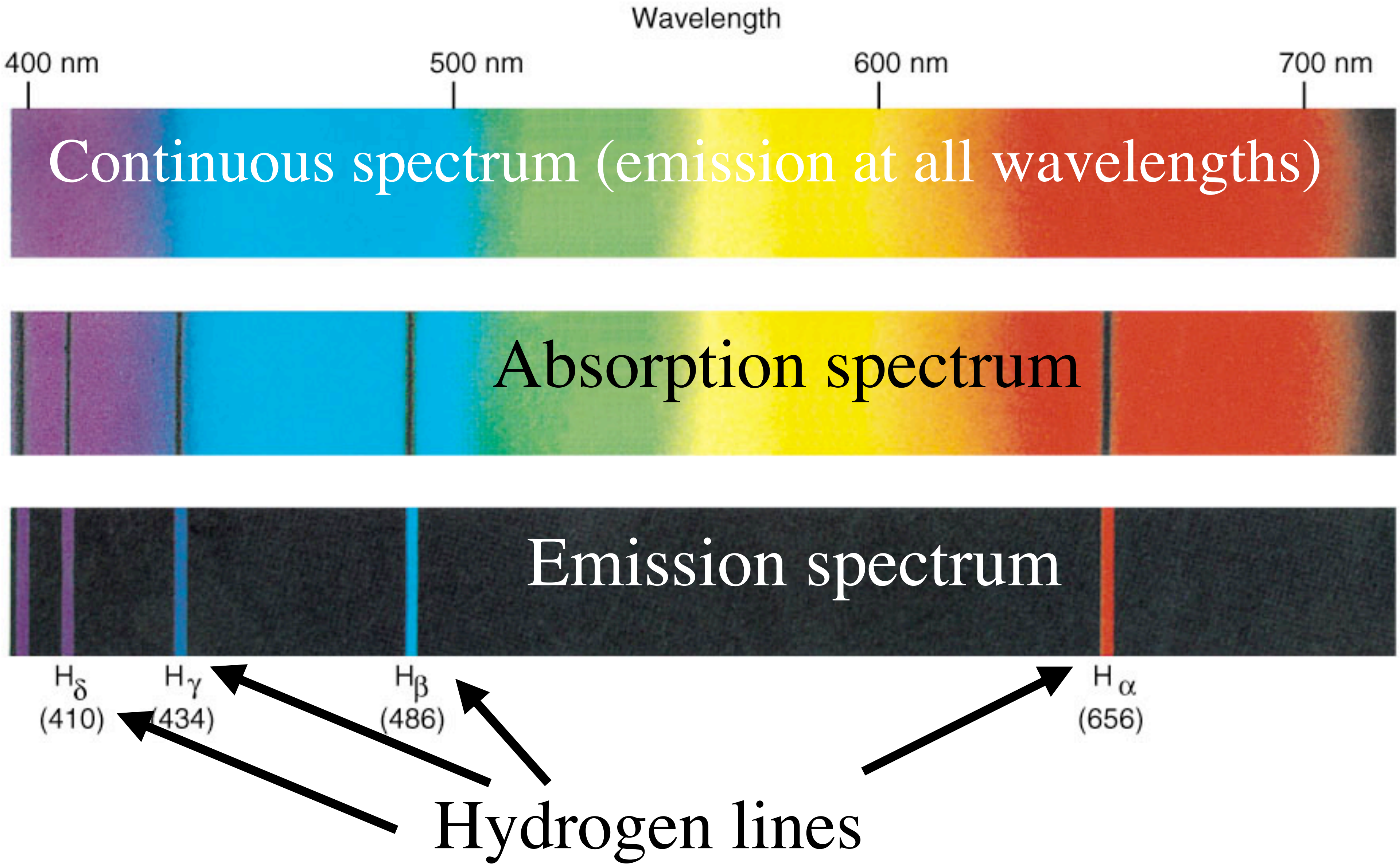
Absorption spectrum

Emission spectrum

Absorption spectrum = radiation missing at some wavelengths

Emission spectrum = radiation occurs only at some wavelengths

Absorption and emission of photons of specific energy by atoms produces spectral lines



Continuous spectrum (emission at all wavelengths)

Absorption spectrum

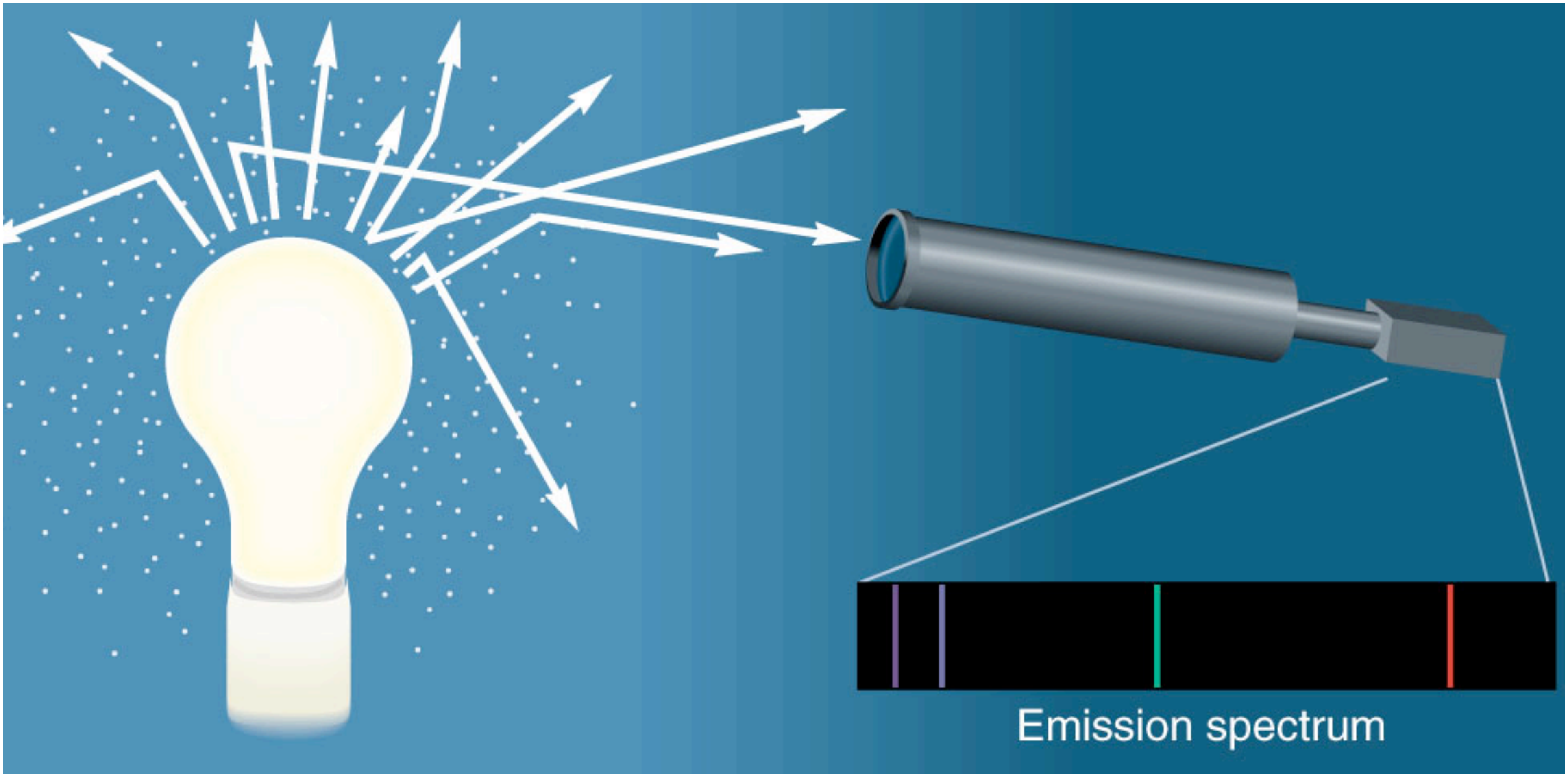
Emission spectrum

Hydrogen lines

Kirchhoff's Laws

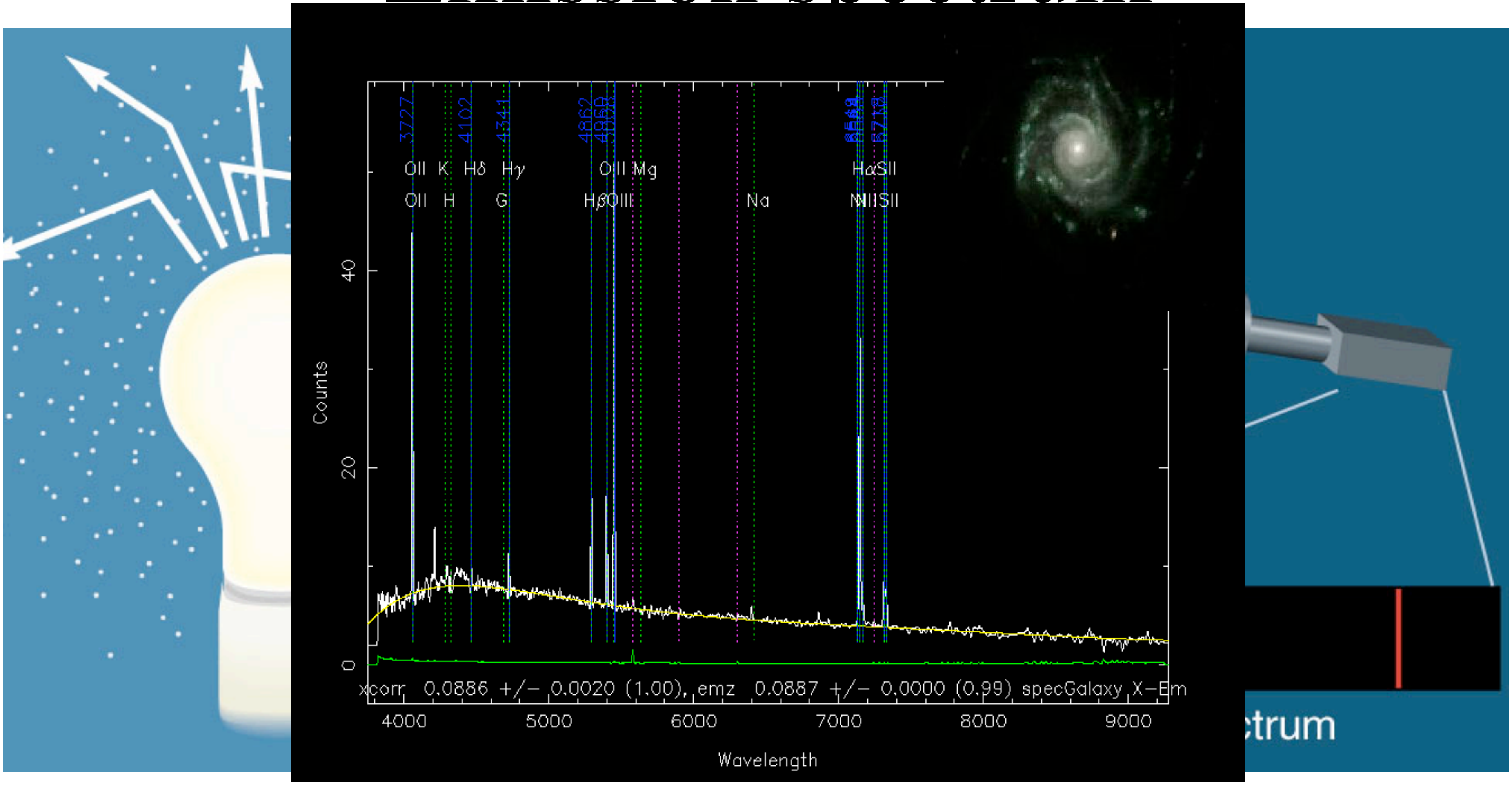
1. A solid, liquid, or dense gas will produce a *continuous* (blackbody) spectrum
2. A low density gas which is excited will produce an *emission* spectrum
3. A continuous spectrum seen through a low-density, cool gas will produce an *absorption* spectrum

Emission spectrum



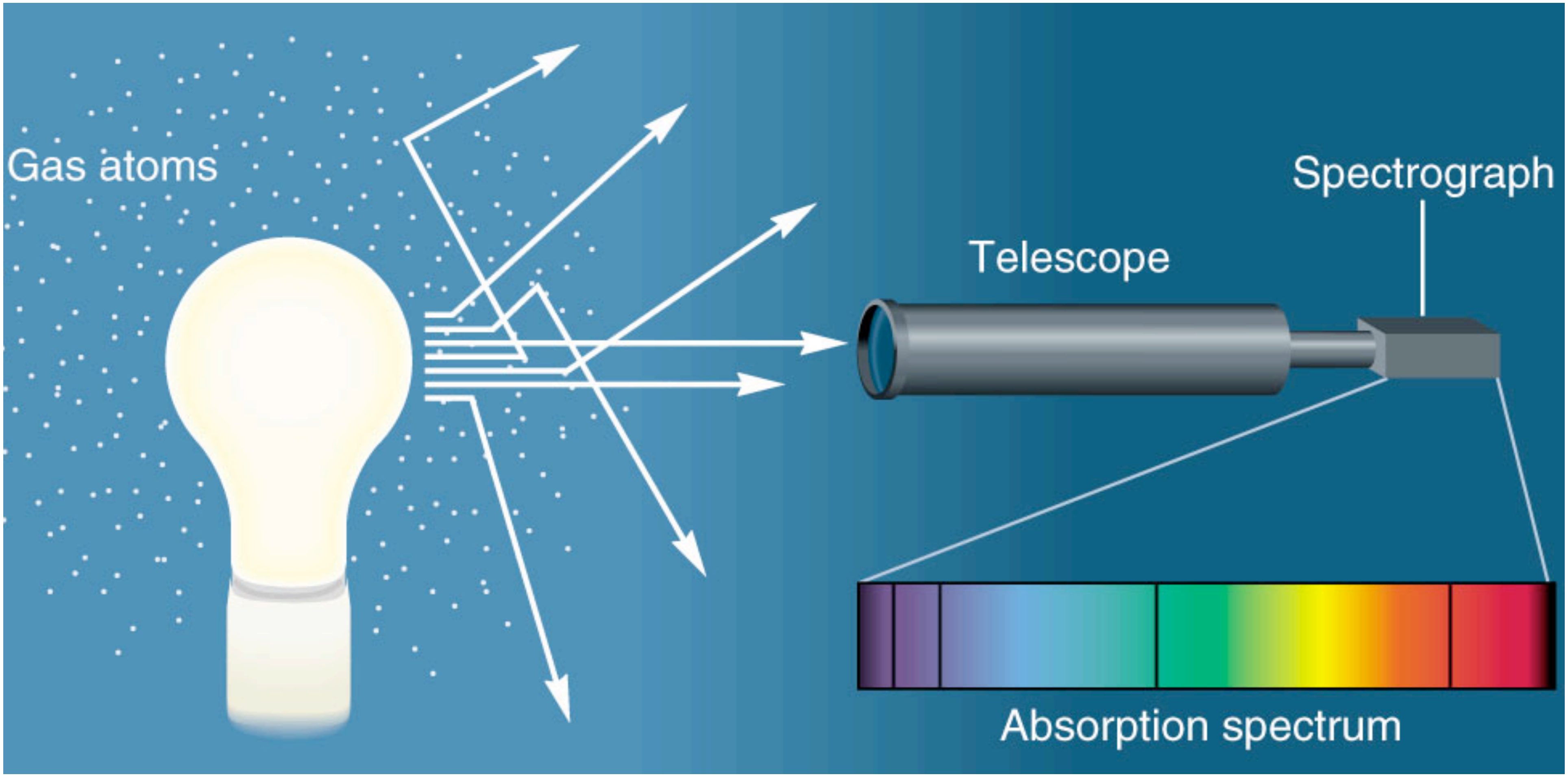
- Telescope with spectrograph, looking at hydrogen gas

