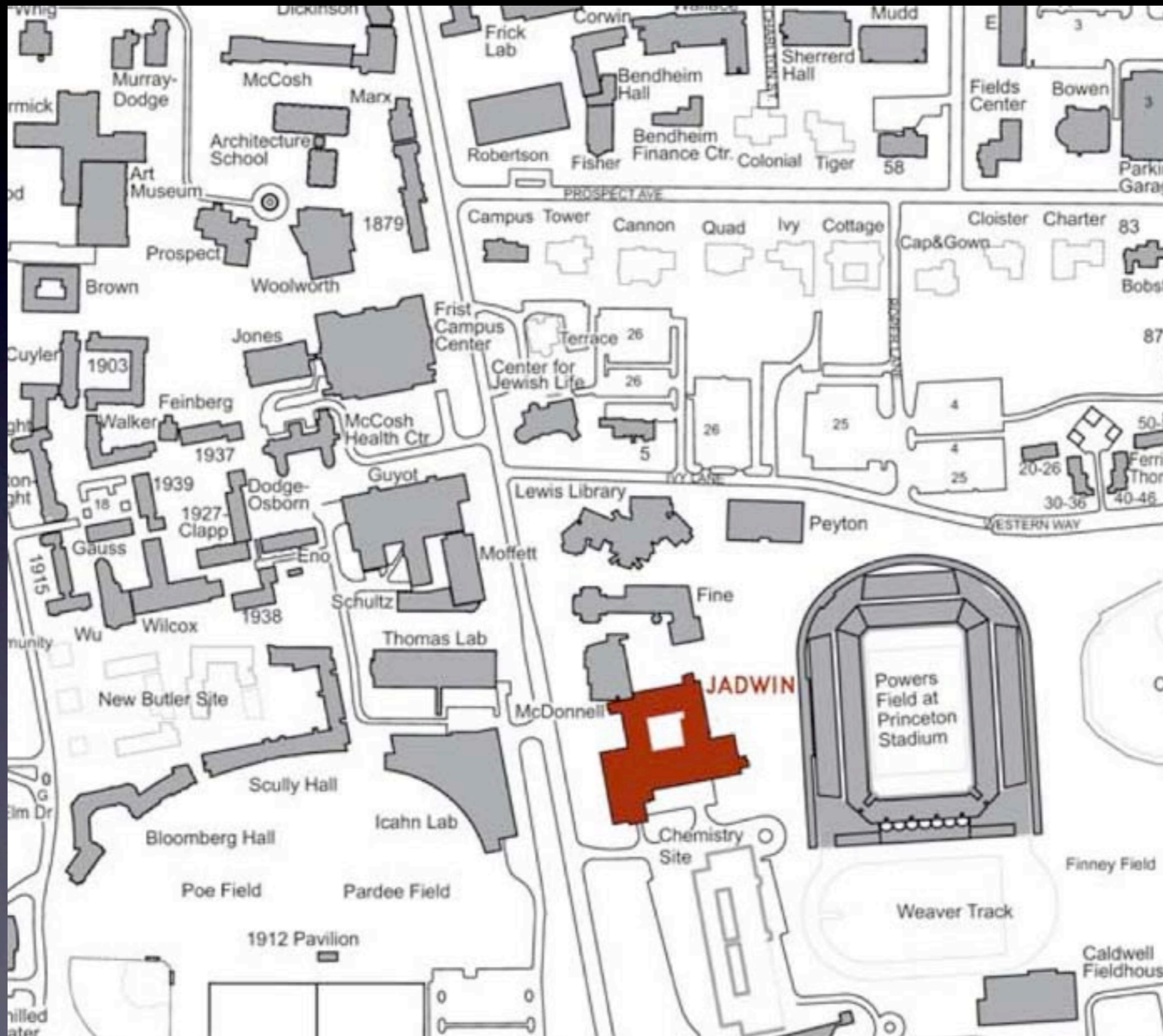
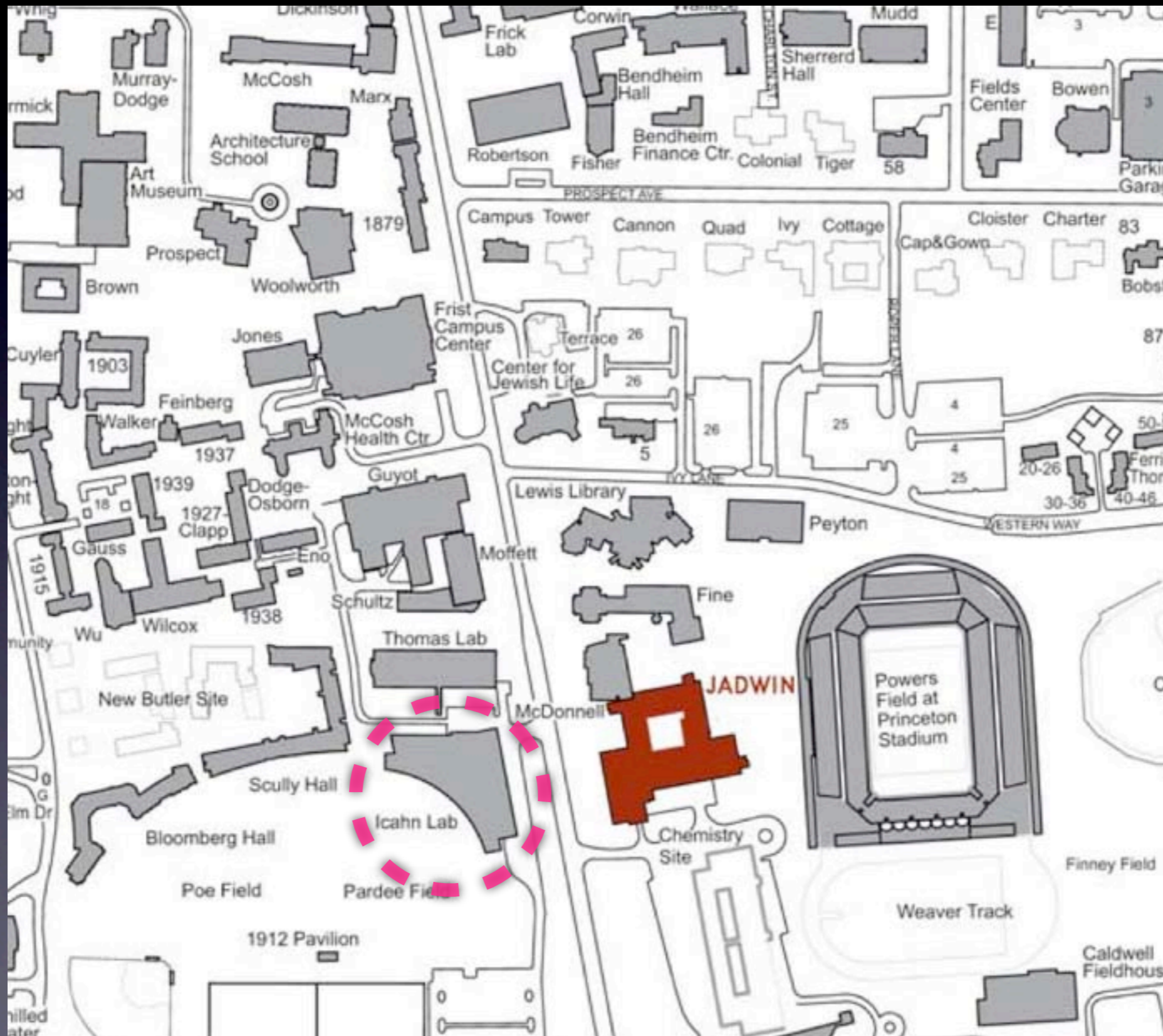


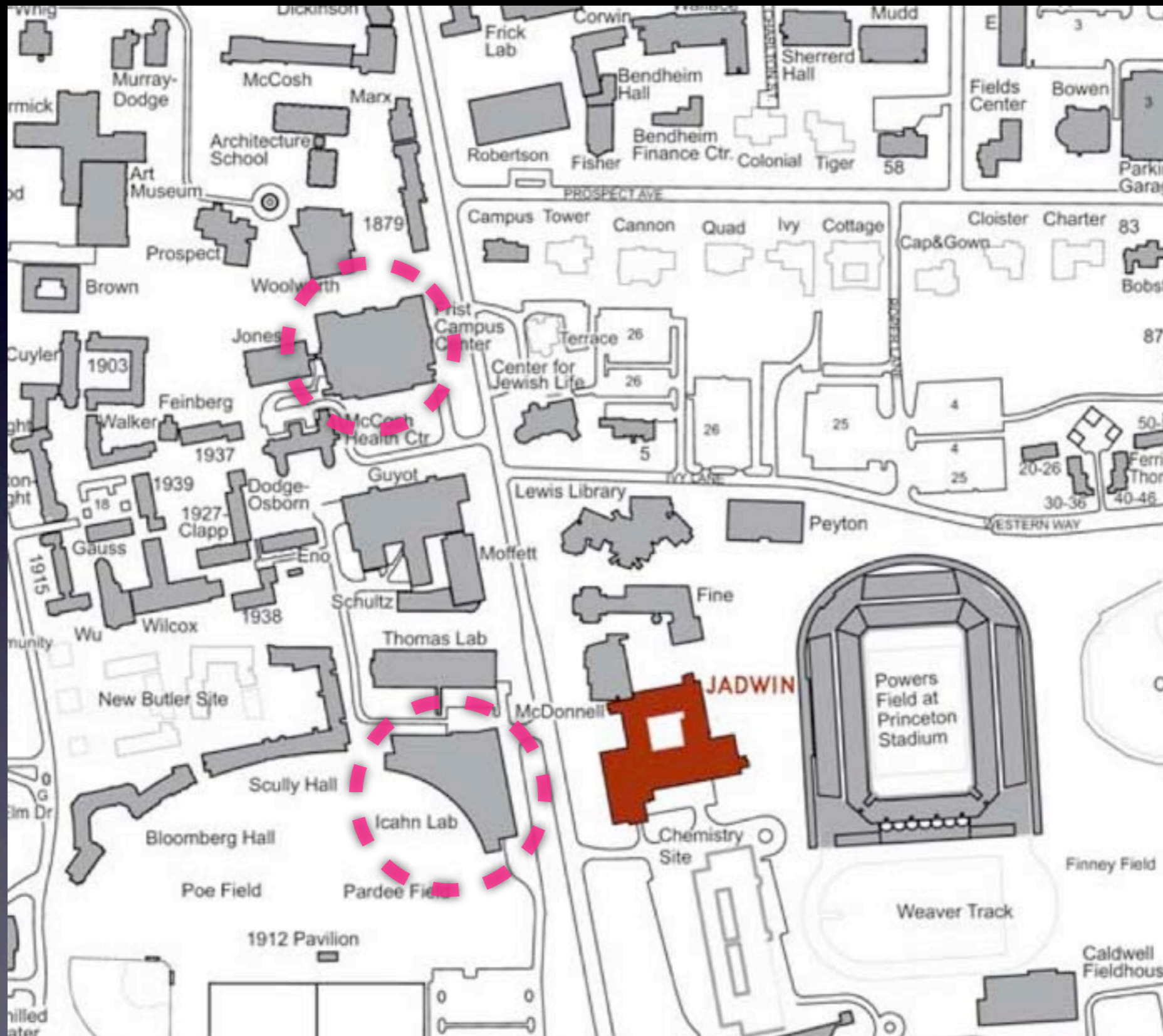
Welcome to Princeton!



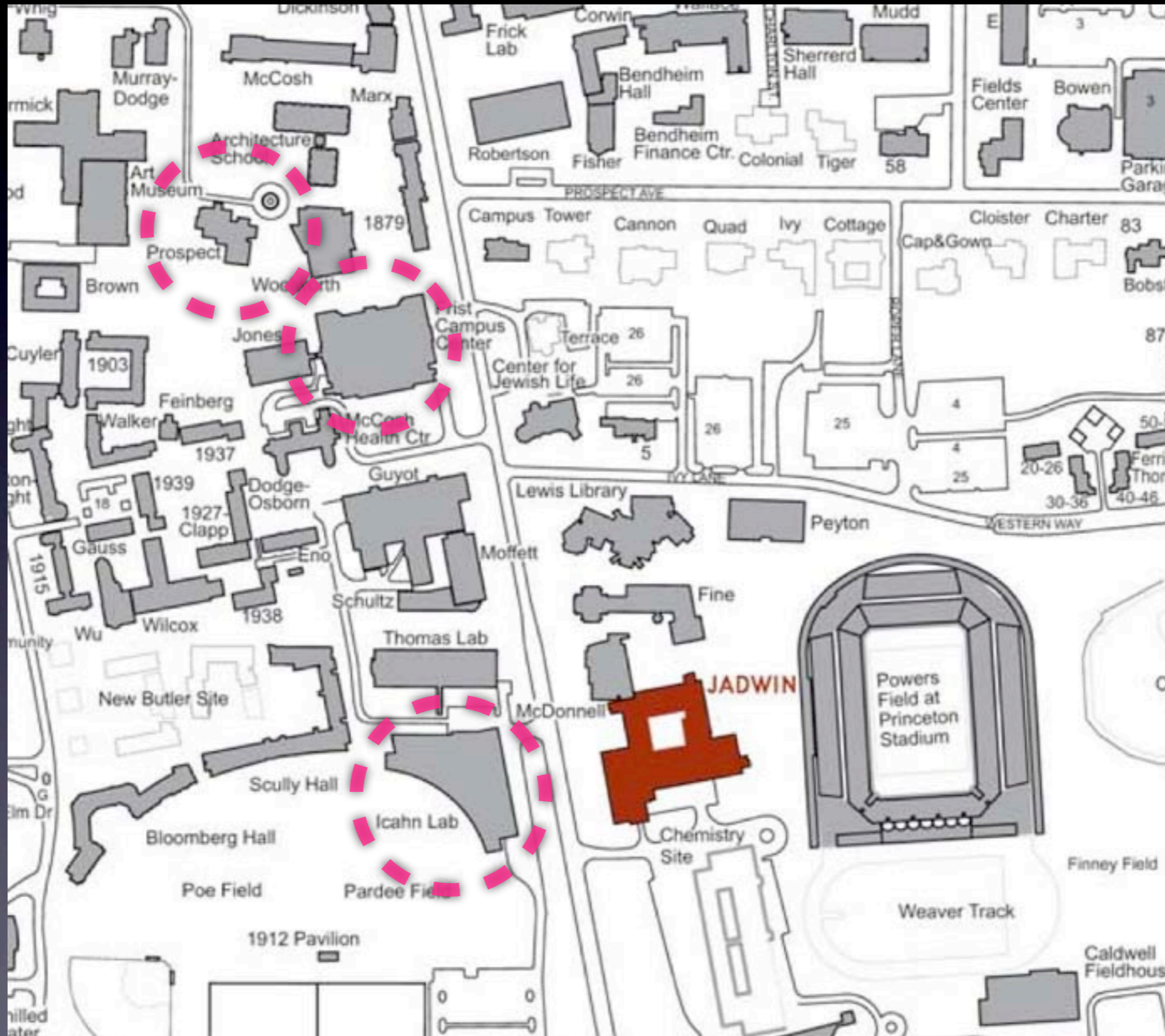
Welcome to Princeton!



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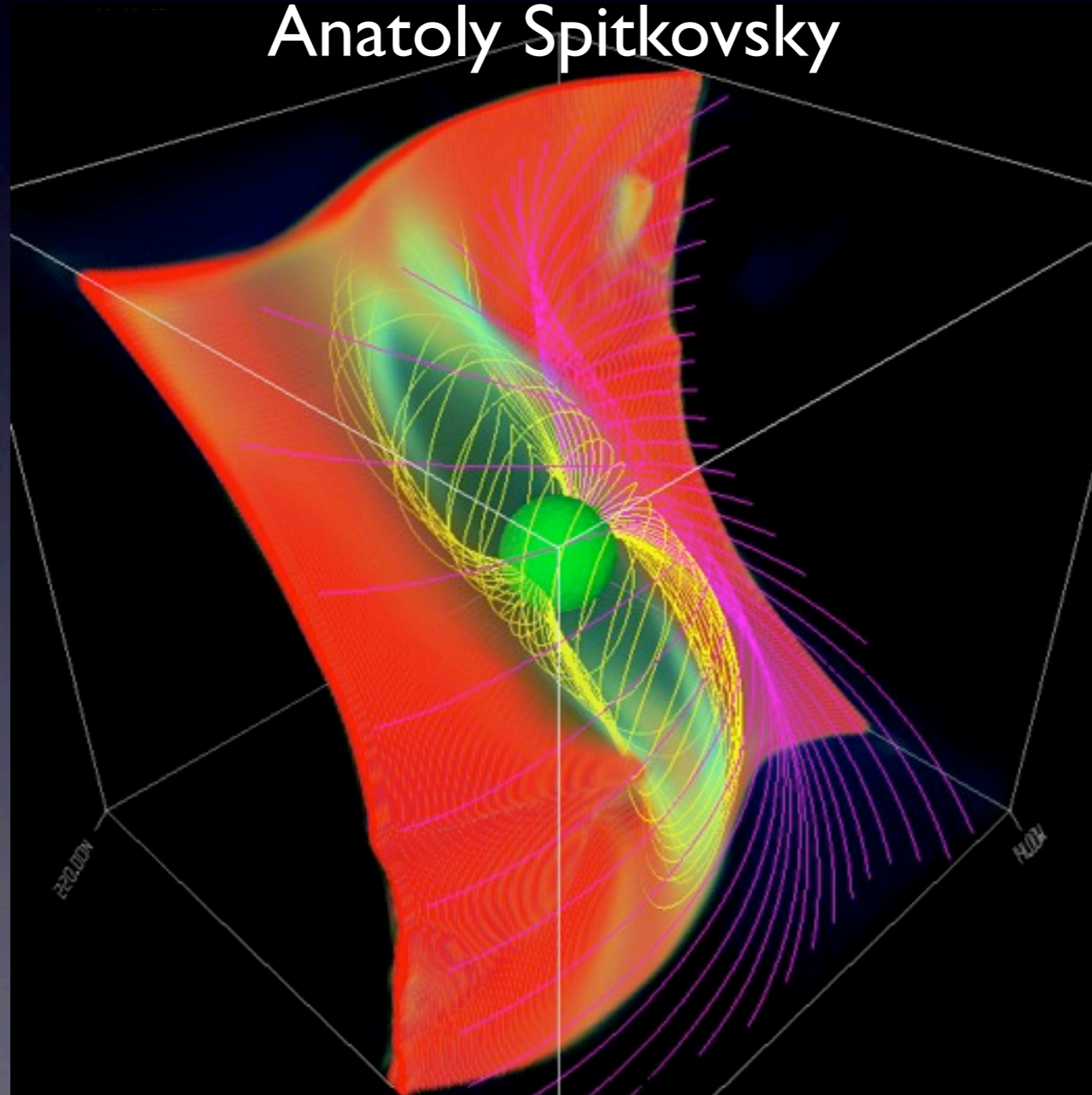


Welcome to Princeton!



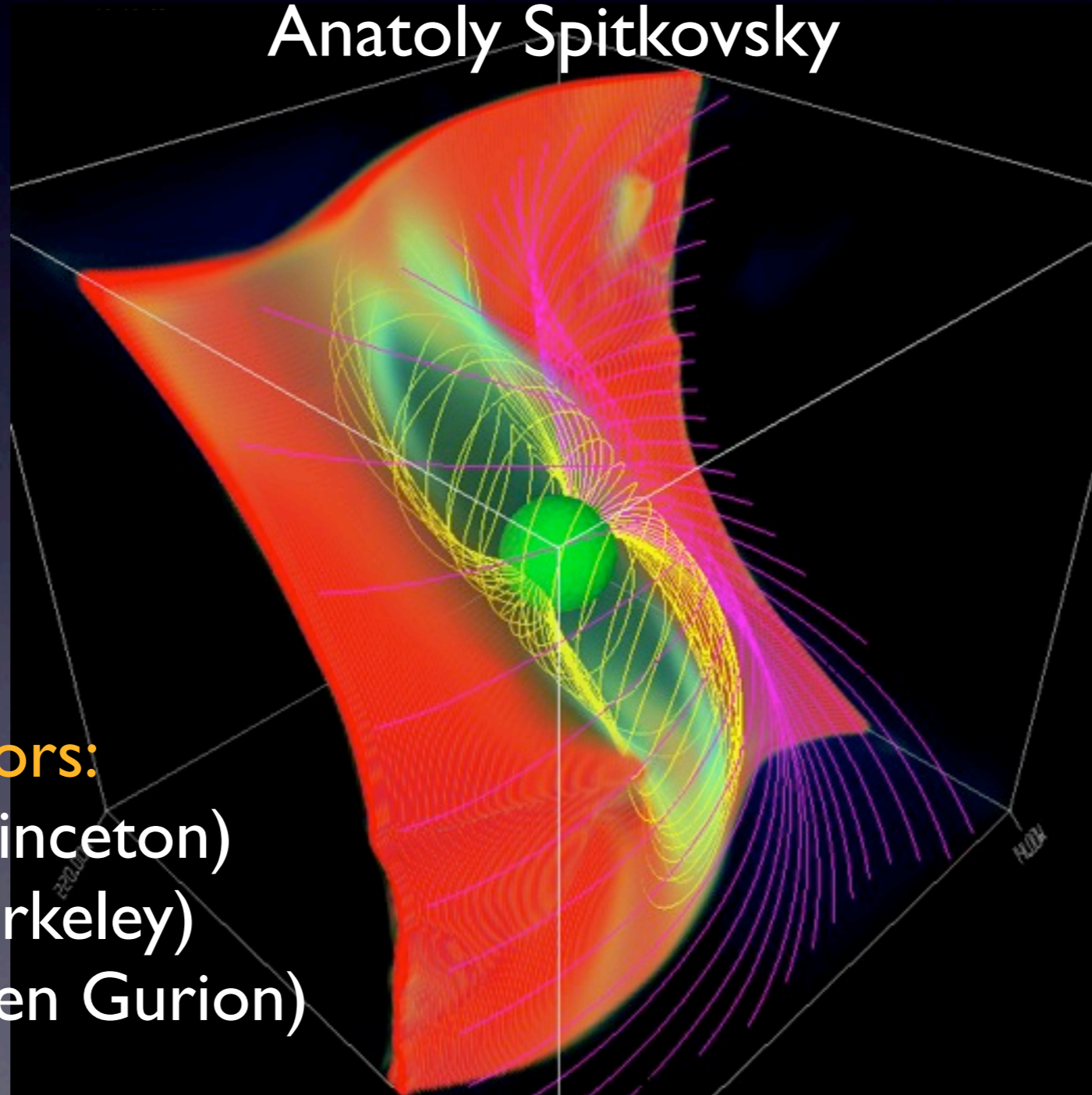
Evolution of magnetized environments: force-free zoology

Anatoly Spitkovsky



Evolution of magnetized environments: force-free zoology

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Collaborators:

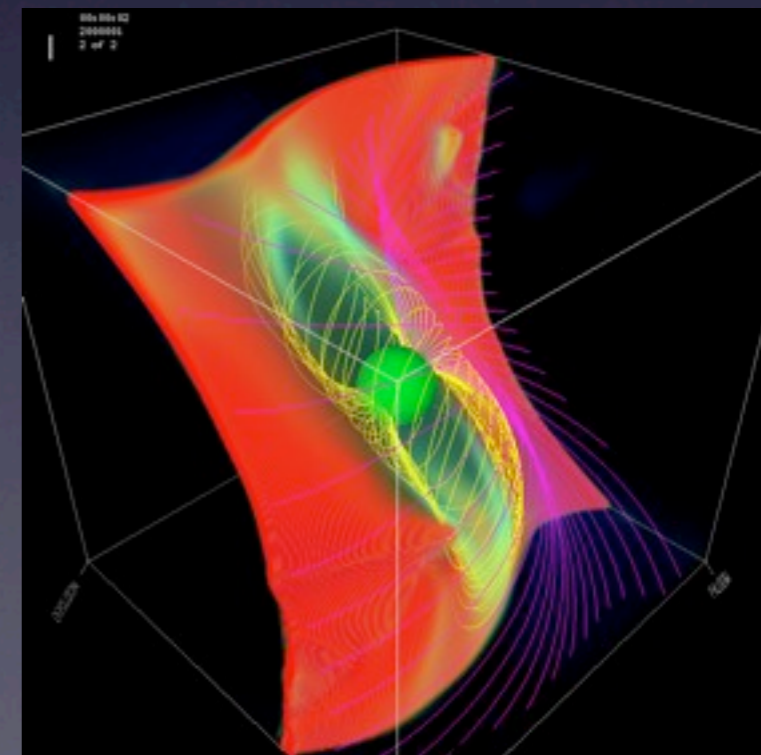
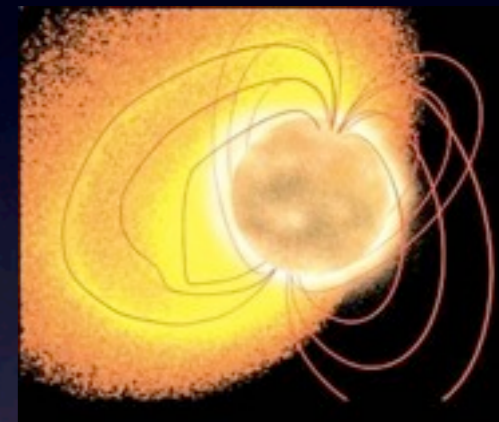
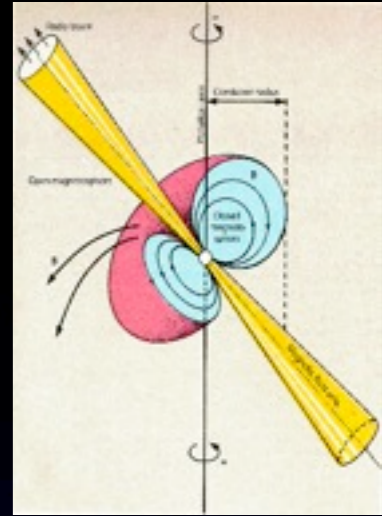
Xuening Bai (Princeton)

Jon Arons (Berkeley)

Yury Lyubarsky (Ben Gurion)

Outline

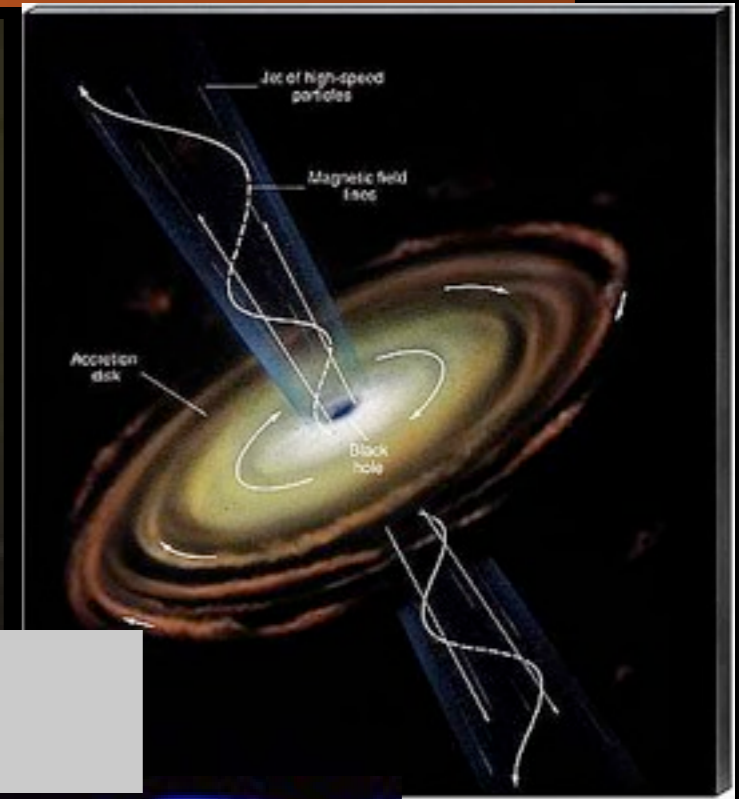
- Magnetically-dominated environments
- Strategies for modeling: force-free approximation
- Behavior of magnetized environments:
 - Pulsars, aligned and oblique
 - Bursting magnetars
 - Coronae of accretion disks
- Gamma-ray emission from pulsars: Fermi
- Conclusions



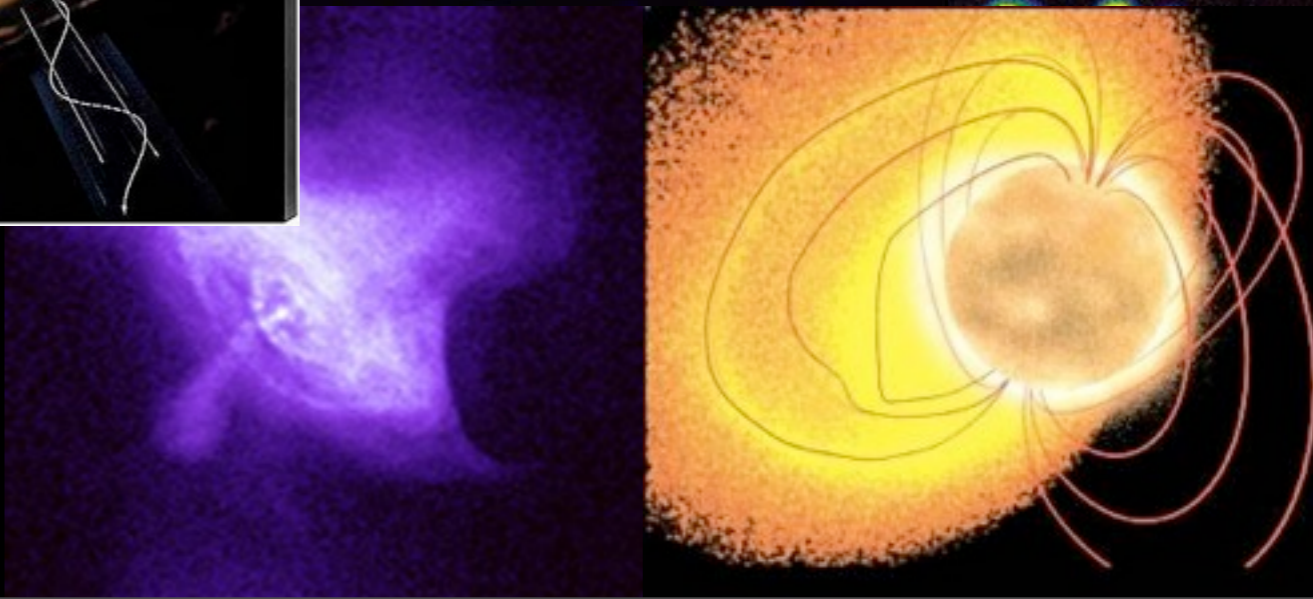
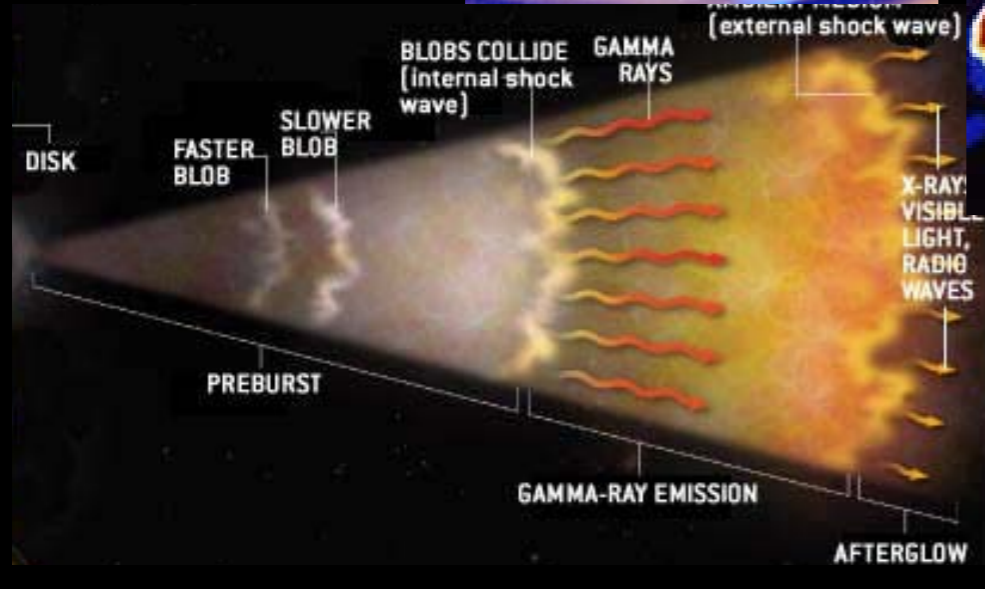
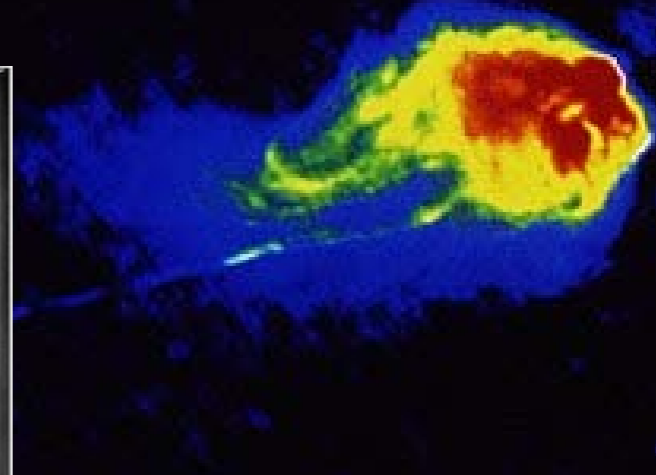
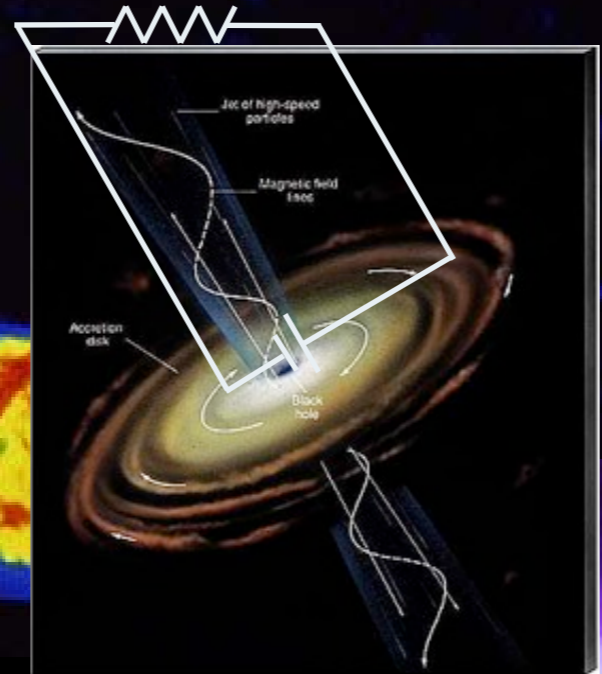
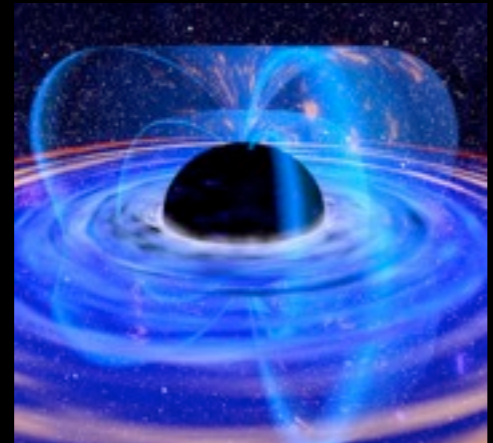
Relativistic outflows in astrophysics

Magnetically dominated environments are usually associated with relativistic flows

- Pulsars + winds, plerions ($\gamma \sim 10^6$)
- Extragalactic radio sources ($\gamma \sim 10$)
- Superluminal expansion (γ - a few)
- Black hole energy extraction
- Gamma ray bursts ($\gamma \sim 100$)
- Magnetars / AXP
- UHE CR

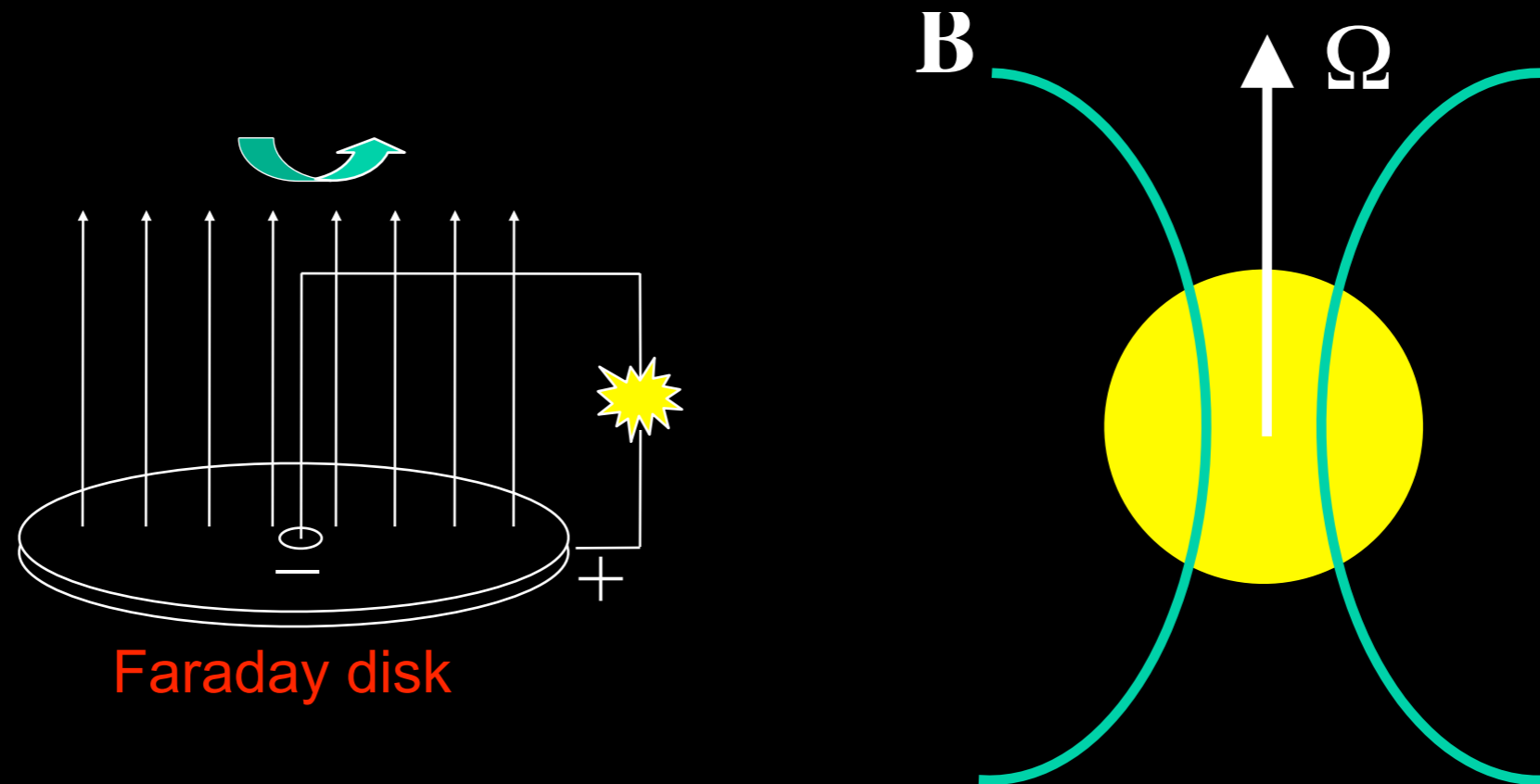


Power source -- rotating magnetized conductors.



