Charged Particle Acceleration in RMHD

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Cosmic Rays

- Relativistic electrons responsible for synchrotron emission at relativistic outflows
  - GRB outflows
  - AGN jets
- Require acceleration mechanism to account for power law electron distributions

Goals

- Evolve a particle distribution passively coupled to a hydromagnetic medium
- To measure the efficiency of charged particles acceleration in different RMHD environments
- Directly observe first or second order Fermi mechanism at work

Theoretical Setting

- A probability distribution function $f(x^\mu, p^\mu)$ evolves according to relativistic Vlasov equation

$$u^\mu \frac{\partial f}{\partial x^\mu} + F^\mu \frac{\partial f}{\partial p^\mu} = 0 \tag{1}$$

$$F^\mu = \frac{e}{m} F^\mu{}_\nu u^\nu \tag{2}$$

- Forcing term comes from Lorentz force law
  - Model described in Kulsrud and Ferrari (1971)
  - B-field frozen in a non-resistive turbulent hydromagnetic medium
  - $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$

Relativistic Magnetohydrodynamics

- Formalism described by Antón et al. (2006), with flat spacetime metric
- Standard setup for solving the system of ideal RMHD equations in conservation form
  - HLL Riemann solver
  - Piecewise linear reconstruction on primitive quantities
  - Method of lines, Runge-Kutta time integration
  - $\nabla \cdot \mathbf{B} = 0$ by Constrained Transport scheme Tóth (2000)
  - Primitive variable recovery described by Noble et al. (2006)

Implementation of Hydro Modules

- Fully modularized coding environment using C and C++
- Hand-written code from scratch with few dependencies
  - Any implementation of the MPI 2 standard
  - HDF5 storage library used for data IO
- Large portions of code are dimension-independent, for solving problems over 1,2,3 dimensional domains with little fuss
- Parallelized to run using domain decomposition on an arbitrary number of cores

Implementation of Charged Particle Modules

- Charged particles are implemented as passive tracers, no feedback into the hydro
- Field values are interpolated to particle positions using either constant or bi/tri-linear interpolation
- Coding makes extensive use of the C++ STL, especially the std::list class template
- Parallelization achieved by shipping particles between domain subgrids

Particle Mover

- Need a solution to the relativistic Lorentz force law

$$\frac{d(\gamma \mathbf{v})}{dt} = \frac{q}{m}(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \qquad \frac{d\gamma}{dt} = \frac{q}{m}(\mathbf{v} \cdot \mathbf{E}) \tag{3}$$

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{q}{m} F^\mu{}_\sigma \frac{dx^\sigma}{d\tau} \qquad \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} = -1 \tag{4}$$

- A solution to these equations is known as a particle mover

Particle mover of Boris (1970)

- Described in Tajima (1986) section 15-4
- Uses second order 'leapfrog' algorithm, x^μ and u^μ for particle orbit are known at staggered times
- Easy to implement, fast execution
- Bad orbit resolution when $\Delta t \frac{\omega_B}{2\pi} > 0.2$
- Used in the TRISTAN Relativistic PIC plasma code

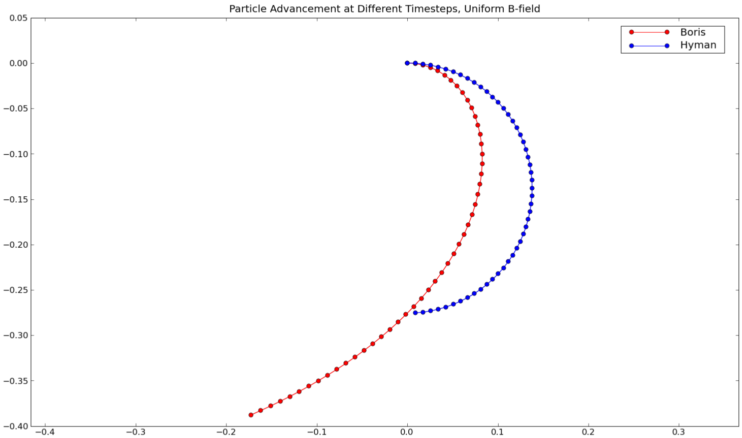
Particle mover of Hyman (1996)

- Explicit solution ${f^{\mu}}_{\lambda}(\tau)$ to Equation (4), parameterized in particle proper time