Emission spectrum



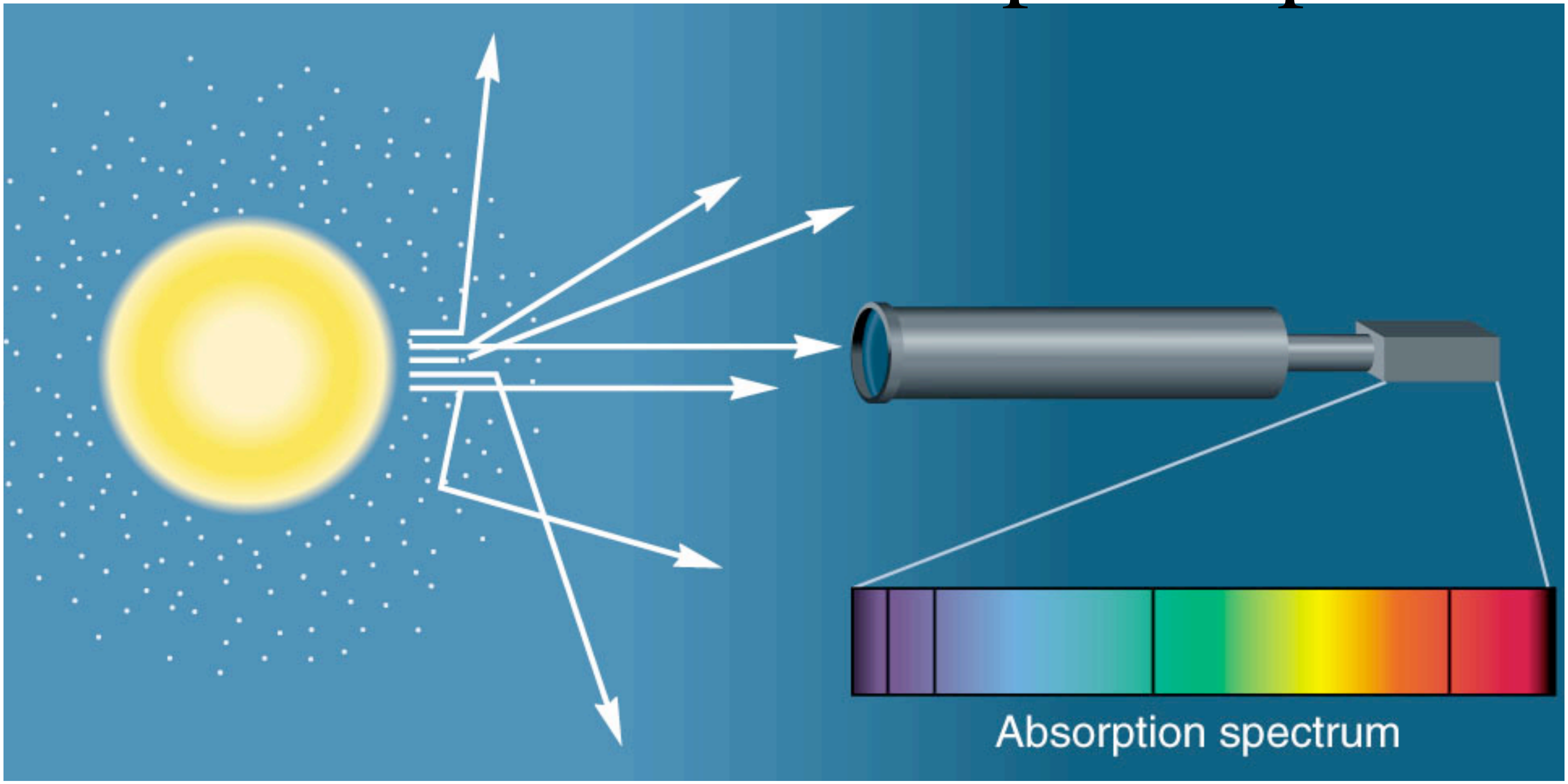
- Telescope with spectrograph, looking at gas in spiral galaxy

Absorption spectrum



Looking toward black body (light bulb), which makes continuous spectrum, through hydrogen gas

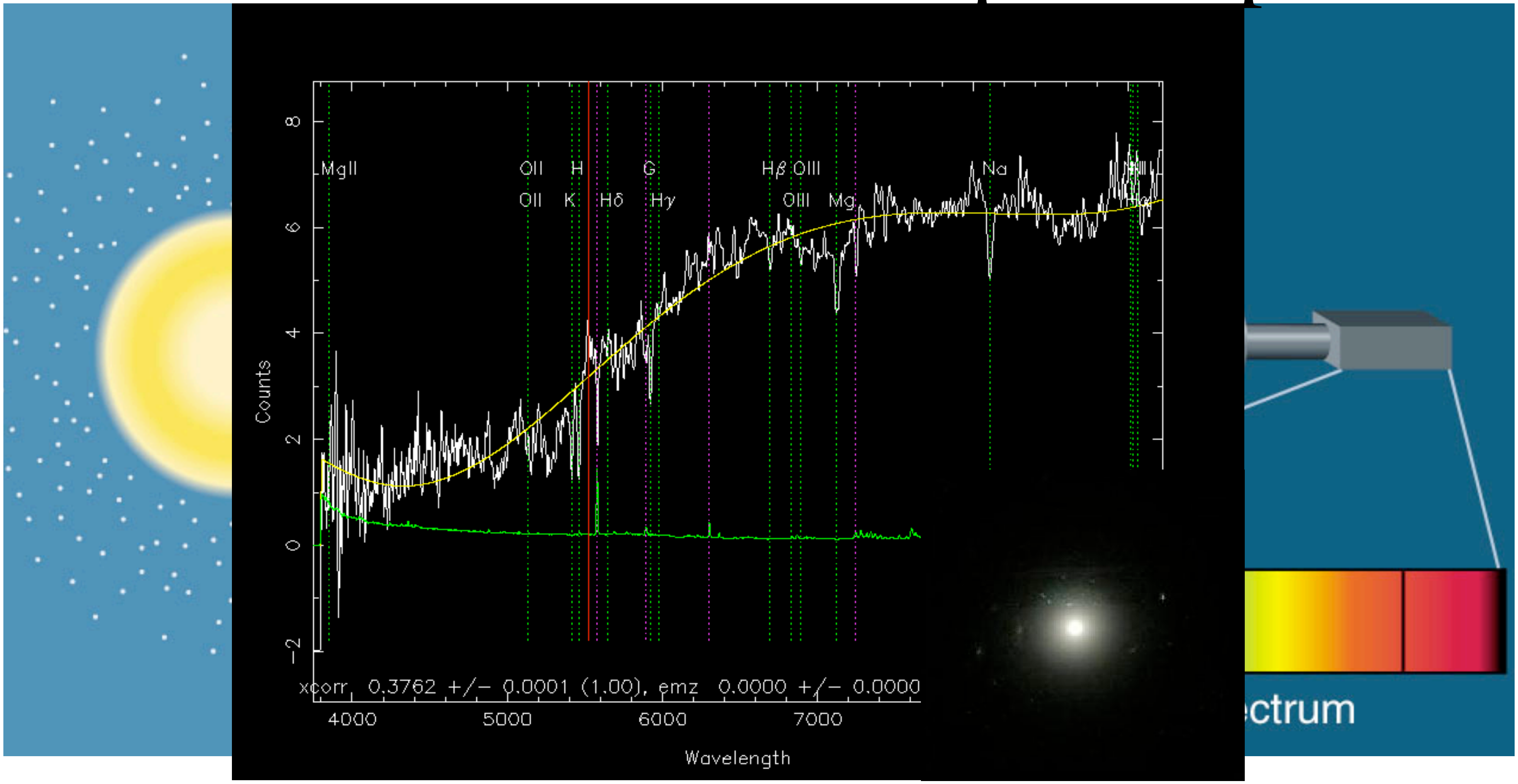
Normal stars have absorption spectra



We see the hot "surface" through the cooler "atmosphere"

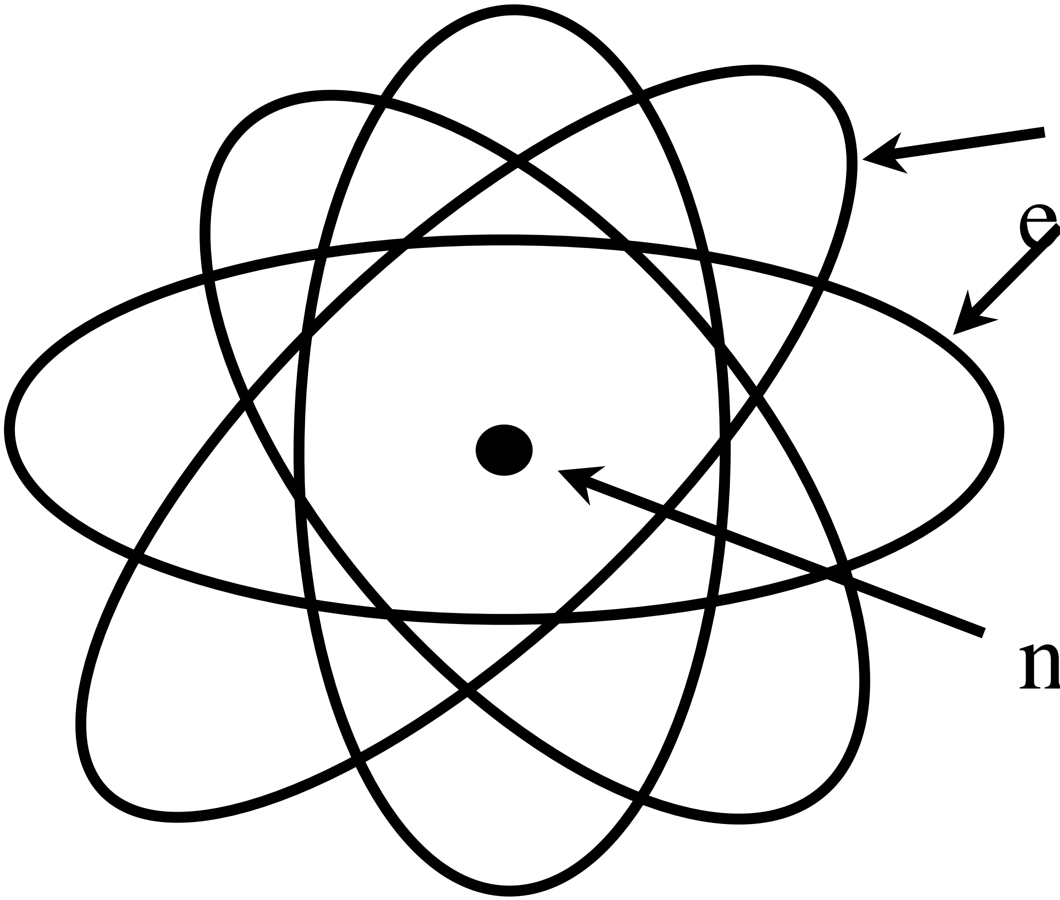
Q: what is spectrum of Moon? Earth?