Unipolar Induction: rotating magnetized conductors

- Alfven (1939), aka Faraday wheel
- Rule of thumb: $V \sim \Omega \Phi$; $P \sim V^2 / Z_0$



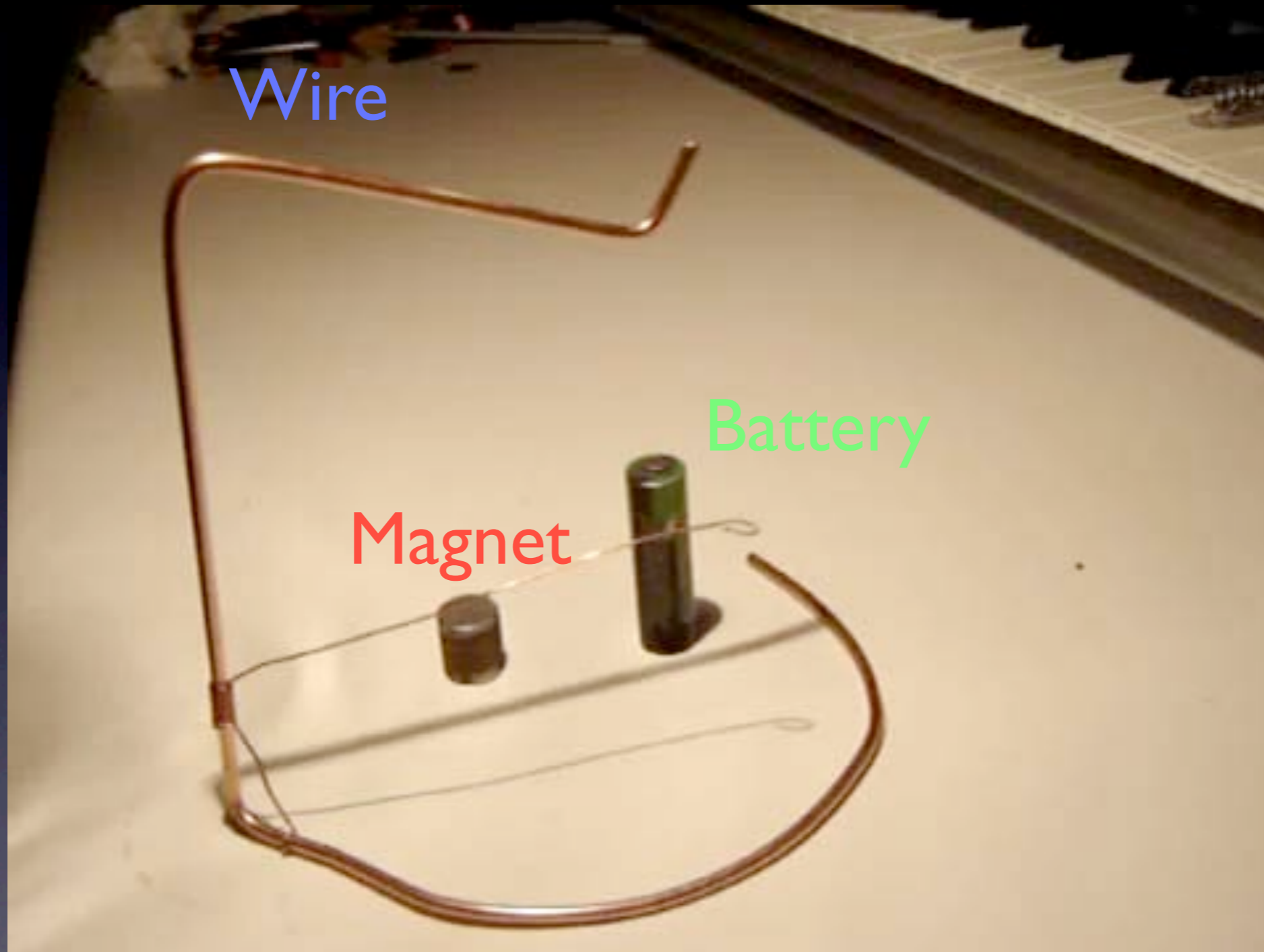
EM energy density \gg particle energy density

Energy is extracted electromagnetically: Poynting flux

Pulsar physics @ home

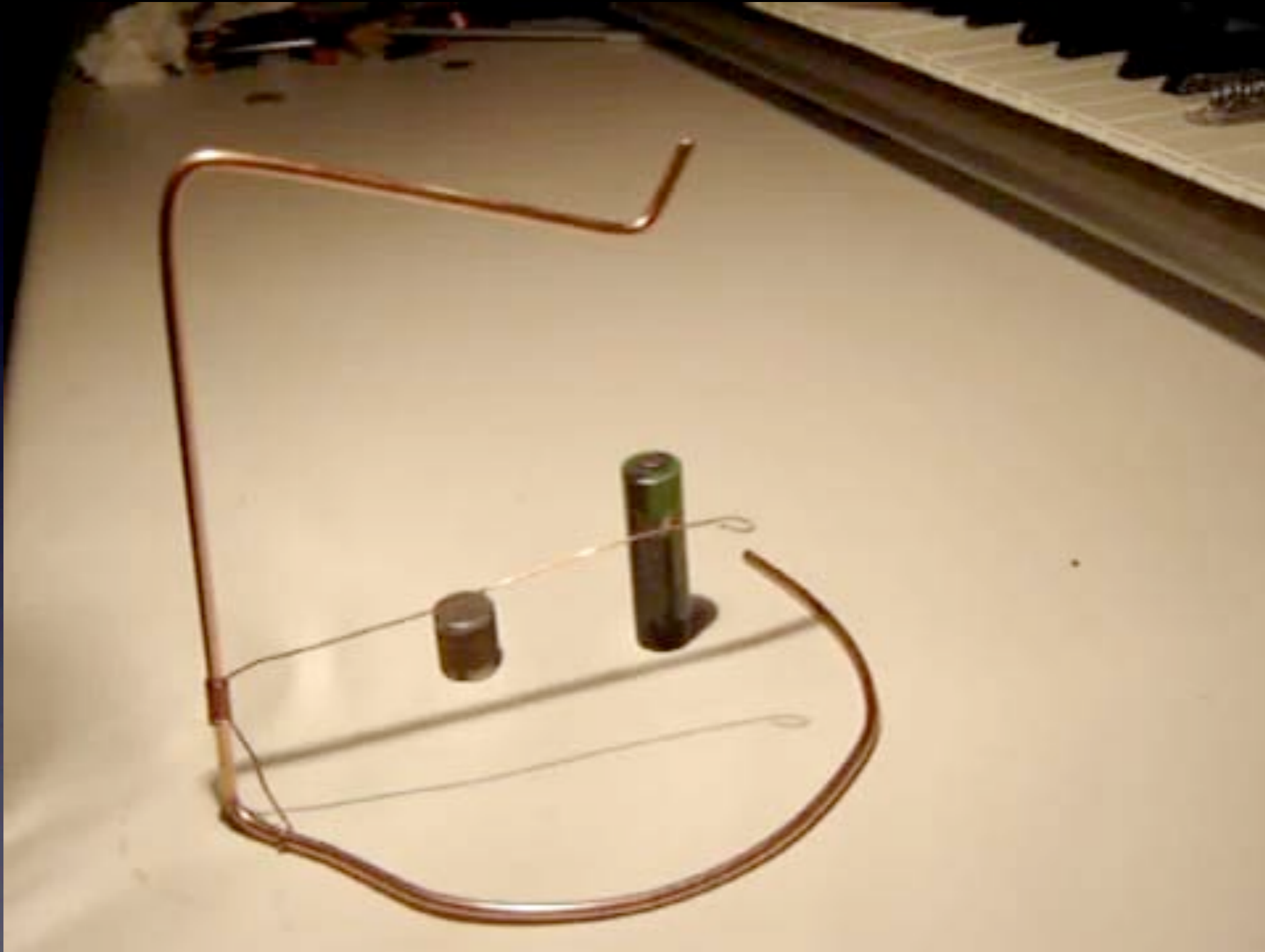
Unipolar induction

Pulsar physics @ home



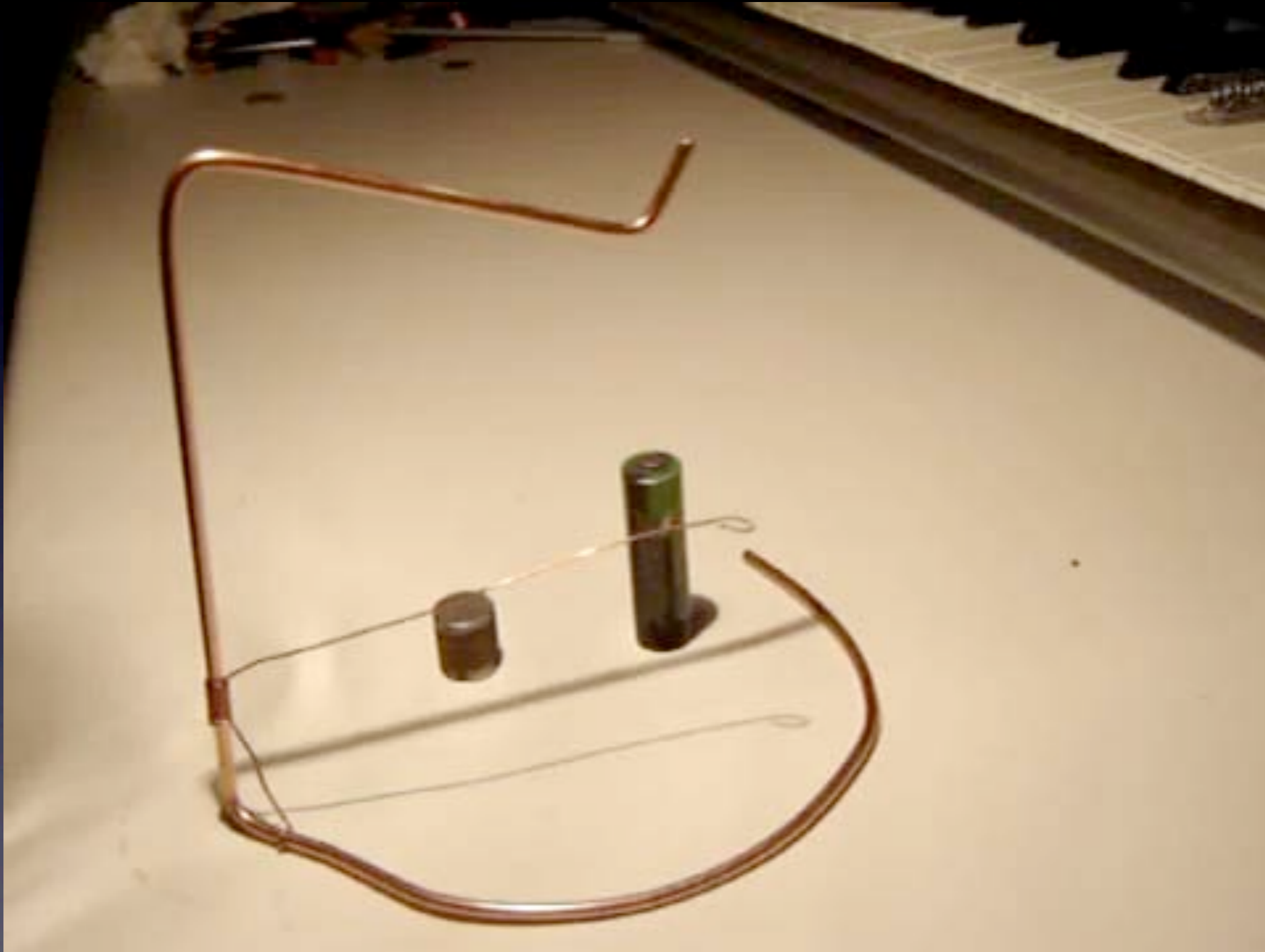
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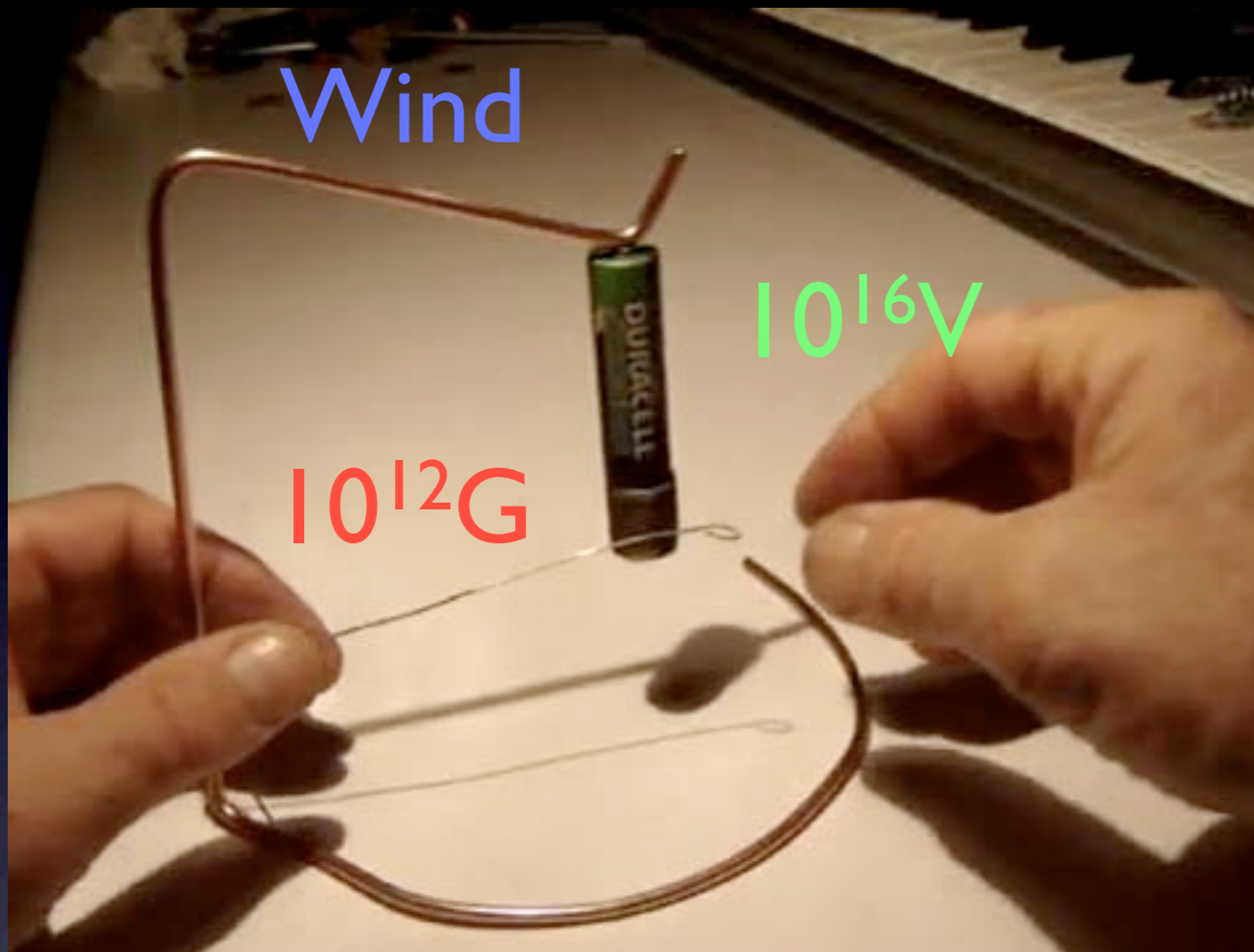
Unipolar induction

Pulsar physics @ home

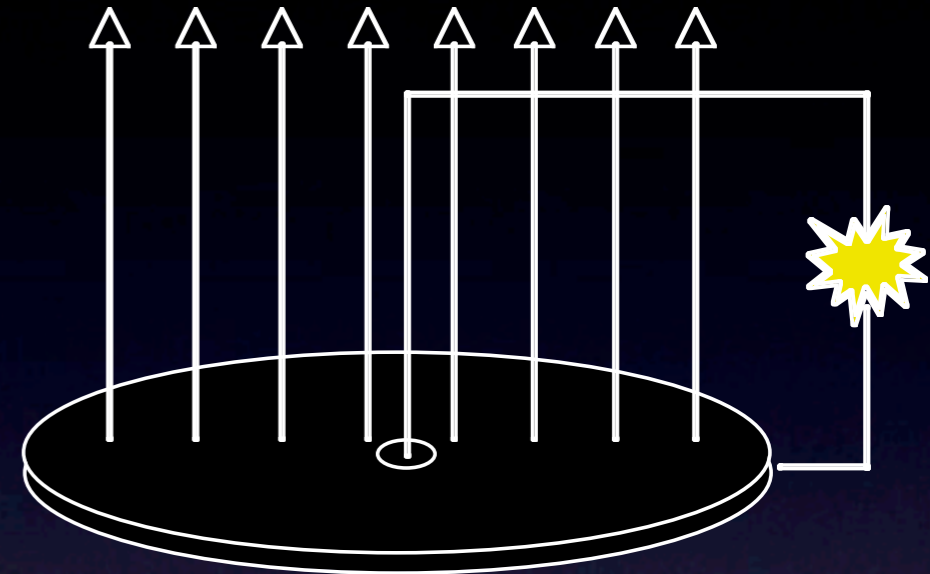


Unipolar induction

Pulsar physics in space



$$\phi_0 = \Omega B a^2 / c$$



Faraday disk

Rule of thumb: $V \sim \Omega \Phi$; $P \sim V^2 / Z_0 = I V$

Crab Pulsar

$$B \sim 10^{12} \text{ G}, \quad \Omega \sim 200 \text{ rad s}^{-1}, \quad R \sim 10 \text{ km}$$

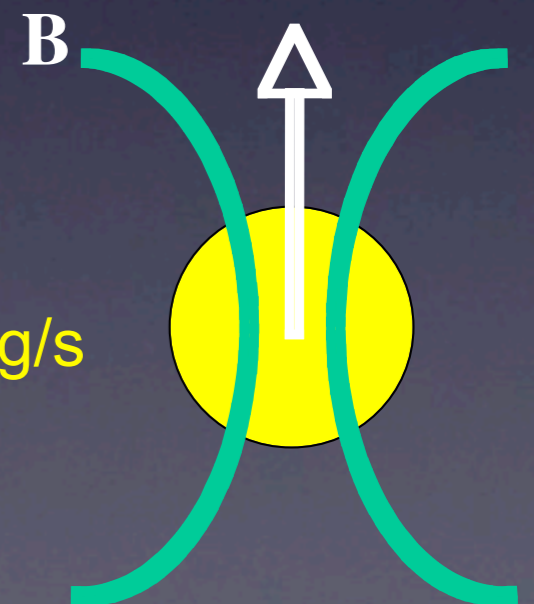
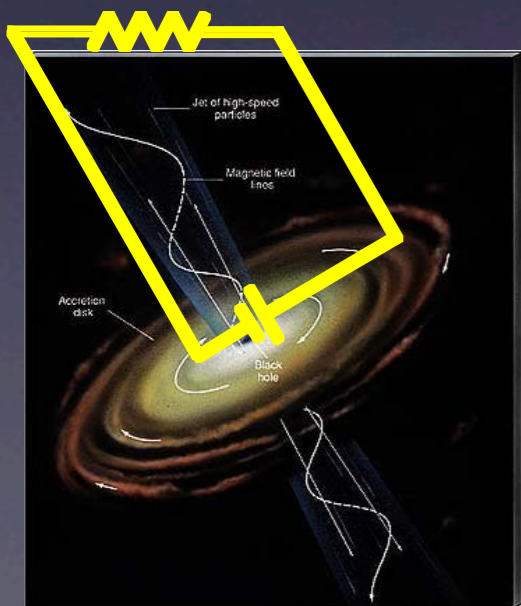
$$\text{Voltage} \sim 3 \times 10^{16} \text{ V}; \quad I \sim 3 \times 10^{14} \text{ A}; \quad P \sim 10^{38} \text{ erg/s}$$

Magnetar

$$B \sim 10^{14} \text{ G}; \quad P \sim 10^{44} \text{ erg/s}$$

Massive Black Hole in AGN

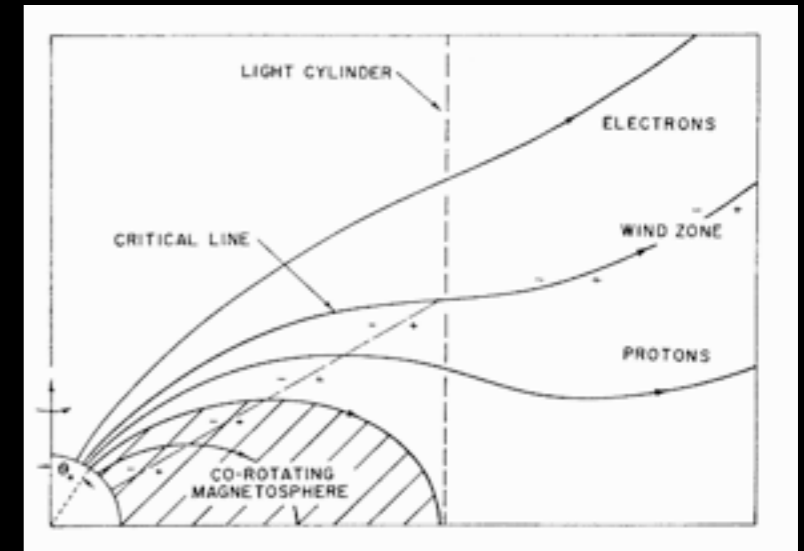
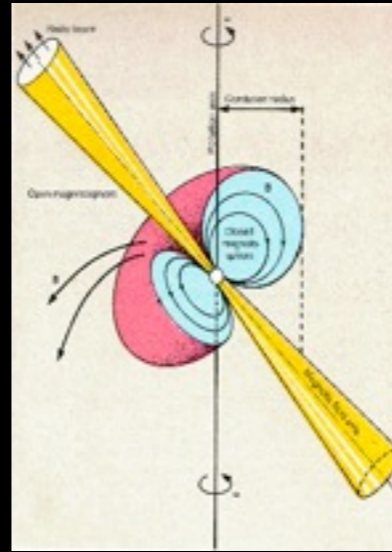
$$B \sim 10^4 \text{ G}; \quad P \sim 10^{46} \text{ erg/s}$$



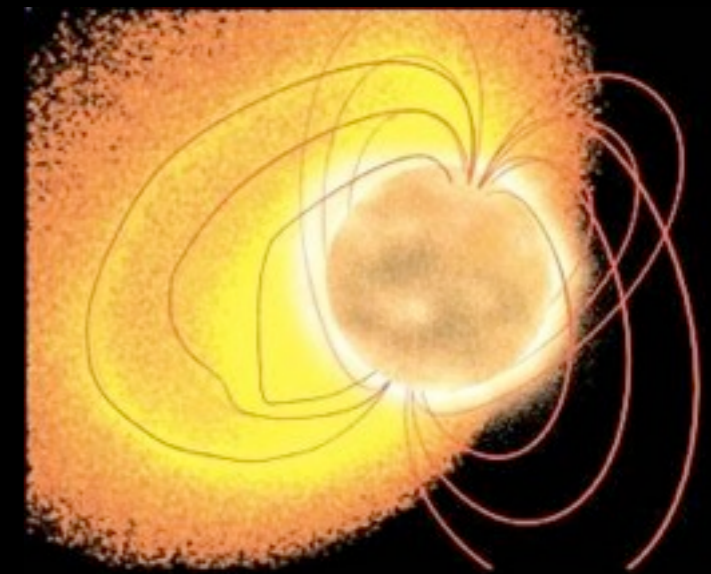
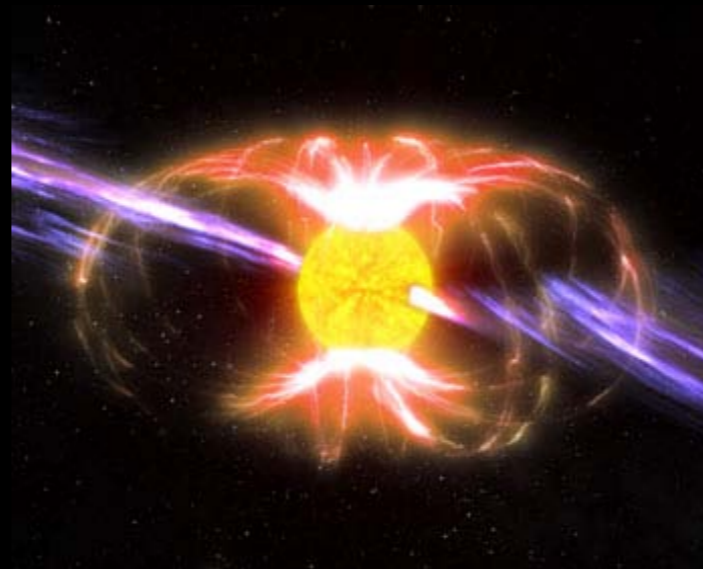
Extreme magnetospheres

A few examples:

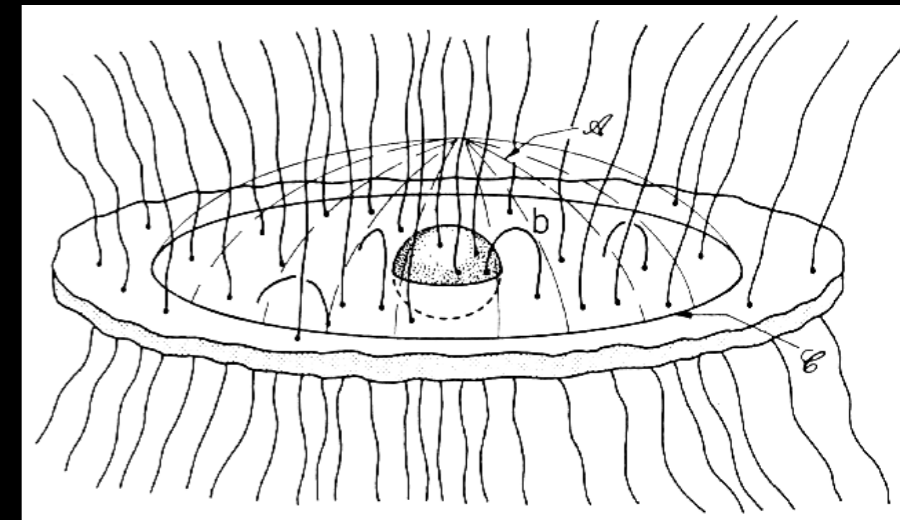
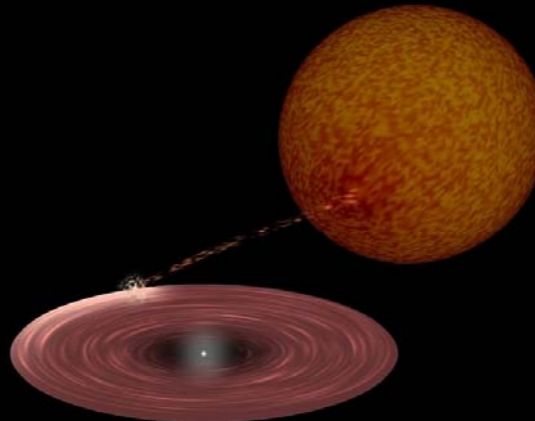
Pulsars



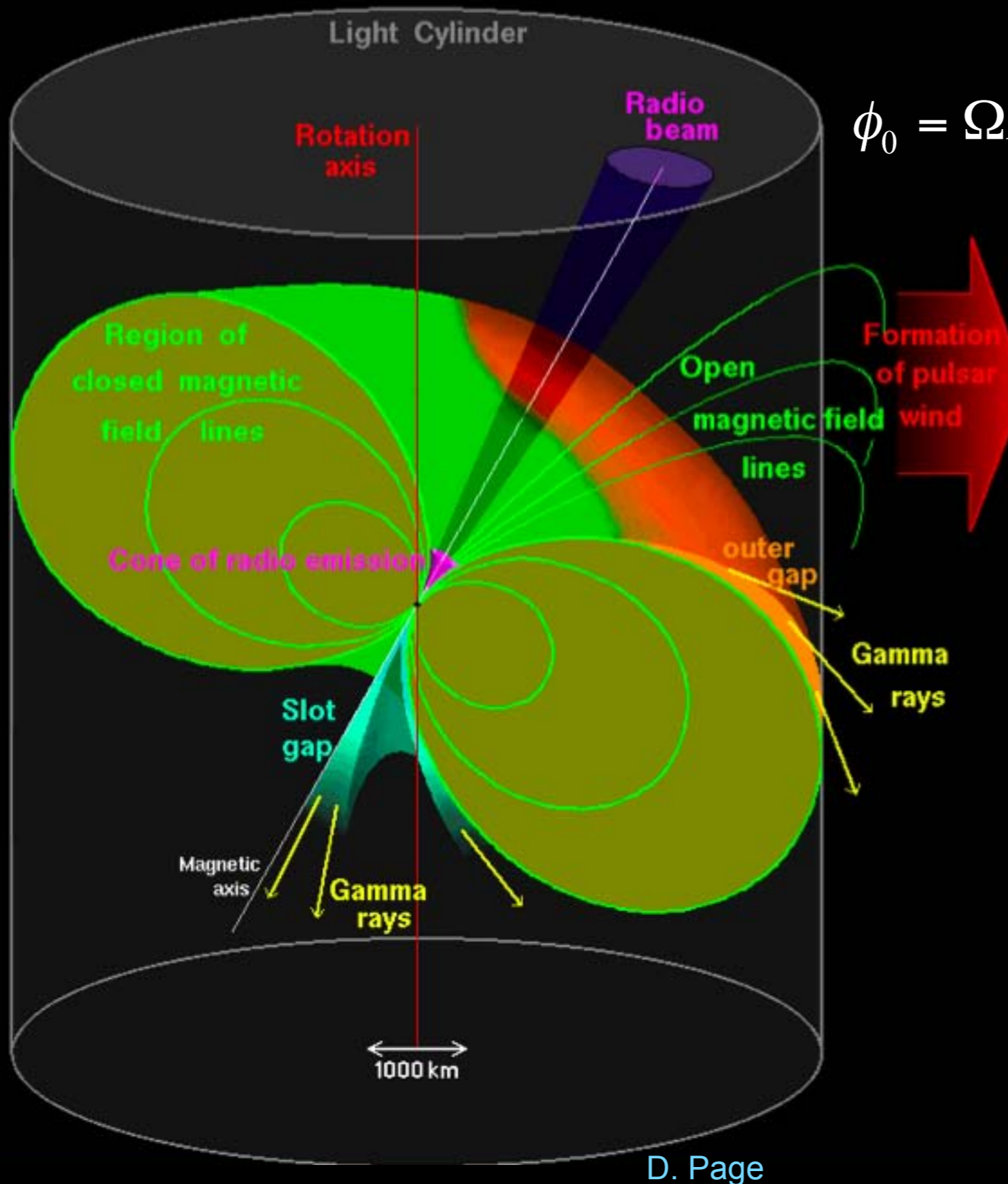
Magnetars



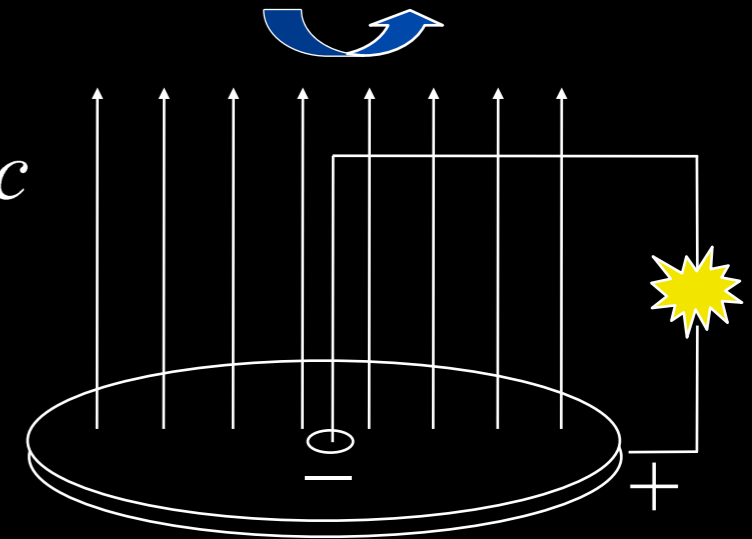
Accretion disks



Pulsar magnetosphere: what do we expect?