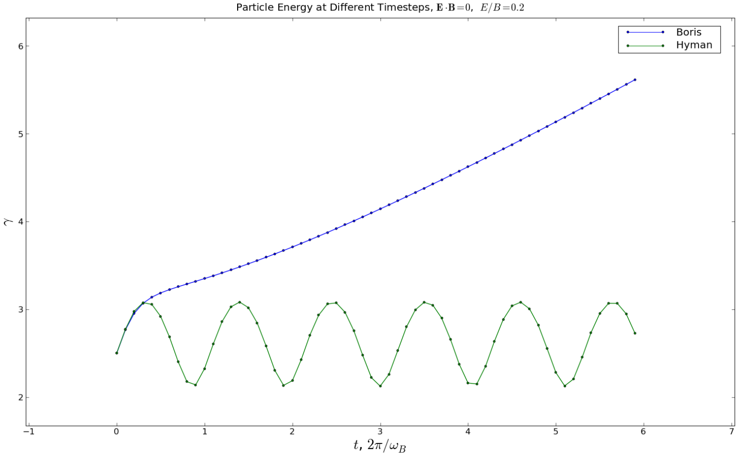
$$x^{\mu}(\tau) = x^{\mu}(0) + {f^{\mu}}_{\lambda}(\tau) u^{\lambda} \tag{5}$$

- Exact description of particle trajectory at any future time
- Need to invert Equation (5) to obtain $\tau(x^0)$, requires an iterative solver

Comparison of Orbit Trajectory



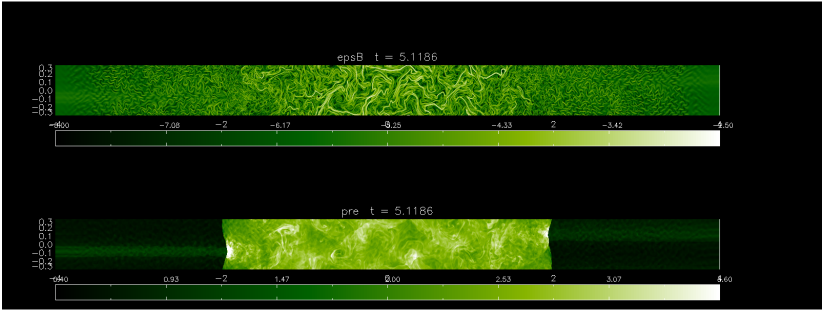
Comparison of Energy Gain



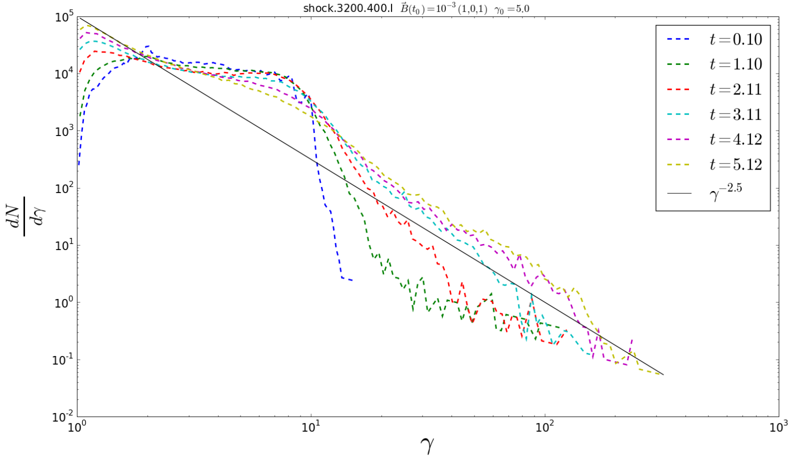
Shock Model

- Meant to be the hydrodynamical counterpart to plasma model described by Sironi and Spitkovsky (2009)
- 2 dimensional domain with converging flow along x-direction
- Equivalent to fluid moving towards a reflecting wall
- Shock propagates outward from the discontinuity
- Turbulence is created by stirring the fluid
  - Velocity perturbations are added at every time step
  - Only within a distance $c \times t$ of the initial discontinuity
  - Weak turbulence upstream of the shock resembles the precurser observed in PIC simulations
  - Strong turbulence downstream in shocked fluid