Normal stars have absorption spectra



Spectrum of elliptical galaxy is sum of all of its stars, so looks like an absorption spectrum

Pattern of spectral lines related to structure of the atom



orbiting electrons

nucleus

Structure of an atom

- Nucleus at center, contains:
  - Protons (positive electric charge)
  - Neutrons (zero charge)
- Electrons (negative charge) orbit nucleus
- Number of protons determines *element*
- Number of neutrons determines *isotope*

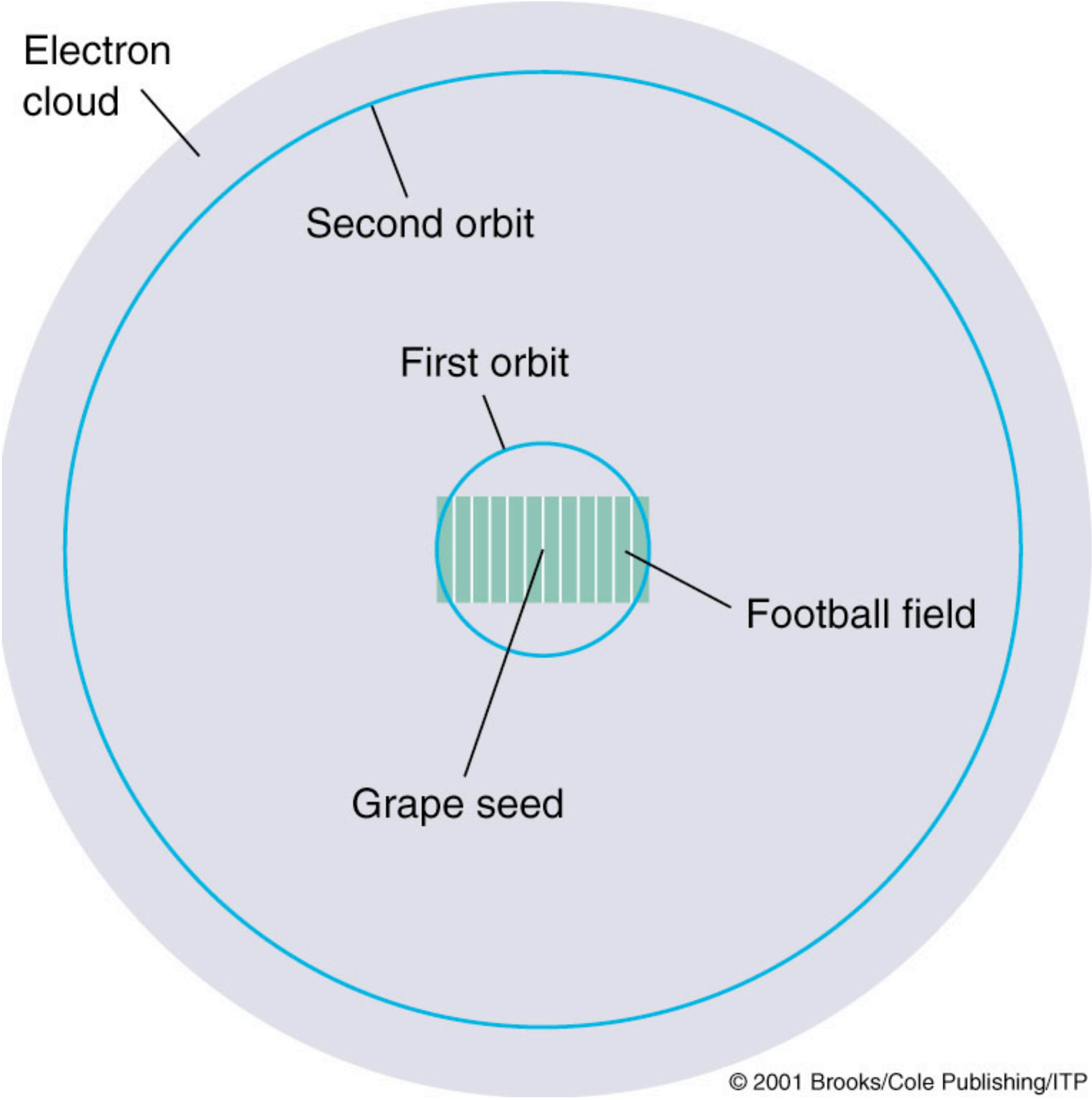
^{1}H = hydrogen

^{2}H = deuterium

^{3}H = tritium

Scale Model of Atom

- Trillion / 1 scale
- Nucleus: grape seed
- First electron orbit: size of football field
- Second orbit: 4.5 football fields



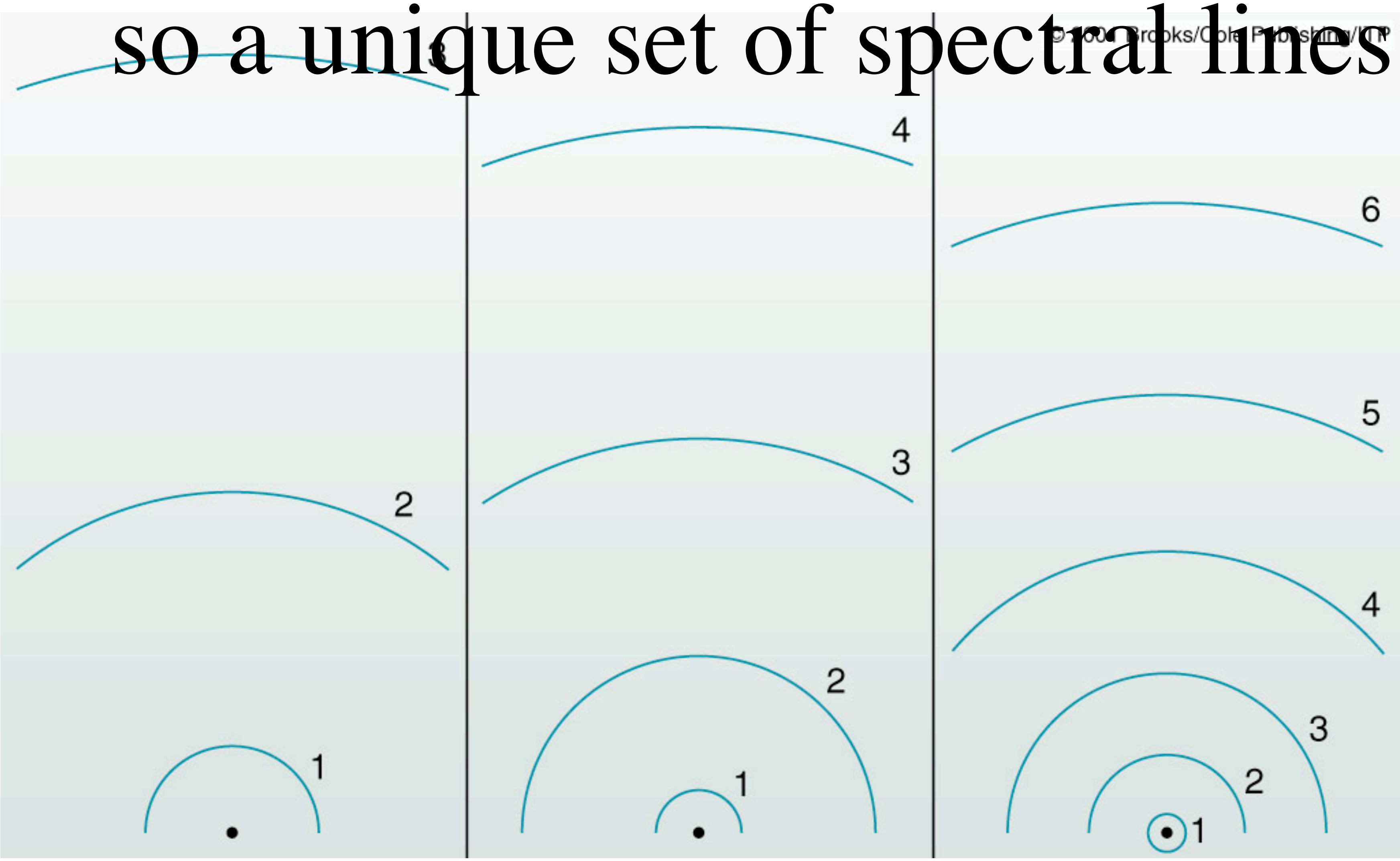
Atoms, ions, and molecules

- An atom is ***ionized*** if
  - One or more electrons are stripped away
    - Happens at high temps, such as in stars
- ***Molecules***
  - Two or more atoms bonded together
    - exist at relatively low temperatures, such as in planets or dense interstellar clouds
      - H_2 H_2O CO_2

Bohr model of the atom

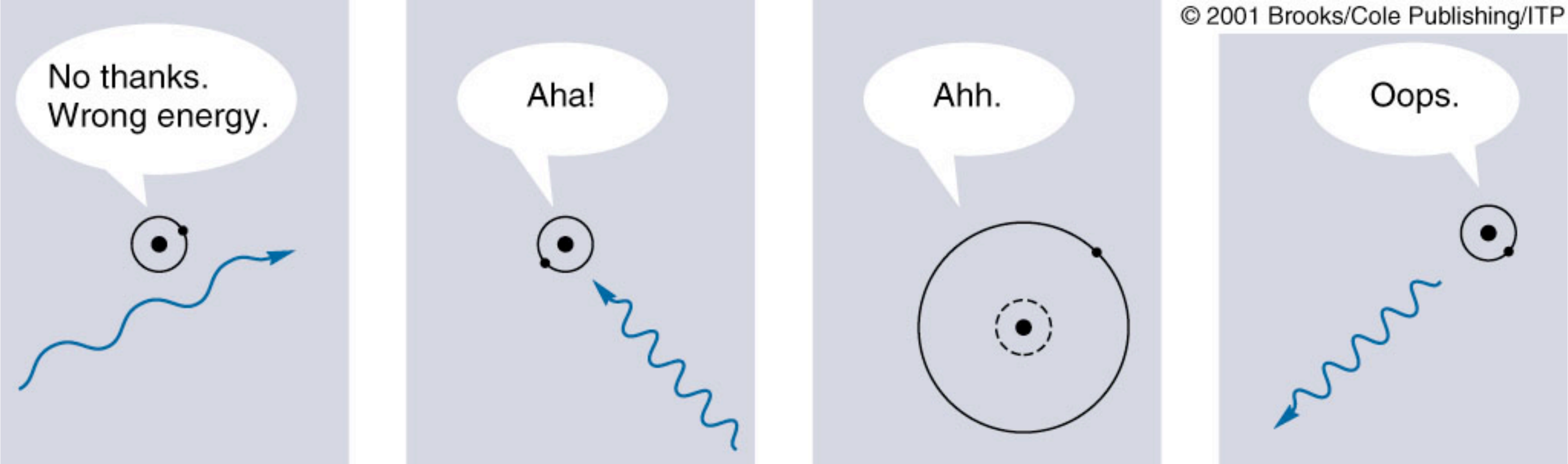
- Structure of atoms governed by the laws of quantum mechanics…

Each element has *unique* orbits so a unique set of spectral lines



Hydrogen Helium Boron

Absorption & Emission of Photons



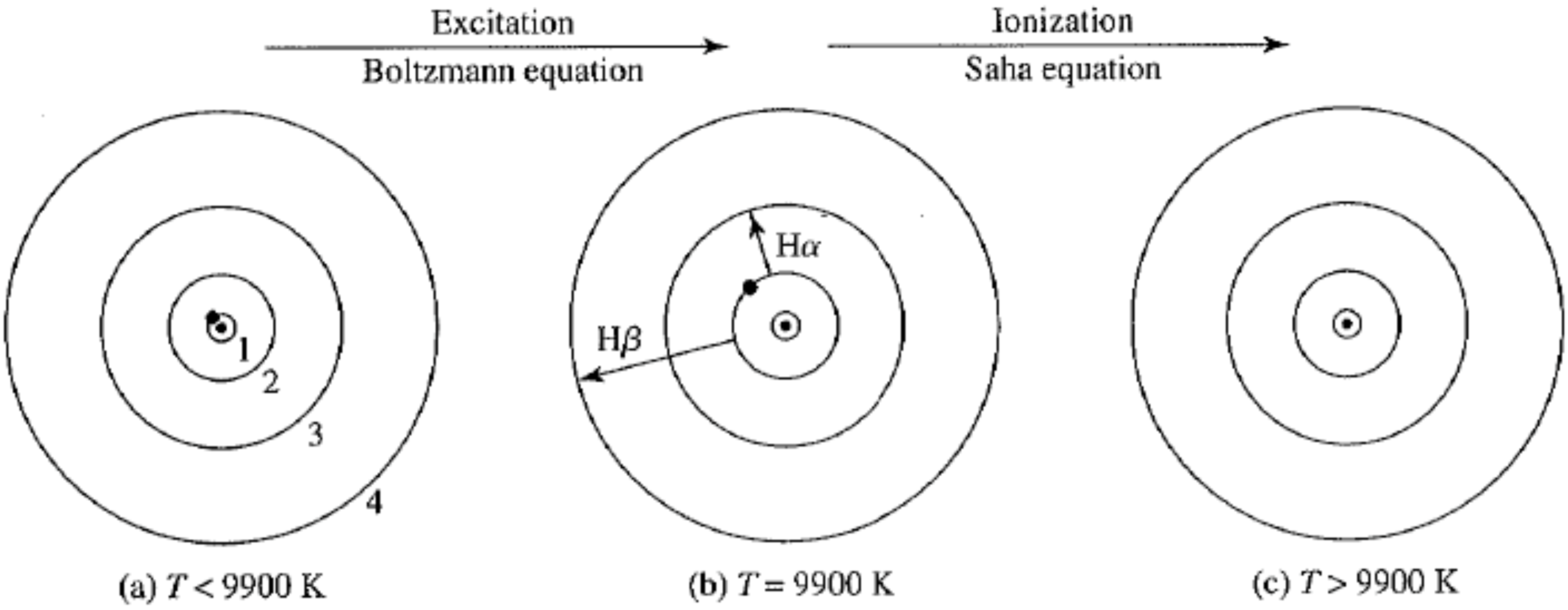
- This photon has *wrong* wavelength (energy)
- Not absorbed

- This photon has *right* wavelength
- Absorbed!!