$$\phi_0 = \Omega B a^2 / c$$



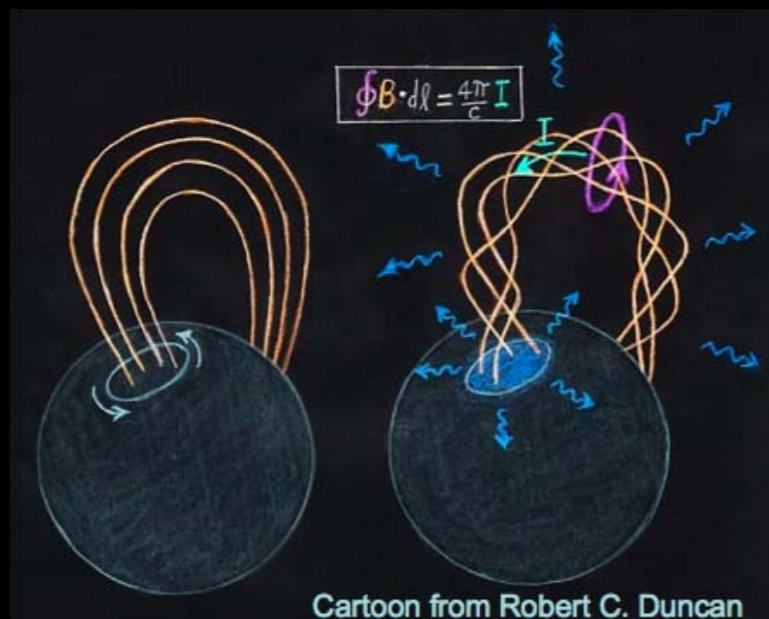
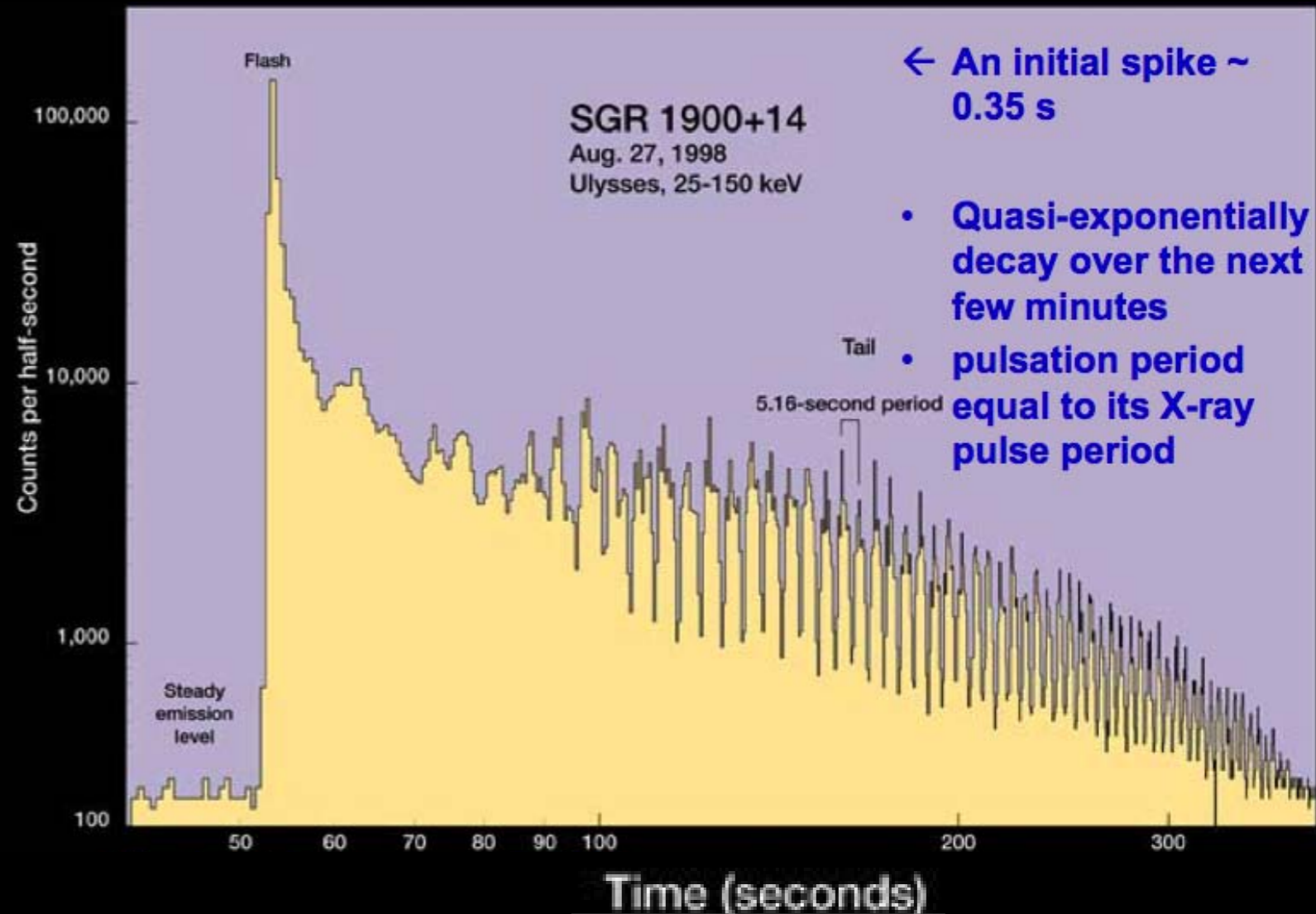
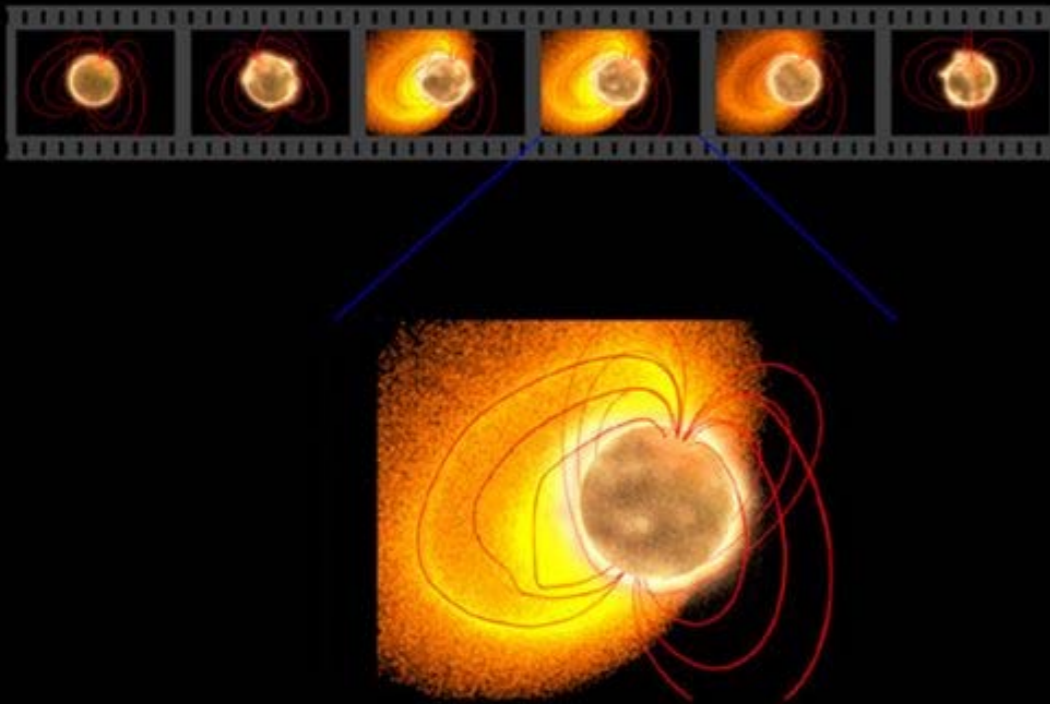
Faraday disk: unipolar induction

- *but pulsars are not in vacuum!*
- *Equator-pole potential difference ($10^{15}V$ for Crab)*
- *Charge extraction from the surface (E field \gg gravity)*
- *Strong magnetization, $\sigma > 10^4$*
- *Corotating zone; Light cylinder*
- *Throwing away toroidal field -- energy loss (Poynting flux)*
- *How do currents modify field?*

D. Page

Magnetars

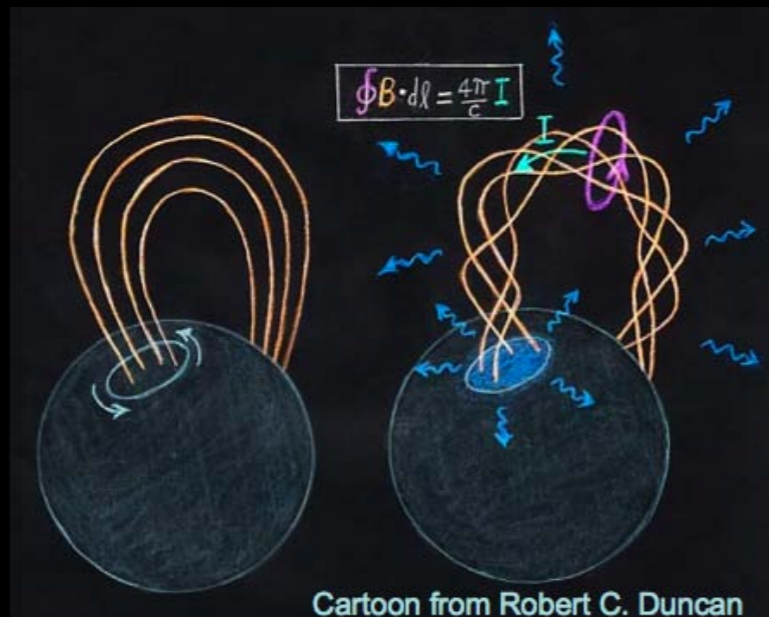
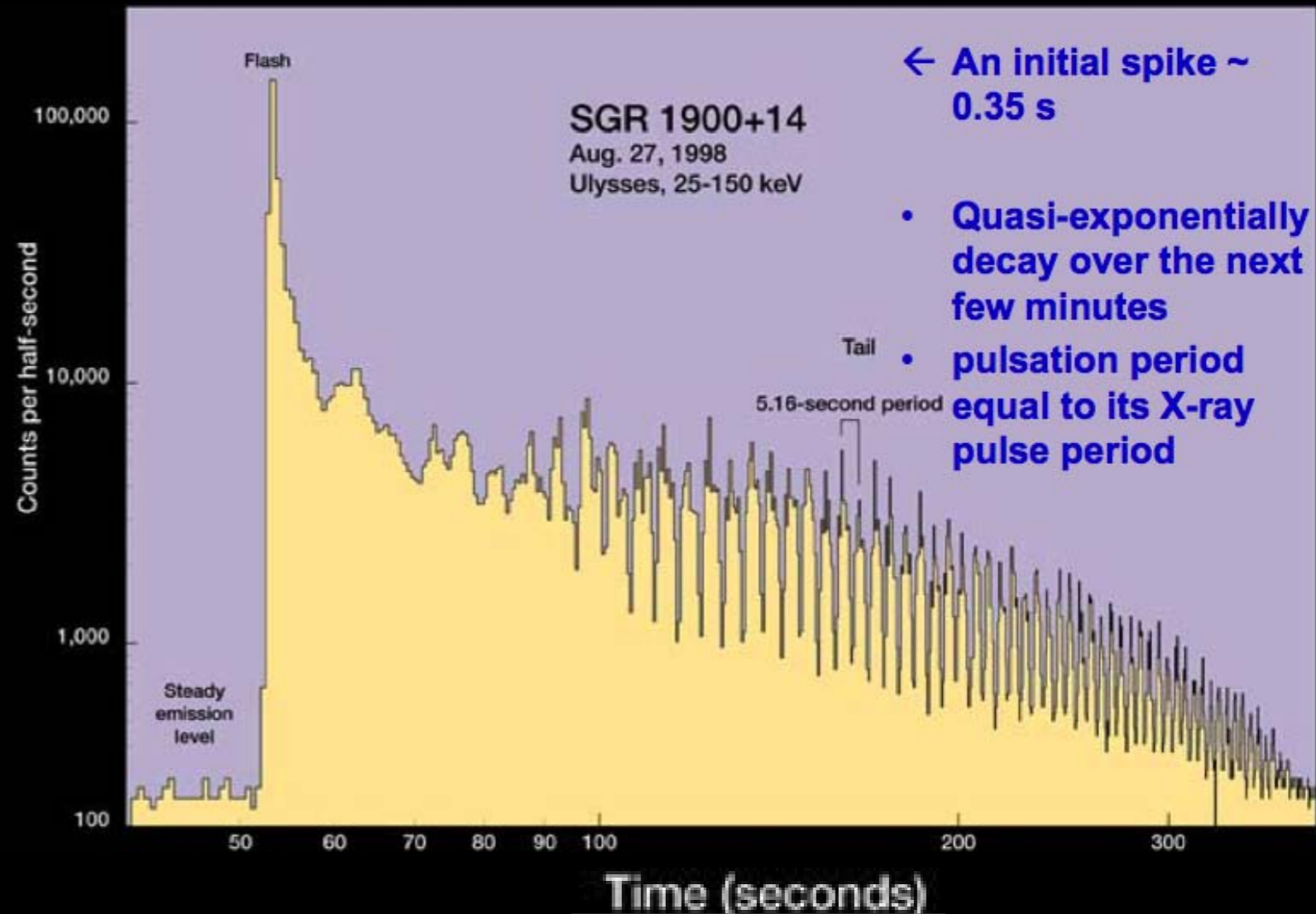
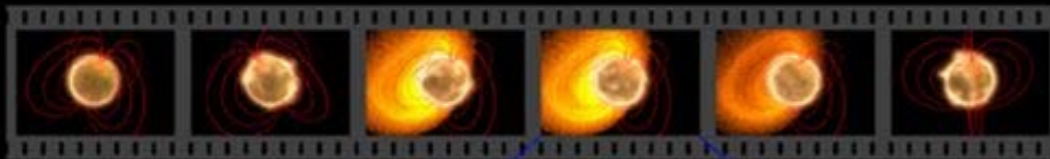
Neutron stars with 10^{15} G field, period 5-10 seconds.
Pulses or bursts of X-rays and gamma-rays ($<10^{41}$ erg/s)



Powered by B field decay. Twisted magnetosphere interpretation (Thompson & Duncan)

Magnetars

Neutron stars with 10^{15} G field, period 5-10 seconds.
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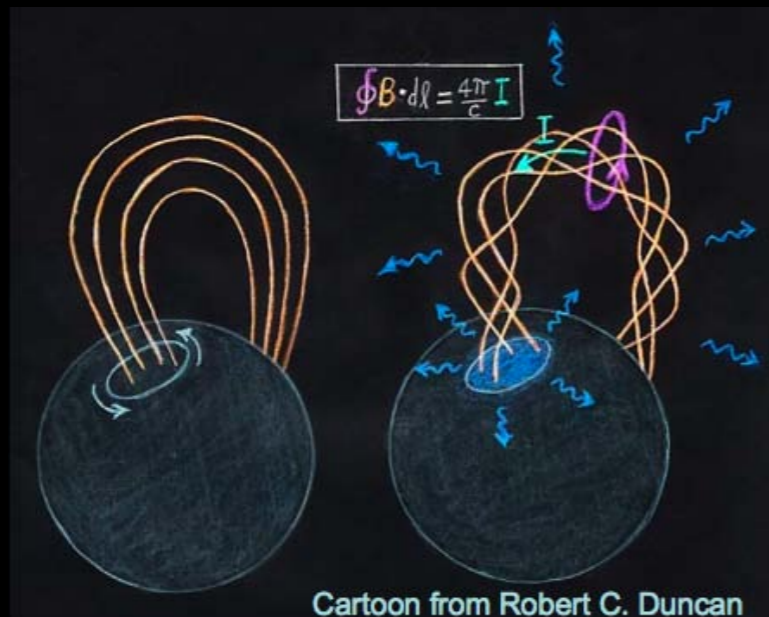
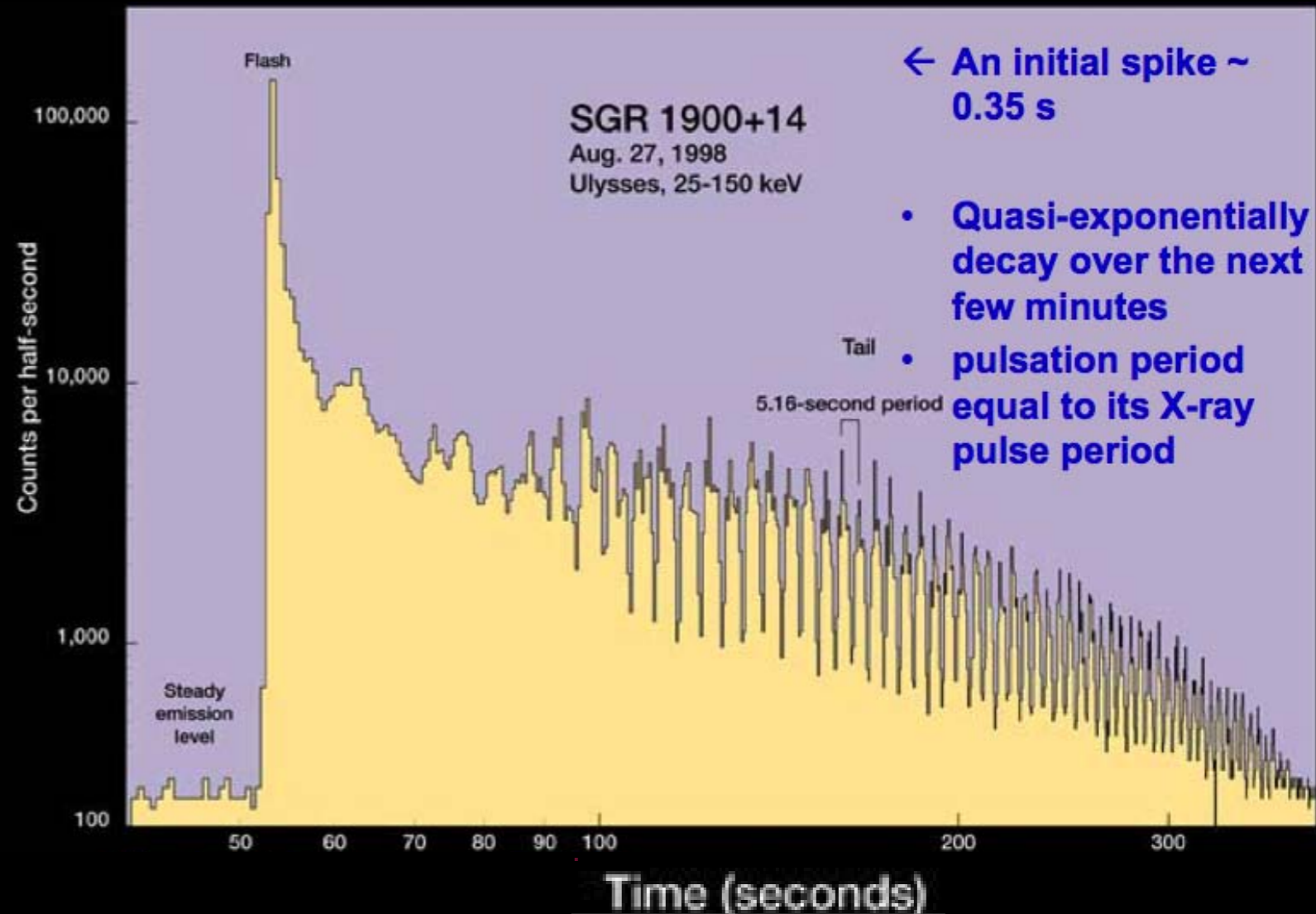
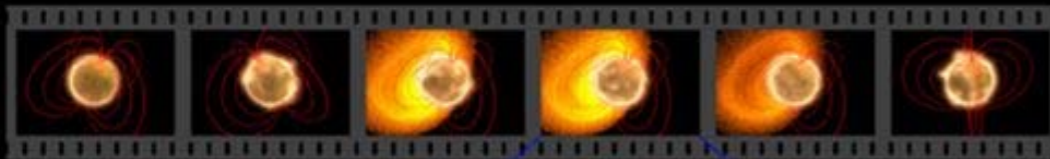


Powered by B field decay. Twisted magnetosphere interpretation (Thompson & Duncan)

What is happening in the magnetosphere?

Magnetars

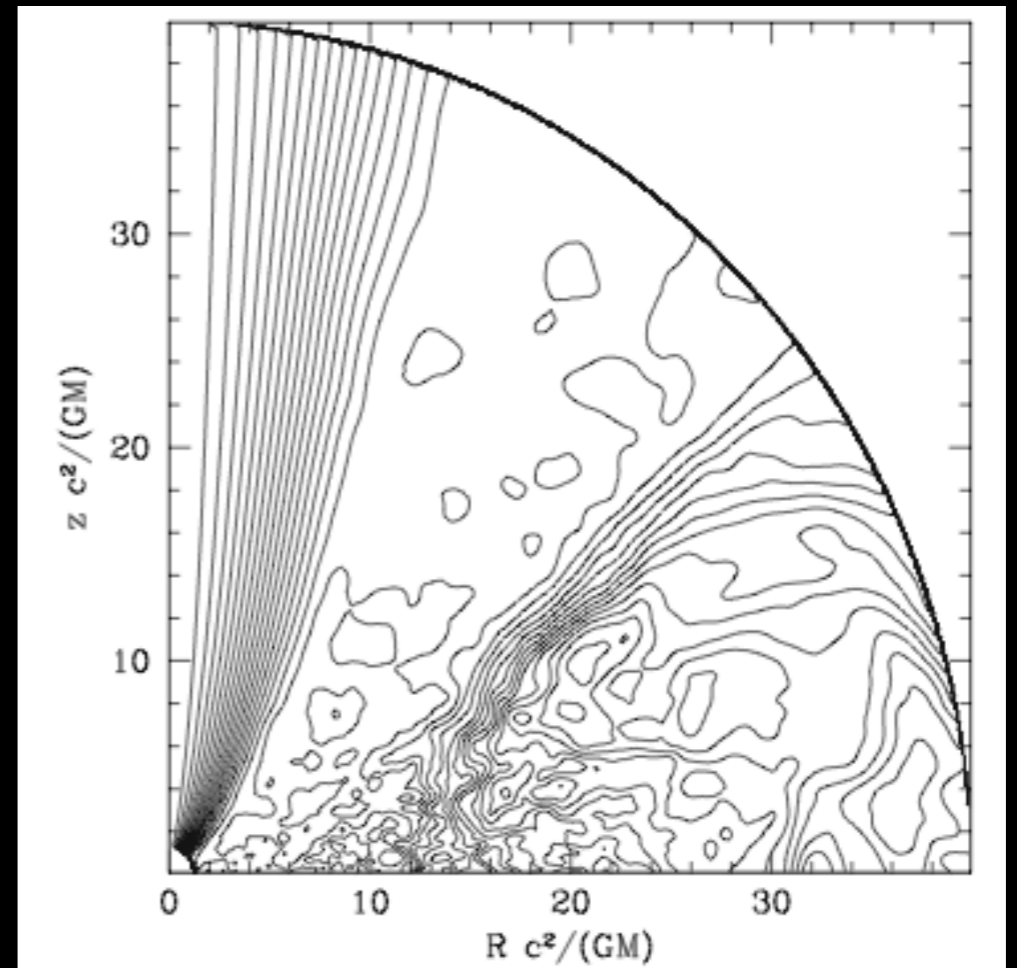
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What is happening in the magnetosphere?

Black hole-disk system

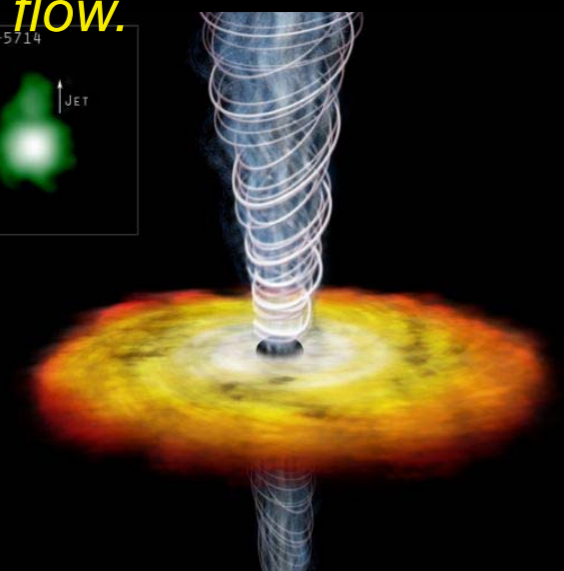


Hawley et al 02

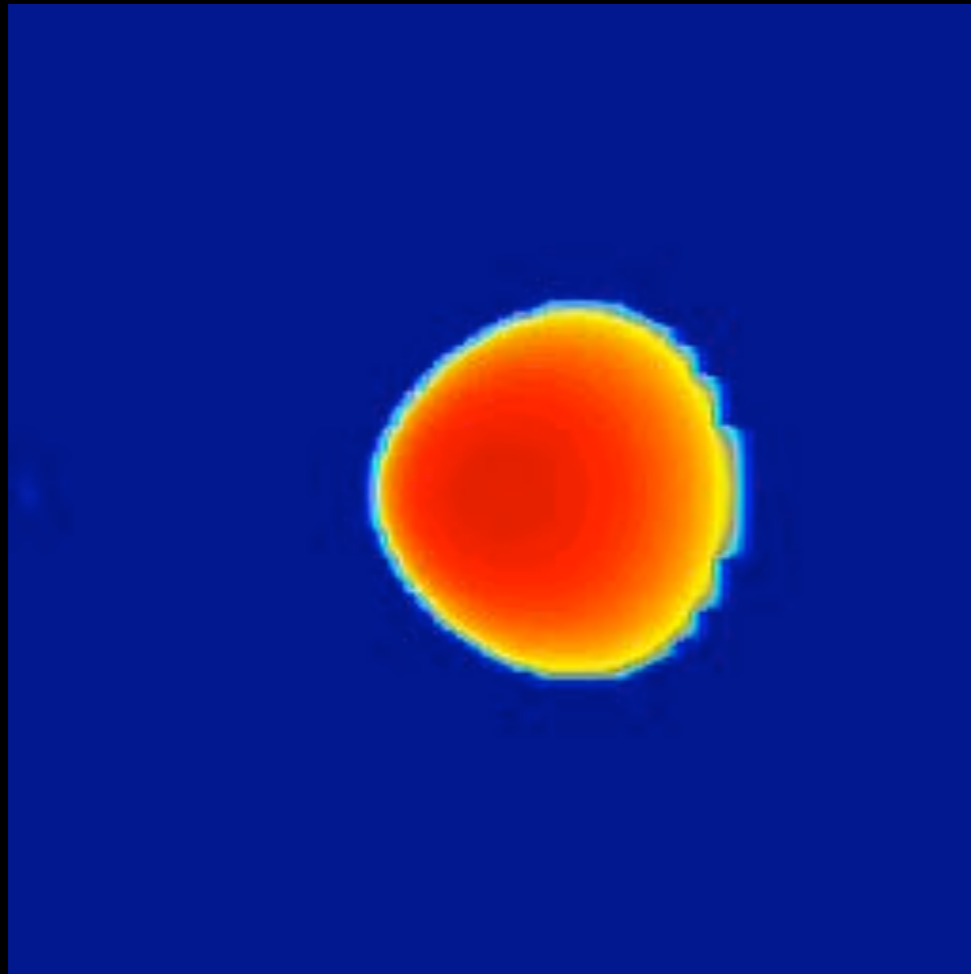
McKinney & Gammie 04

Interaction between magnetically dominated and “normal” flow.

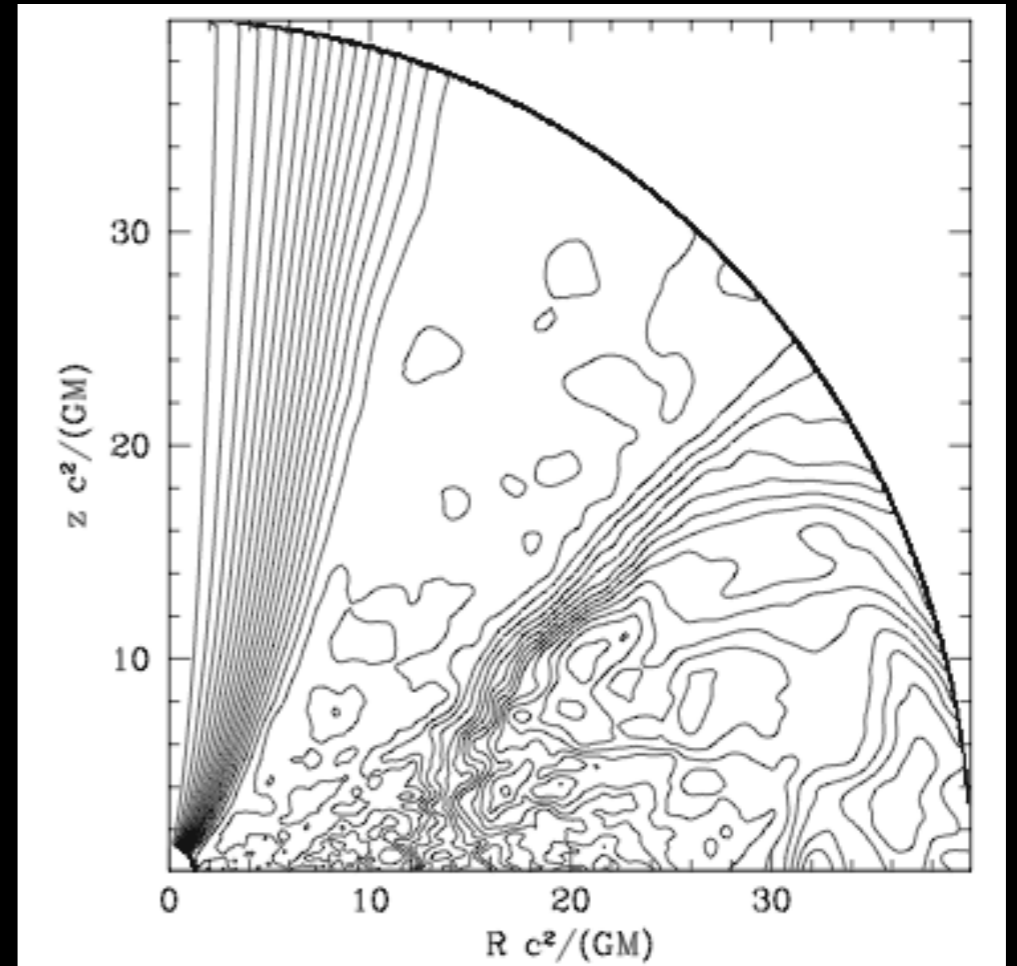
Magnetic extraction of rotational energy from black hole is associated with jet formation. Jet and corona are magnetically dominated.



Black hole-disk system



Hawley et al 02

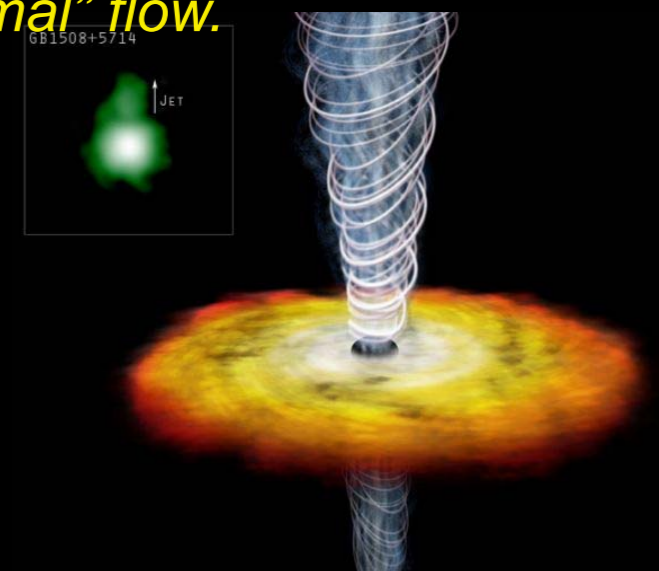


McKinney & Gammie 04

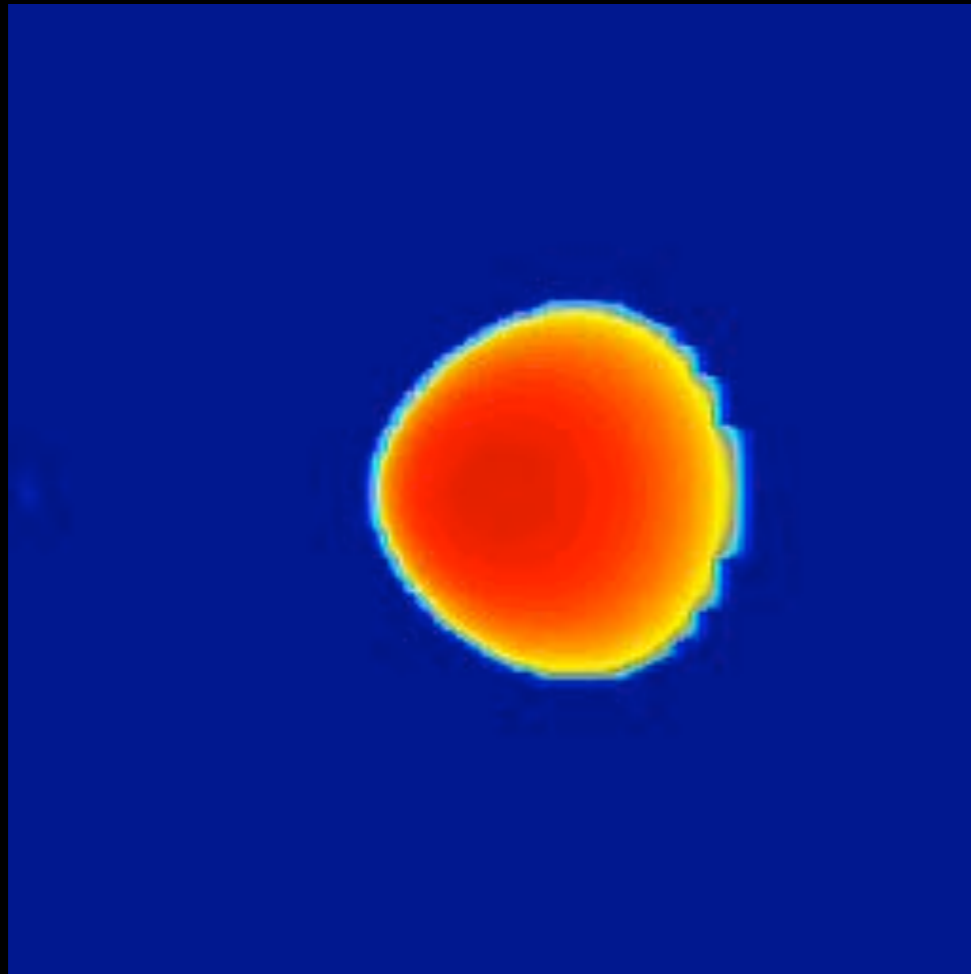
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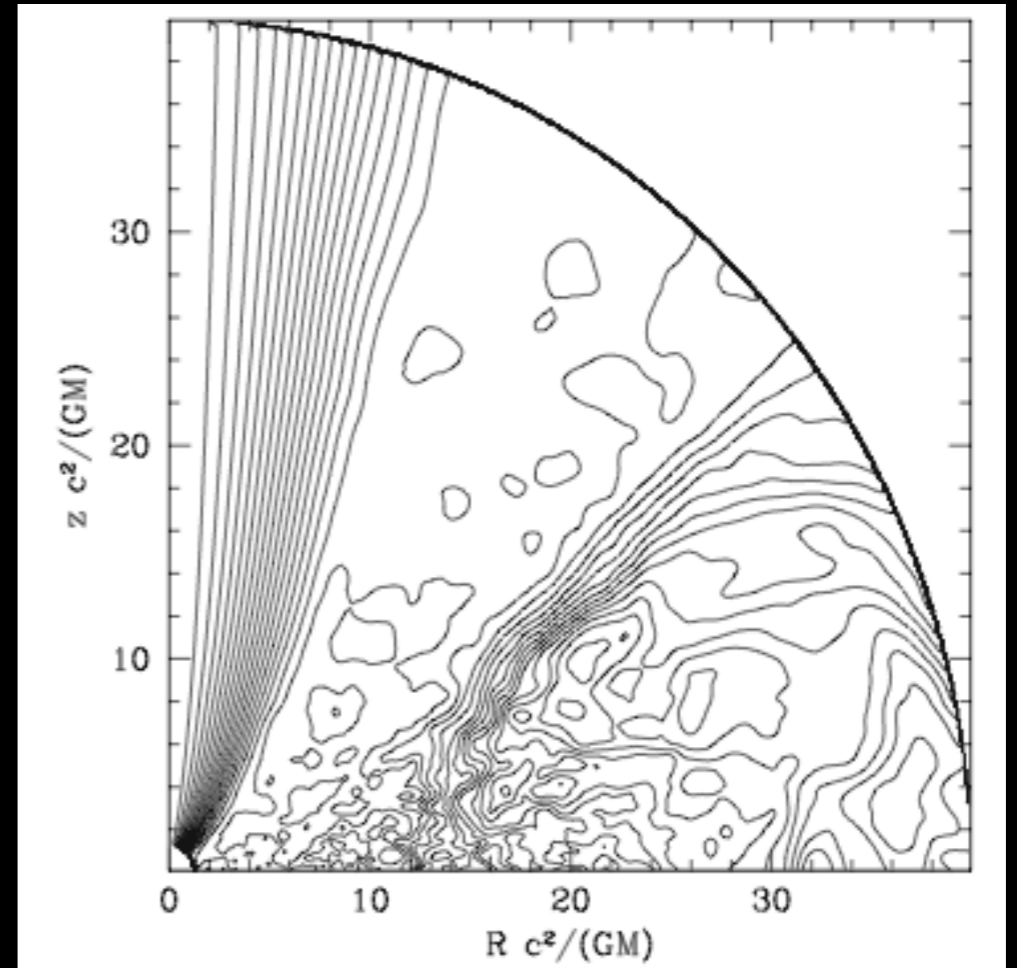
Do we understand the behavior of the corona + jet?



Black hole-disk system



Hawley et al 02

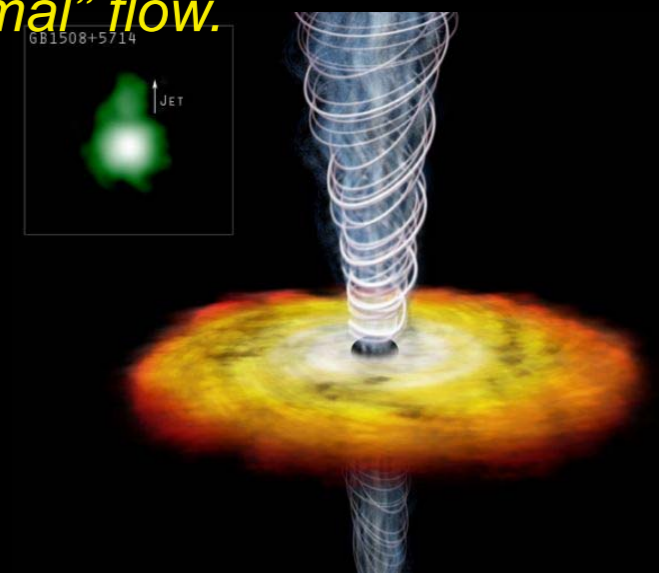


McKinney & Gammie 04

Interaction between magnetically dominated and "normal" flow.

Magnetic extraction of rotational energy from black hole is associated with jet formation. Jet and corona are magnetically dominated.

Do we understand the behavior of the corona + jet?



Open questions:

- *What is the magnetospheric structure of a magnetized rotating conductor in the presence of plasma?*
- *What is the rate of energy loss?*
- *What are the properties of the wind/outflow?*

We need to be able to solve self-consistent dynamics of plasmas in strong EM fields. Difficult to do both analytically and numerically.