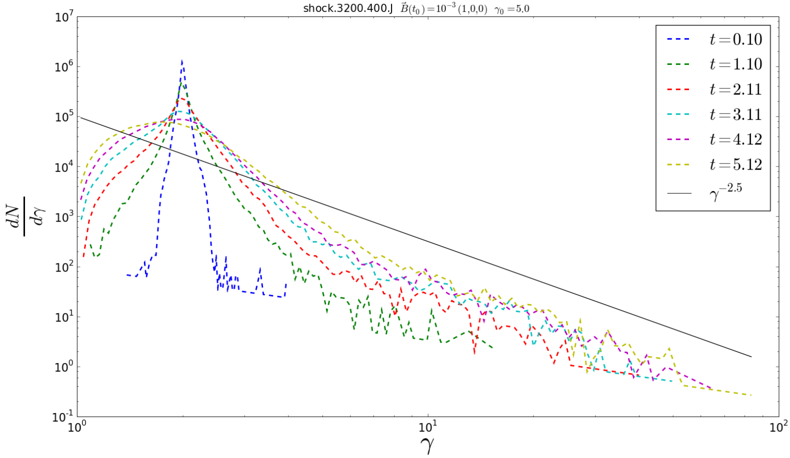
What the Shock Looks Like



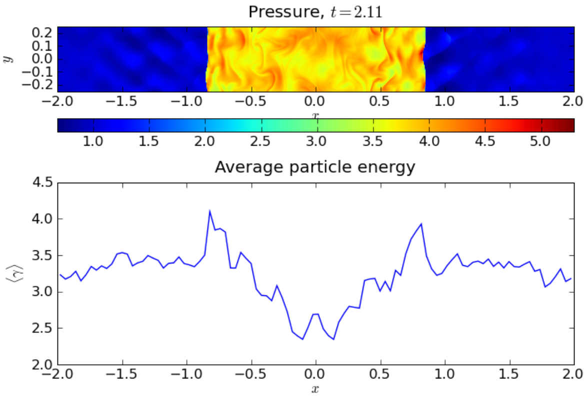
Magnetic Field in Shock Plane



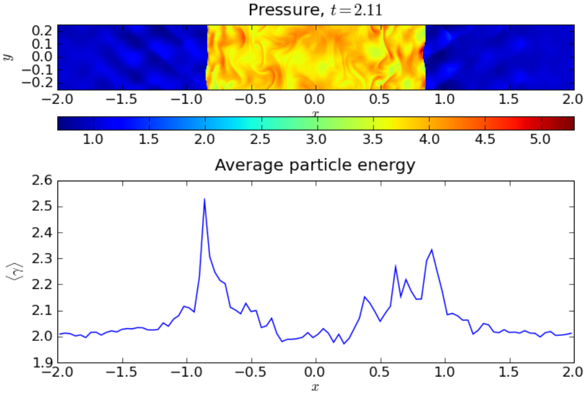
No Magnetic Field in Shock Plane

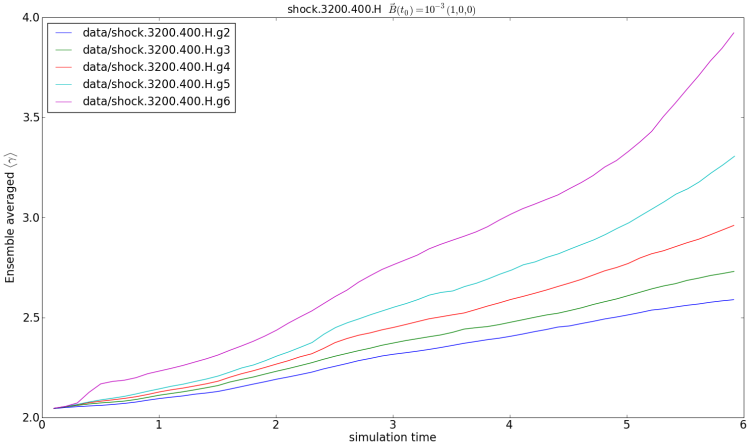


Magnetic Field in Shock Plane



No Magnetic Field in Shock Plane

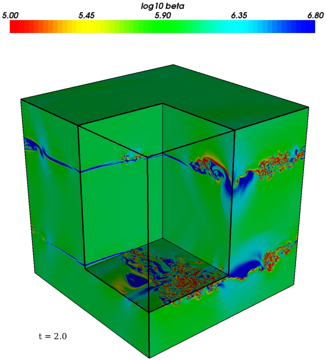



shock.3200.400.H $\vec{B}(t_0) = 10^{-3}(1,0,0)$
data/shock.3200.400.H.g2
data/shock.3200.400.H.g3
data/shock.3200.400.H.g4
data/shock.3200.400.H.g5
data/shock.3200.400.H.g6
Ensemble averaged $\langle\gamma\rangle$
simulation time
4.0
3.5
3.0
2.5
2.0
0
1
2
3
4
5
6