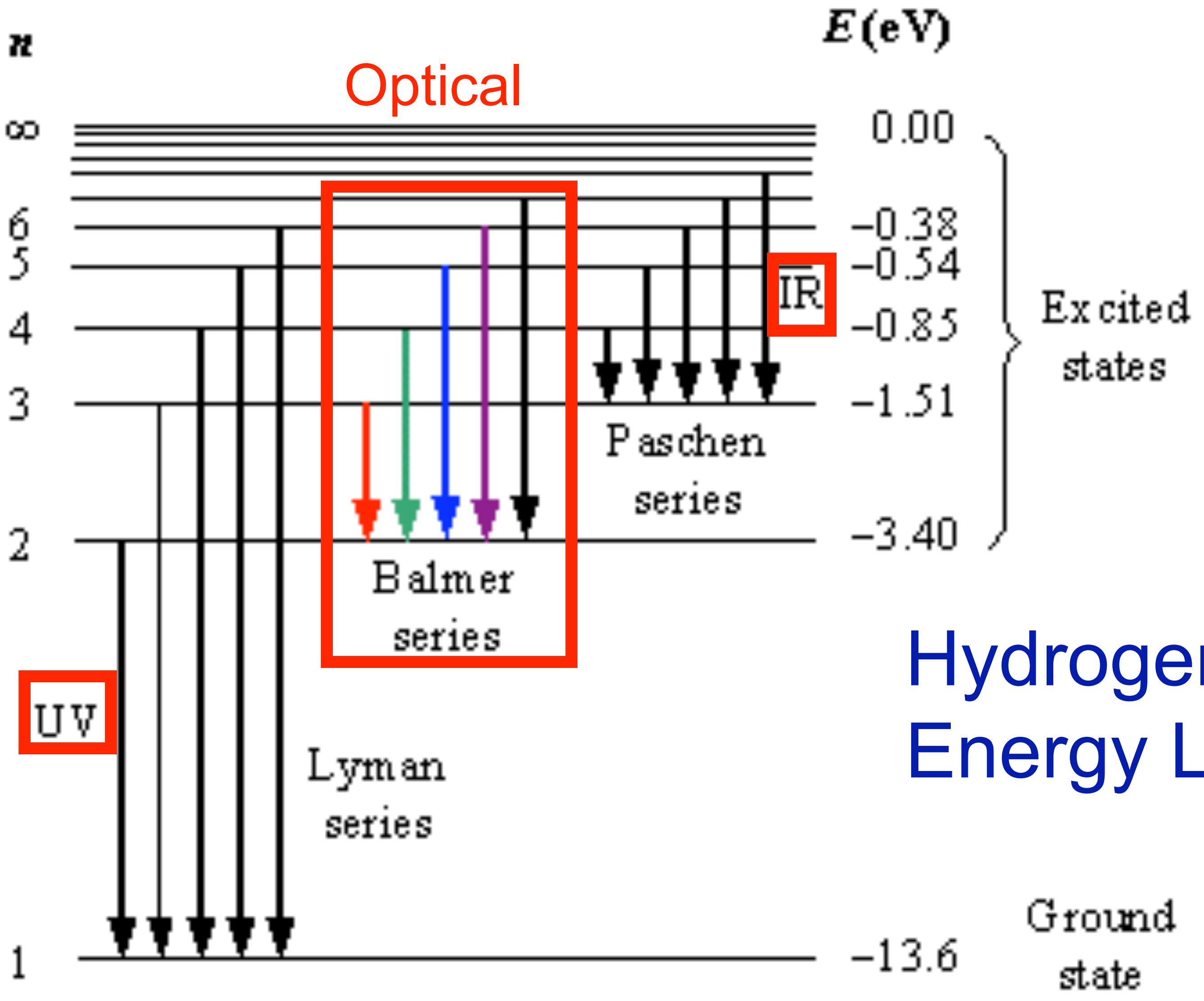
- The *excited* atom (electron in a bigger orbit)

- Excited atom emits photon, electron drops to lower orbit

Spectral types as a temperature sequence



Hydrogen Atom Energy Levels

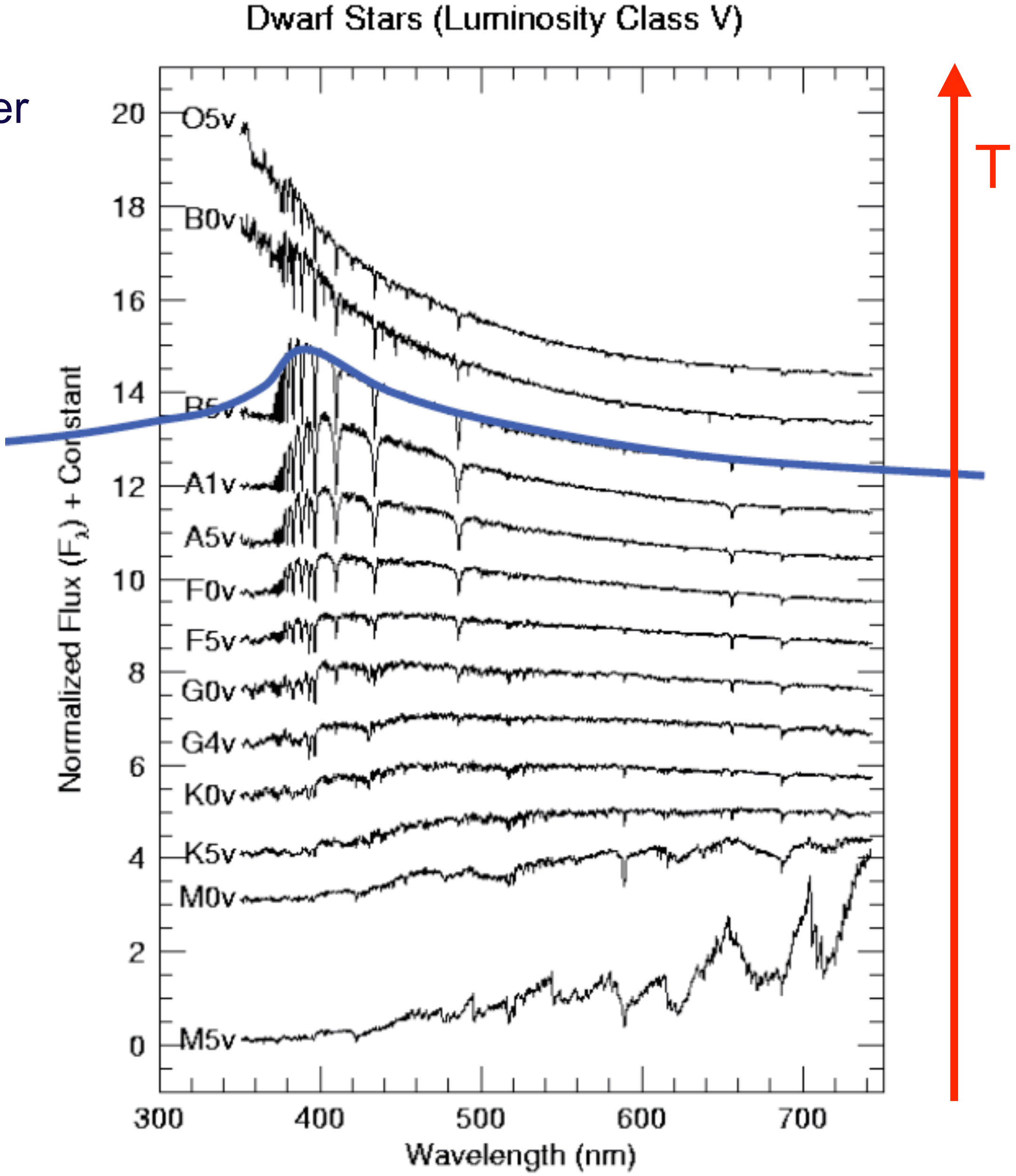


Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

- With decreasing temperature, the continuum (BB) peak shifts to longer wavelengths (Wien's law)

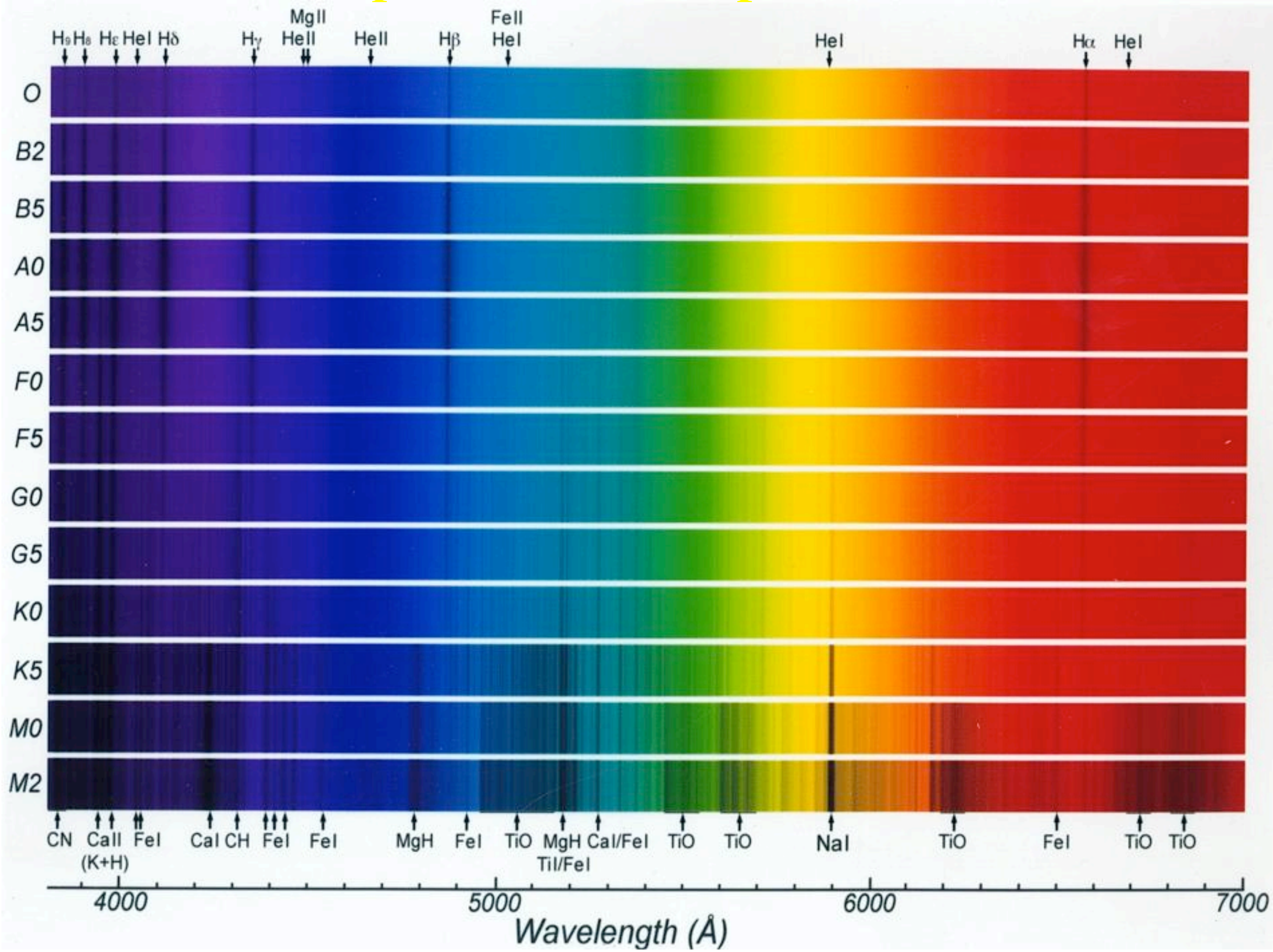
- Since all stars have roughly the same composition, line strength traces mostly the photospheric temperature

…or, two ways to probe the stellar surface T…

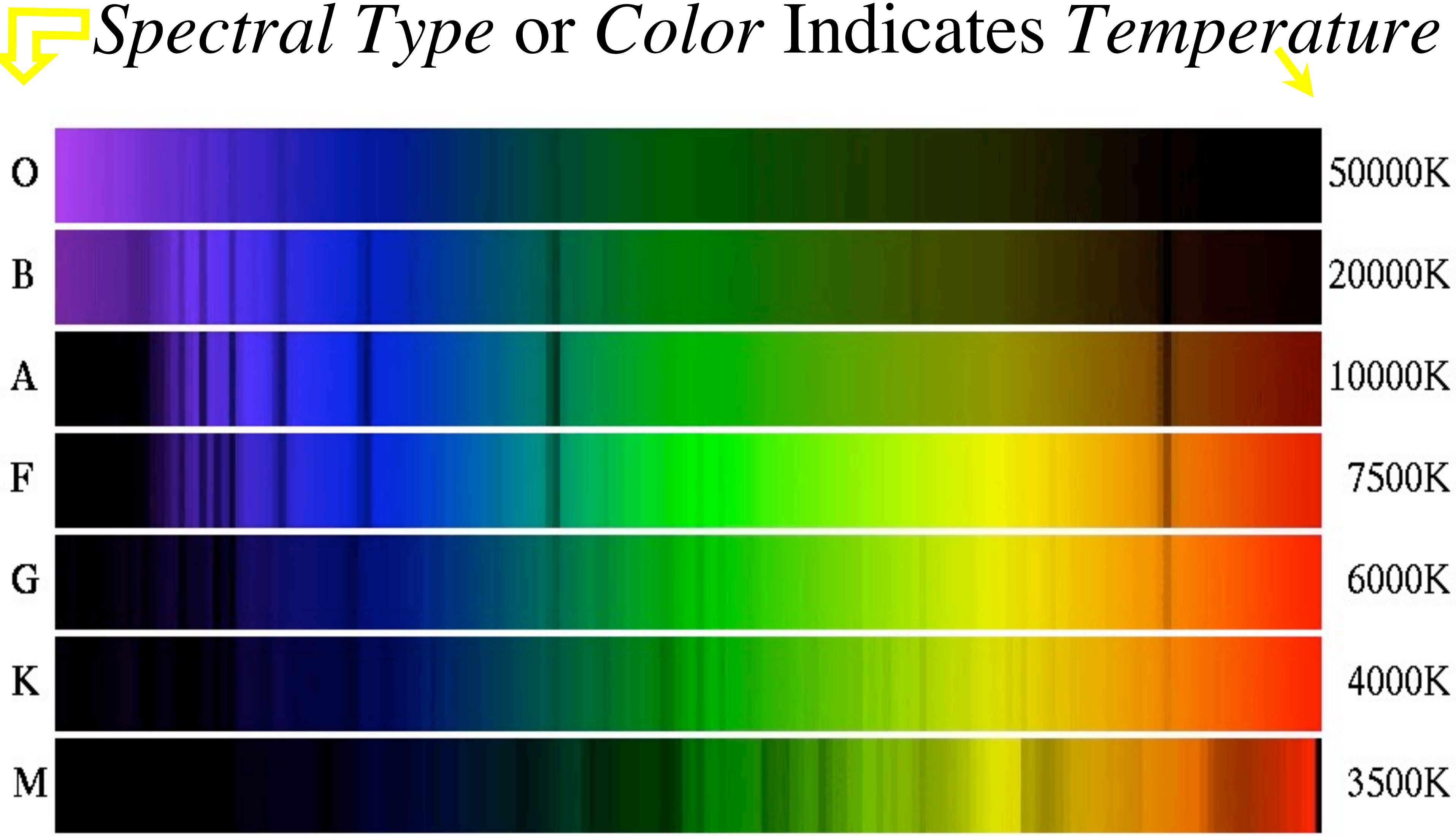


BUT, WHICH ARE THE PHYSICAL PRINCIPLES?