Conditions:

$$v \simeq c, \quad v_a \simeq c,$$

Equations:

$$\nabla_{\beta} \left(T_{(m)}^{\alpha\beta} + T_{(f)}^{\alpha\beta} \right) = 0$$

$$\nabla_{\beta} {}^*F^{\alpha\beta} = 0$$

$$\nabla_{\alpha} (n u^{\alpha}) = 0$$

$$F_{\nu\mu} u^{\mu} = 0 \quad - \text{perfect conductivity}$$

$$T_{(f)}^{\alpha\beta} = F^{\alpha\gamma} F^{\beta}_{\gamma} - \frac{1}{4} (F_{\mu\nu} F^{\mu\nu}) g^{\alpha\beta}$$

*-stress-energy-momentum of
electromagnetic field*

$$T_{(m)}^{\alpha\beta} = w u^{\alpha} u^{\beta} + p g^{\alpha\beta}$$

-stress-energy-momentum of matter

from S. Komissarov

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$$T_{(m)}^{\alpha\beta} = w u^\alpha u^\beta + p g^{\alpha\beta}$$

-stress-energy-momentum of matter

Advantages:

- 1) *Allows adiabatic transfer of energy and momentum between the electromagnetic field and particles;*
- 2) *Allows dissipation at shocks;*
- 3) *All wave speeds below c .*

Disadvantages:

- 1) *Complexity;*
- 2) *Difficult to solve if*

$$\rho c^2 \ll E^2 + B^2$$

Conditions:

$$v \simeq c, \quad v_a \simeq c,$$

Equations:

$$\nabla_\beta \left(T_{(m)}^{\alpha\beta} + T_{(f)}^{\alpha\beta} \right) = 0$$

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Equations:

$$\nabla_\mu T_{(f)}^{\nu\mu} = 0$$

$$\nabla_\beta {}^*F^{\alpha\beta} = 0$$

$$F_{\mu\nu} {}^*F^{\mu\nu} = 0$$

$$F_{\mu\nu} F^{\mu\nu} > 0$$

or

$$\mathbf{E} \cdot \mathbf{B} = 0$$

$$B^2 - E^2 > 0$$

(Komissarov 2002)

Advantages:

- 1) *Simple hyperbolic system of conservation laws (linearly degenerate fast and Alfvén modes);*
- 2) *Well suited for “force-free” magnetospheres of black holes and neutron stars;*

Disadvantages:

- 1) *Does not allow adiabatic transfer of energy and momentum between the electromagnetic field and particles;*
- 2) *Does not allow dissipation;*
- 3) *Fast wavespeed equals to c (subsonic);*
- 4) *Often breaks down;*

Conditions:

$$T_{(m)}^{\alpha\beta} \ll T_{(ef)}^{\alpha\beta}$$

Equations:

$$\nabla_{\mu} T_{(f)}^{\nu\mu} = 0$$

$$\nabla_{\beta} {}^*F^{\alpha\beta} = 0$$

$$F_{\mu\nu} {}^*F^{\mu\nu} = 0$$

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Force-free equations

Full RMHD equations become stiff for high magnetization

$$mn \frac{\partial \gamma \vec{v}}{\partial t} = \rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} \approx 0$$

Derive dynamical set of equations by ignoring particle inertia but retaining plasma charges and currents.

Force-free equations

Full RMHD equations become stiff for high magnetization

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Derive dynamical set of equations by ignoring particle inertia but retaining plasma charges and currents.

$$\frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} - \frac{4\pi}{c} \vec{j}$$

$$\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\rho \vec{E} + \frac{\vec{j}}{c} \times \vec{B} = 0$$

$$\frac{\partial}{\partial t} \vec{E} \cdot \vec{B} = 0$$

Force-free equations

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“Force-free MHD”

Gruzinov 99, Blandford 01

Force-free equations

Full RMHD equations become stiff for high magnetization

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“Force-free MHD” Gruzinov 99, Blandford 01

Where is plasma? Assumed to flow with $\vec{E} \times \vec{B}$ velocity, but velocity along the field is undefined. Plasma provides only charges and currents, no inertia.

Hyperbolic eqs. Use electromagnetic solvers to advance the system in time.

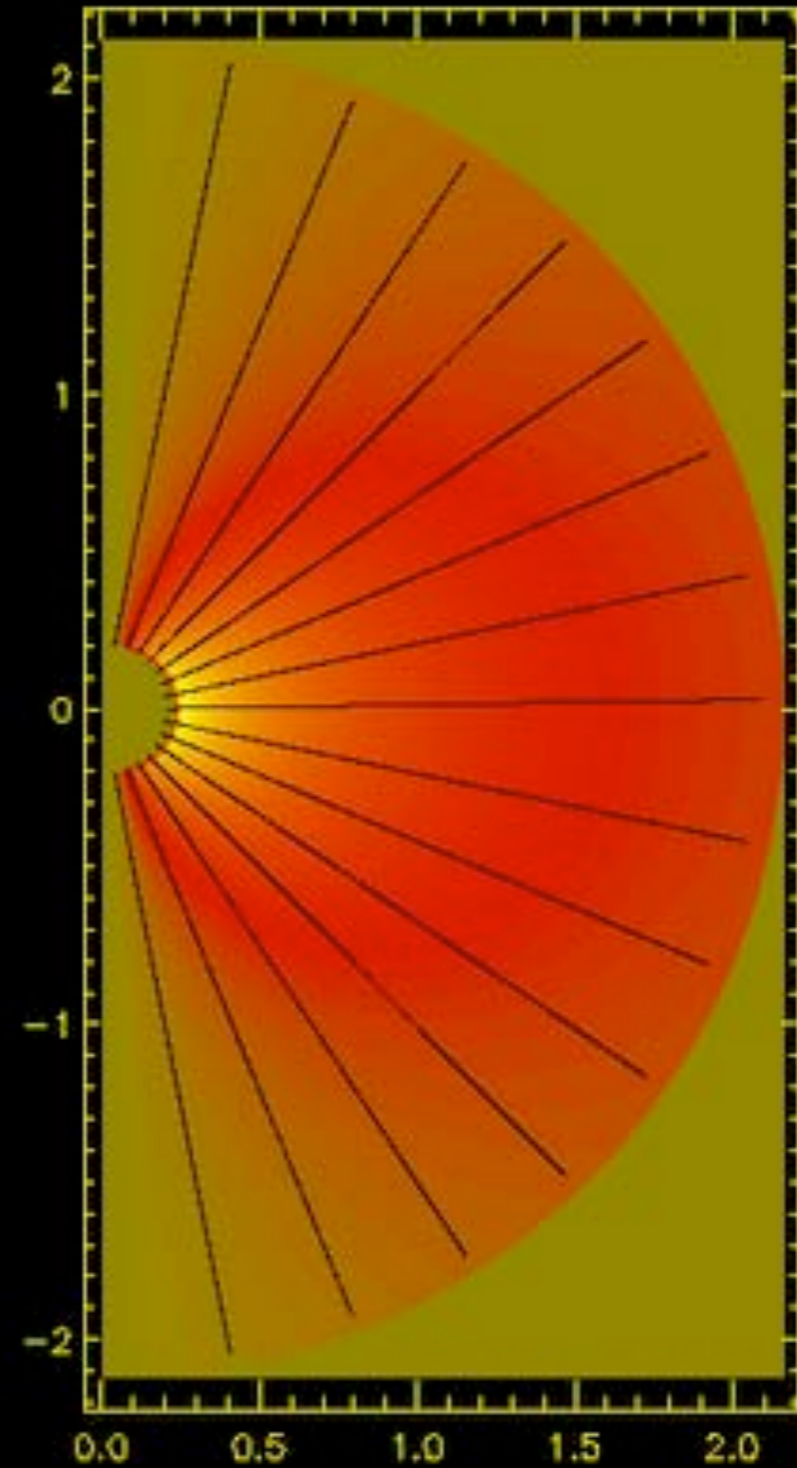
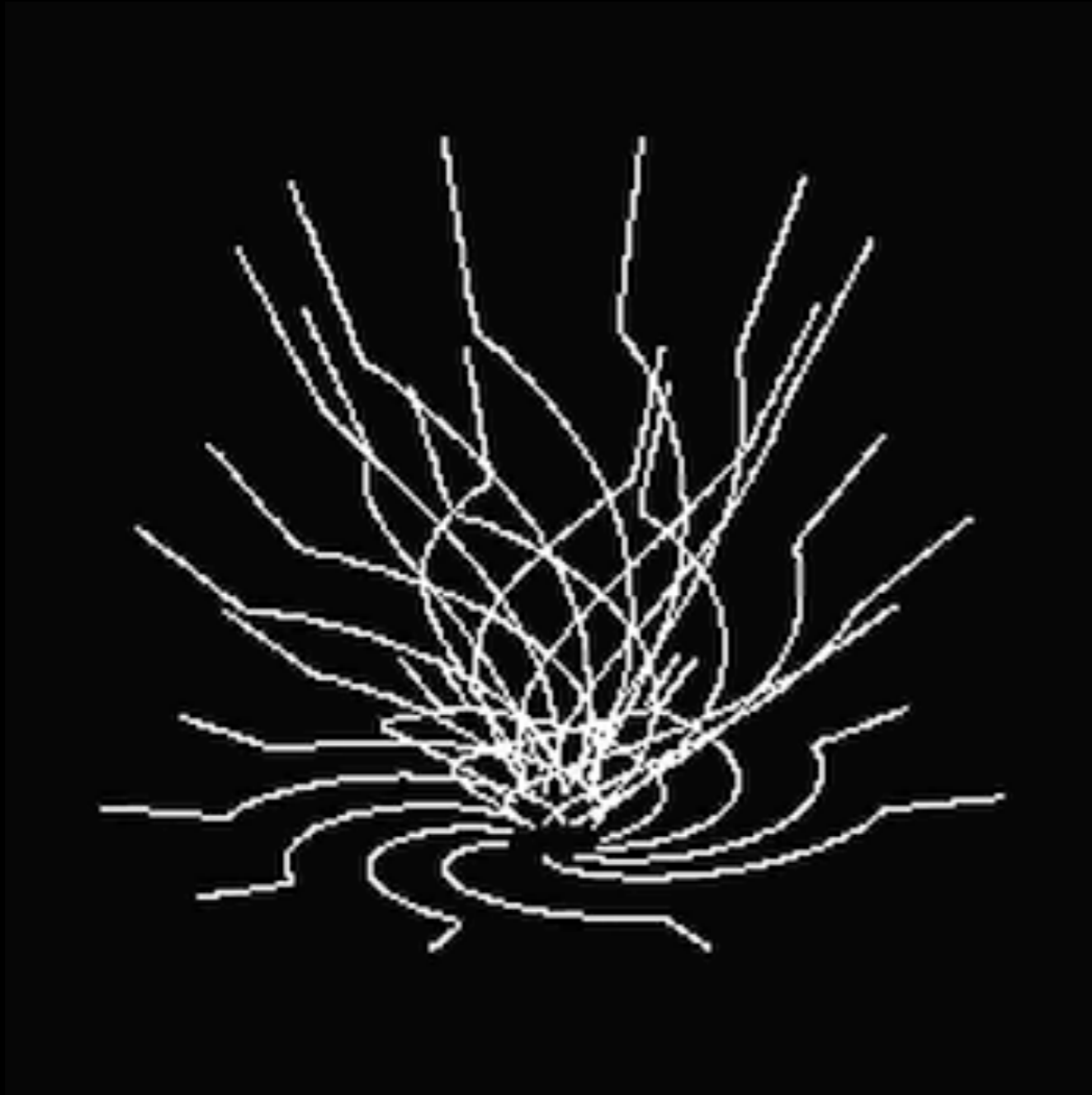
Monopole magnetosphere: time-dependent solution

Monopolar field, torsional Alfen wave
polarizes the medium with space charge

Reproduces Michel solution ('73), nothing
special at light cylinder, Poynting energy
loss.

Monopole magnetosphere: time-dependent solution

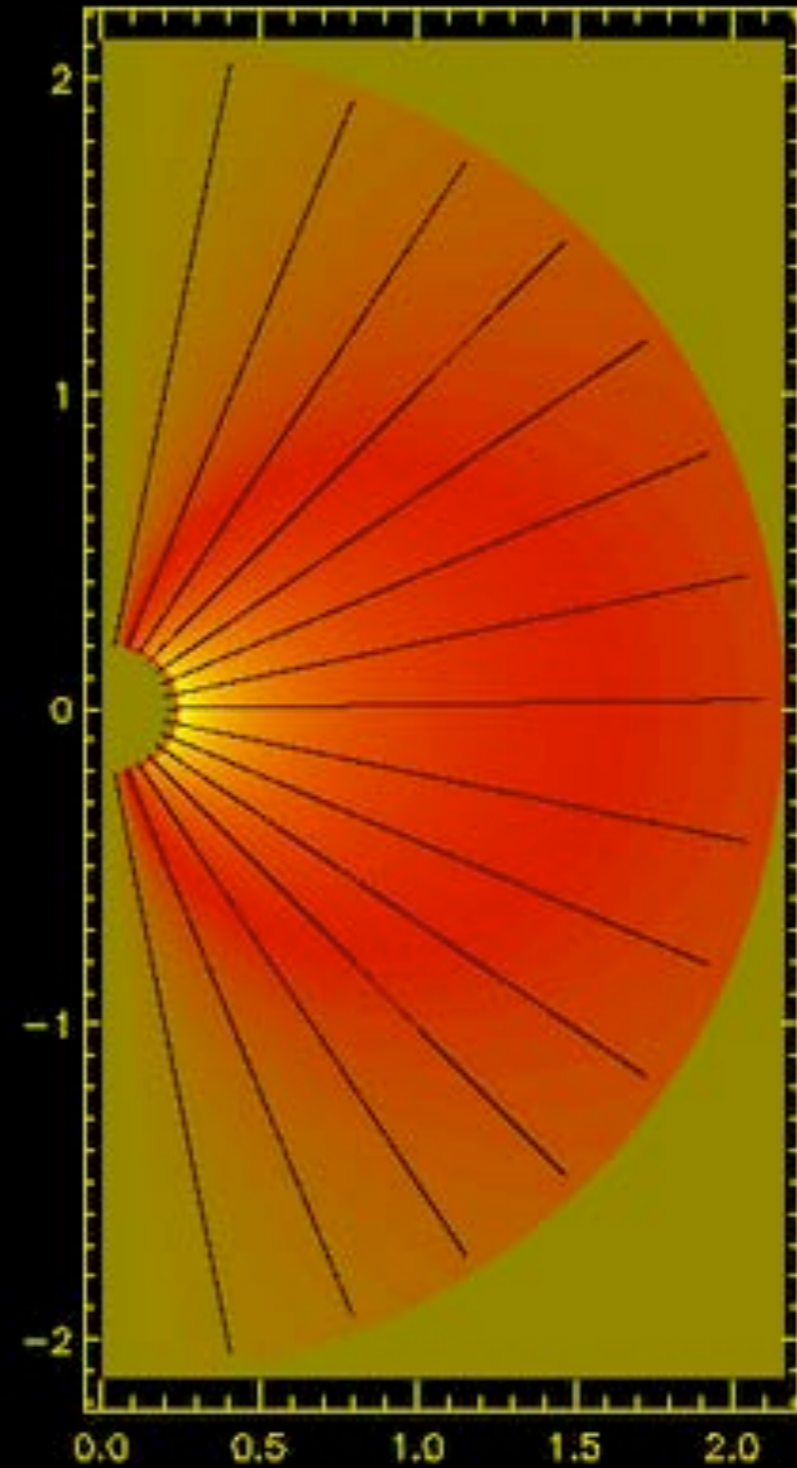
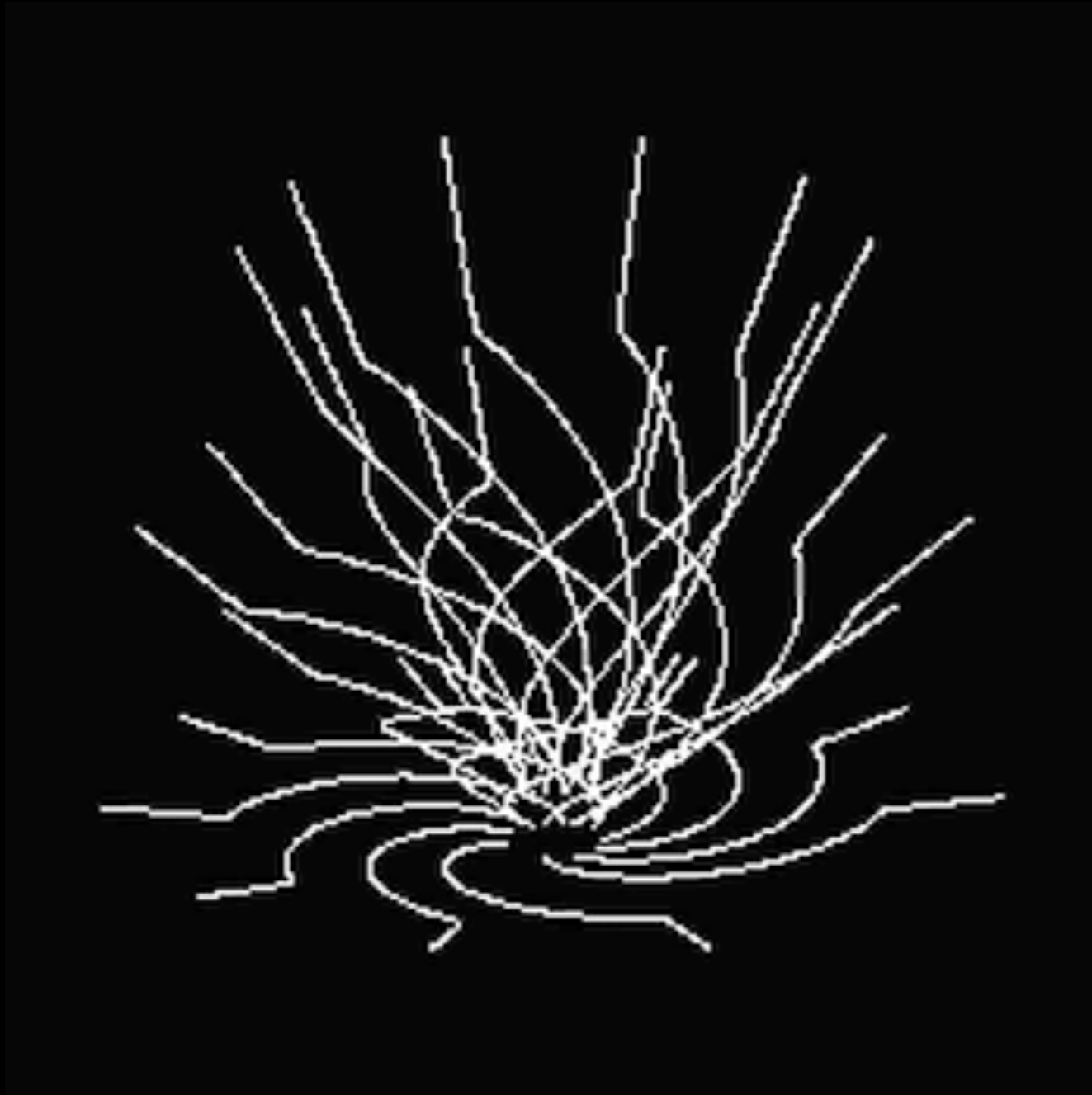
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Structure of magnetosphere: time-dependent solution

*Toroidal
field*

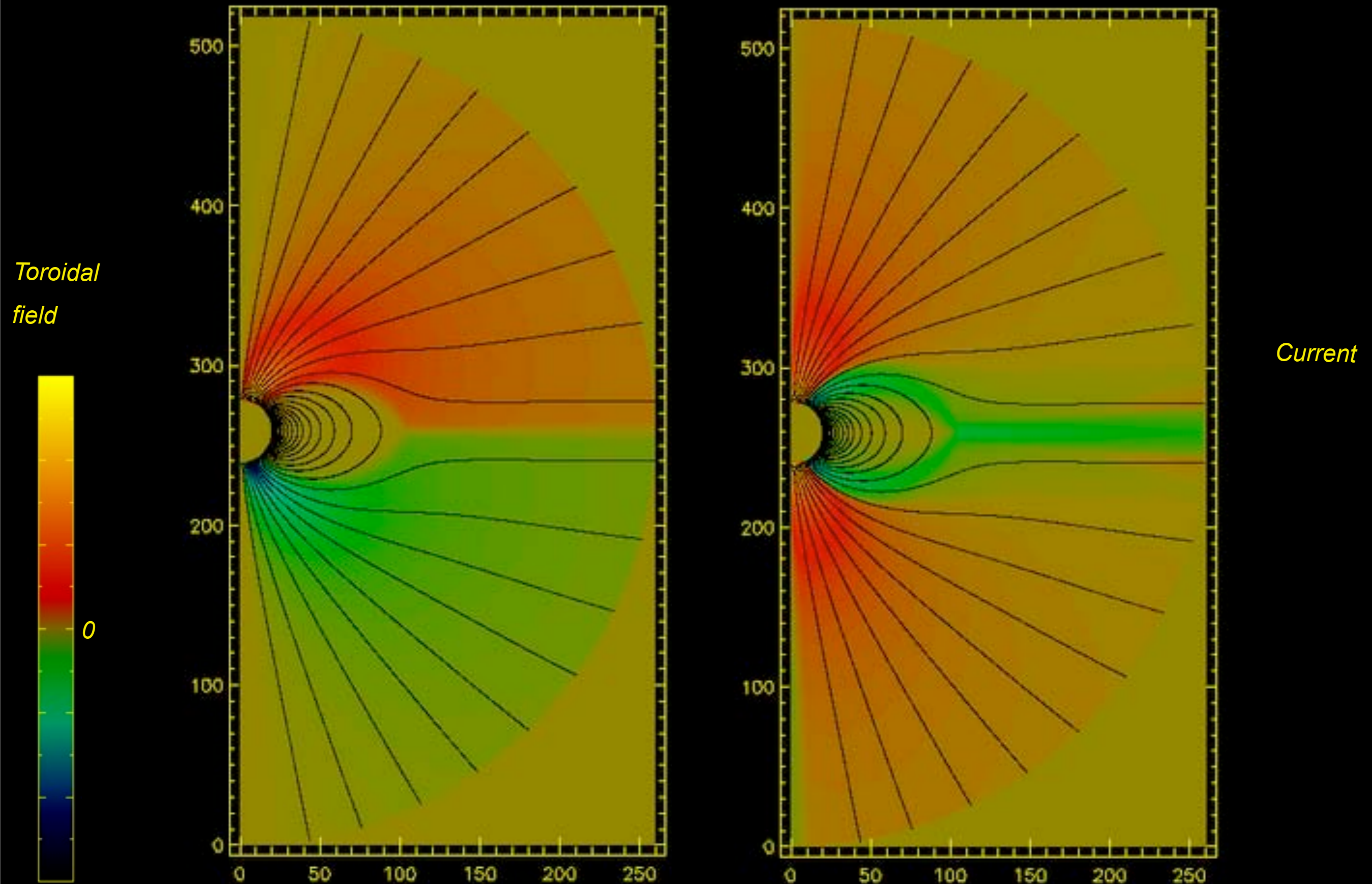
Current



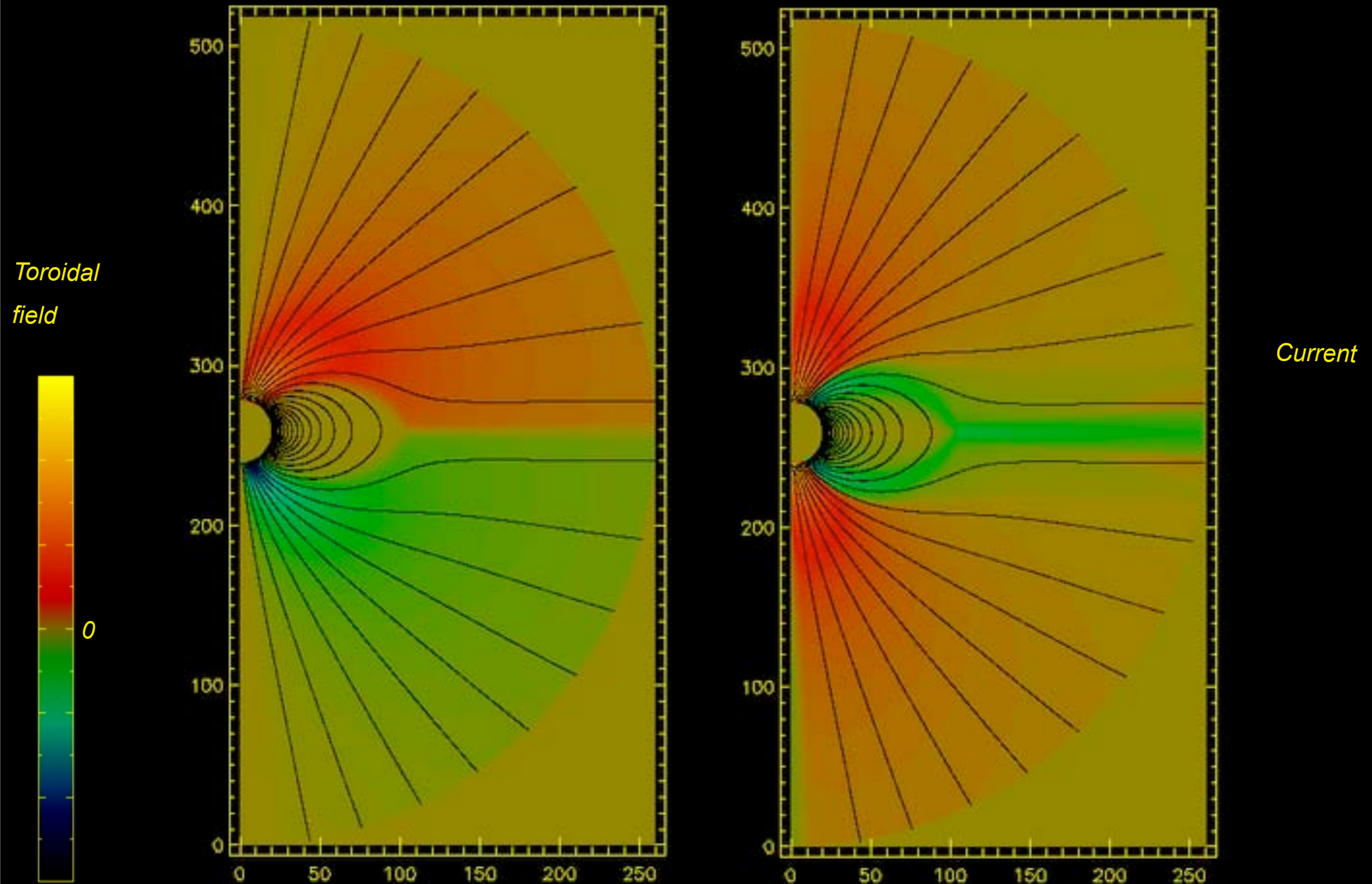
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A.S. (2005)

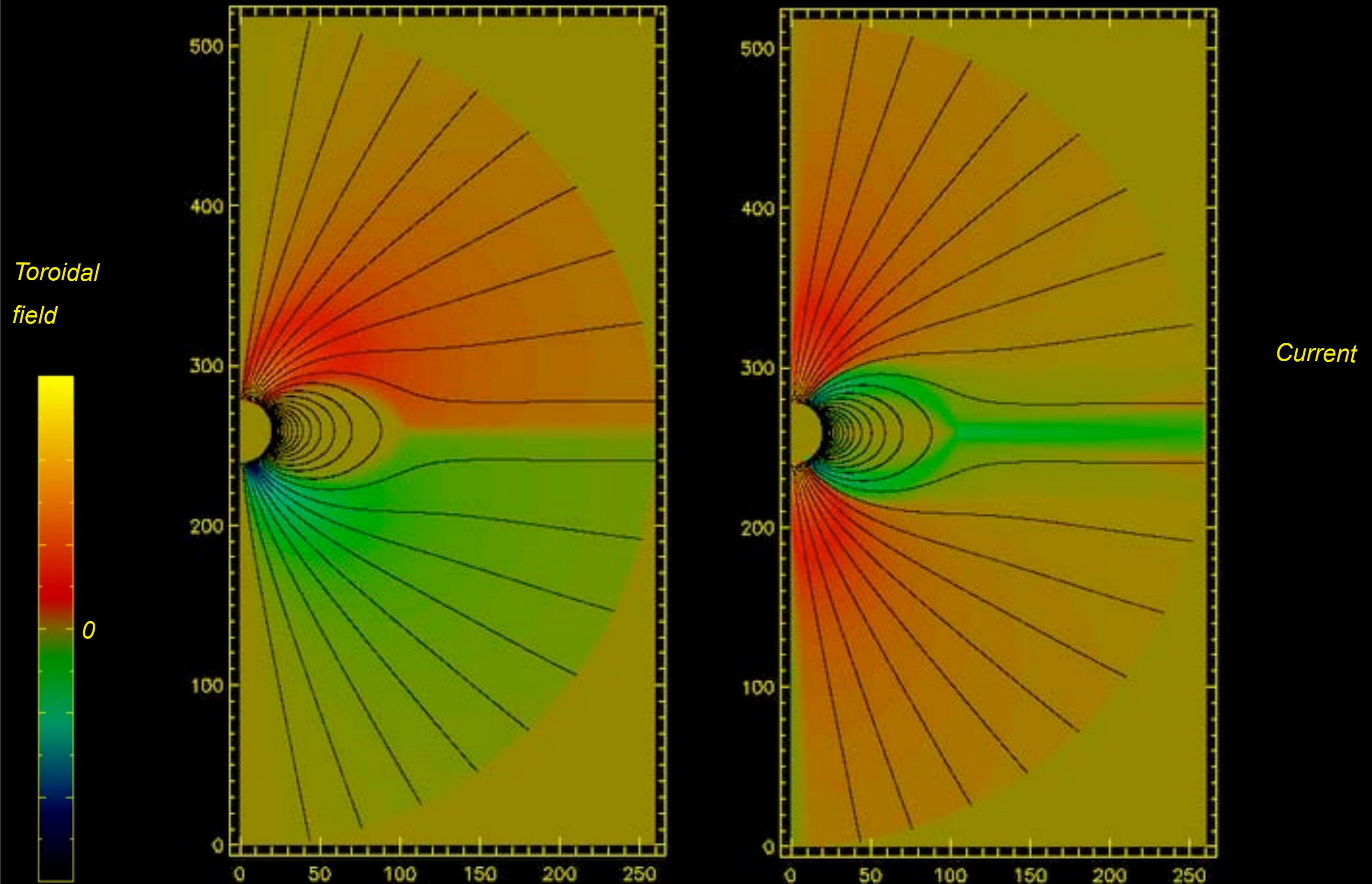
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Structure of magnetosphere: time-dependent solution



Limits of applicability of force-free system

a) $E < B$ (physical limit) Drift velocity should be $< c$.

Not enforced by the original system of equations -- need resistivity

b) $B \neq 0$ (numerical and philosophical limit)

Spontaneous current sheet formation is a natural property of magnetized flows. In current sheets, force-free approximation breaks down. Resistivity helps maintain physical solutions.

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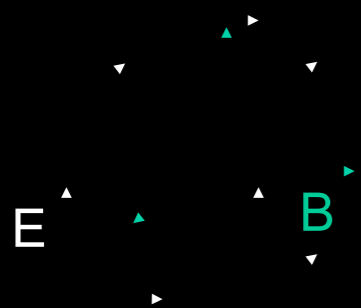
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Numerical method: finite-difference time-domain (FDTD) method for

Maxwell's equations; E and B staggered in space

(Yee mesh);

No numerical resistivity, but dispersive and oscillatory at discontinuities. Can add diffusion.



Other methods can be used too

(McKinney 06 conservative; Komissarov 05 Godunov)

(recent results by Contopoulos with the same method)

Structure of magnetosphere: time-dependent solution

Time dependent force-free relativistic MHD approximation (long term evolution).

Properties of the solution:

- Spontaneous formation of equatorial current sheet.
- Reconnection necessary to reach LC
- Y-point (inside LC)
- Field is divergent at Y-point
- Field is zero in the equatorial plane
- Asymptotically -- split monopole
- Closed zone expands to LC over 10 period timescale.