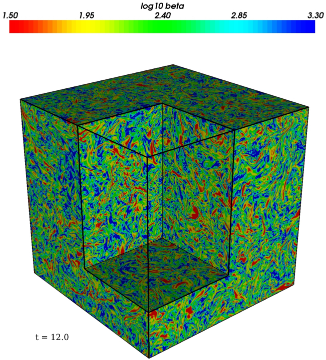
Turbulence Model

- Described in Zhang et al. (2009)
- Tangential velocity discontinuity in periodic 3 dimensional domain
- Kelvin-Helmholtz instability is excited at the shearing layer
- Turbulence cascade from large to small scales until saturation
- Turbulence decays as it slowly loses kinetic energy to grid viscosity

What Kelvin-Helmholtz Looks Like

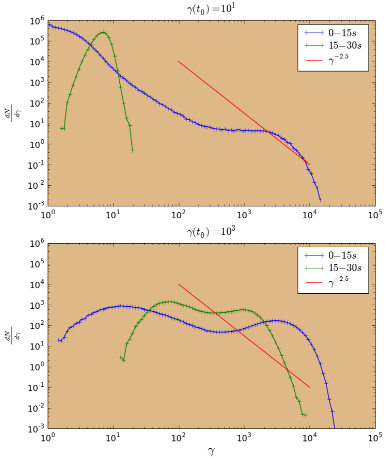


Wind blowing over water creates waves in ponds, lakes and oceans. This process is the same as what causes the ripples you see when you blow over a cup of coffee or tea and is the result of a fluid behaviour called the Kelvin-Helmholtz instability. This instability causes ripples to grow at the interface between shearing fluids, taking energy from the bulk shear flow and eventually generating turbulence as the ripples grow and become non-linear. The pictures above are from a computer simulation of the Kelvin-Helmholtz instability and resulting turbulence in a relativistic magnetized fluid performed at NYU (on the 2nd floor of Meyer Hall).



The color represents the strength of magnetic field in the box with red representing stronger field. The simulation shows the amplification of a weak magnetic field due to the stretching and folding of field lines by the turbulent velocity field generated by the instability. The resulting amplified magnetic field allows for the generation of synchrotron radiation by spinning relativistic electrons. This radiation is thought to illiminate astronomical sources such as Active Galactic Nuclei and Gamma-ray Bursts which can shine as brightly as 100,000,000,000,000,000 stars.

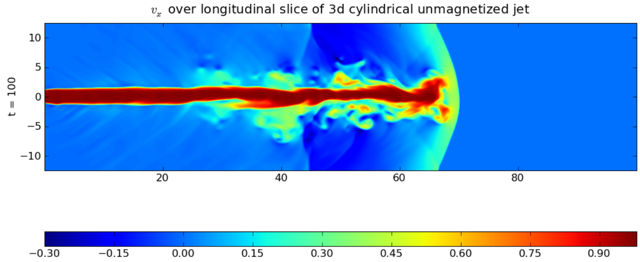
Particle Acceleration Properties



Jet Model

- Described in Mignone et al. (2009)
- Initial model consists of uniform ambient medium
- Circular jet nozzle placed on left wall consists of
  - Small differential rotation
  - Toroidal magnetic field
  - Pressure profile to ensure no momentum flux in radial direction (satisfies Bernoulli equation)
  - Perturbations added to radial velocity component

What Cylindrical Jets Look Like



Summary

- Implementing charged particles into MHD simulations is practical
  - Computationally inexpensive
  - Easy to code
  - Numerically reliable
- Power-law type distributions in particle energy are naturally realized
- The efficiency of Diffusive Shock Acceleration can be directly observed to increase with higher velocities of the bulk flow

That's it!

- Thank you for listening
- Questions / discussion welcome

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