Examples of stellar spectra



Spectral Type or *Color* Indicates *Temperature*



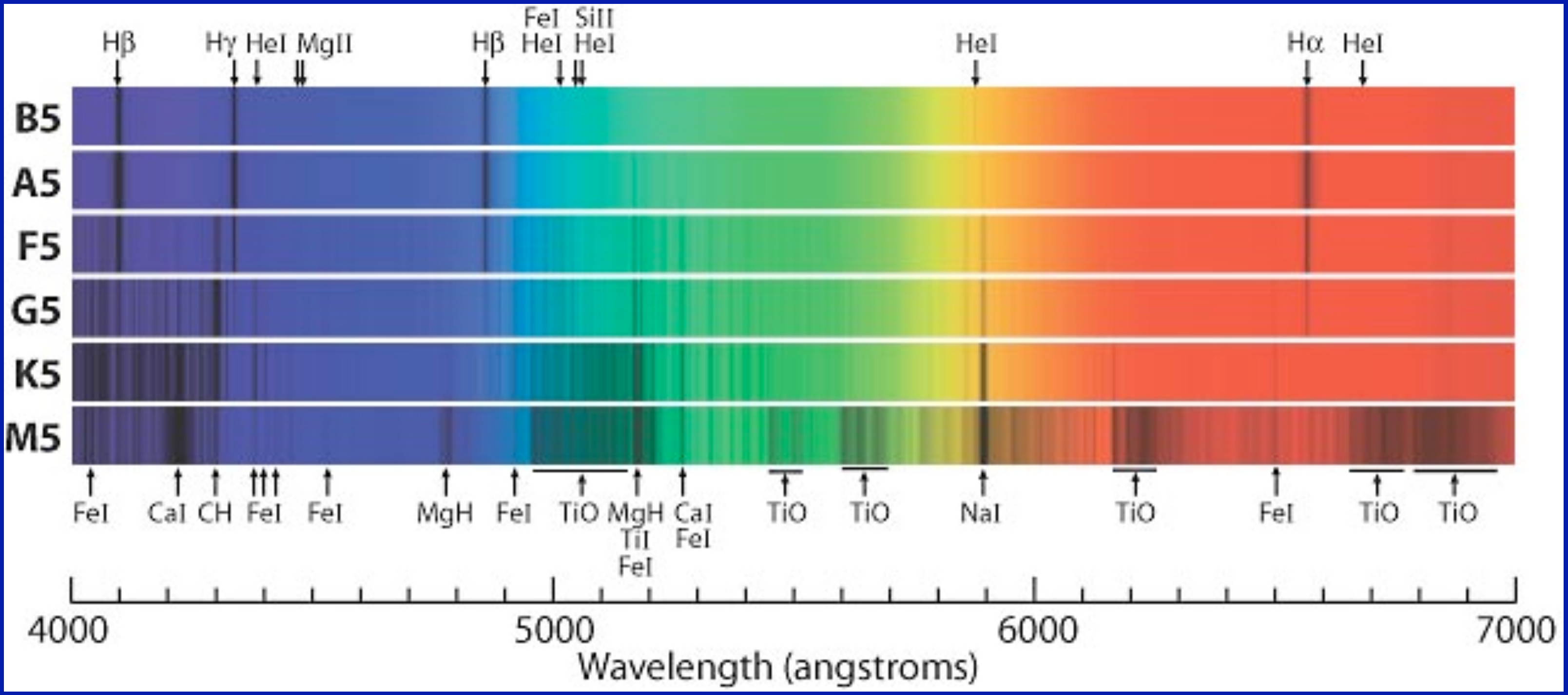
Spectral classification of stars

Stars show various patterns of absorption lines:

- some have strong Balmer lines (Hydrogen, the $n_{low}=2$ series), but some don't (e.g., the Sun)