Spindown:

$$\dot{E} = \frac{\mu^2 \Omega^4}{c^3} = c B_{LC}^2 R_{LC}^2$$

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$$\dot{E}_{vac} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \theta$$

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A.S. (2006)

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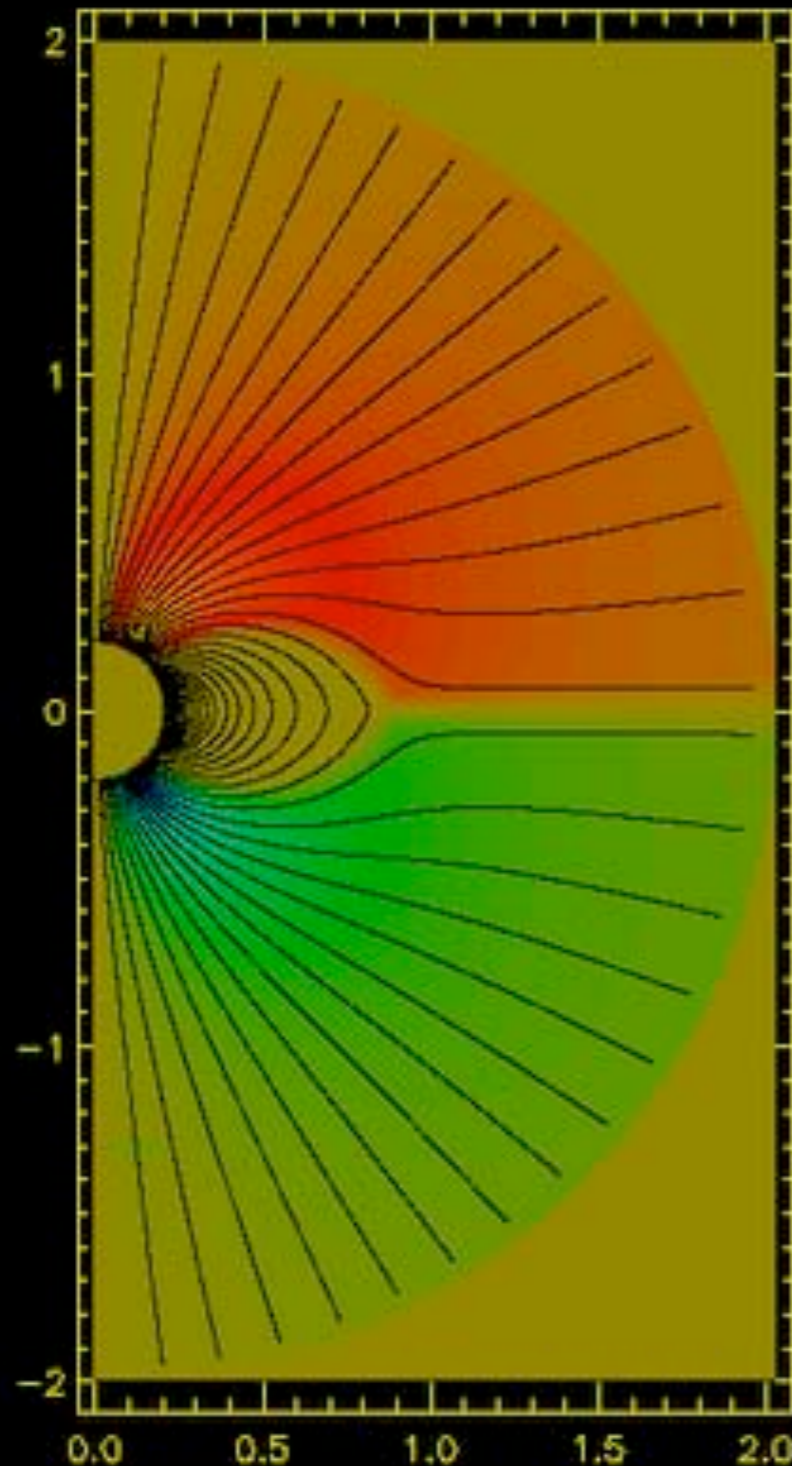
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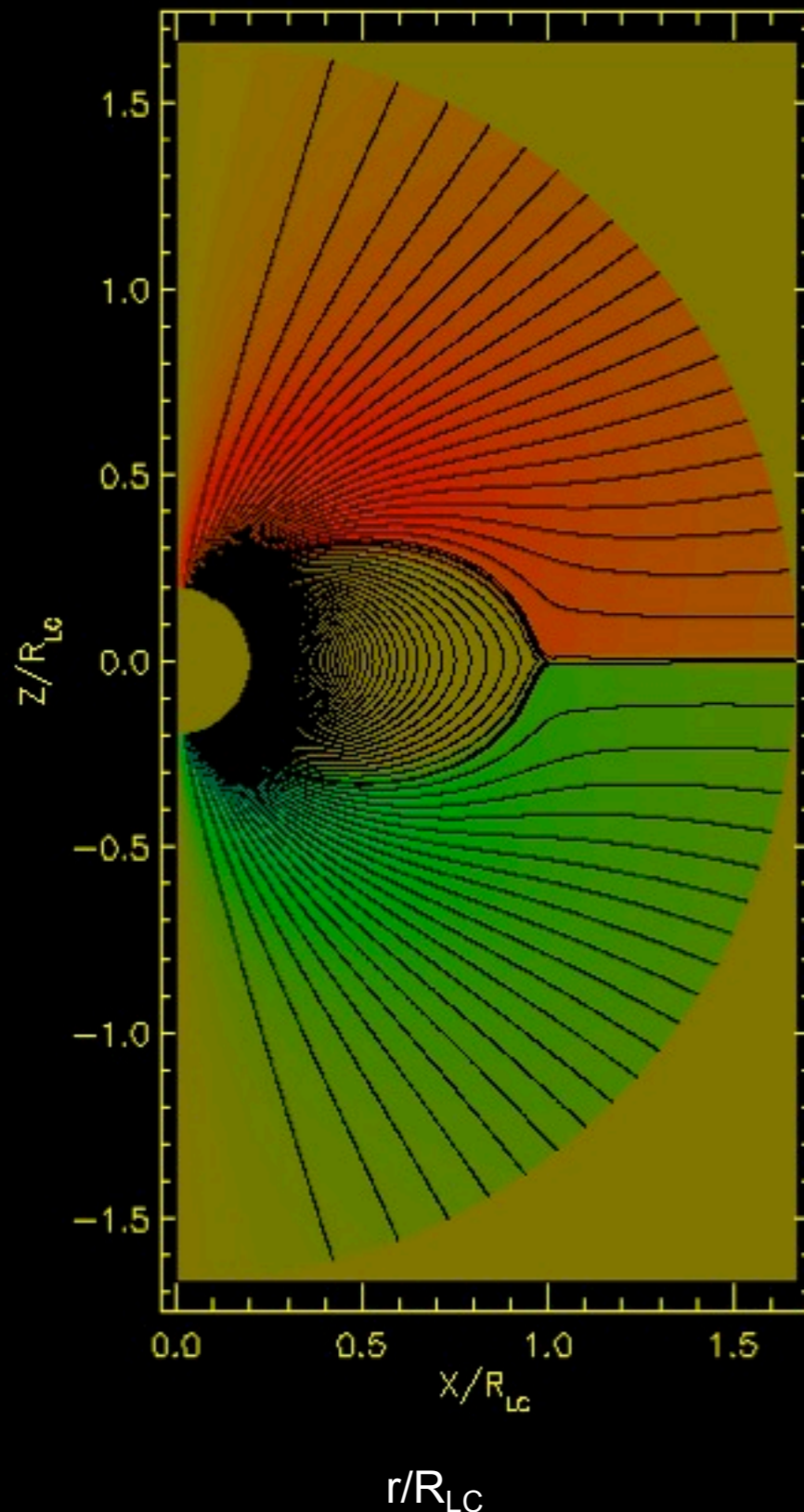
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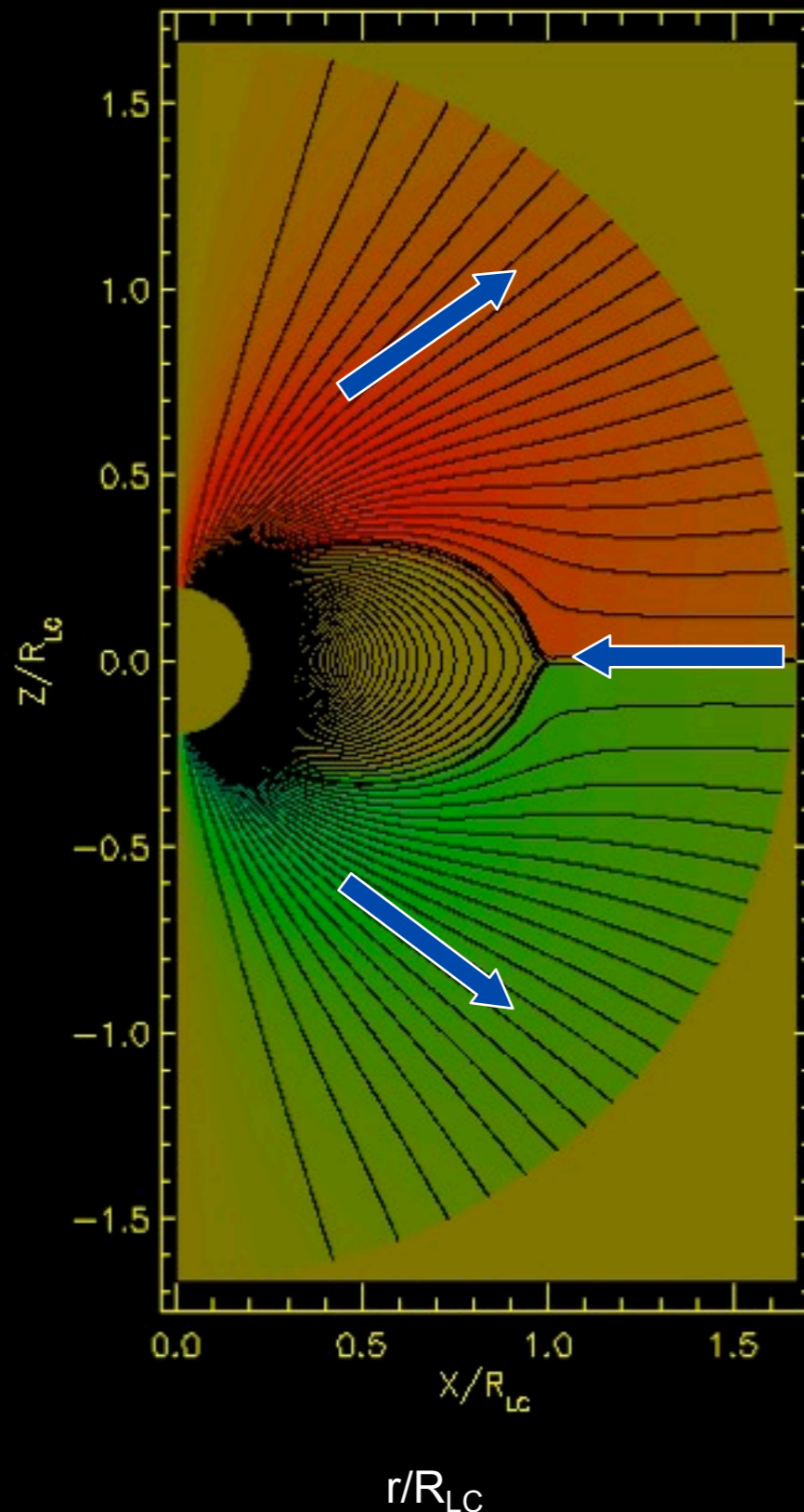
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A.S. (2006)



40 years of pulsar magnetospheres

- August 1967 -- discovery by Jocelyn Bell and Tony Hewish

- 1969 -- Goldreich-Julian model

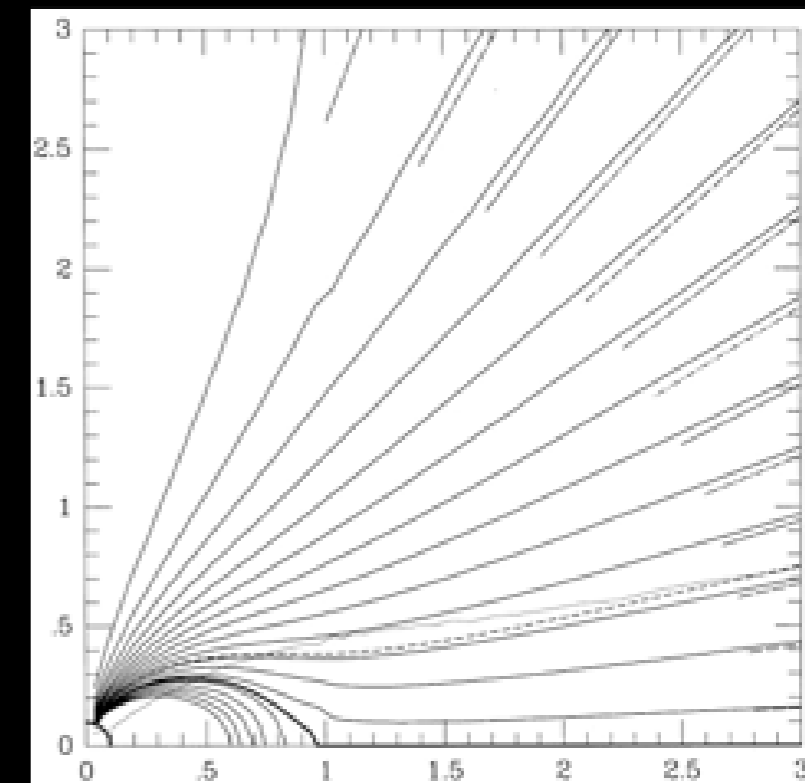
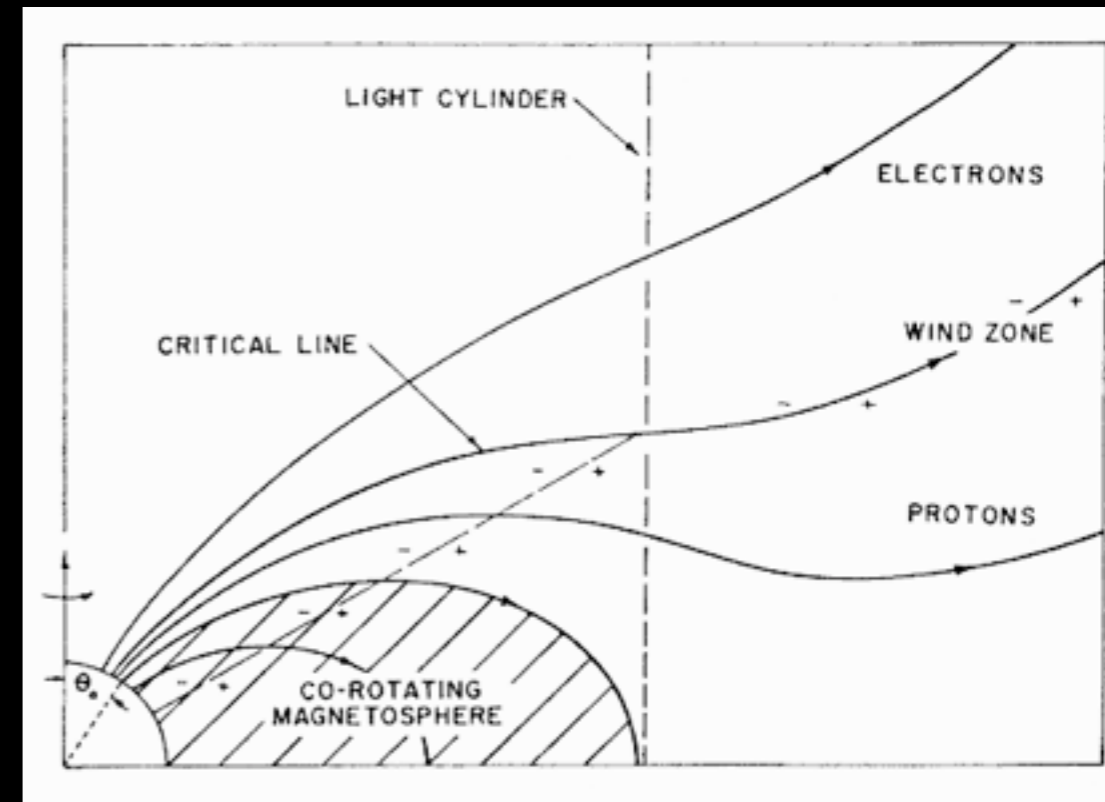
- 1970-s “pulsar equation”, pair formation, particle acceleration, geometrical emission Models (key players: Ruderman, Michel, Arons)

Magnetospheric shape unsolved even for aligned rotator.

- 1999 -- Contopoulos Kazanas Fendt Time-independent aligned magnetosphere (numerical solution of “pulsar equation”)

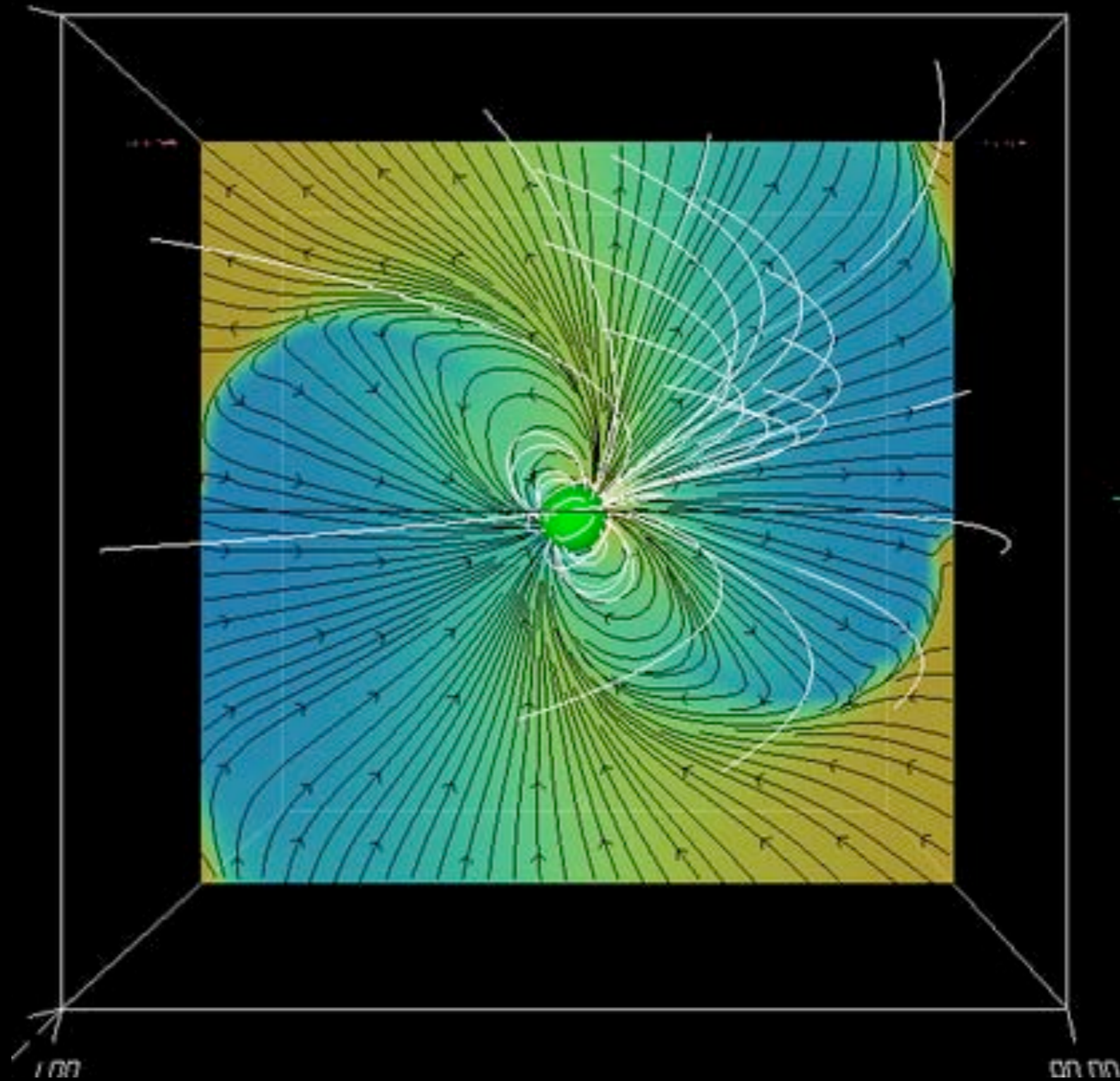
- 2003+ time-dependent numerical models (force-free + MHD). Good agreement with steady model. (McKinney; Komissarov, AS)

What's left is the oblique rotator.

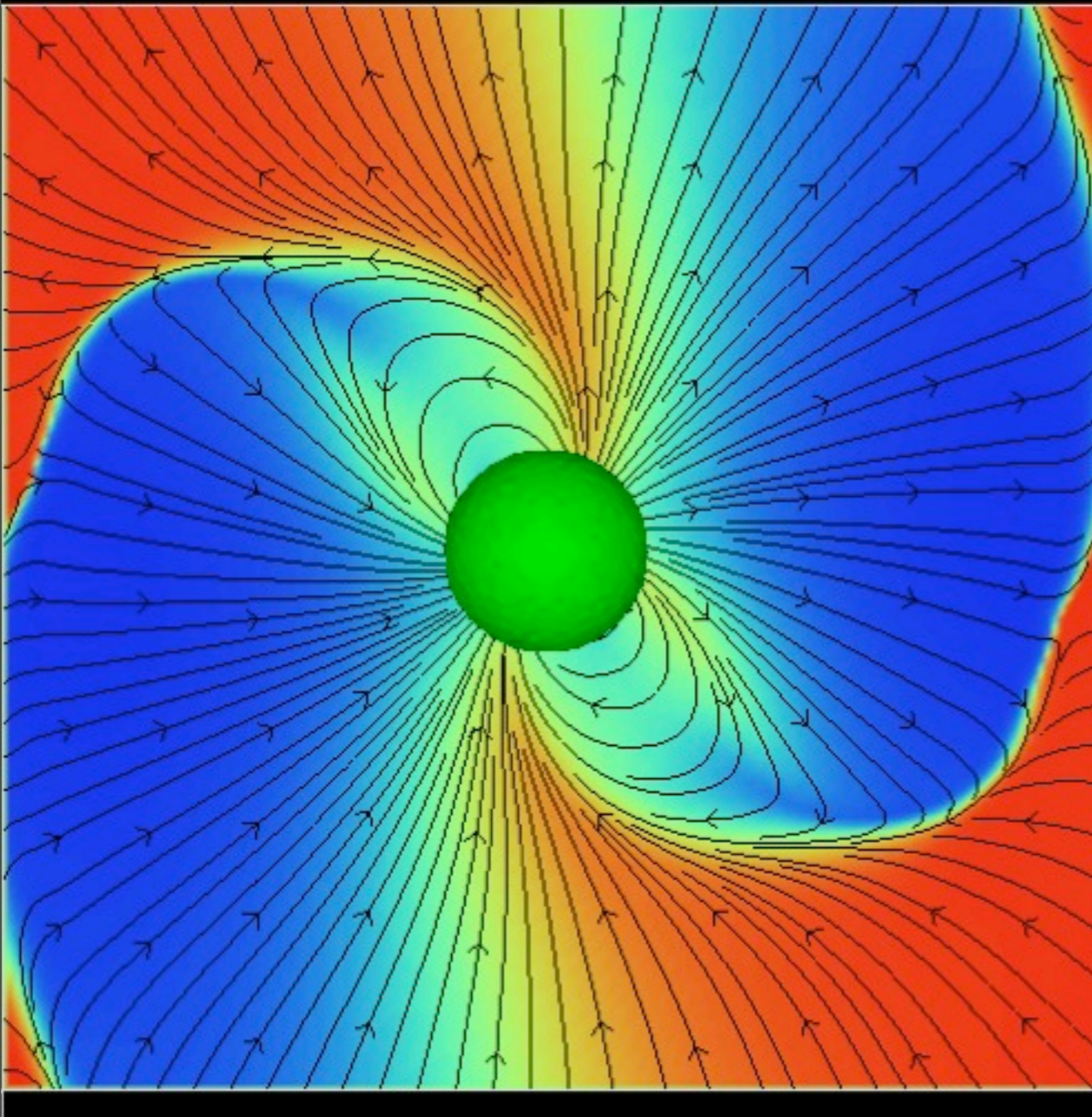


3D force-free magnetosphere: 60 degrees inclination

3D force-free magnetosphere: 60 degrees inclination



Meanwhile in the rotating frame: 60 degrees inclination



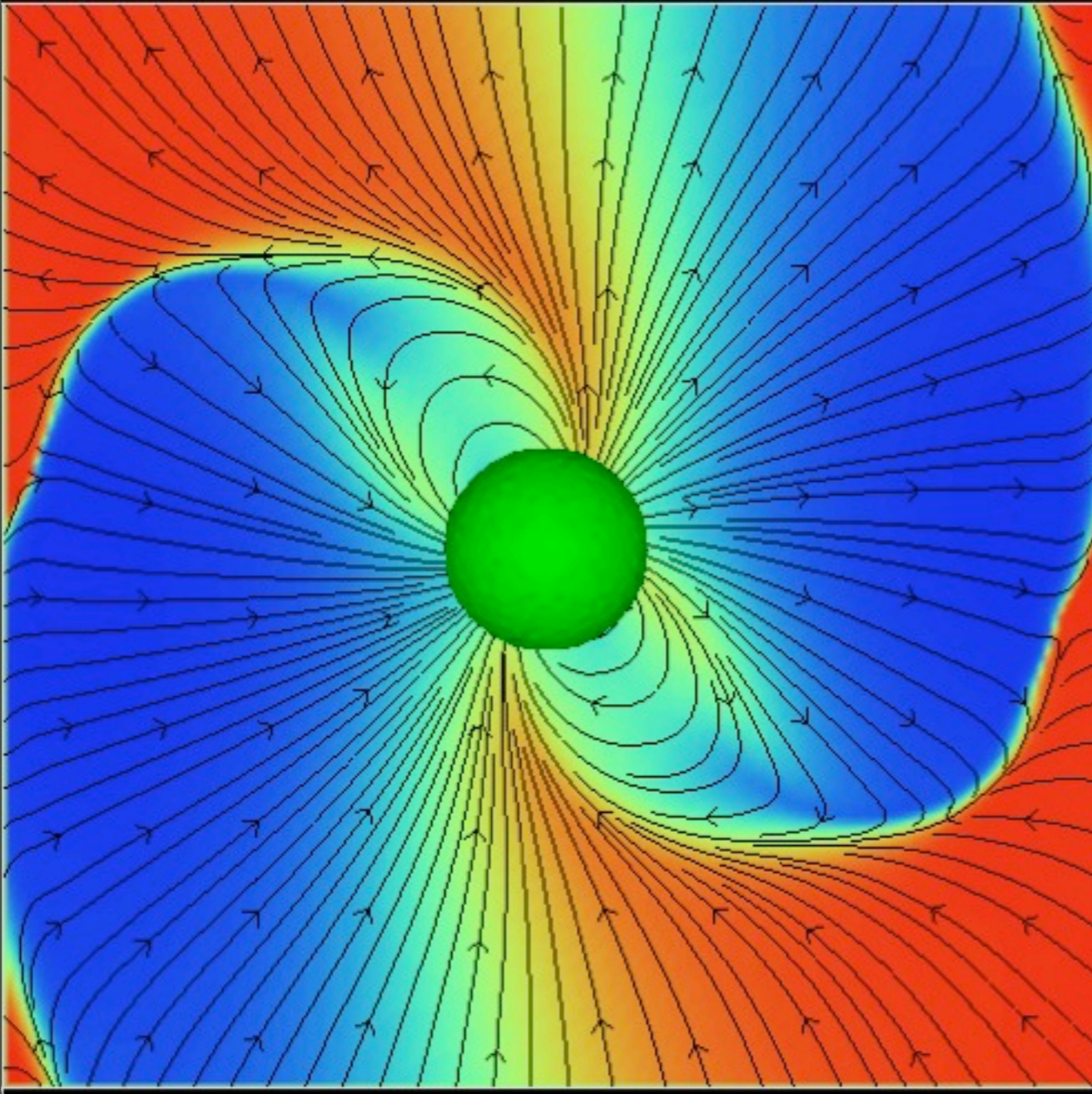
0.00E

159.1

Magnetic field, plane of μ - Ω

Current density, plane of μ - Ω

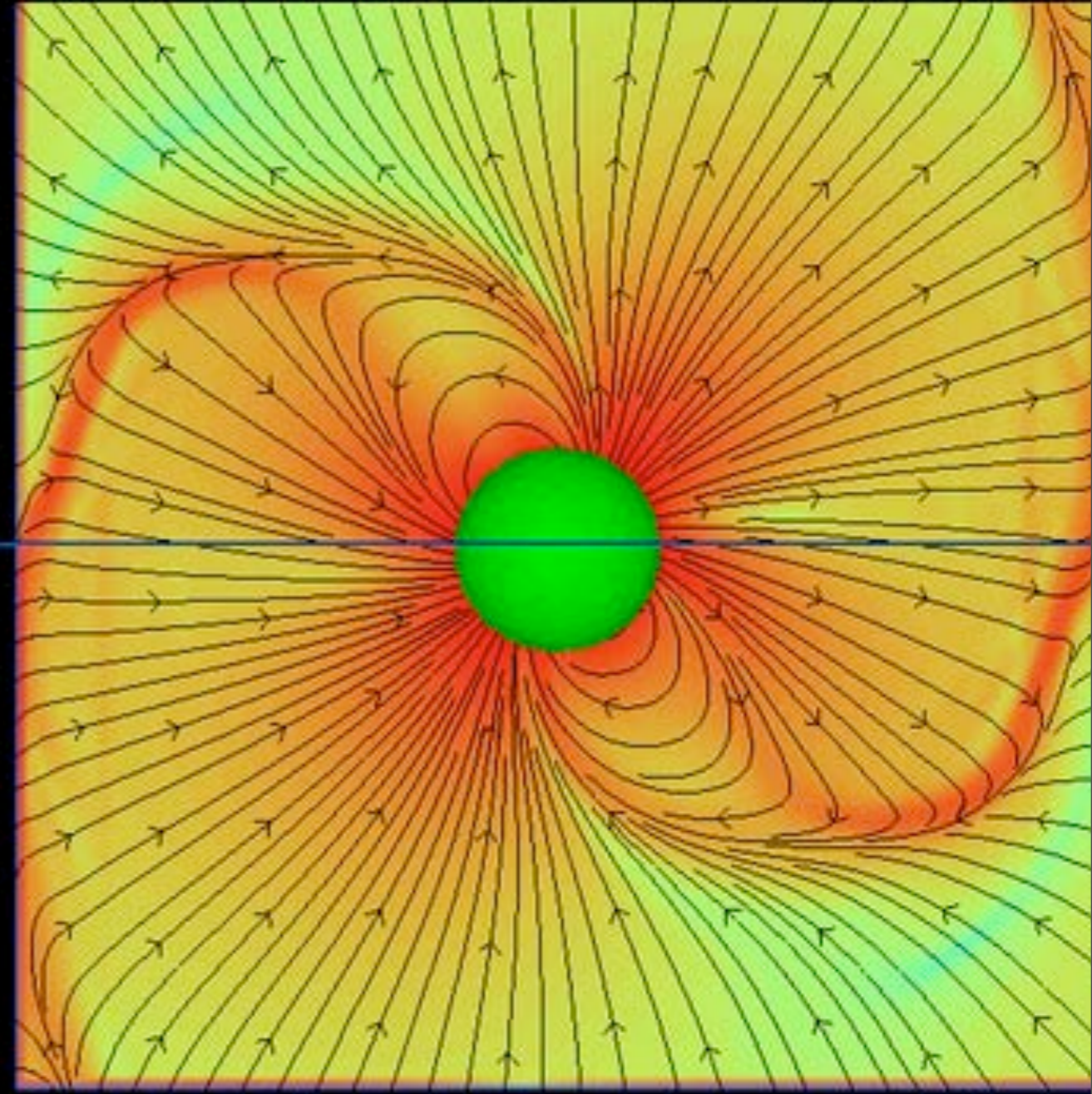
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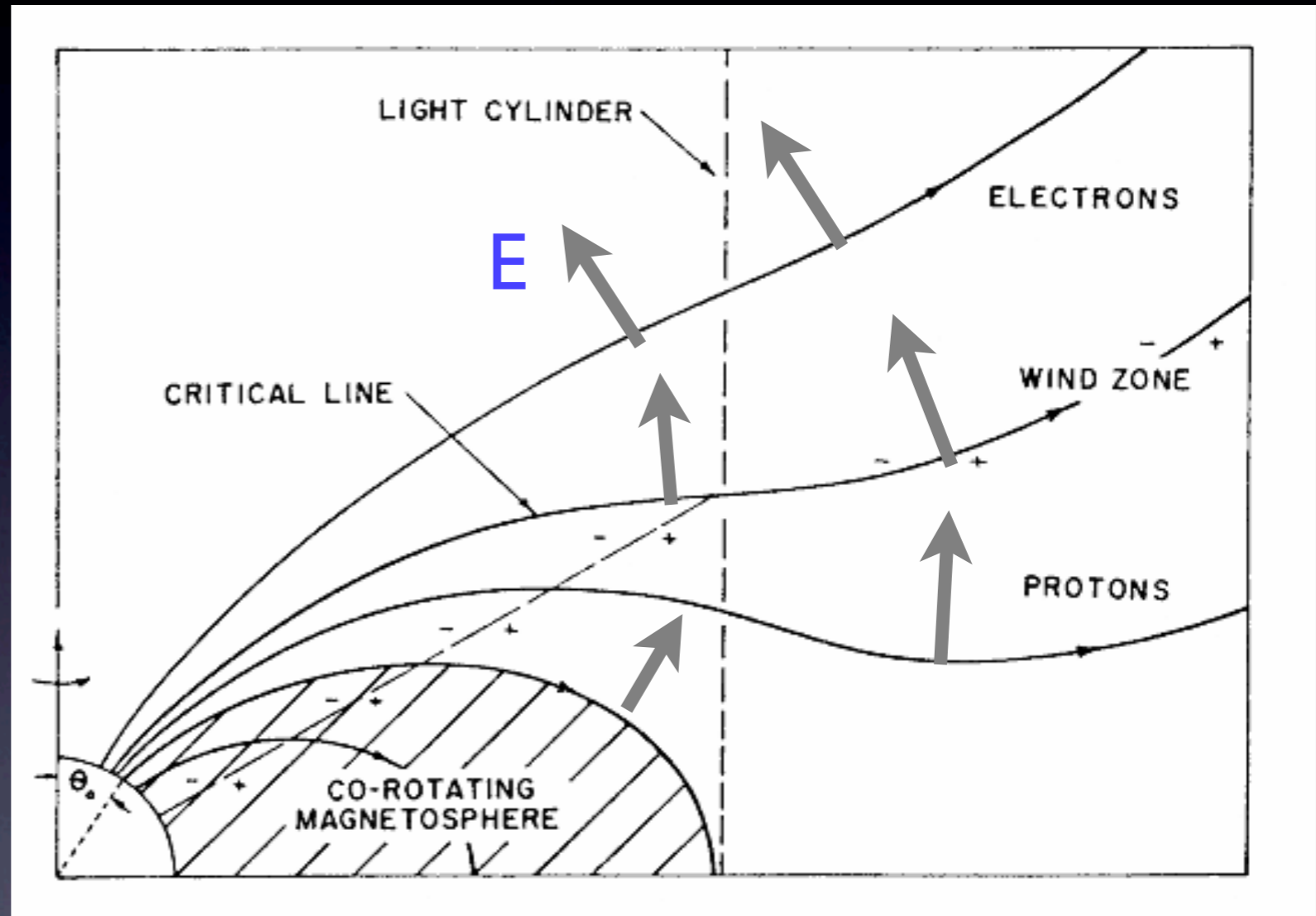
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Pulsars: energy loss

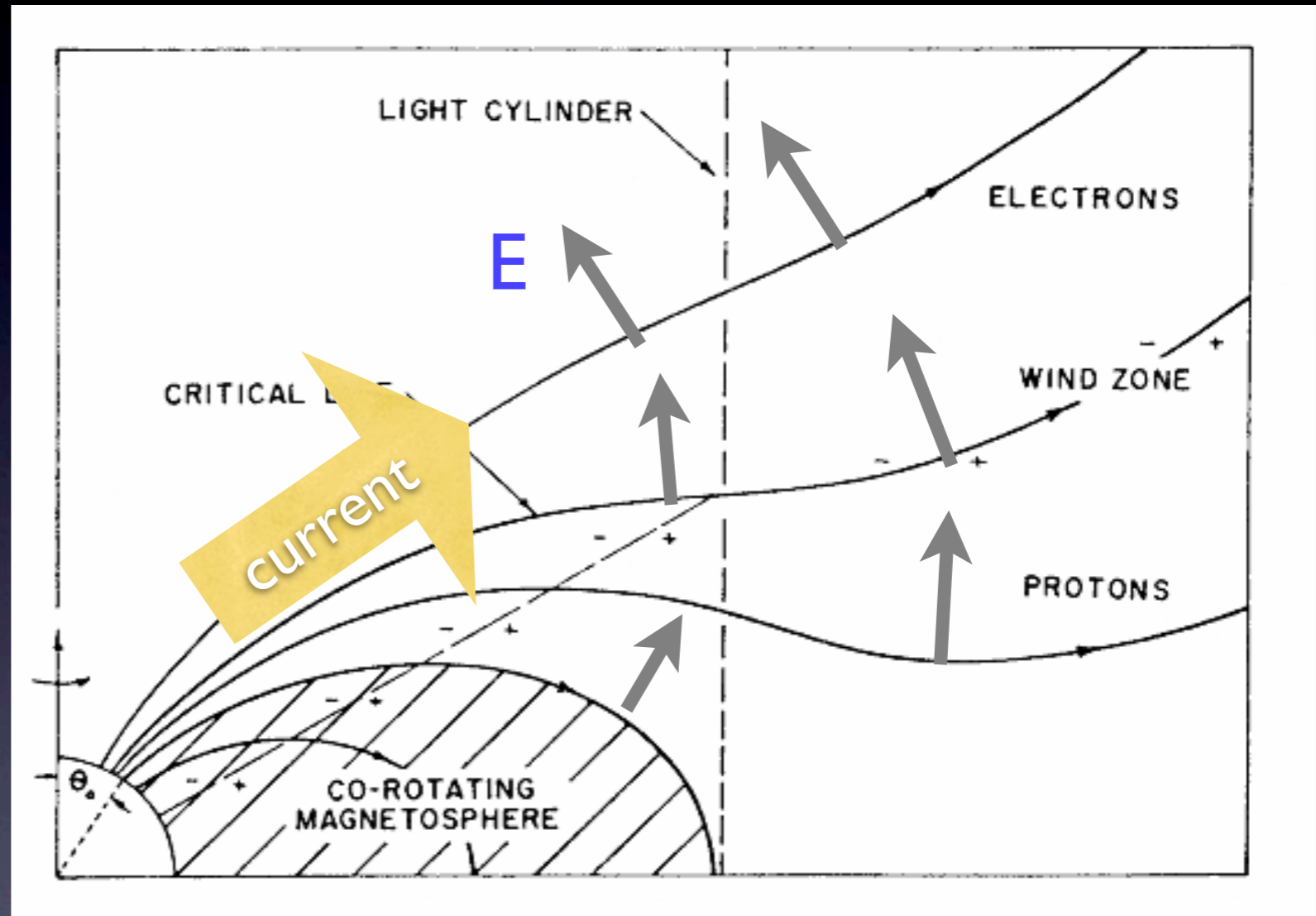
- Corotation electric field
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- $E \times B \rightarrow$ Poynting flux
- Electromagnetic energy loss