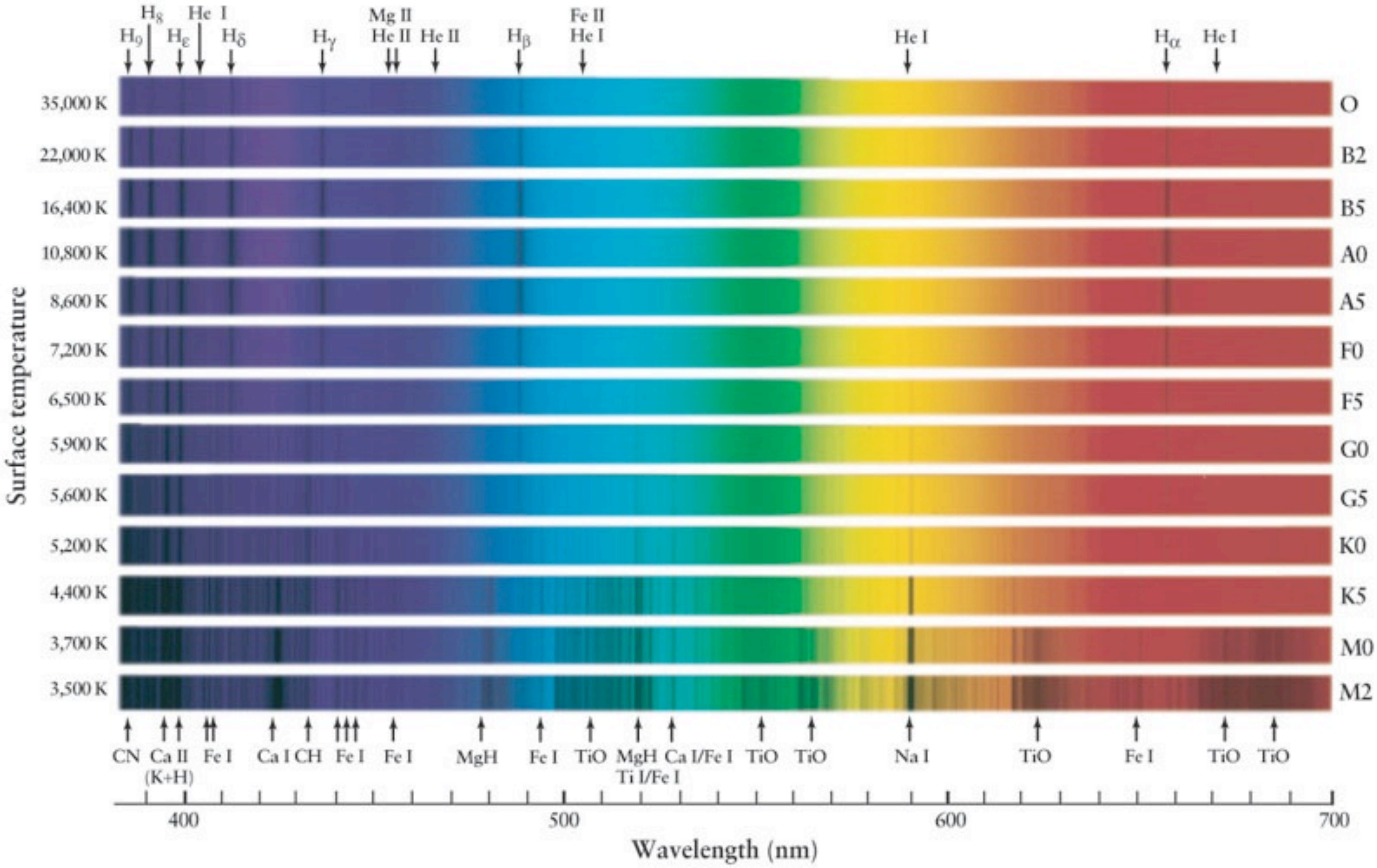
- some show strong lines of Ca, Fe, Na

- some show lines from molecules such as TiO, MgH



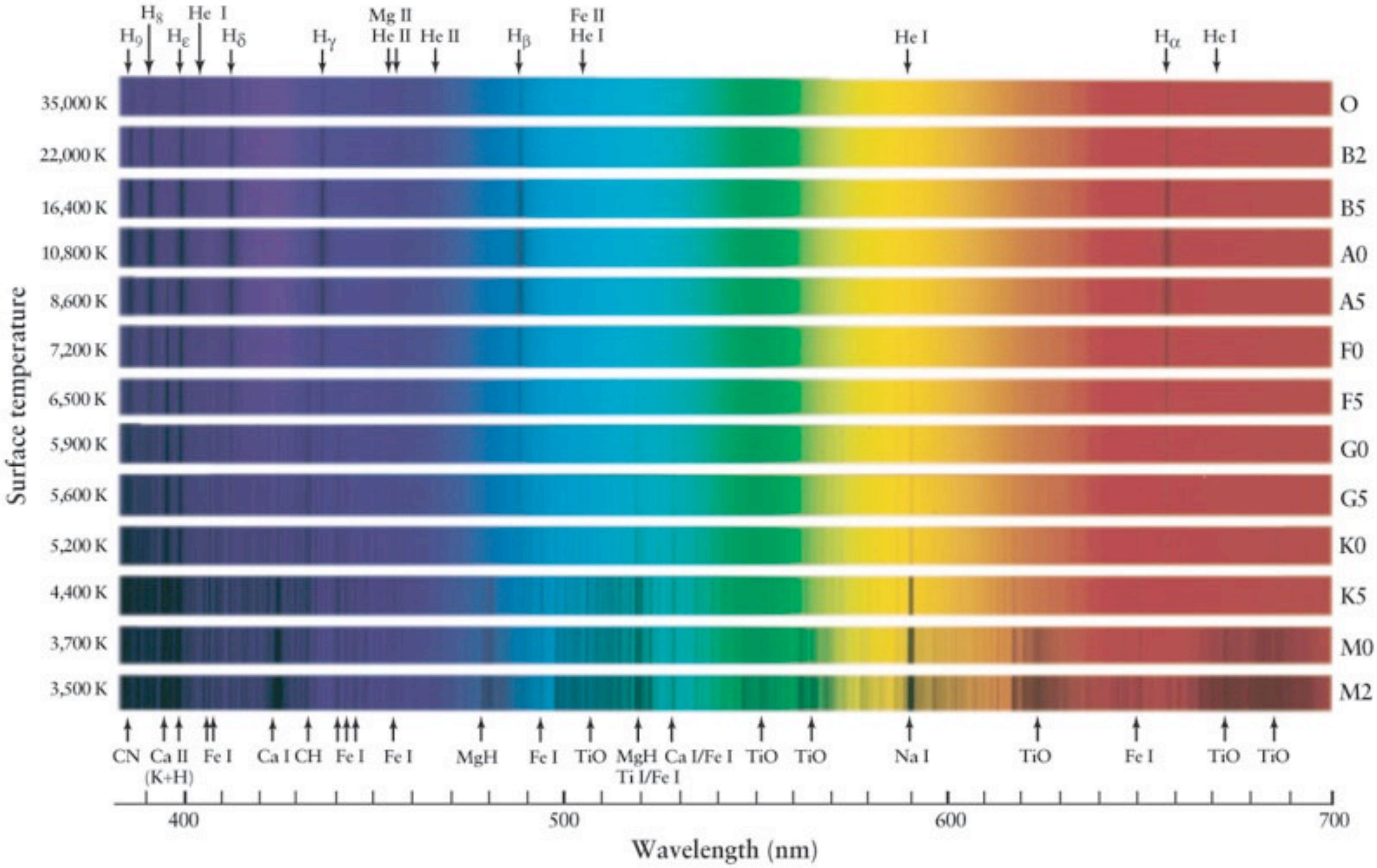
Line strength vs spectral type

He I lines: strongest in B2 stars



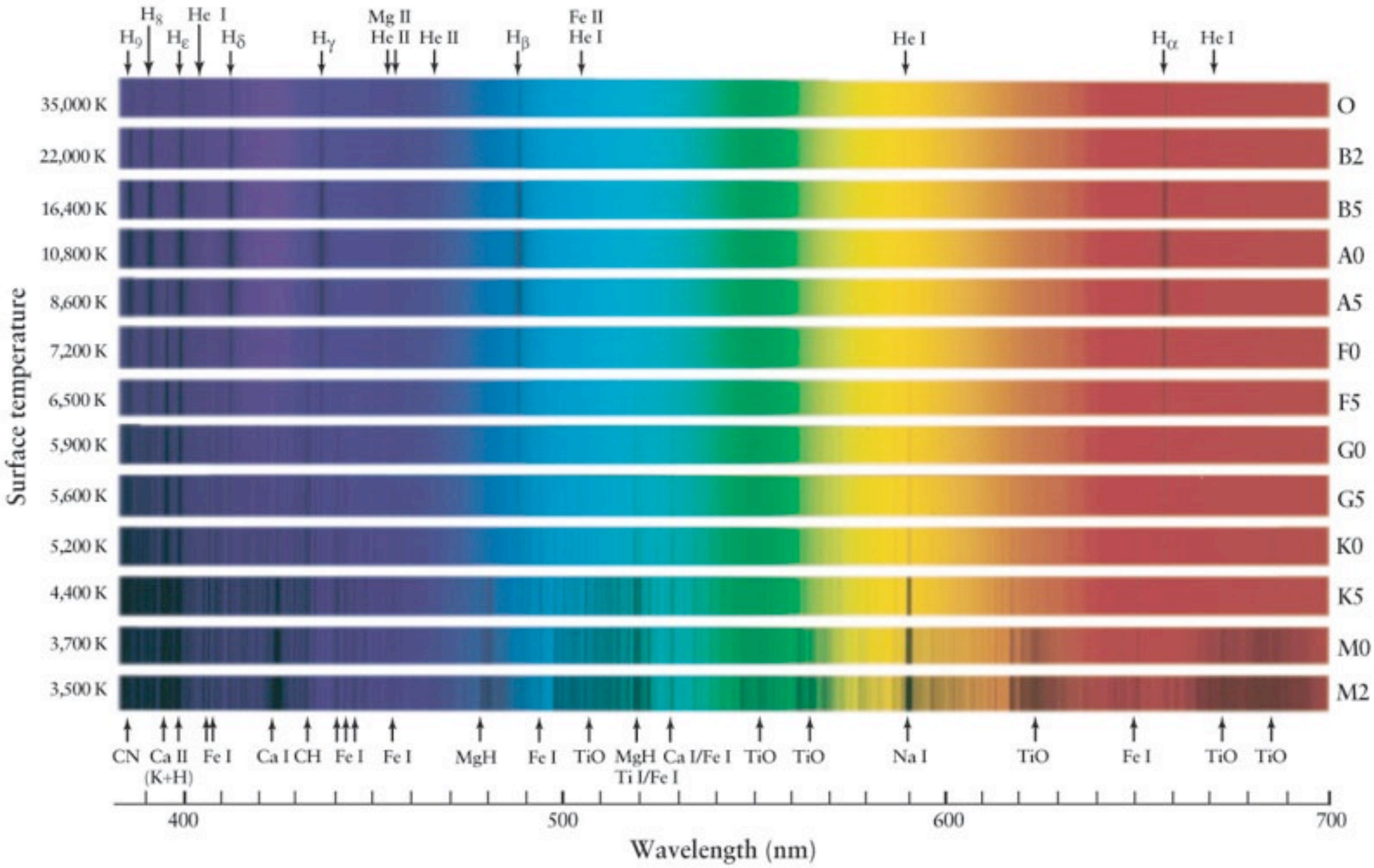
Line strength vs spectral type

H I Balmer lines: strongest in A0 stars



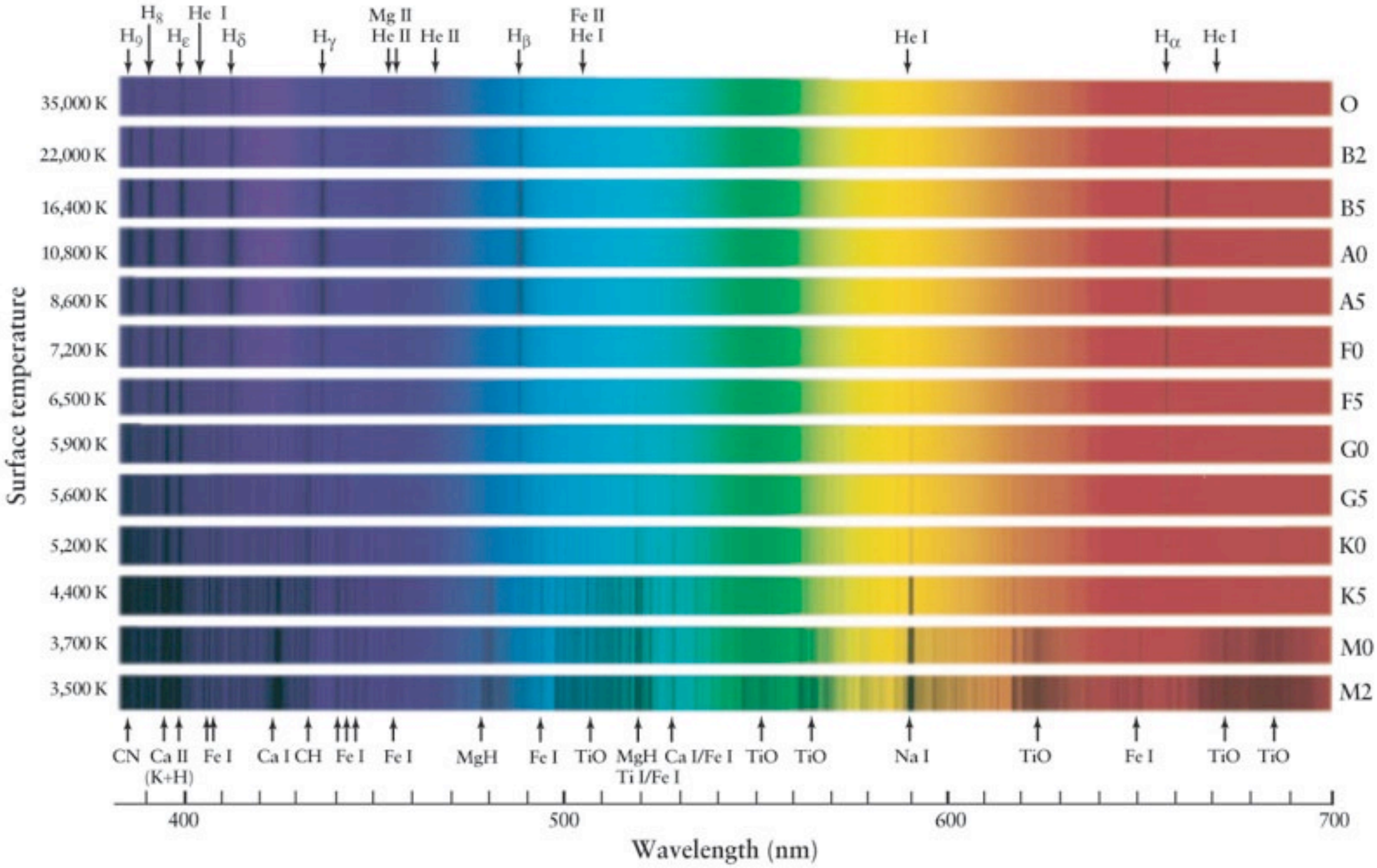
Line strength vs spectral type

Ca II H and K lines: strongest in K0 stars



Line strength vs spectral type

Molecular absorption bands (TiO, VO): strongest in M stars



The physics of spectral lines

Boltzmann equation predicts the number $N_{a,b}$ of atoms in a given excitation state with energy $E_{a,b}$ and degeneracy $g_{a,b}$:

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b - E_a)/kT}.$$

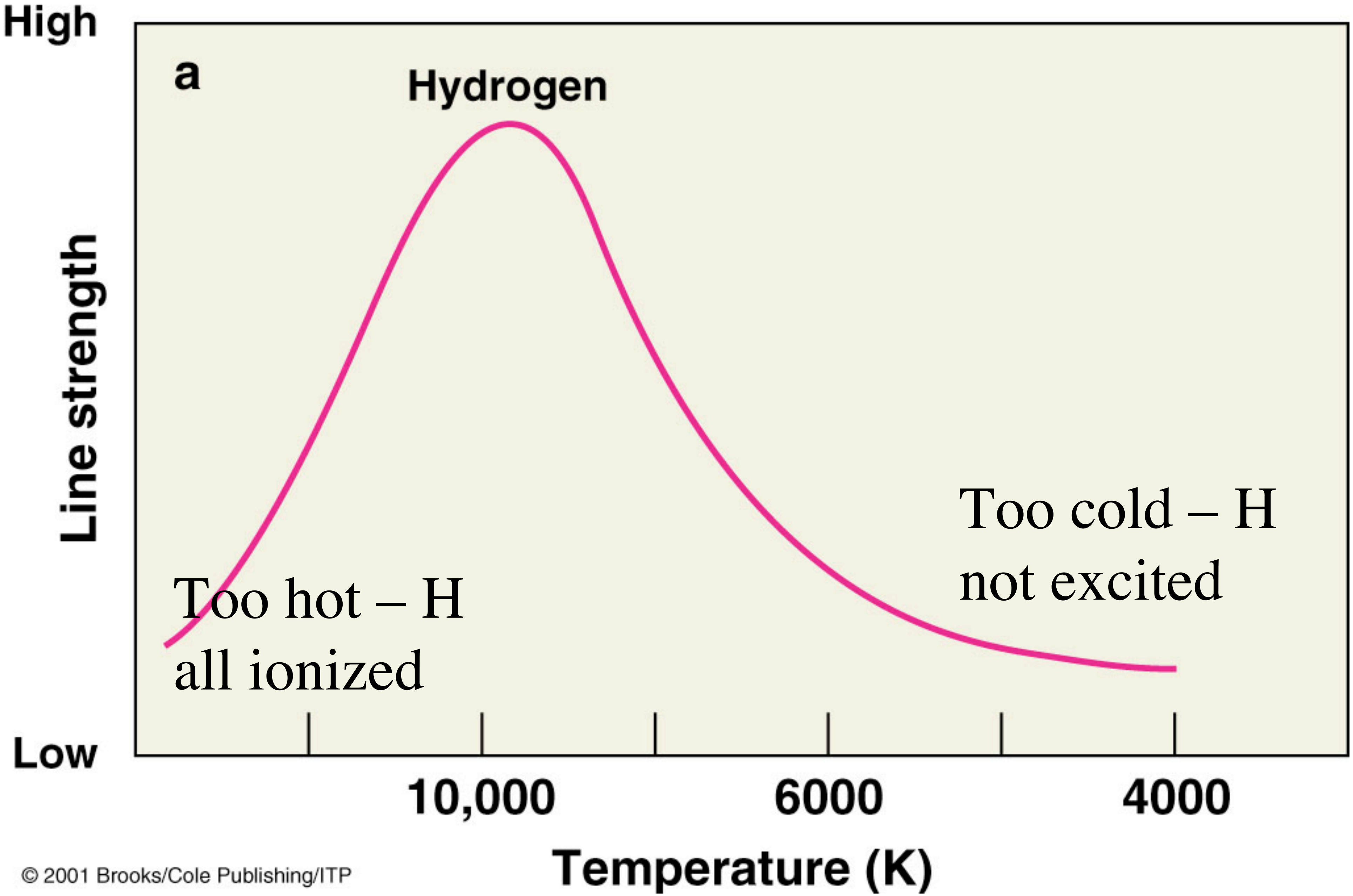
Saha equation predicts the number $N_{i,i+1}$ of atoms in a given ionization state with ionization energy χ_i:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.$$

$$Z = \sum_{j=1}^{\infty} g_j\, e^{-(E_j - \enclose{circle}{E_1})/kT}.$$

Partition function: sum of the number of ways to arrange the atomic electrons, weighted by the Boltzmann factor

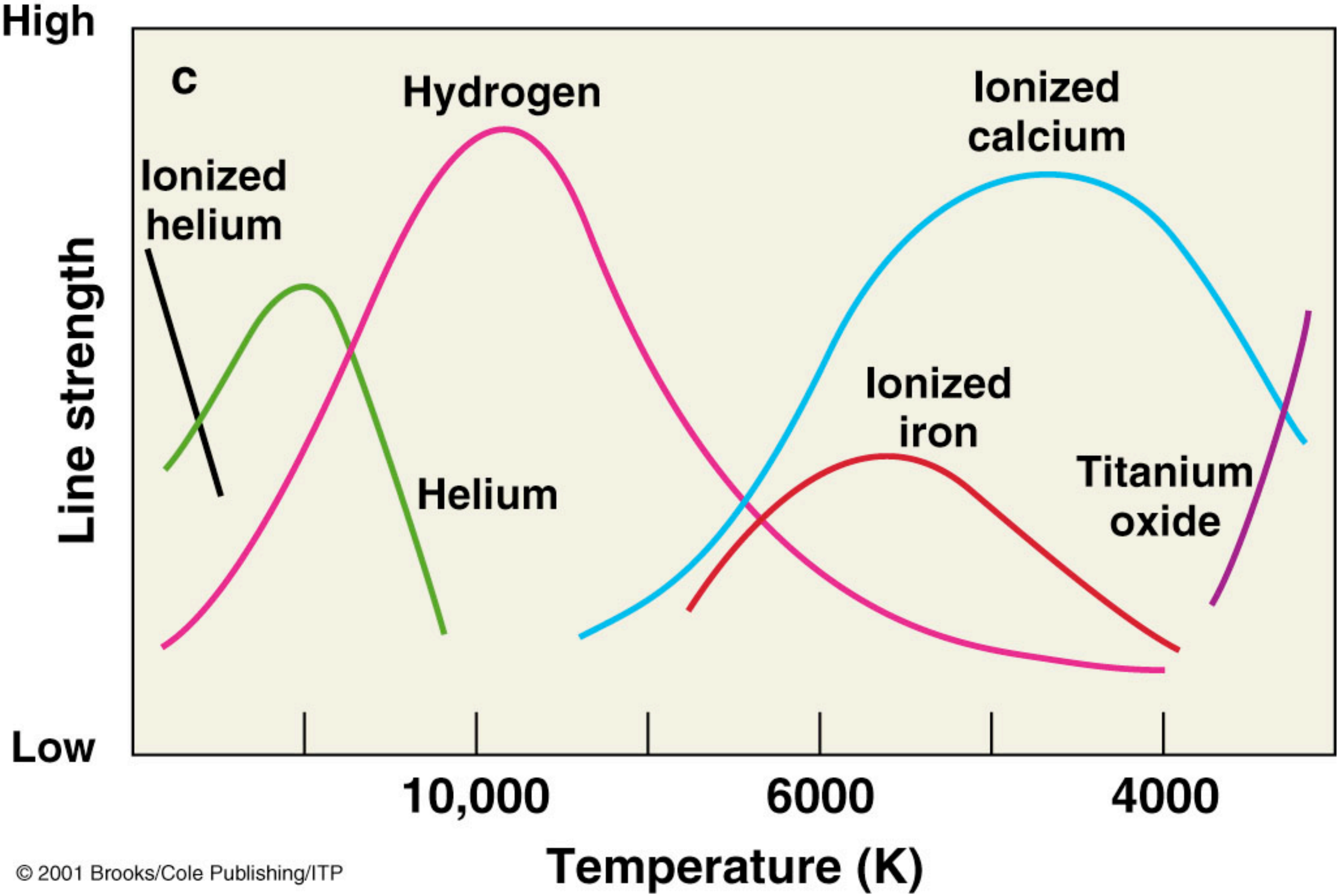
Line Strengths vs Temperature



Too hot – H
all ionized

Too cold – H
not excited

Line Strengths vs Temperature



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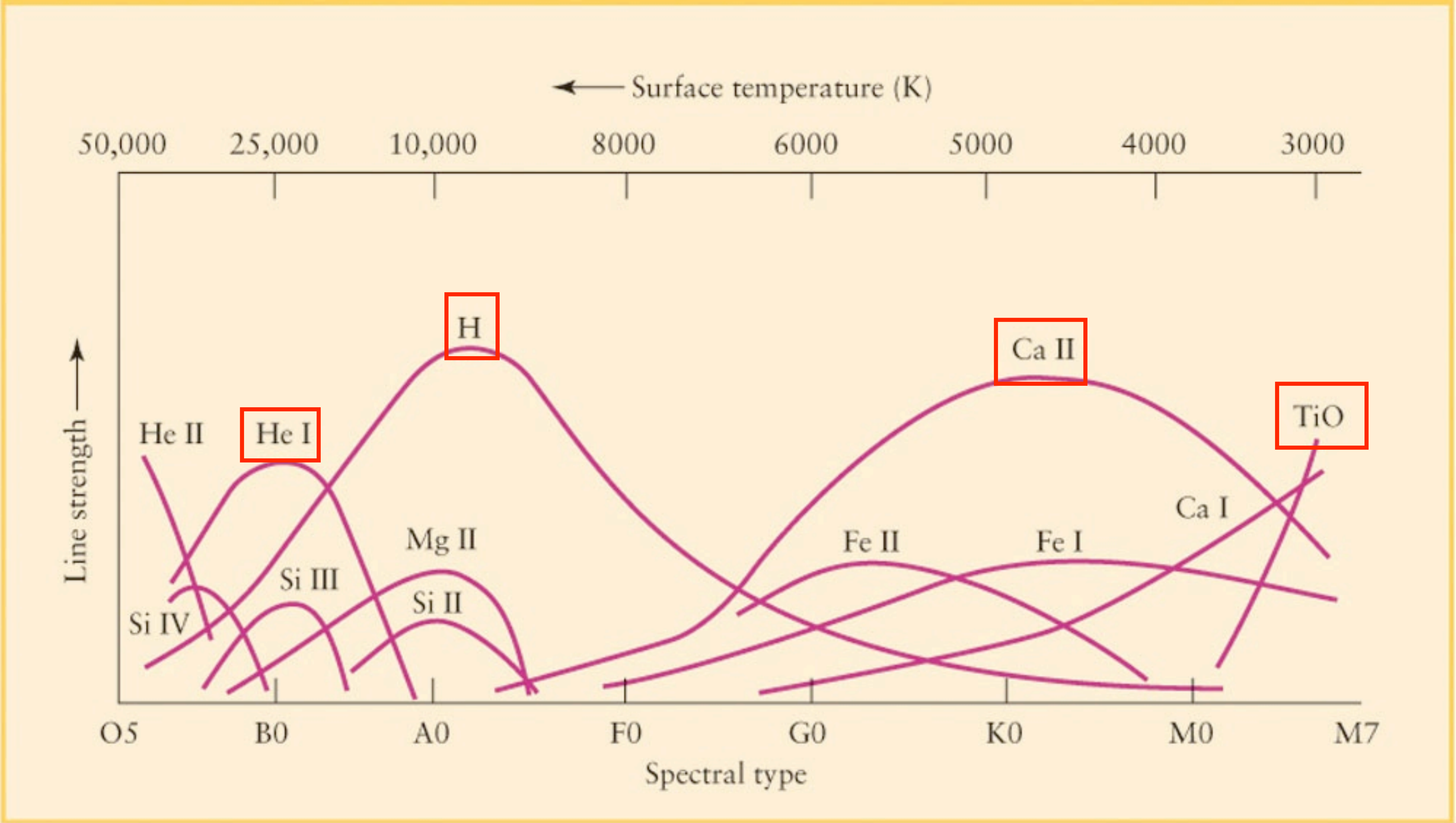
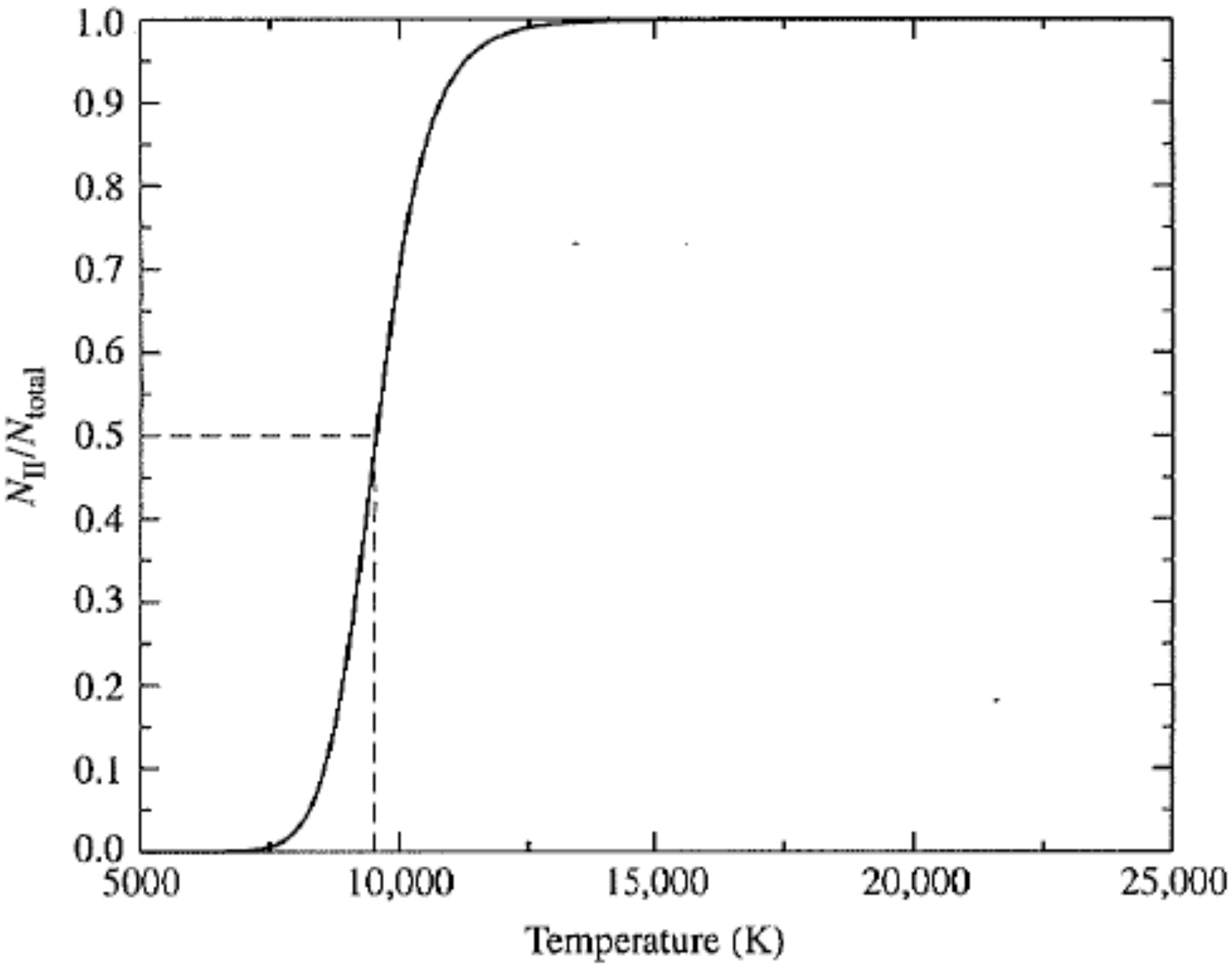
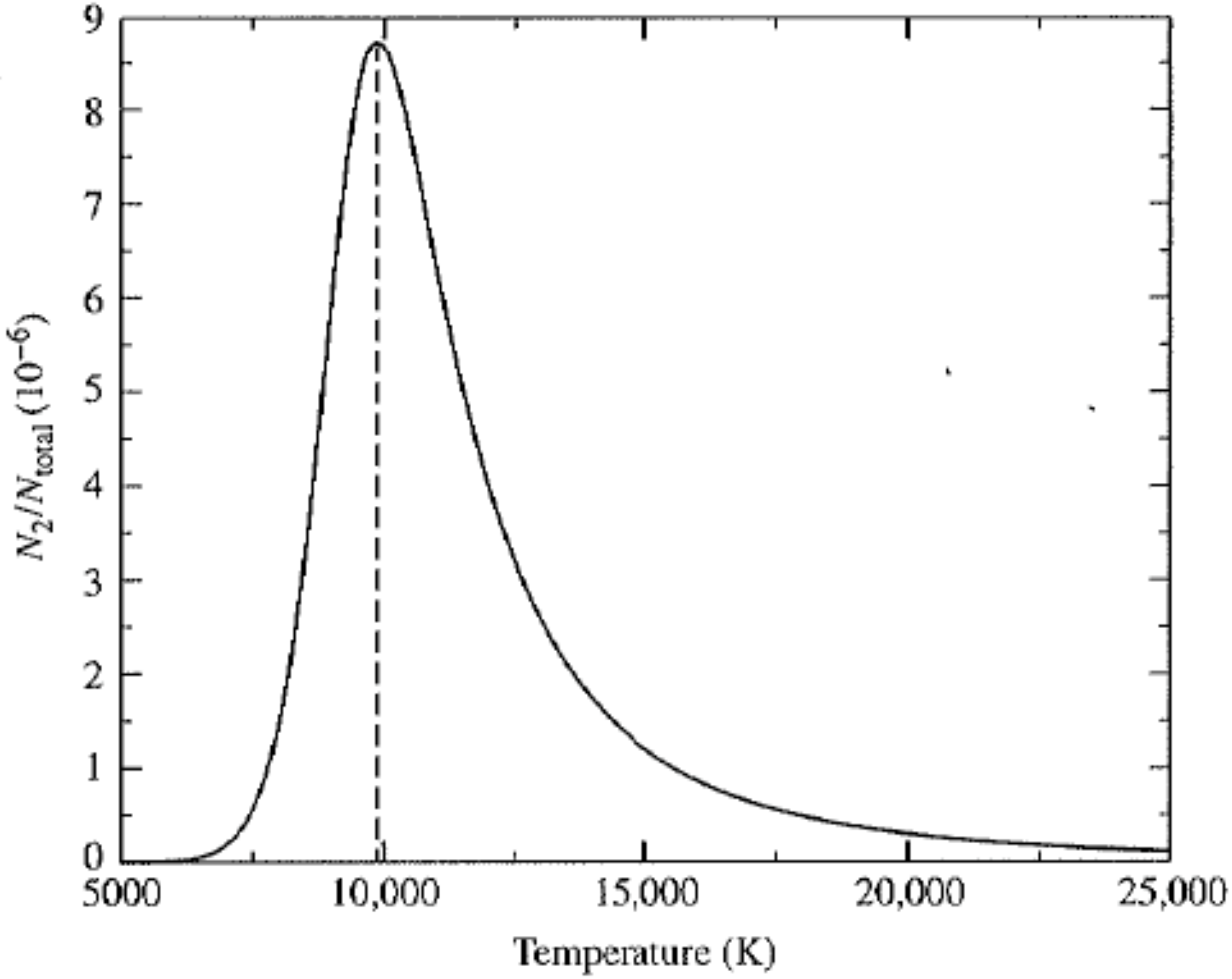
In other words: what determines the temperature range in which a given atom produces prominent absorption lines in the visible part of the spectrum?



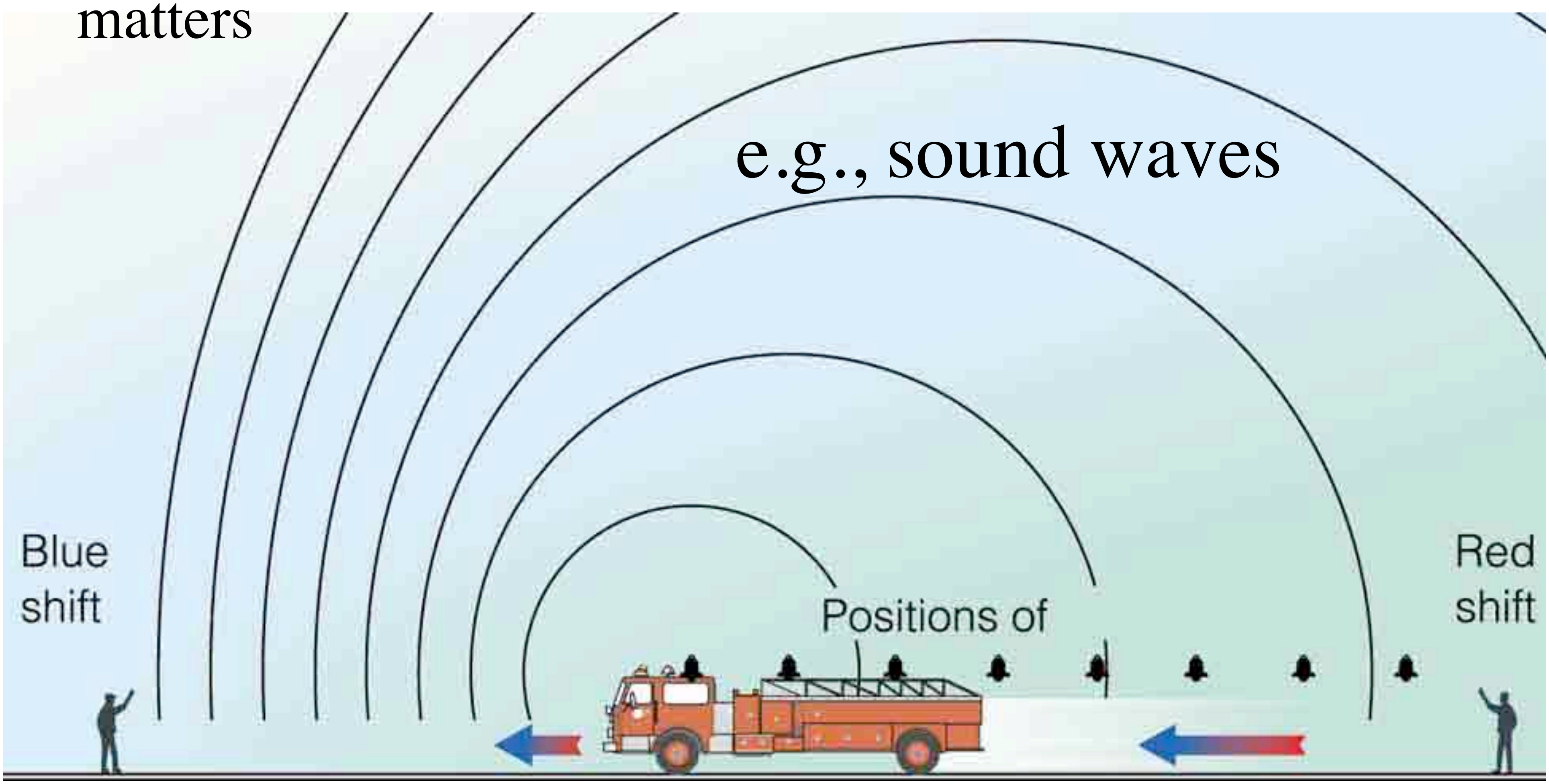


N_{II}/N_{total}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
5000
10,000
15,000
20,000
25,000
Temperature (K)