Goldreich & Julian 1969

Pulsars: energy loss

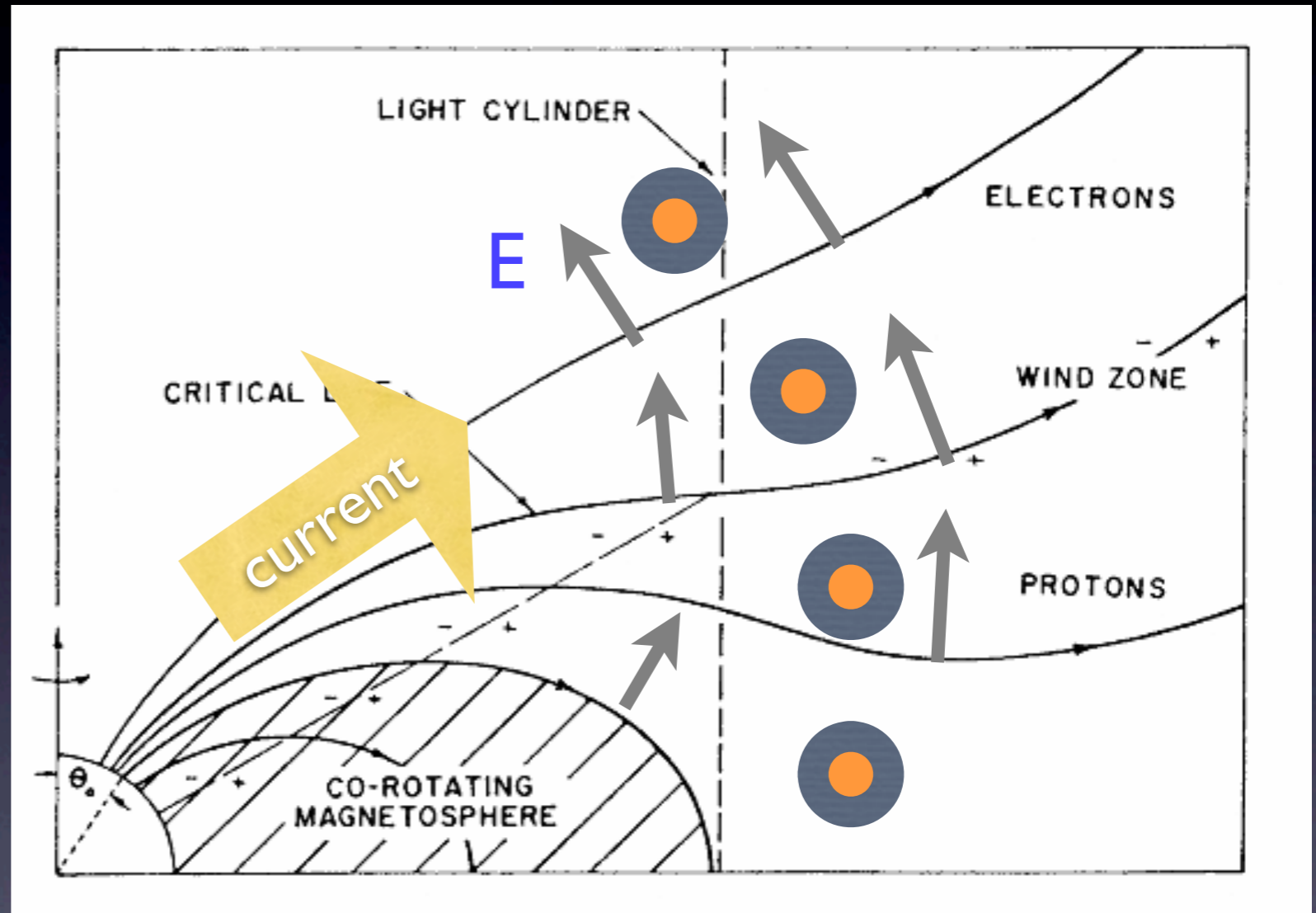
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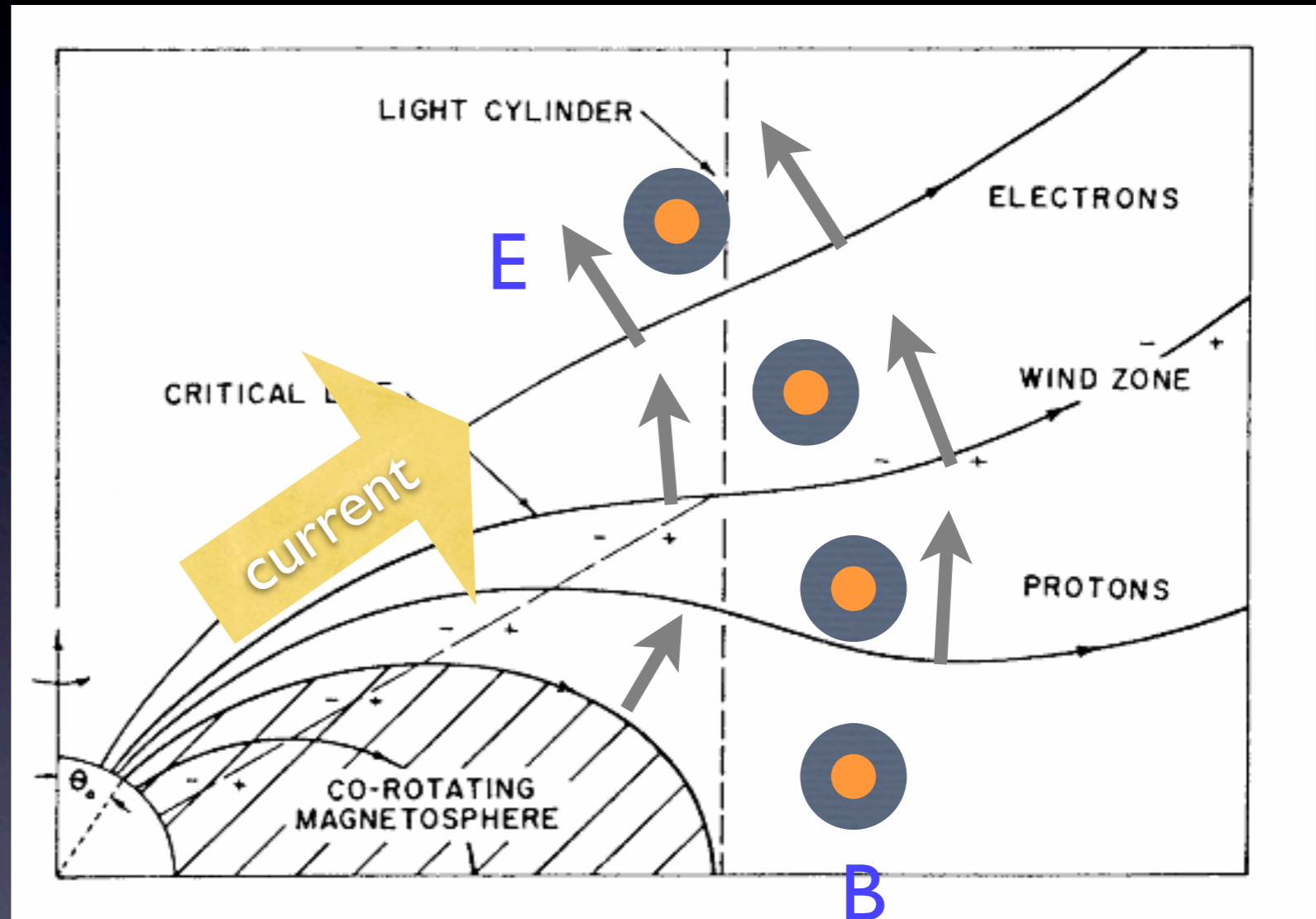
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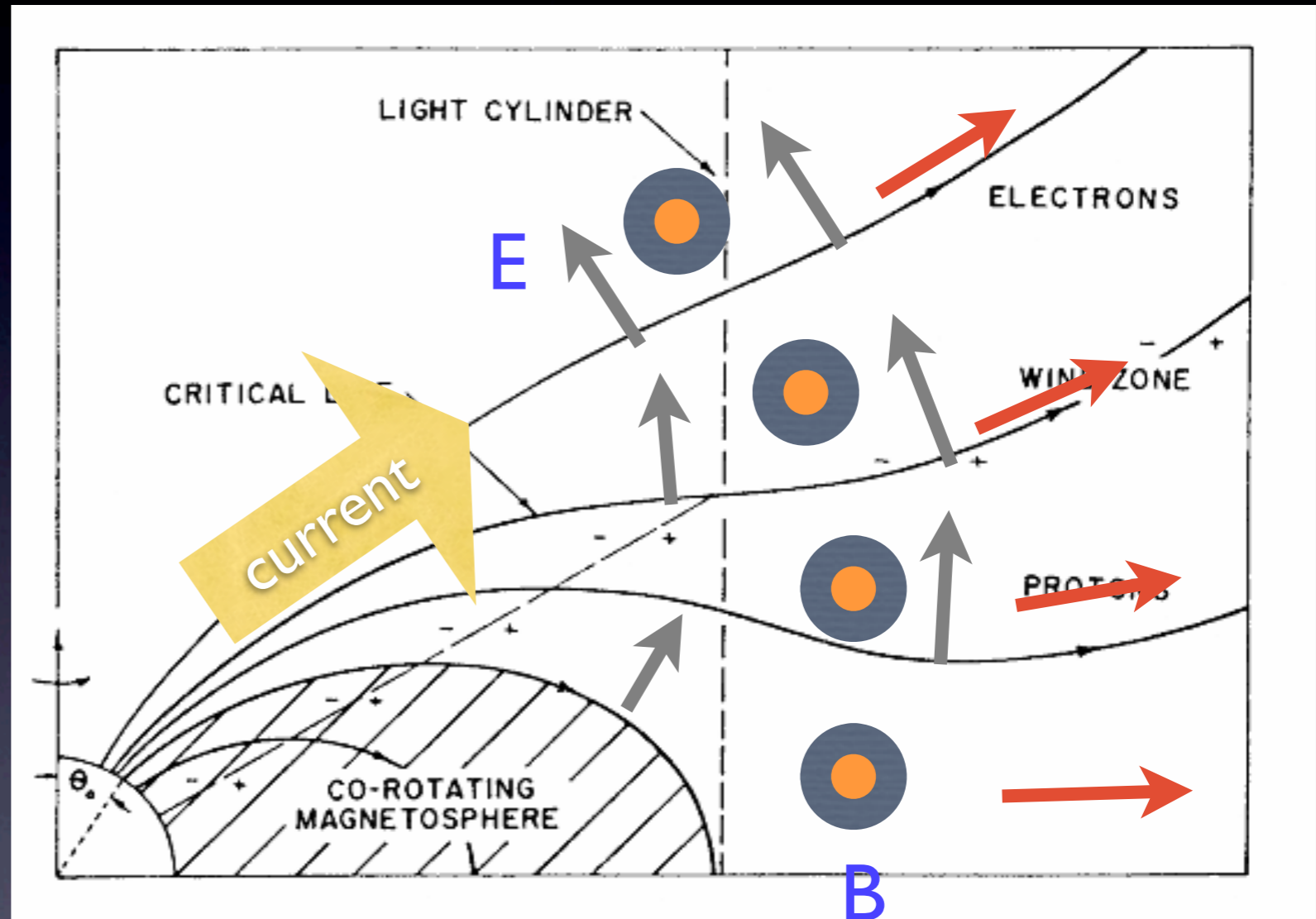
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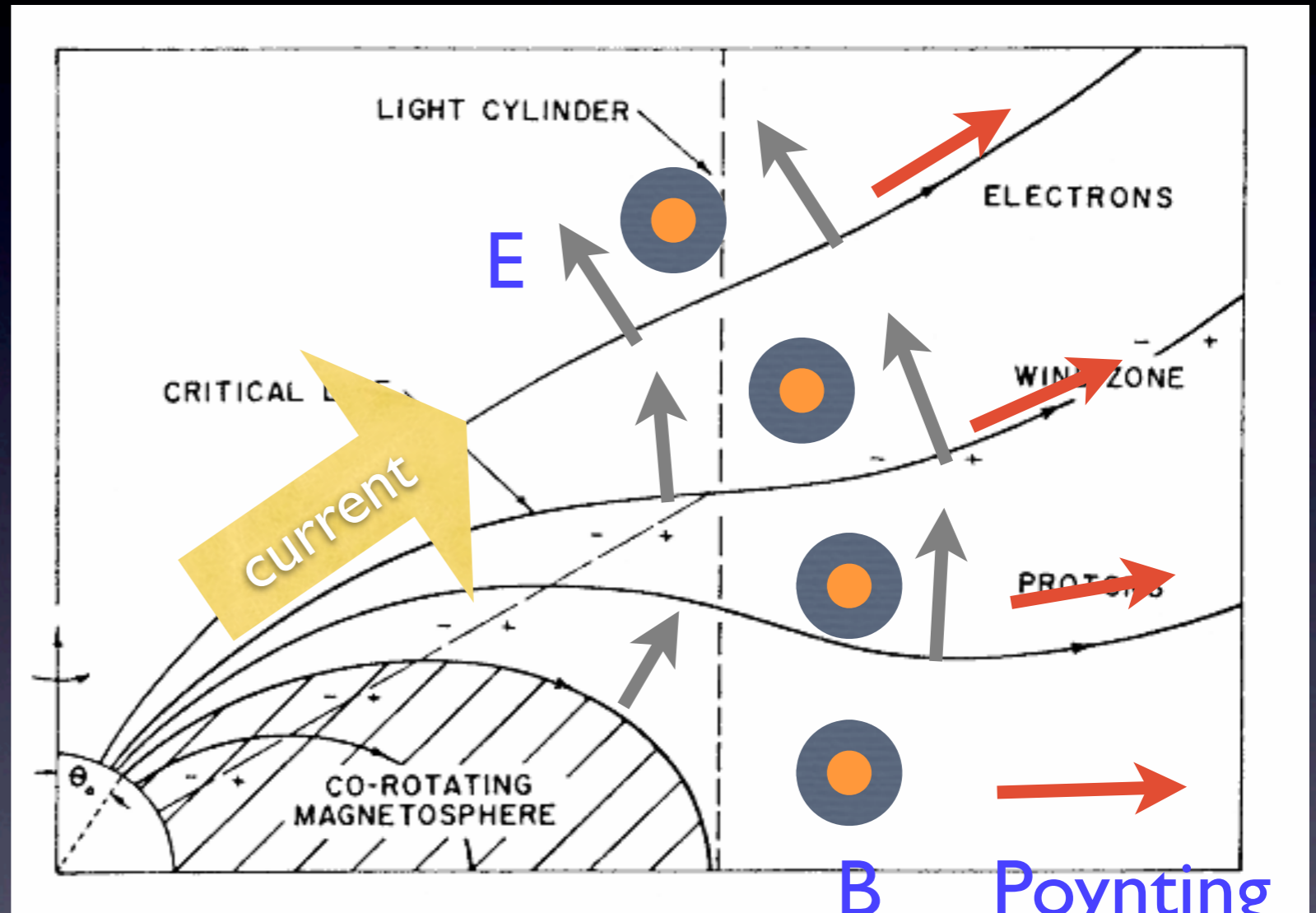
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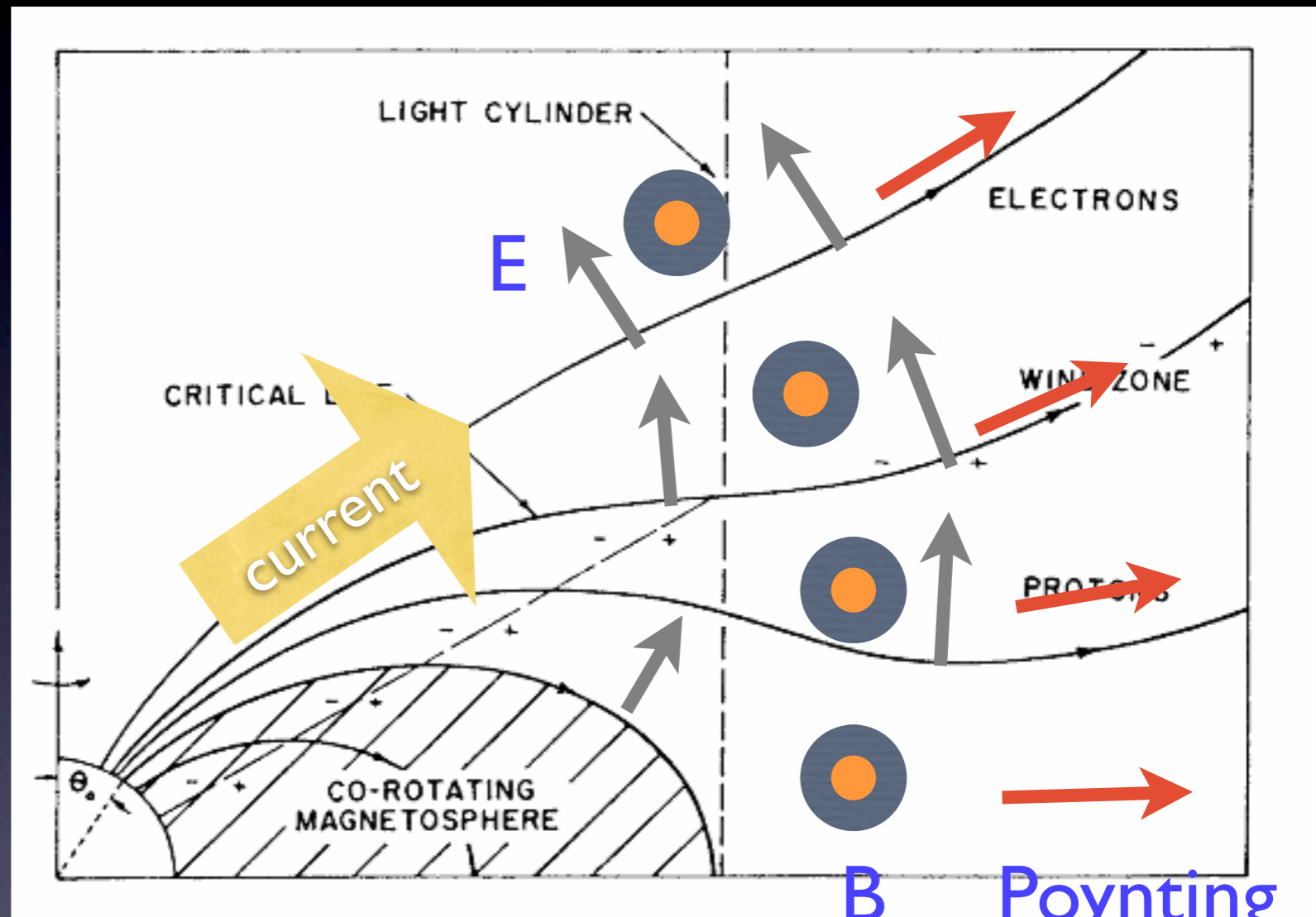
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B **Poynting**
Goldreich & Julian 1969

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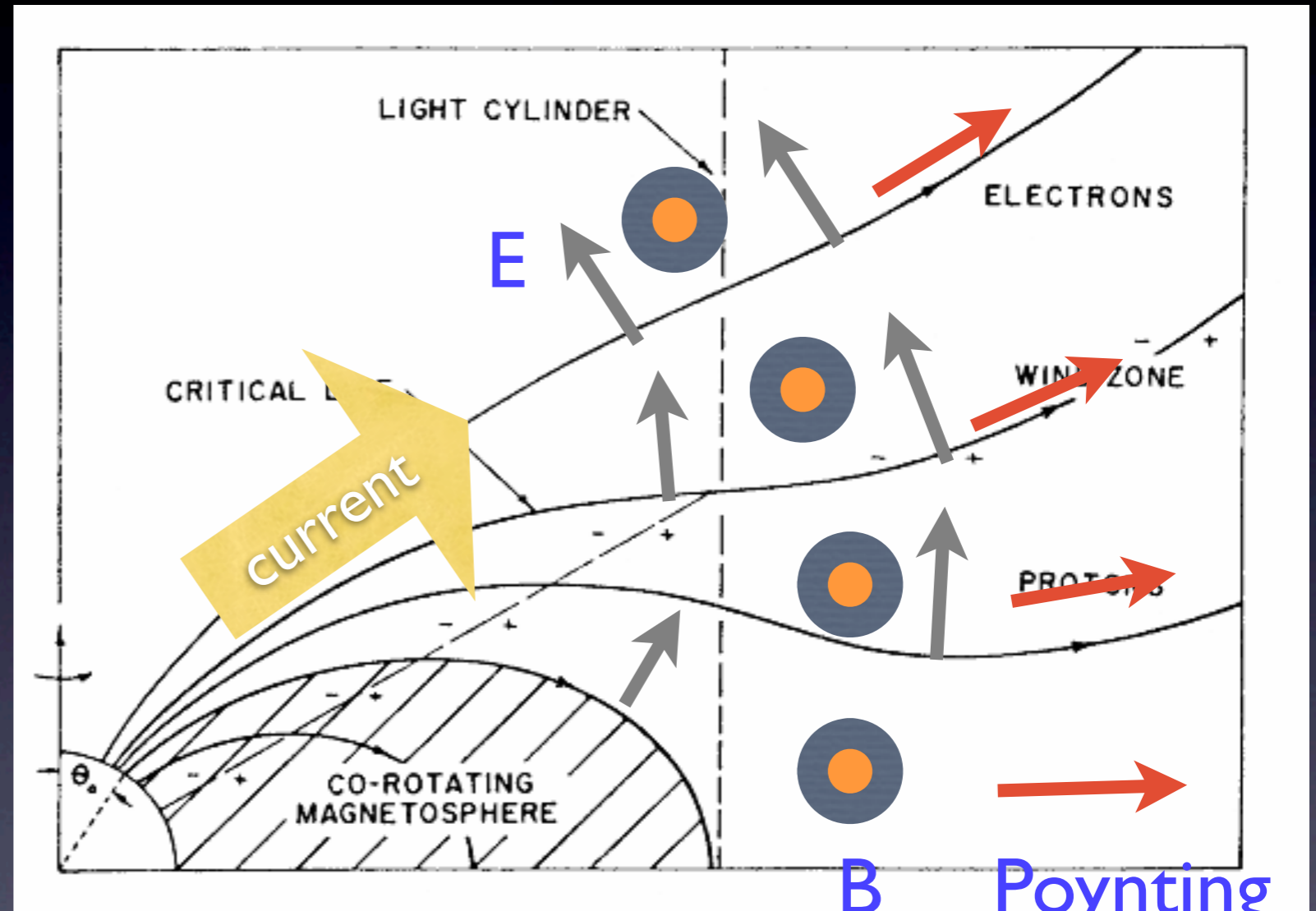
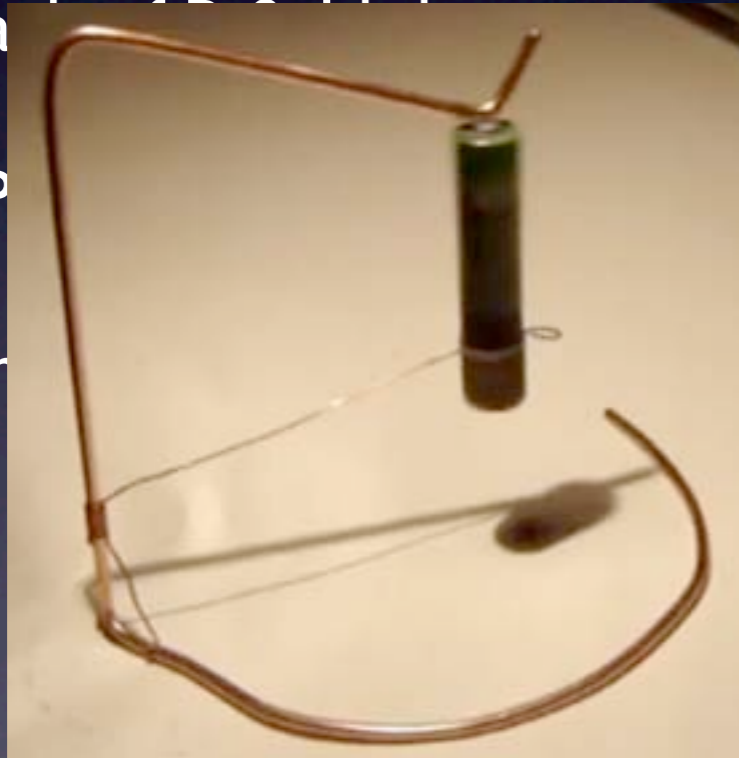


Goldreich & Julian 1969

Radiator in Fermi band is tapping into this energy flux

Pulsars: energy loss

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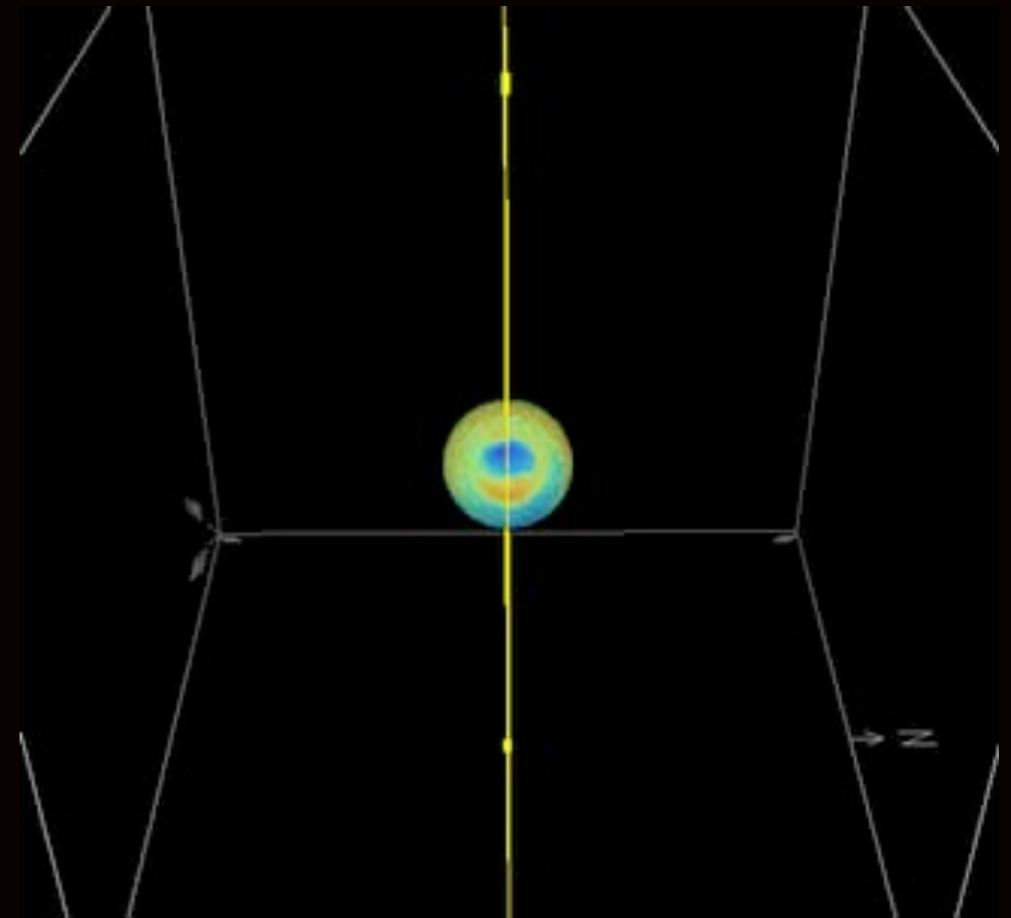
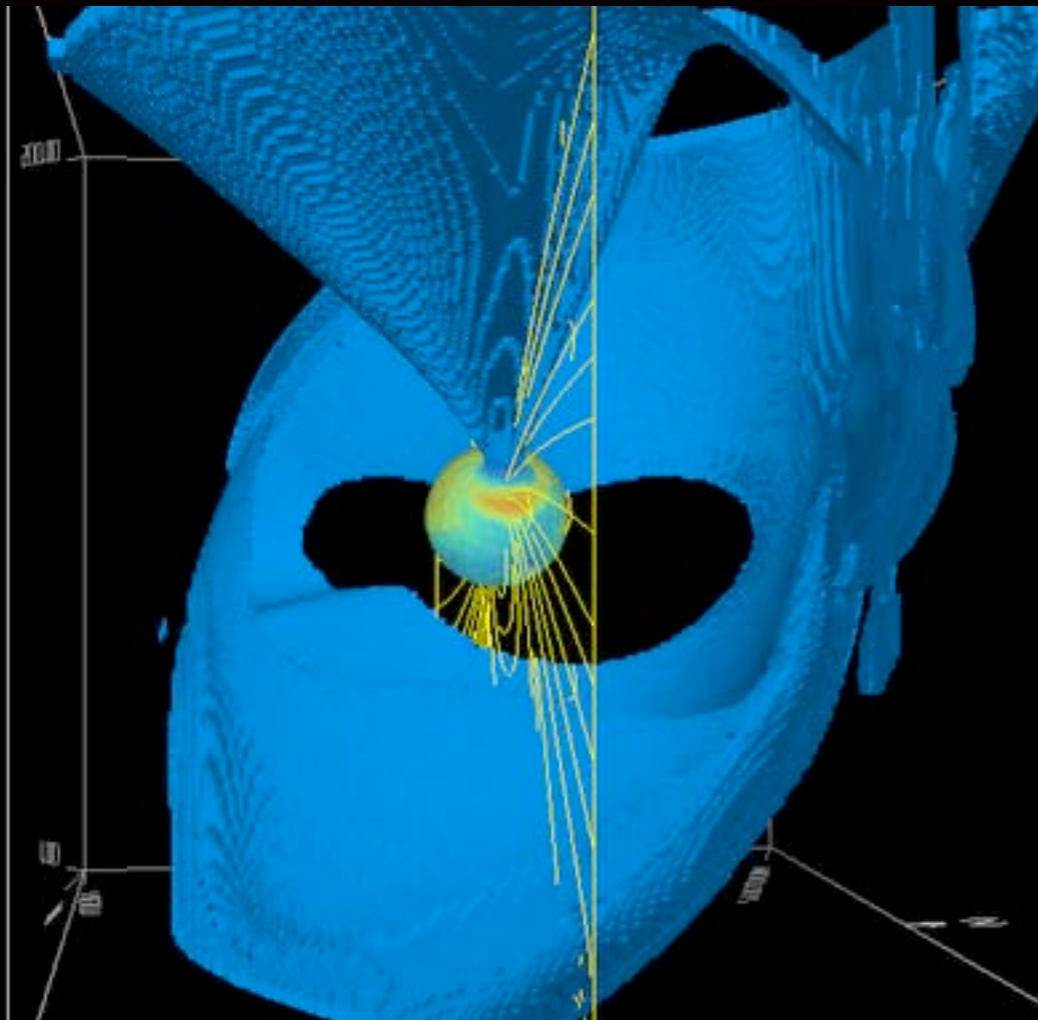
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3D solution: flux surfaces

Inclination affects the current structure and open flux tube geometry. Need to determine open/closed flux. Gruzinov (2005) found an invariant on field lines:

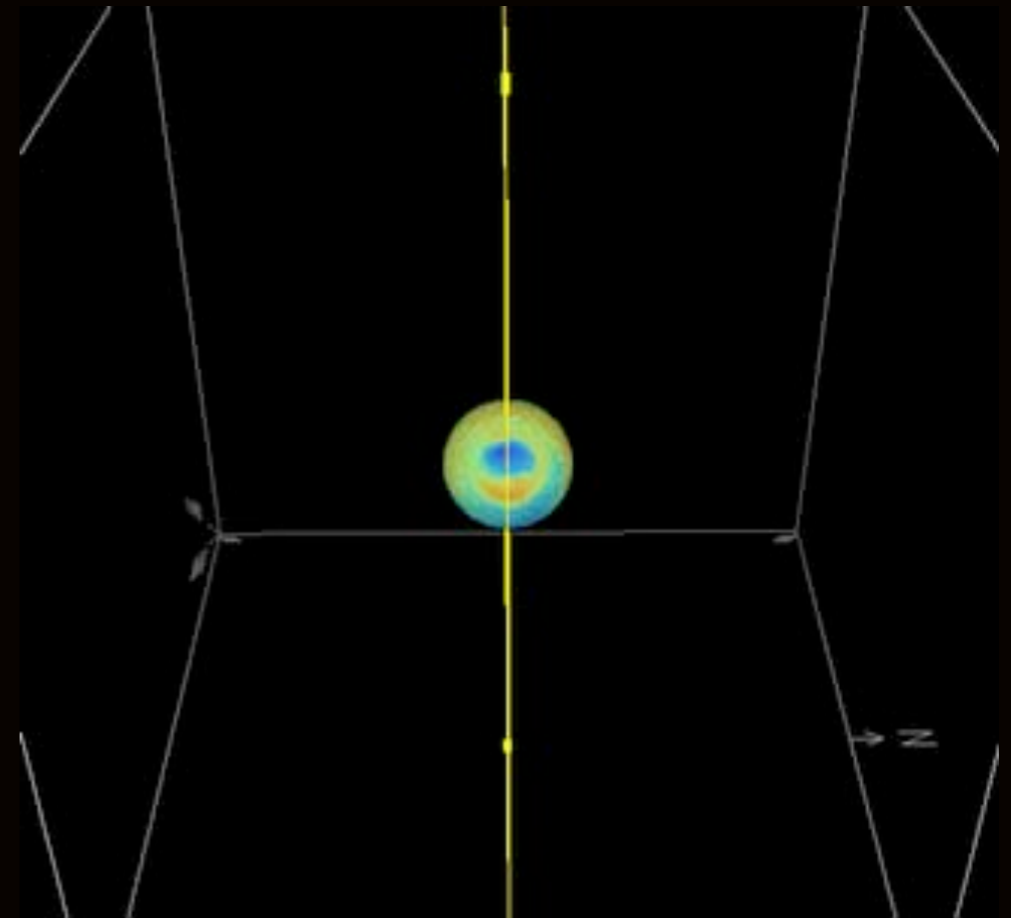
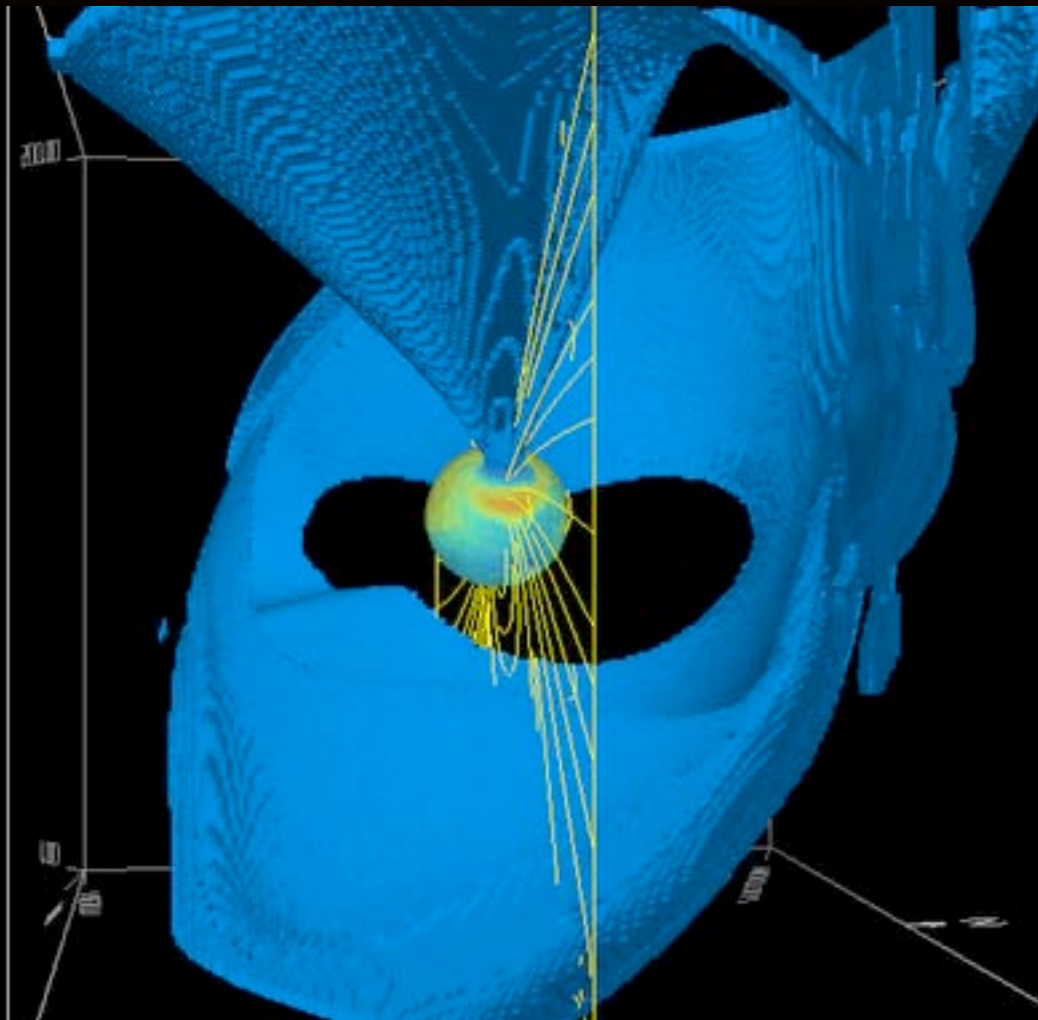
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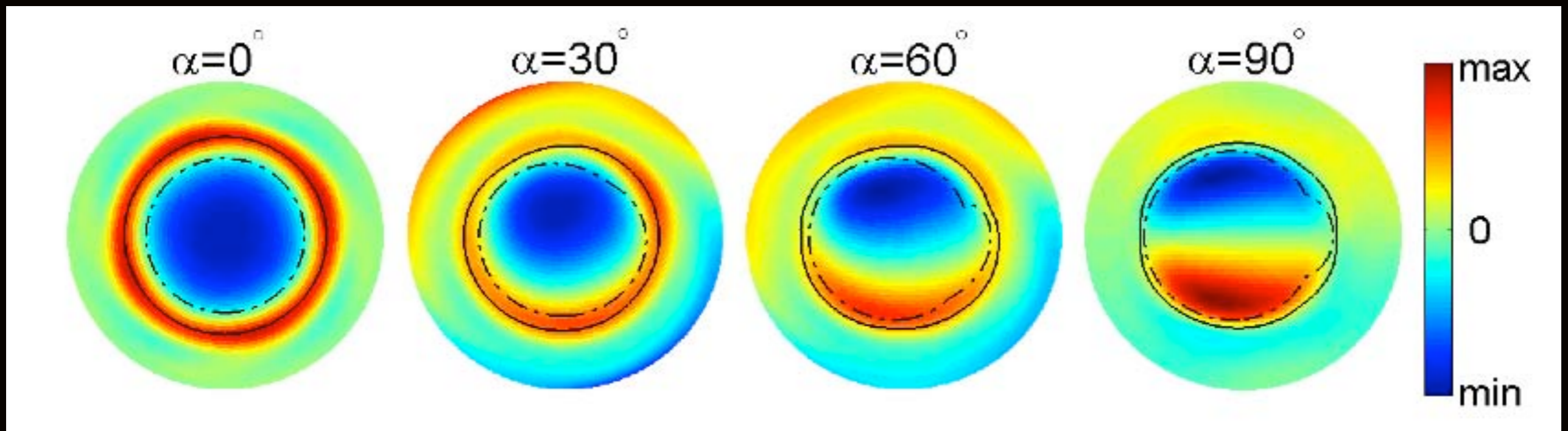


asymptotic split-monopole is ideal for caustic formation

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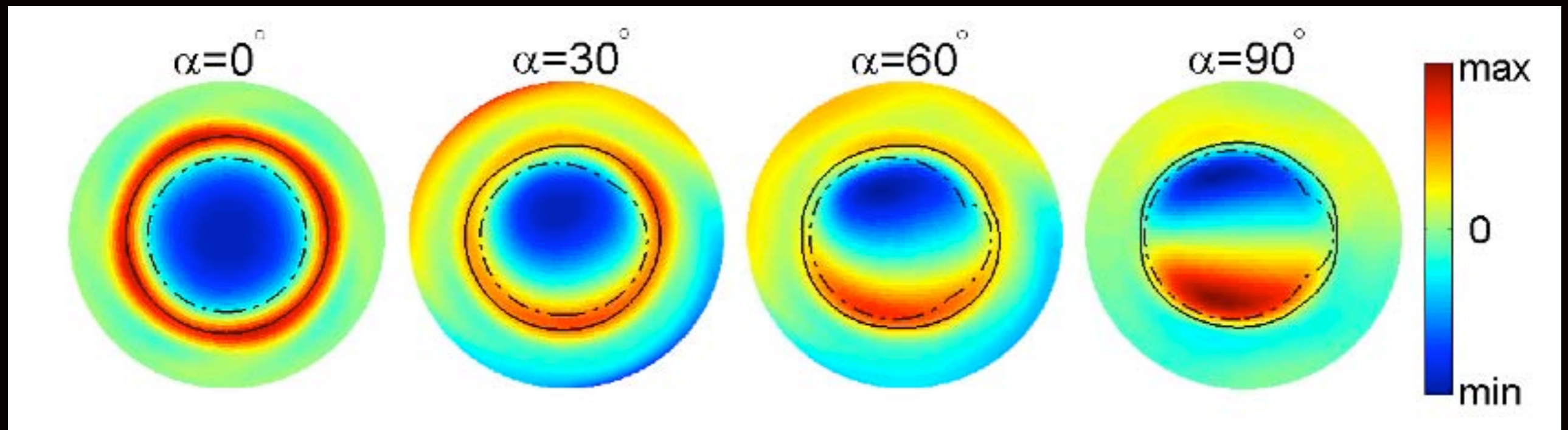