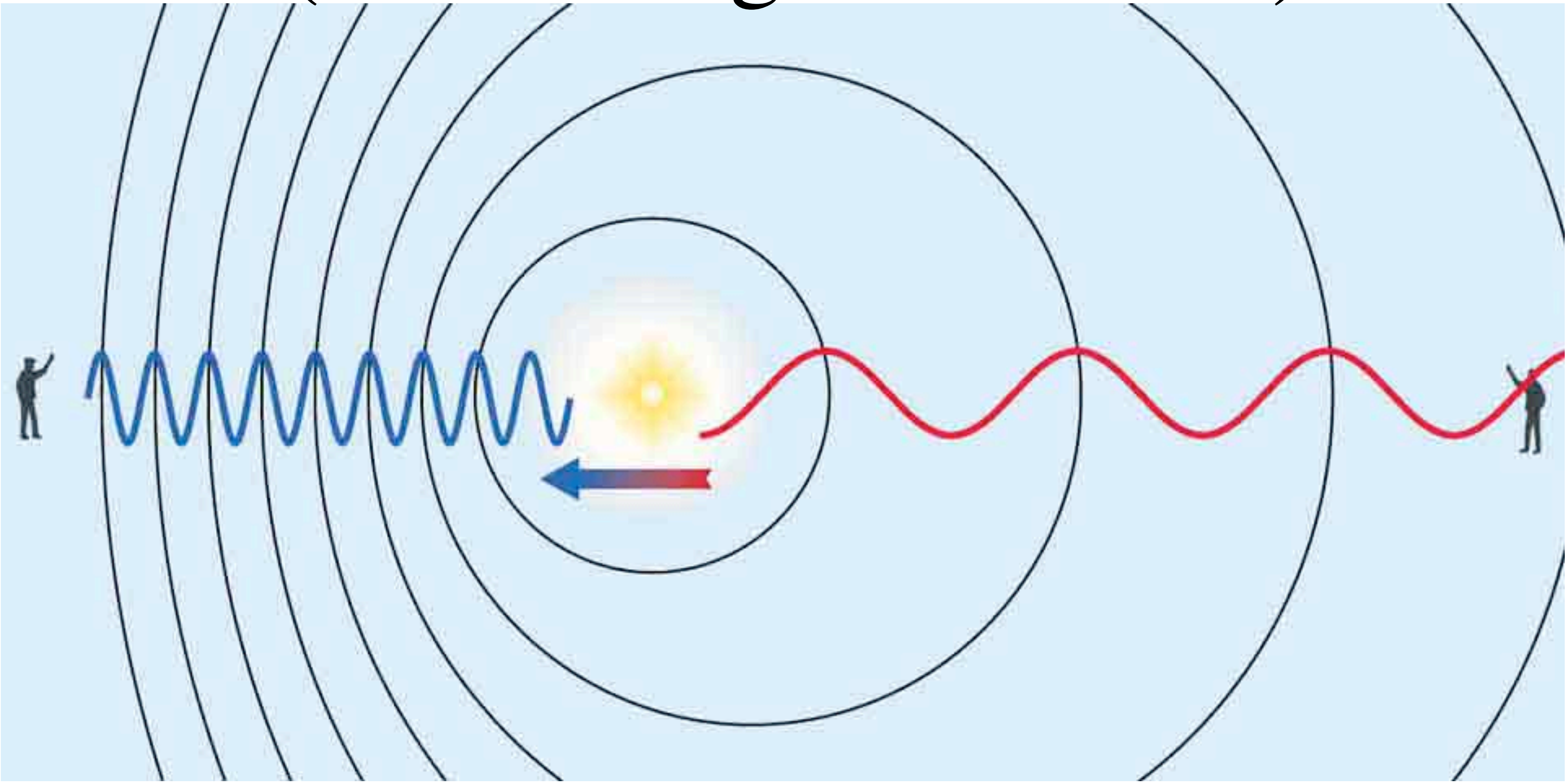

$N_2/N_{\text{total}}\ (10^{-6})$
0
1
2
3
4
5
6
7
8
9
5000
10,000
15,000
20,000
25,000
Temperature (K)

Doppler Effect

- Apparent change in wavelength caused by motion of object
- only motion directly towards or away from observer matters



Doppler Effect (electromagnetic waves)

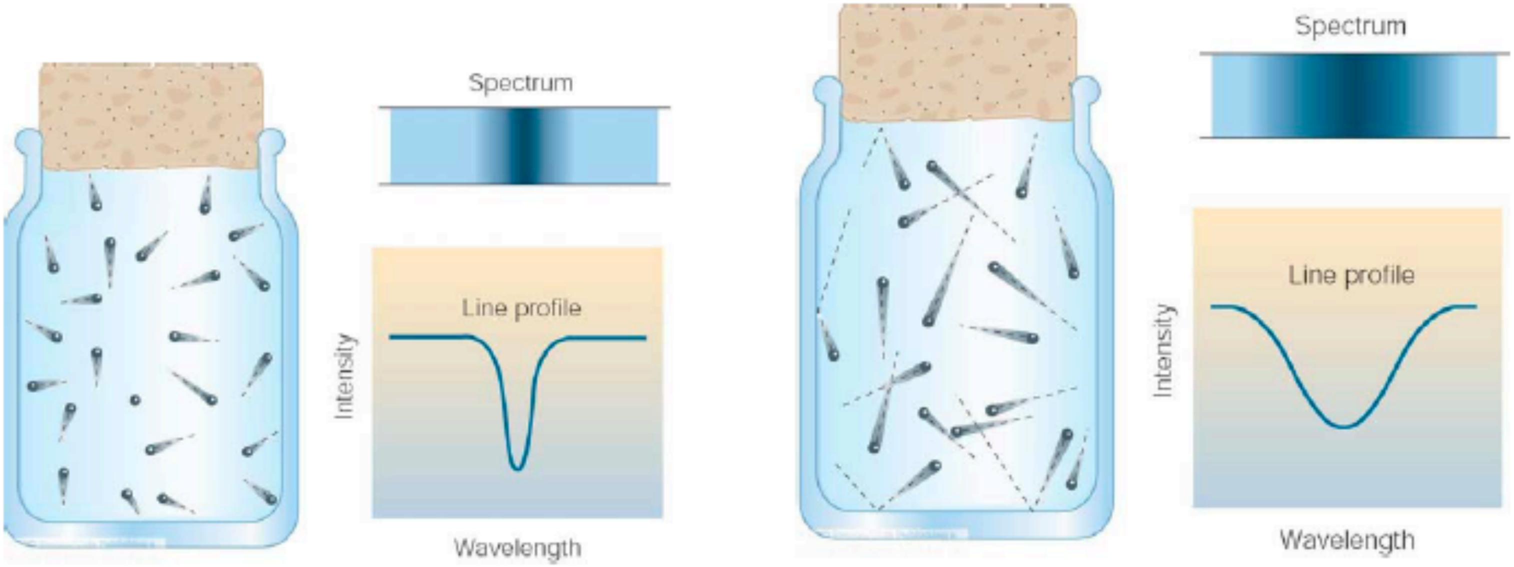


Doppler Effect

- Object moving towards observer
  - → Radiation shifted towards *shorter* wavelengths
  - → **Blue shift**
- Object moving away from observer
  - → Radiation shifted towards *longer* wavelengths
  - → **Red shift**
- For non-relativistic motion ($v << c$), $\Delta\lambda/\lambda = v/c$ Allows us to measure the speed of an object (star, planet, galaxy) towards or away from us

Line Broadening

Higher Temperatures
⇔Higher thermal velocities
⇔ broader lines



Doppler Broadening is usually the most important broadening mechanism.

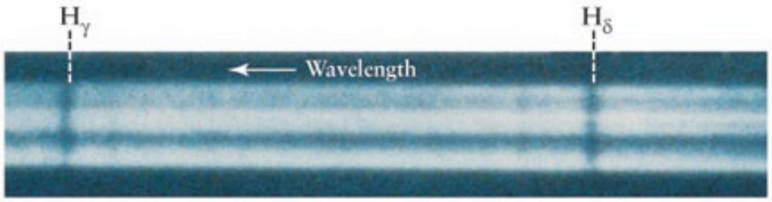
From the spectrum to the stellar radius

By comparing the spectra of stars with the same surface temperature but different luminosities:

- a giant star has narrow Balmer absorption lines

- a main-sequence star has broad Balmer absorption lines

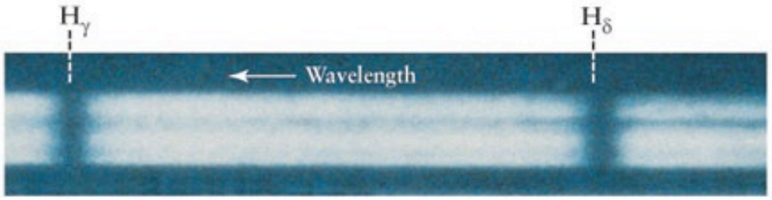
B8



Supergiant

(a) A supergiant star has a low-density, low-pressure atmosphere: its spectrum has narrow absorption lines

B8



Main-sequence

(b) A main-sequence star has a denser, higher-pressure atmosphere: its spectrum has broad absorption lines

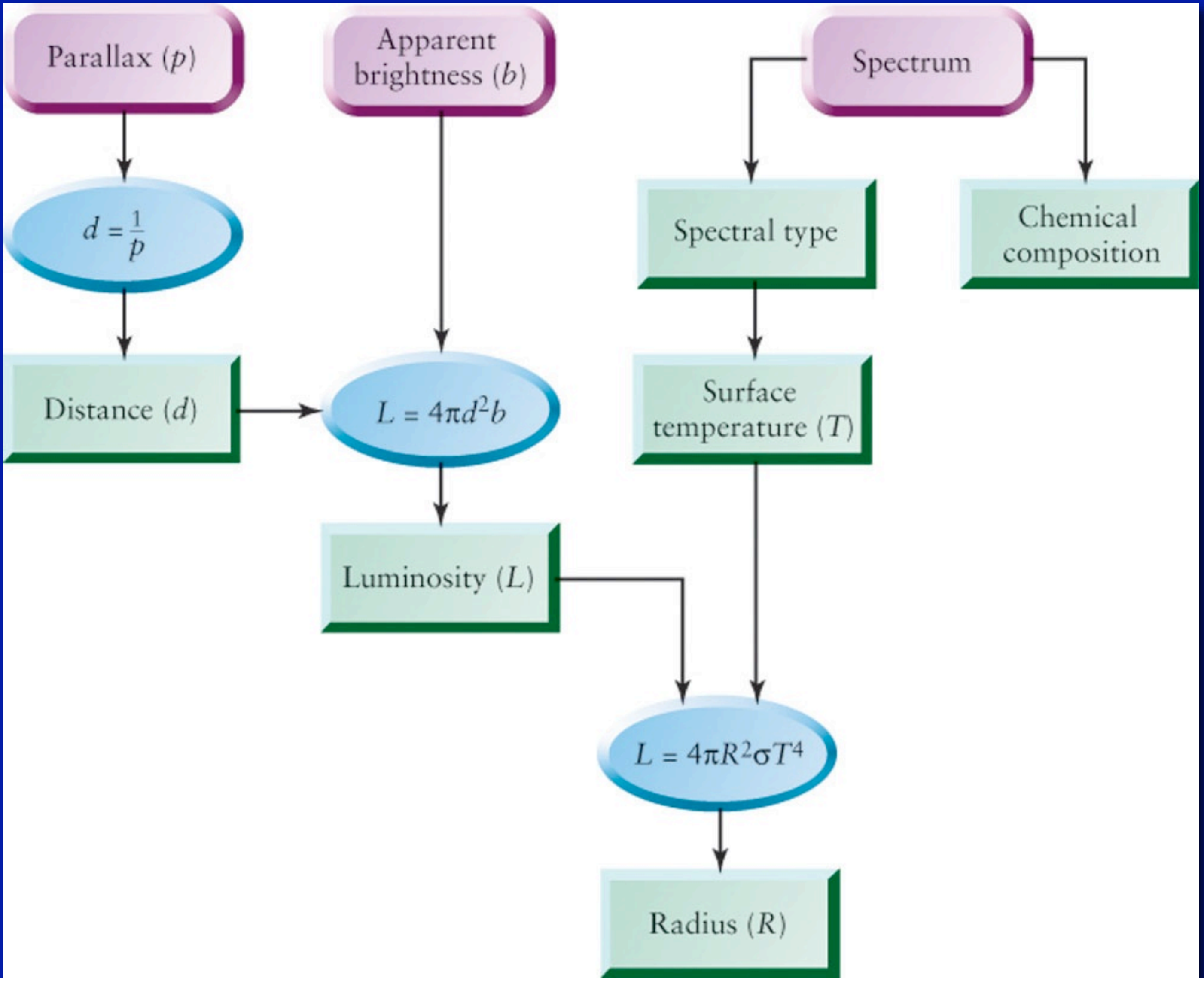
Broader lines IF larger surface pressure (or density) IF larger surface gravity $g = GM/R^2$ IF smaller radius

So what can we learn from spectra?

- Chemical composition
- Temperature
- Density
- Degree of ionization
- Radial velocity

… A lot!

From Observed Quantities to Physical Quantities of Stars



What do we learn from spectral lines?

- Abundances: if T is fixed, Na lines stronger if Na is more abundant
- Photospheric temperature of the star, from the prevalent lines
- The stellar radius, from the line width
- From the Doppler effect: radial velocity of the star (Doppler shift) and temperature/random velocity of emitting atoms/ions (Doppler broadening)