Bai & AS 09

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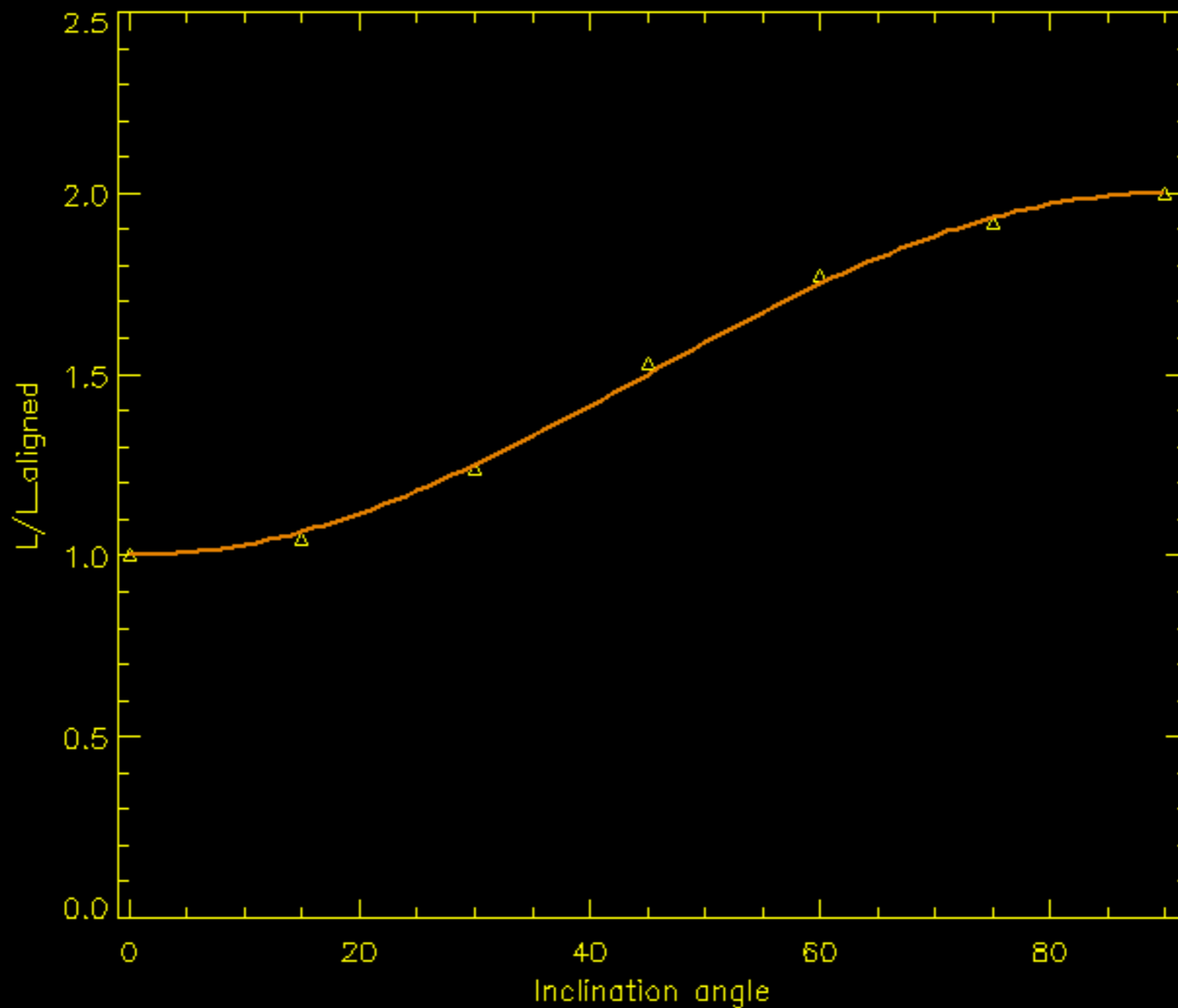
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Bai & AS 09

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Pulsar spindown



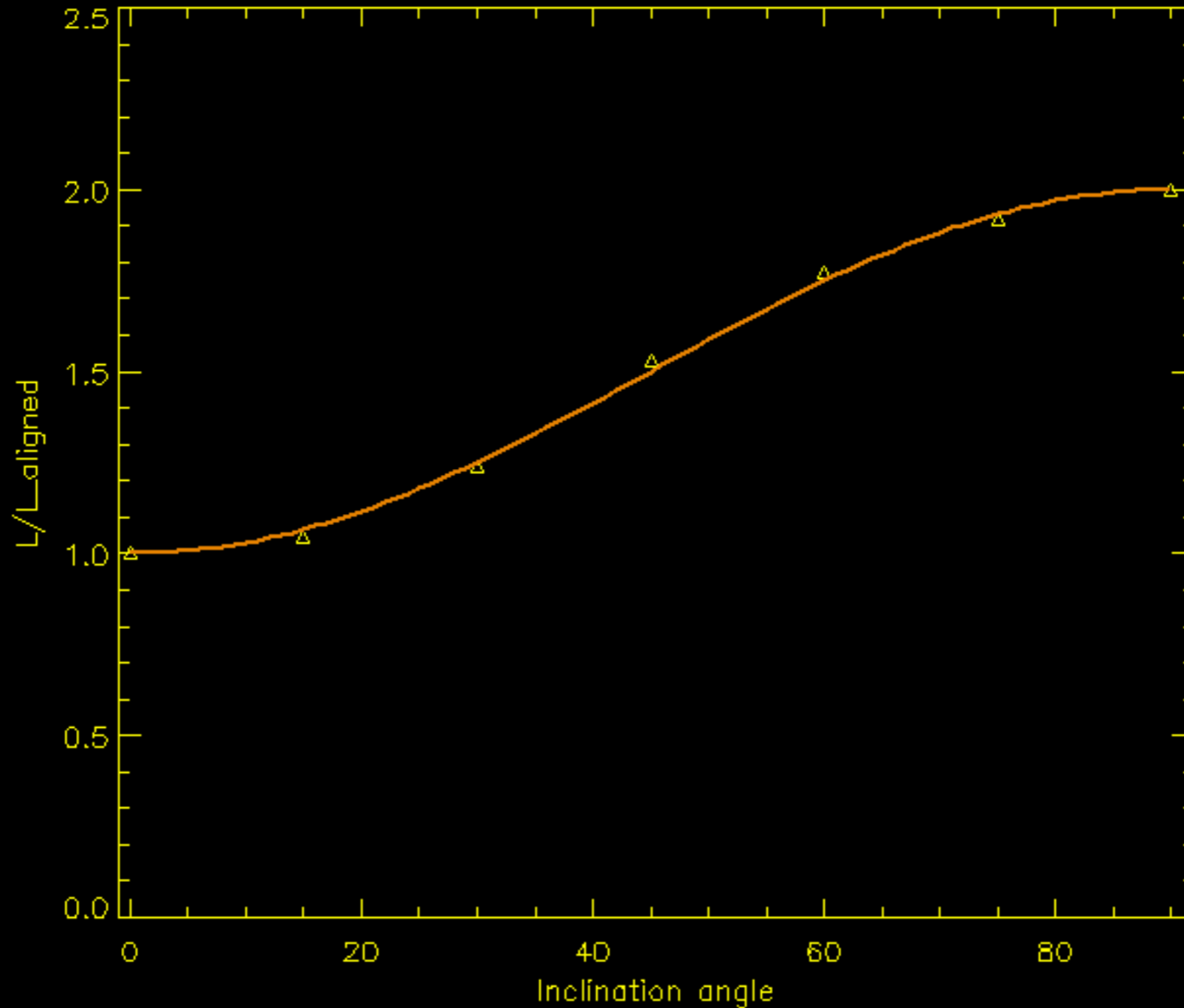
Spindown of oblique rotator

$$\dot{E} \approx \frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \theta)$$

Vacuum formula

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There are books that conclude that 90 degree rotator does not spin down at all...

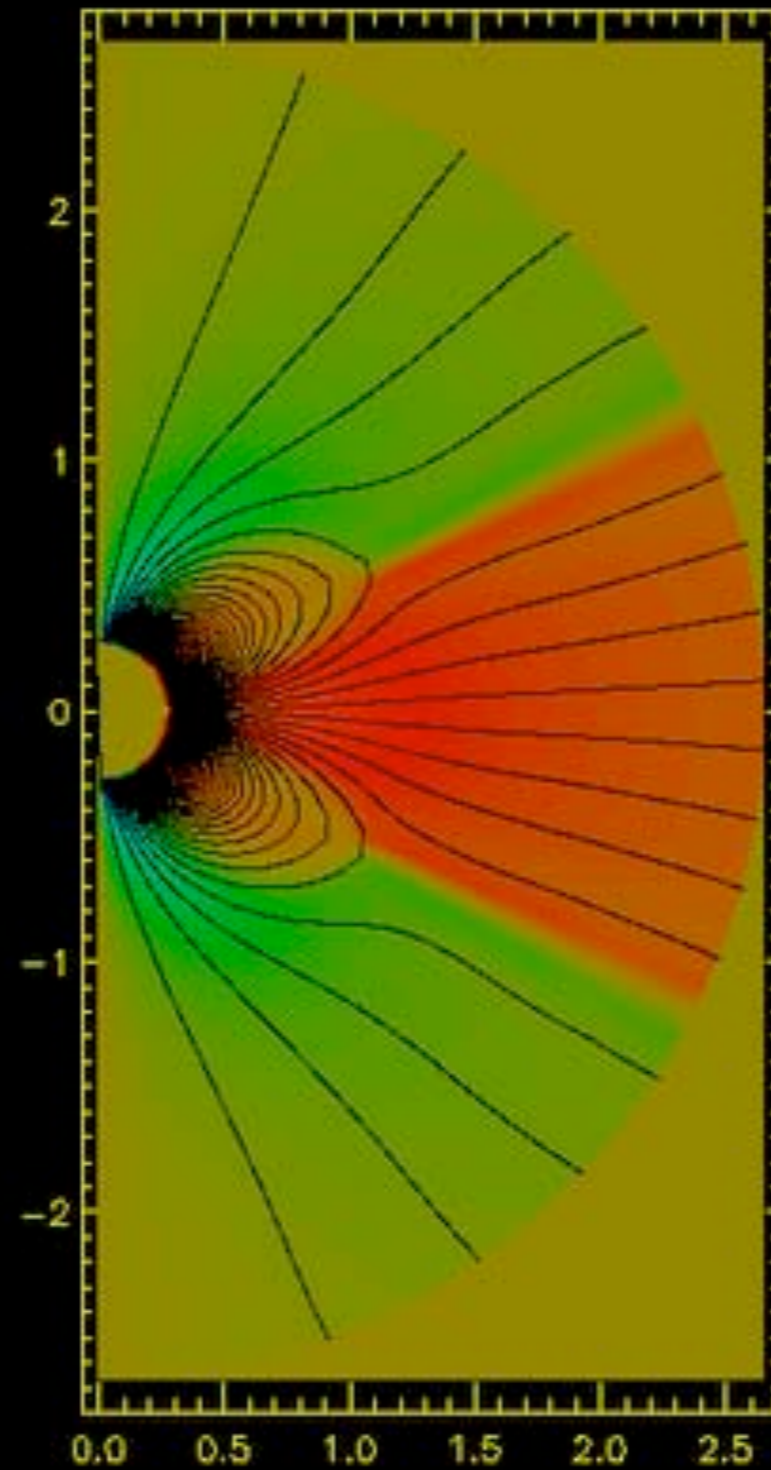
Force-free zoology

Quadrupole spindown



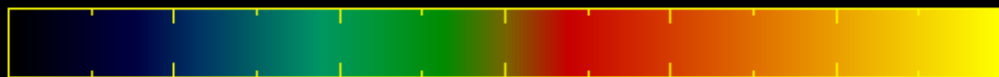
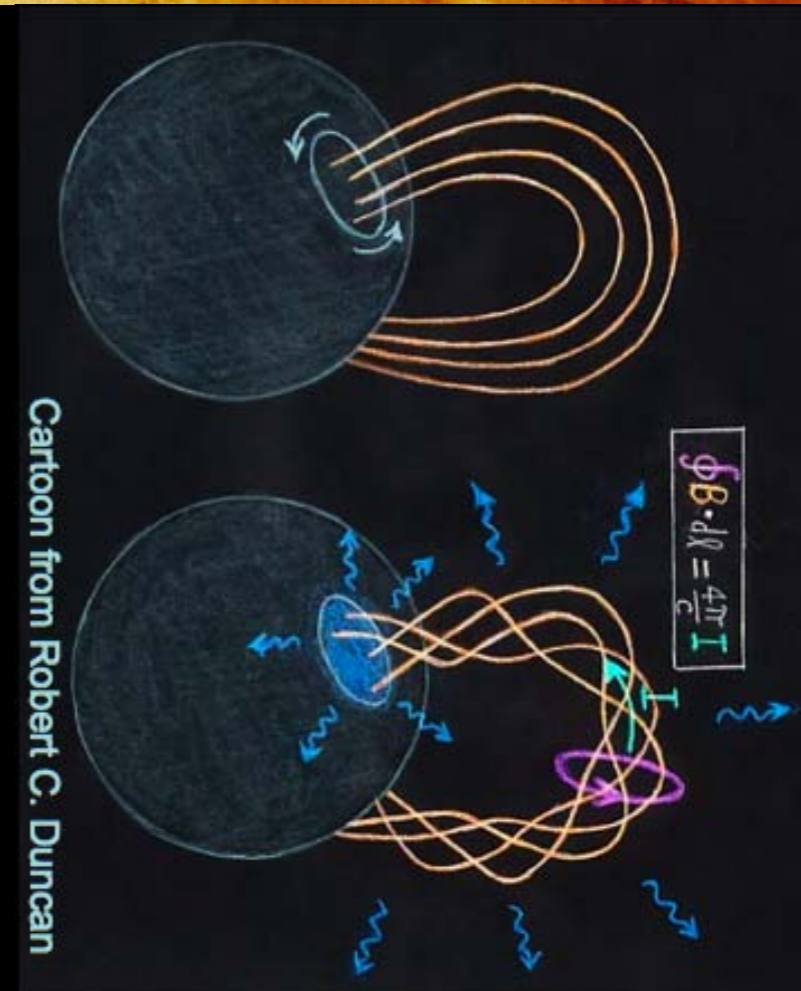
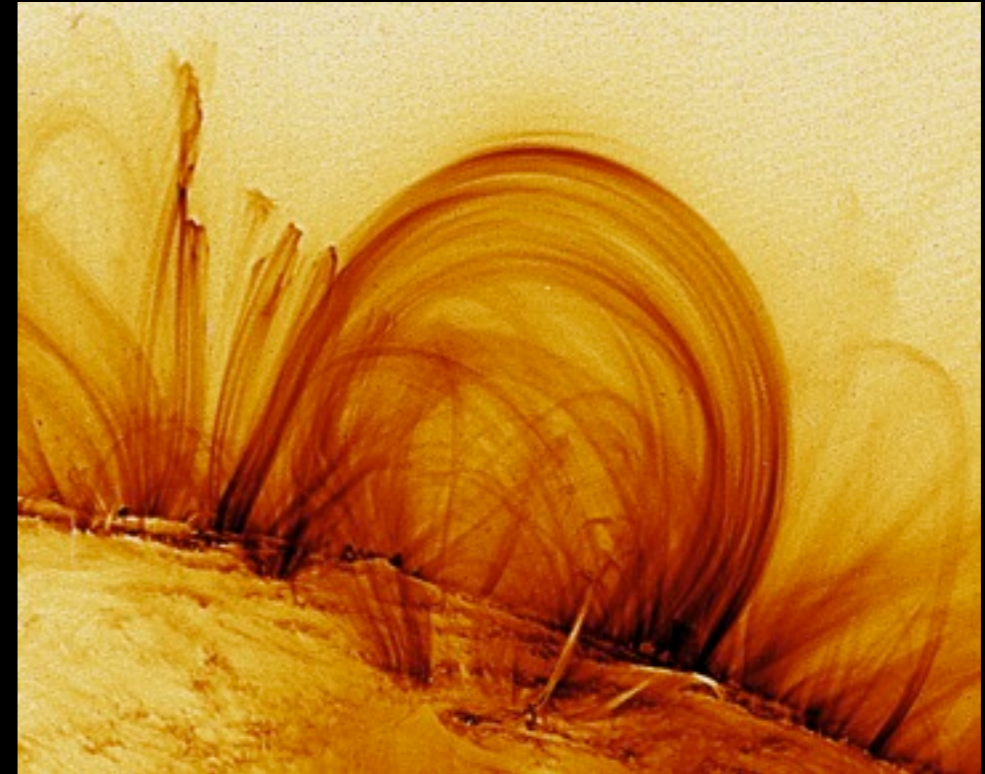
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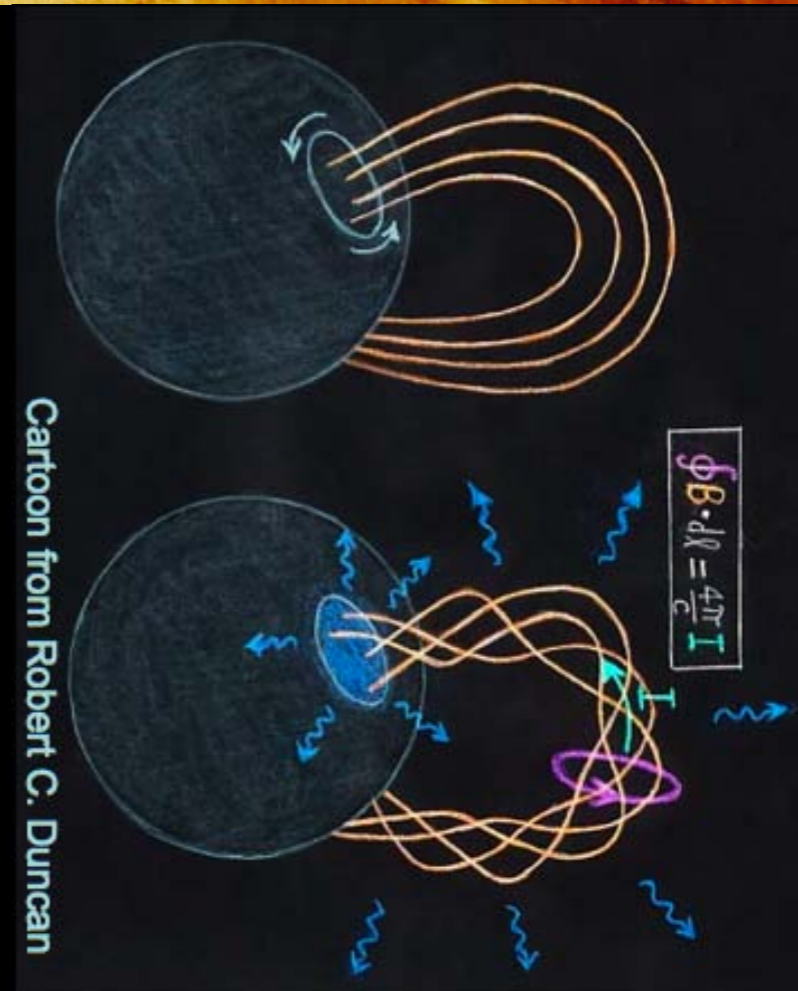
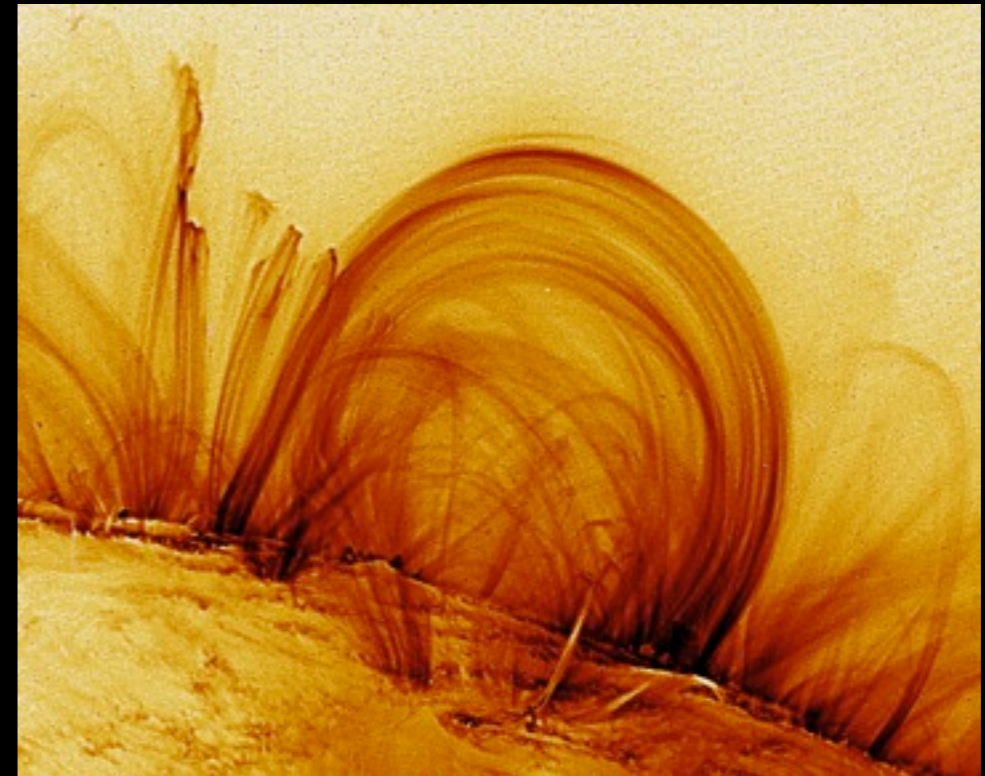
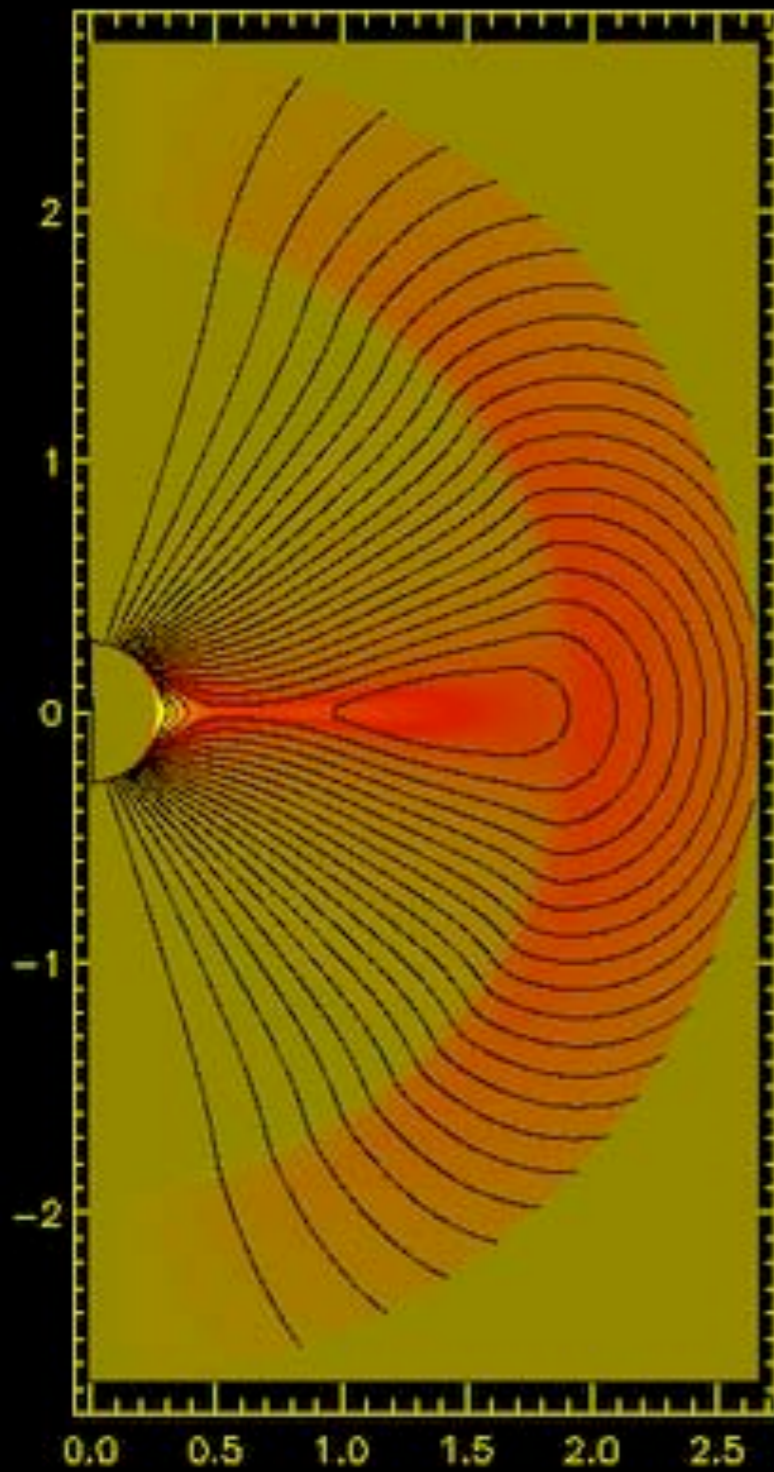
Force-free zoology

Magnetar starquake



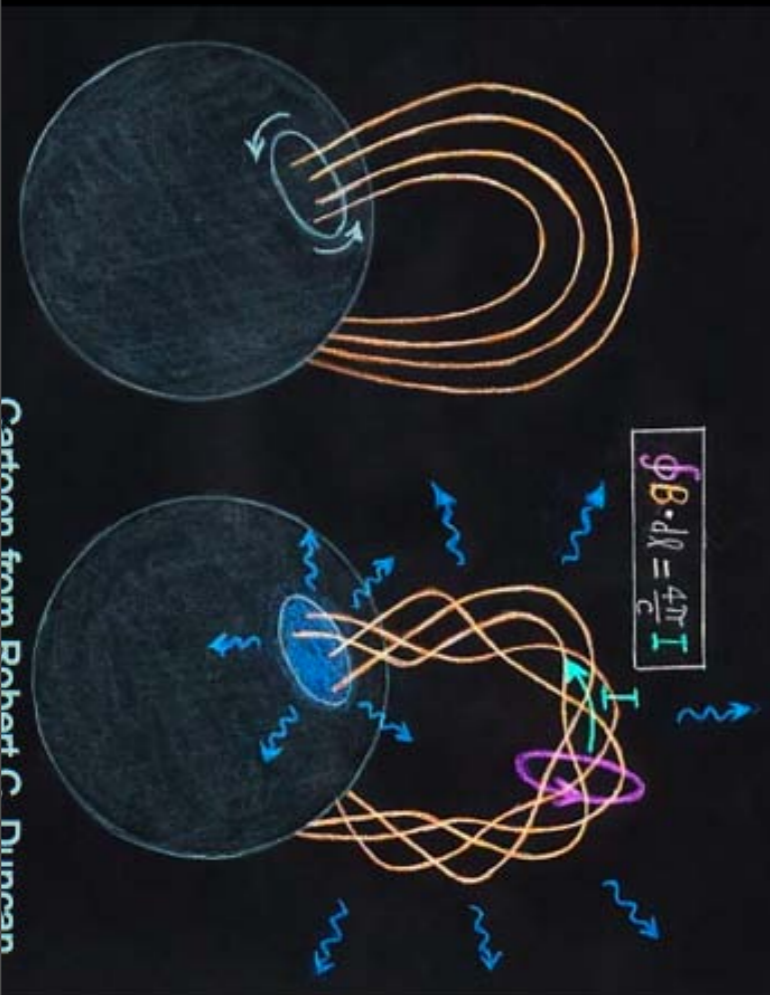
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Magnetospheres of magnetars

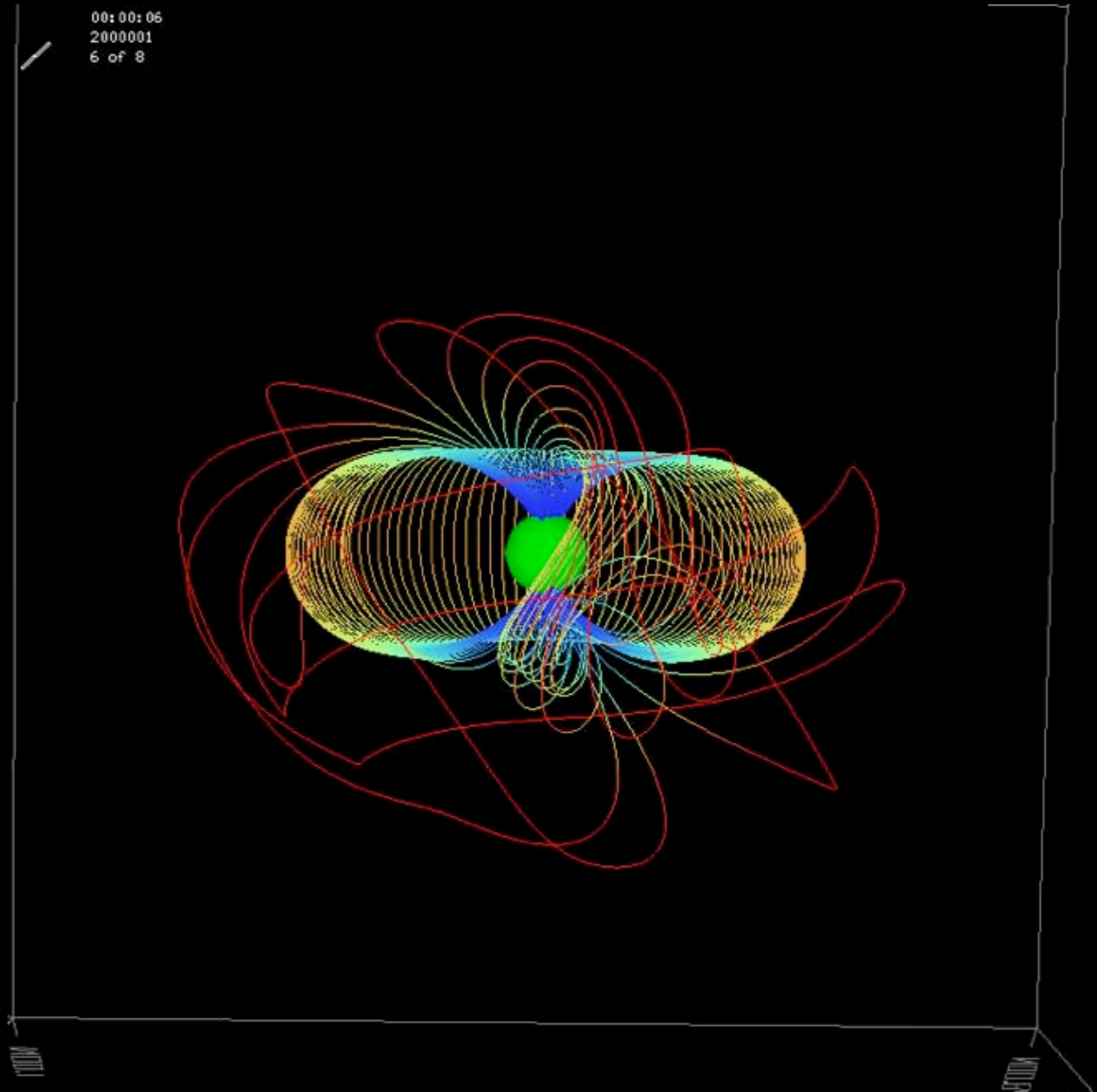
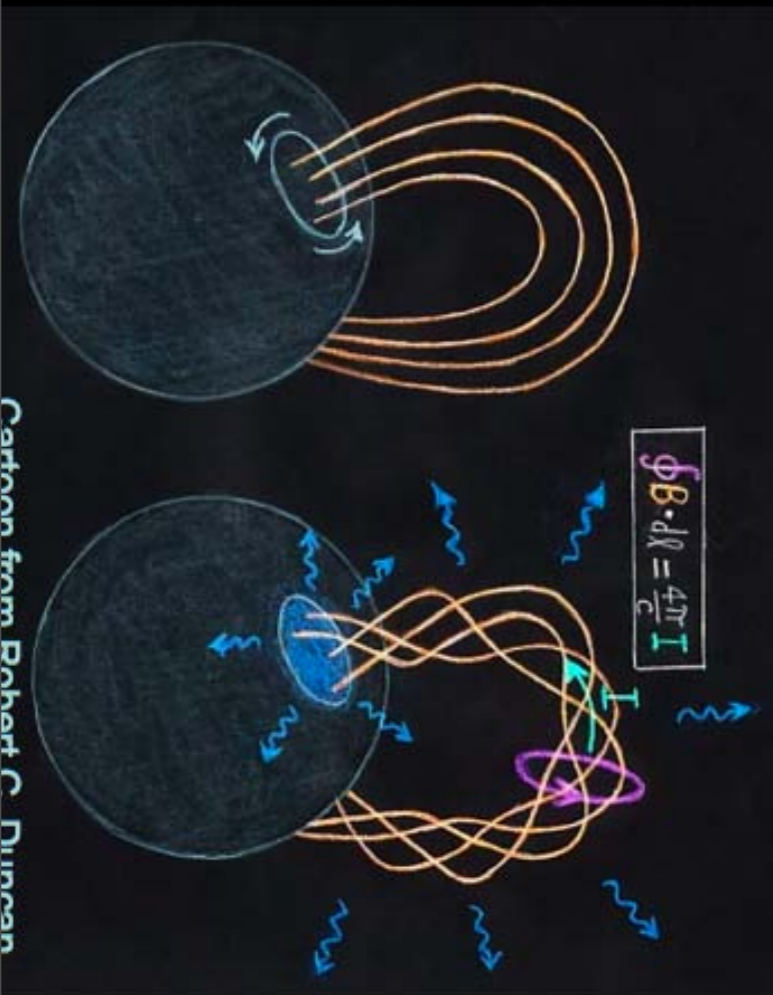
Magnetar starquake



Cartoon from Robert C. Duncan

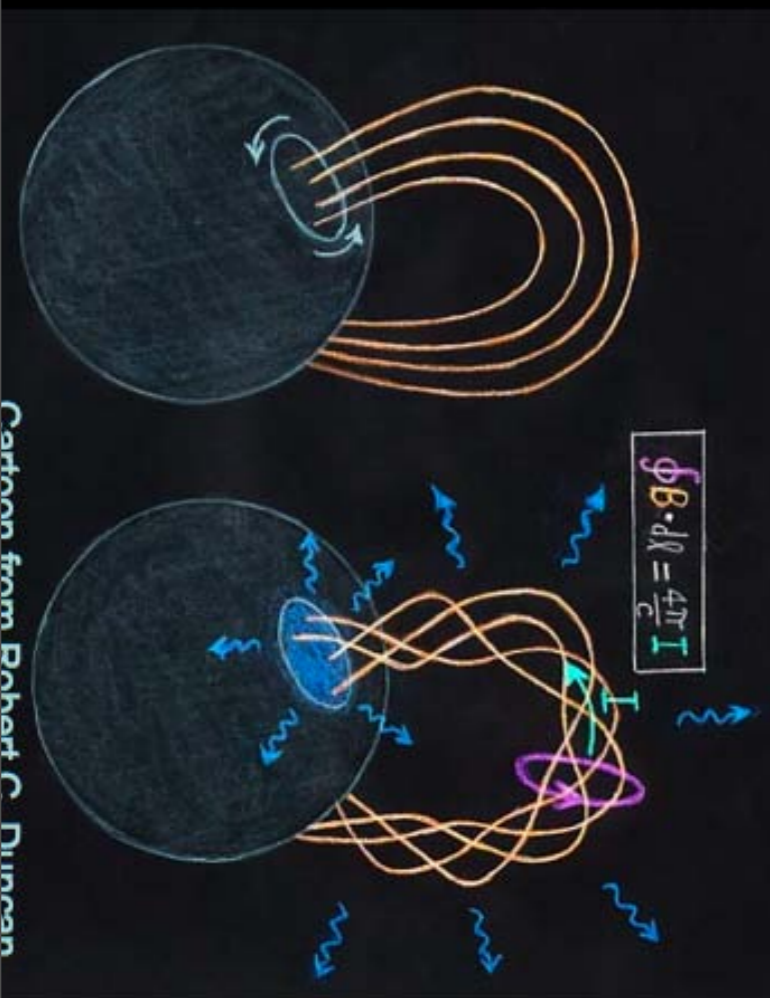
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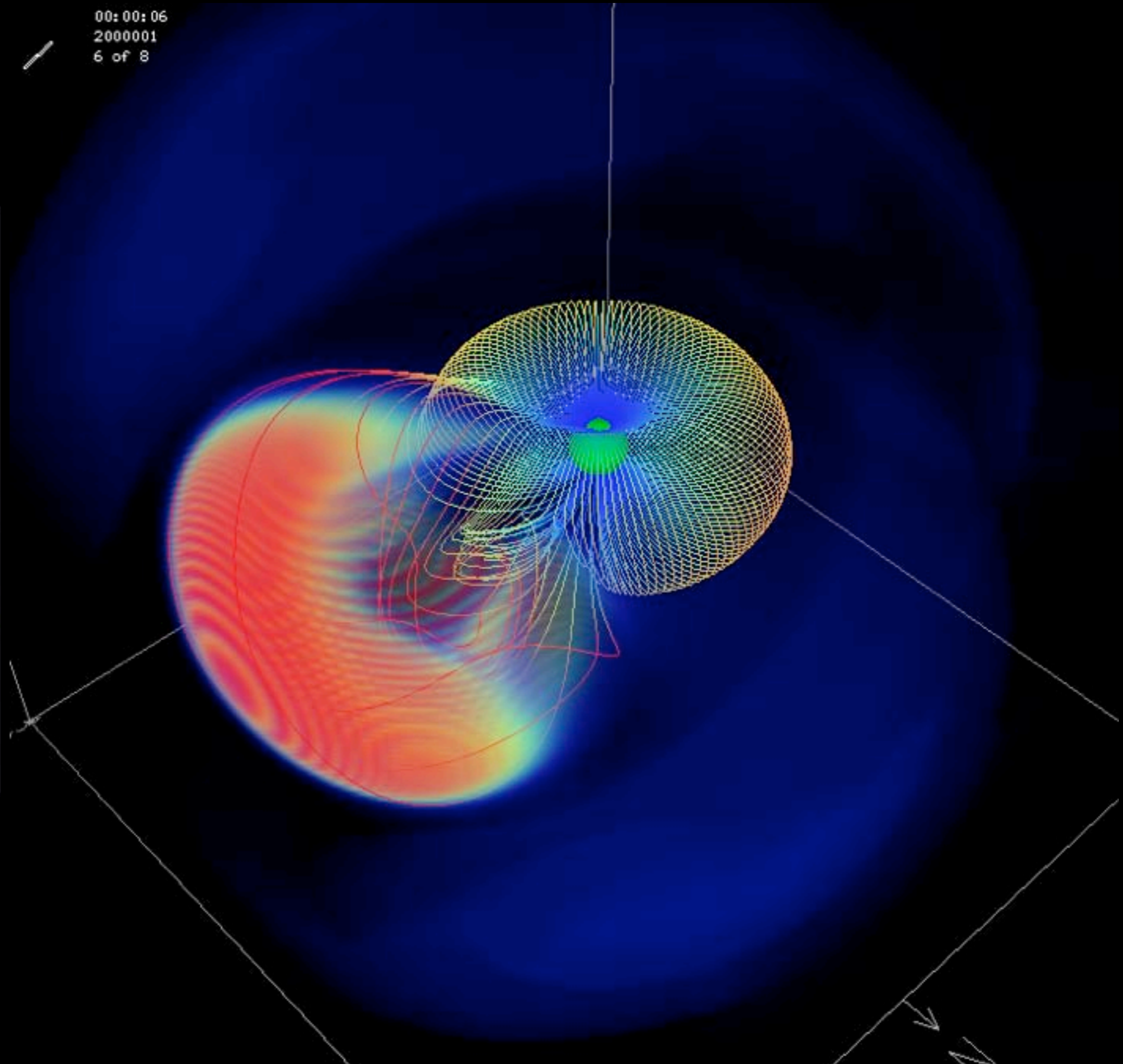
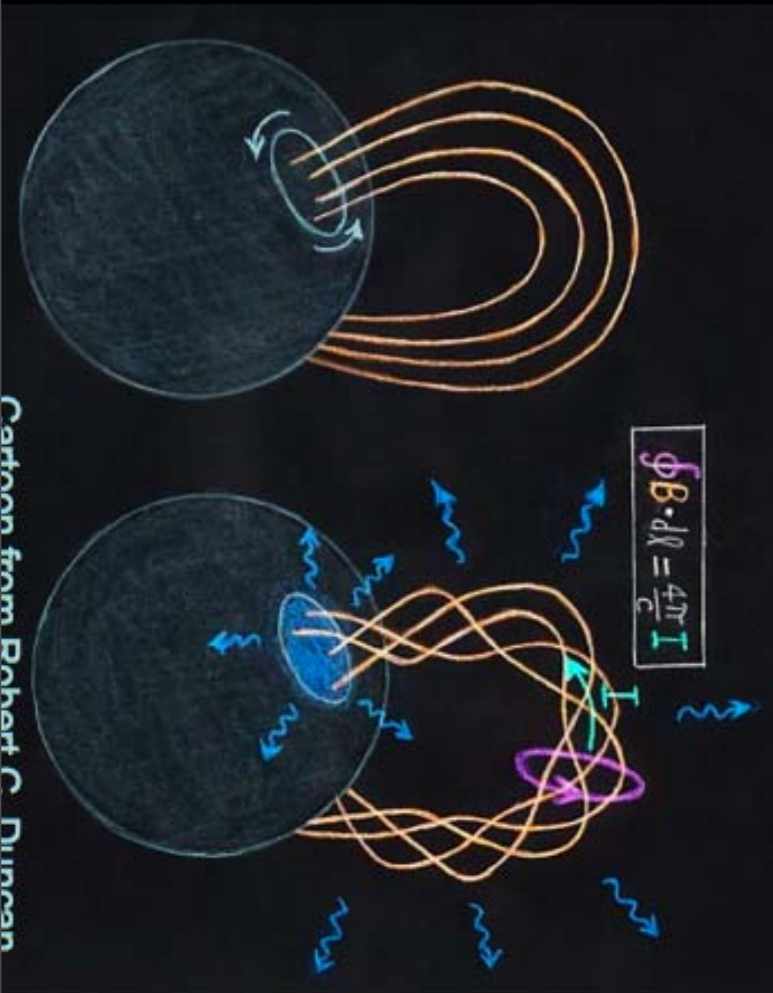


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6 of 8

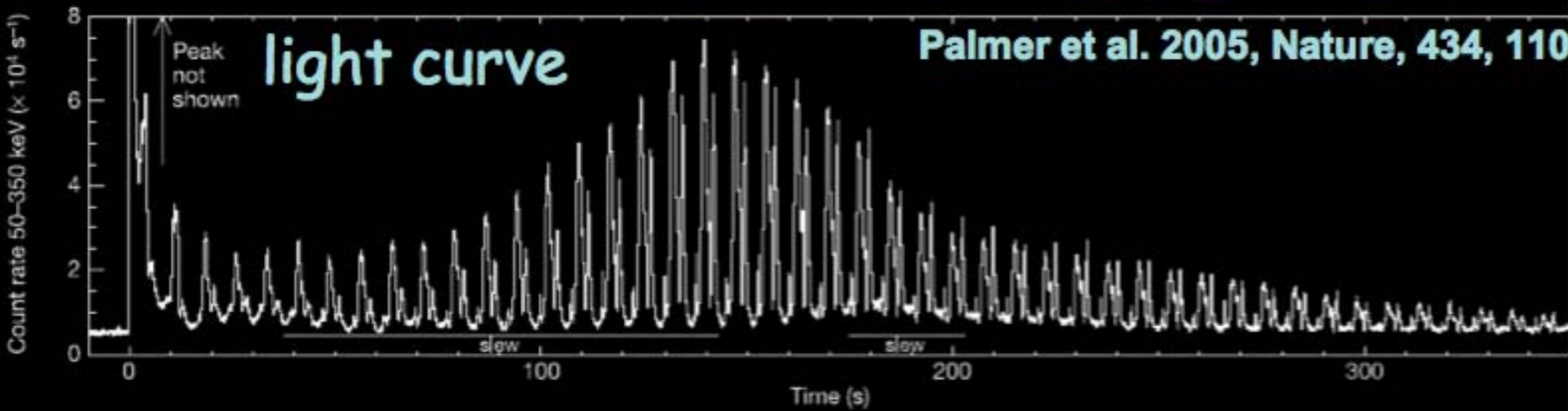


Magnetospheres of magnetars

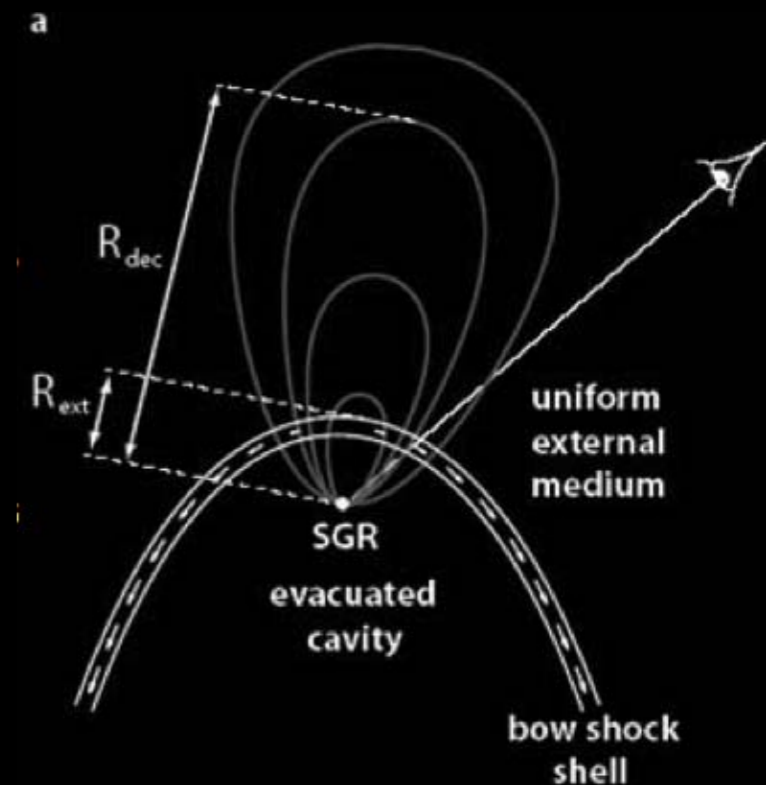
Magnetar starquake

SGR 1806-20 Dec 27, 2004

- **BAT: (15-350 keV) coded aperture imager on Swift**
- **the flare powered an expanding radio nebula**



Brightest object in the sky!

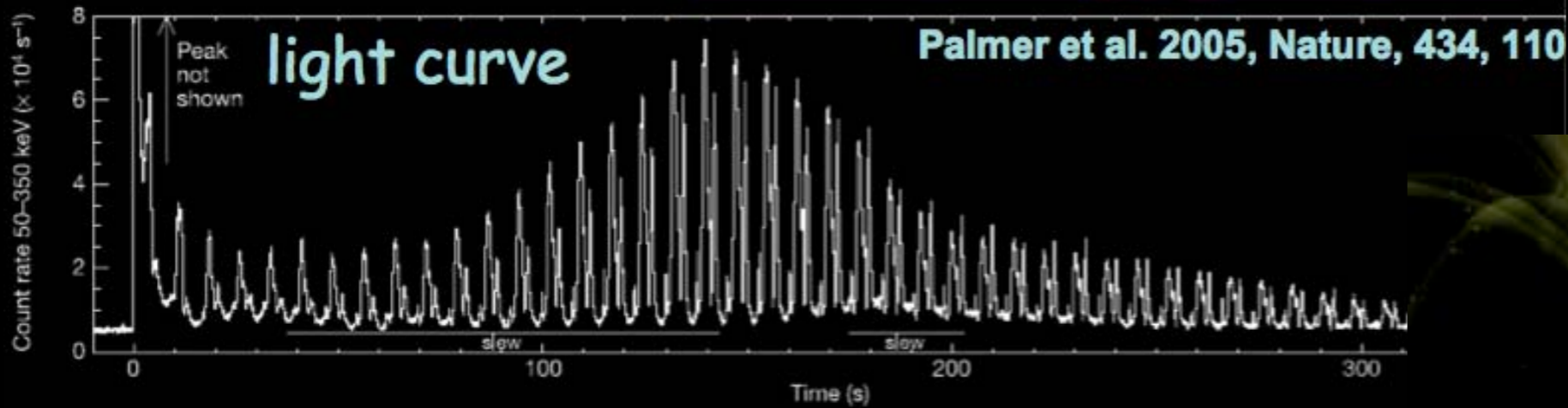


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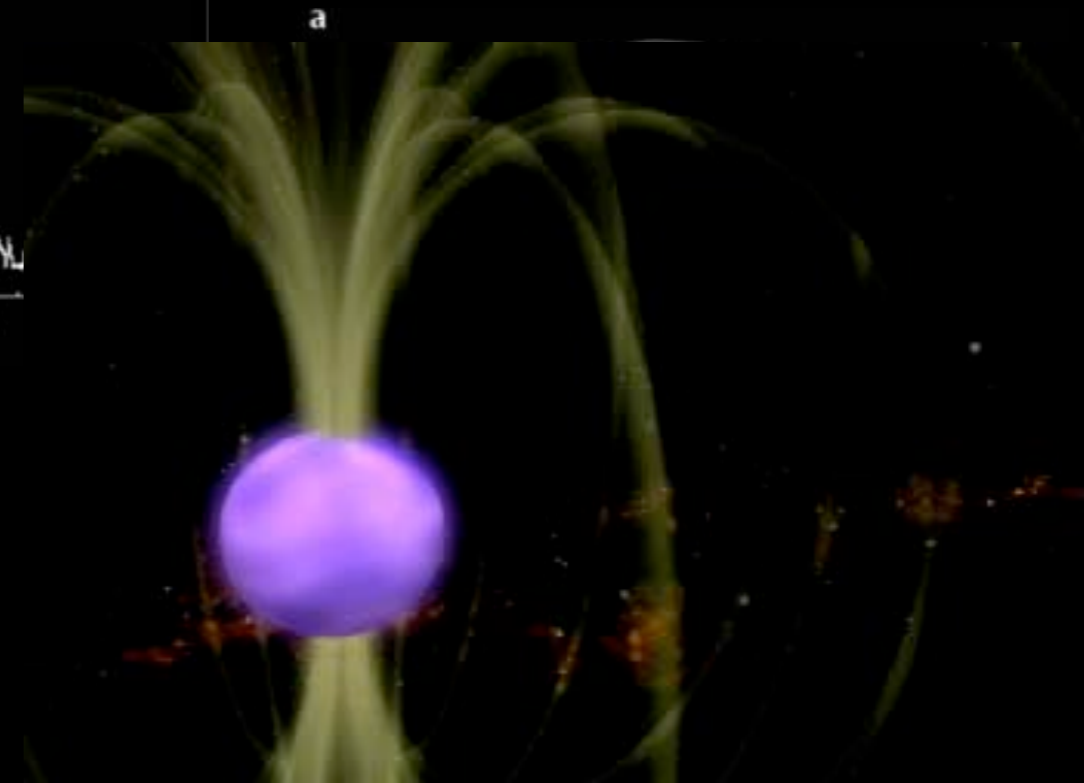
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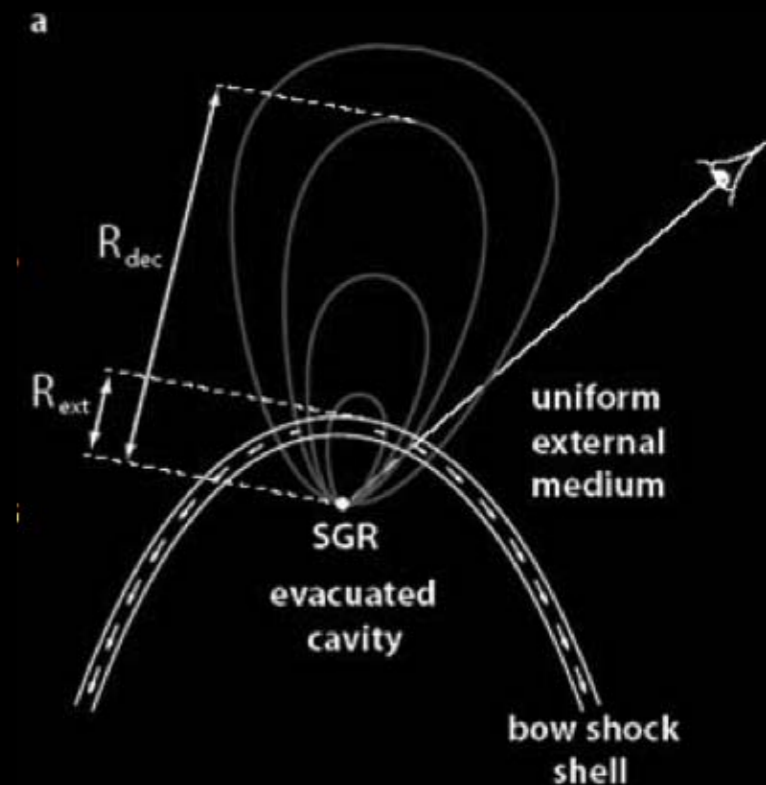
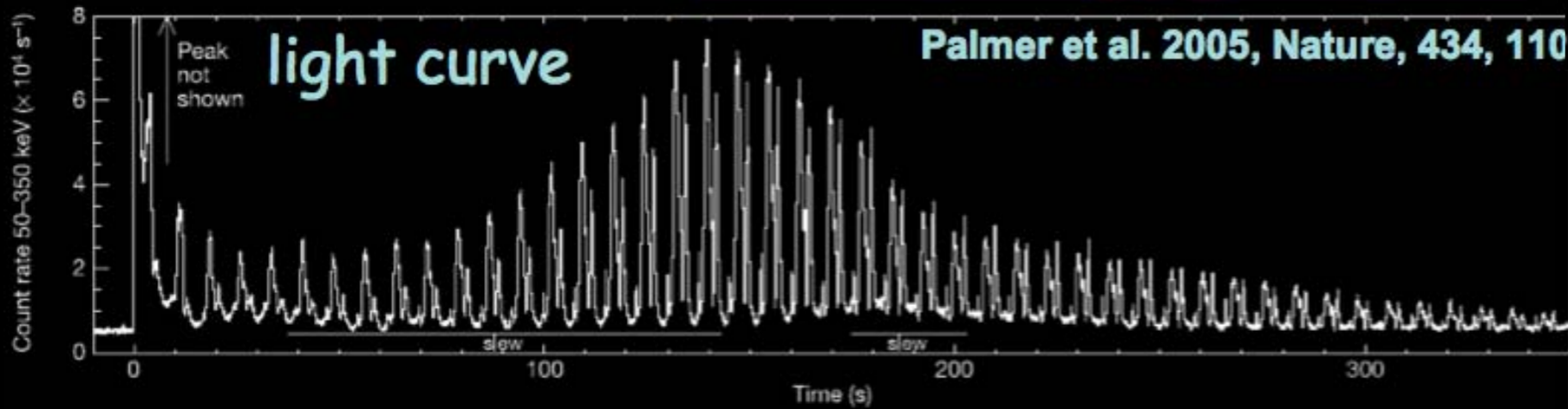
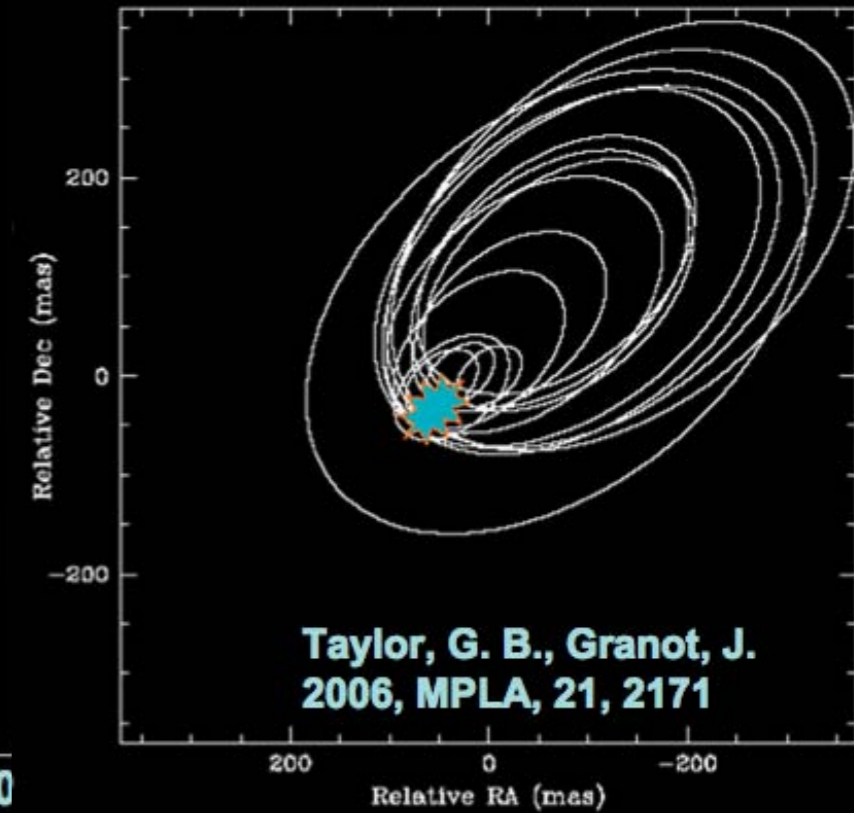


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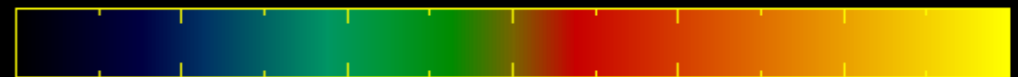
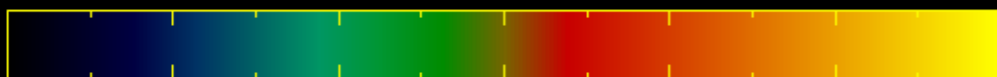
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Ejection of one-sided shells fits well with radio observations of afterglow of giant flares from magnetars.

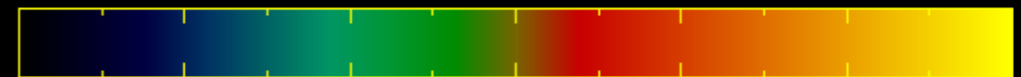
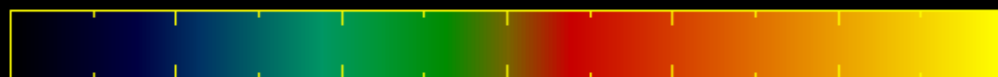
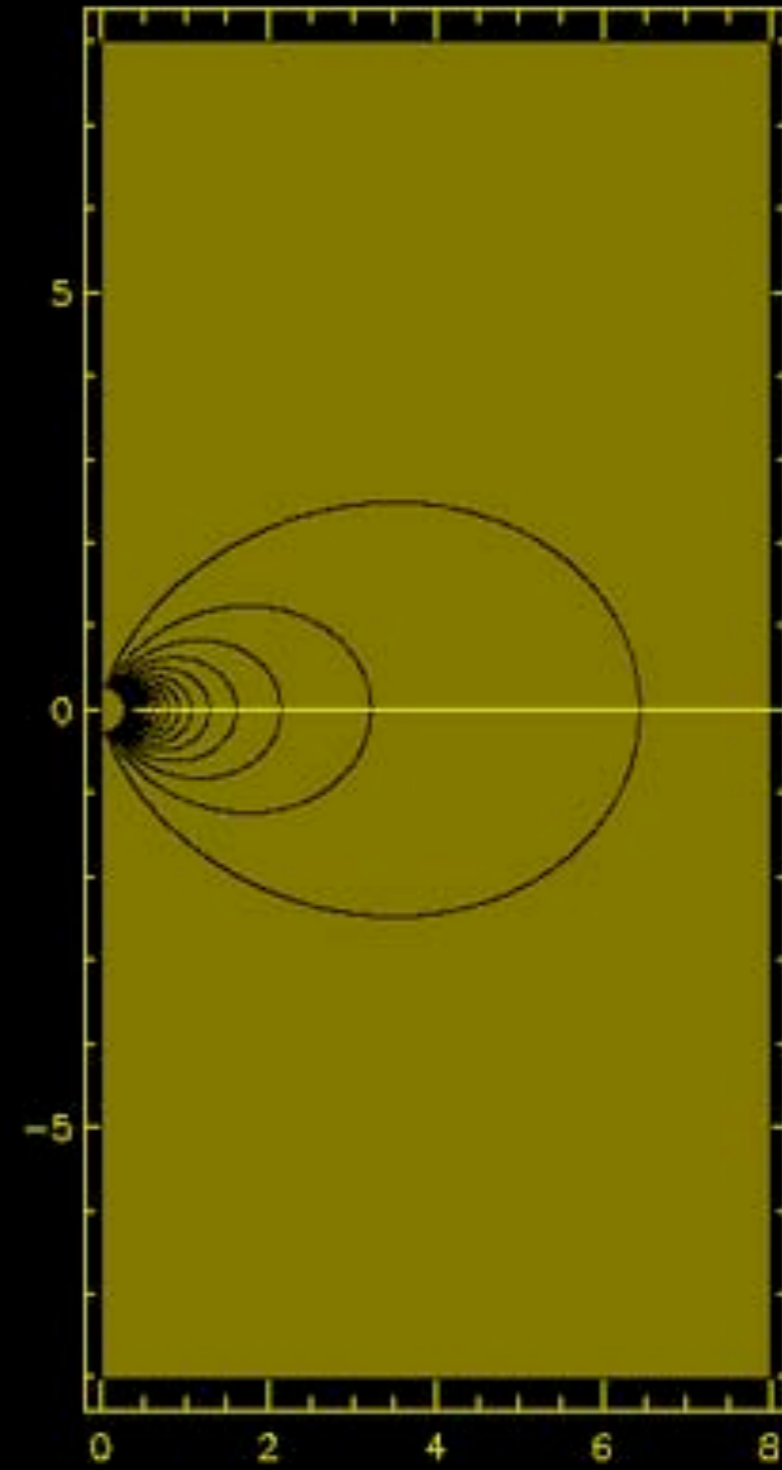
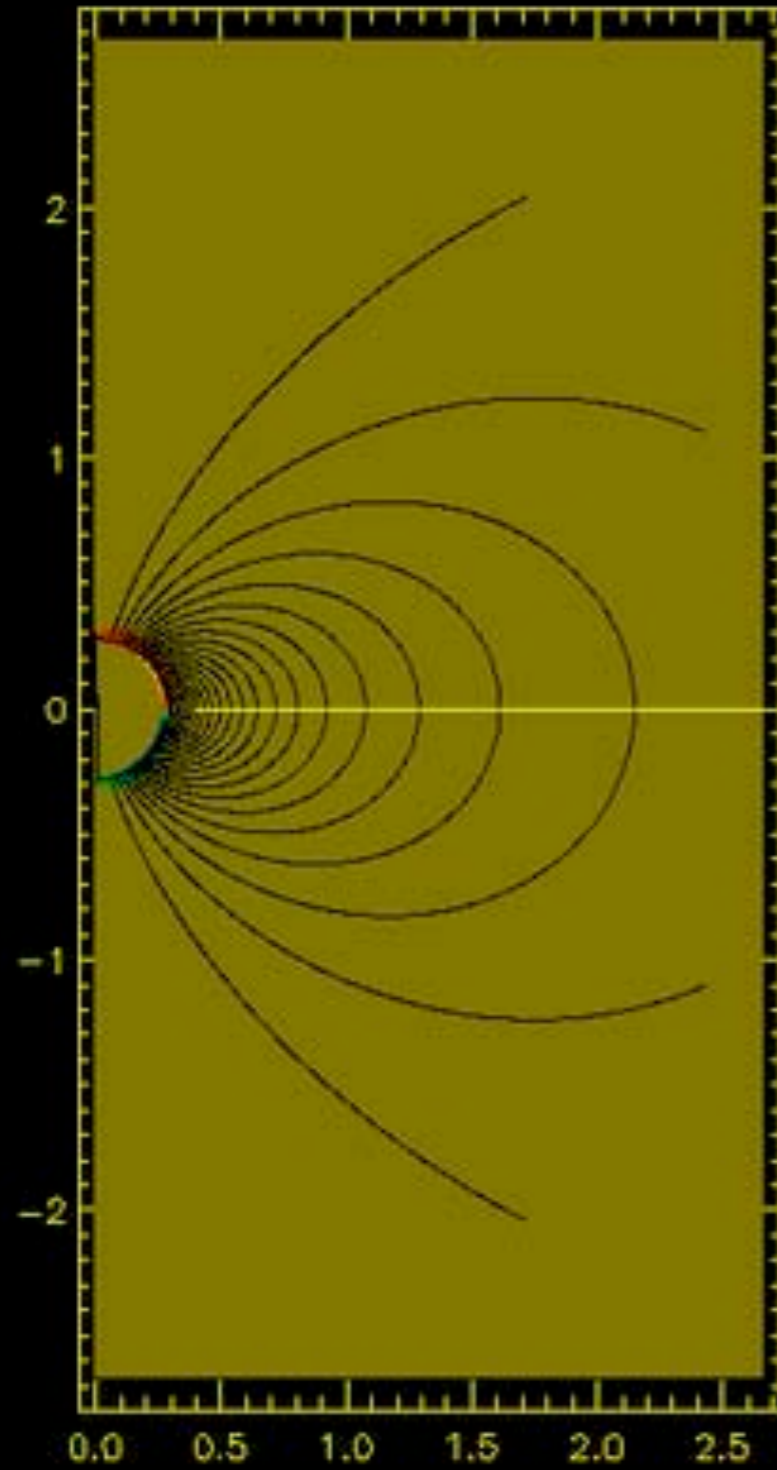
Force-free zoology: disks

Star+disk



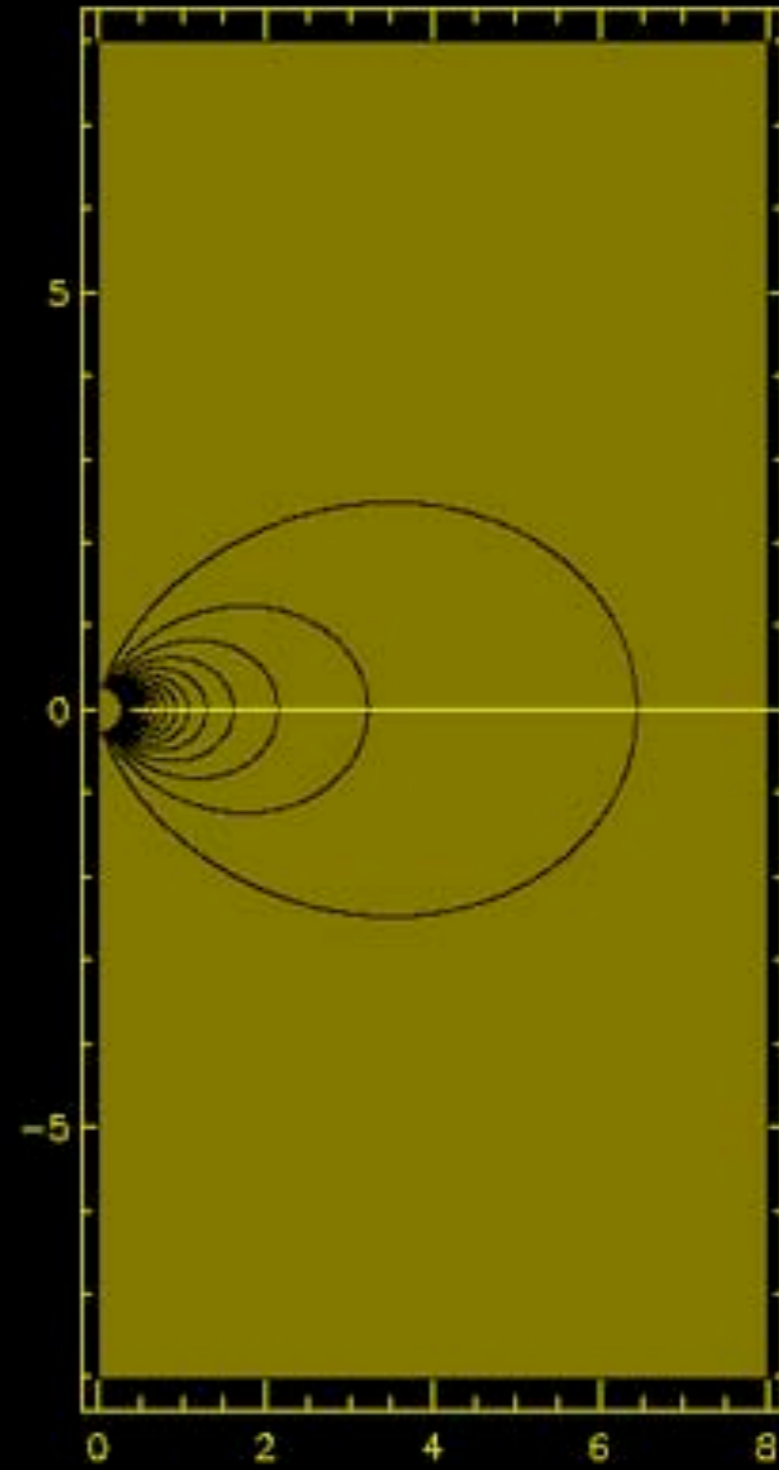
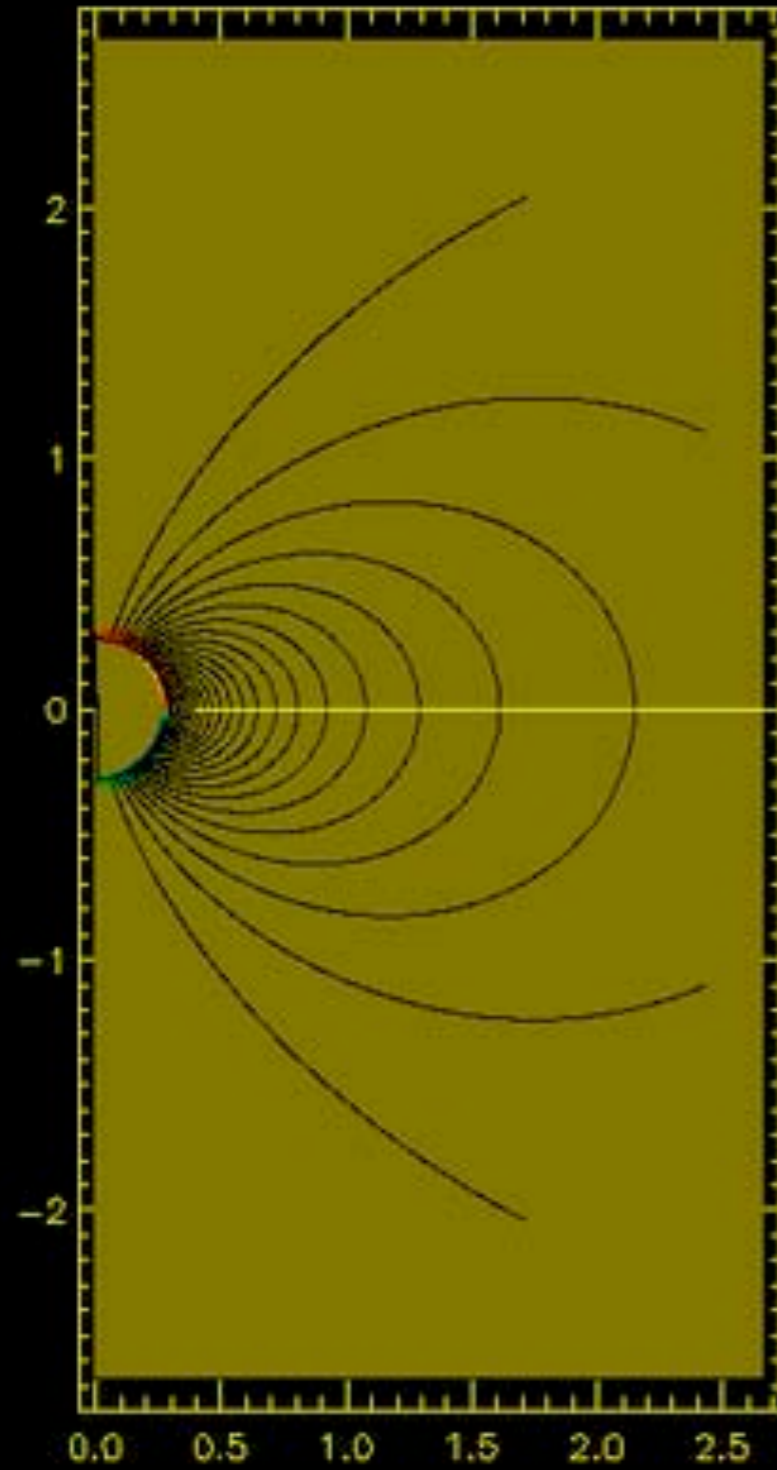
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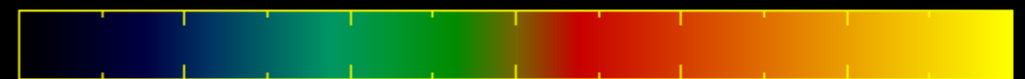
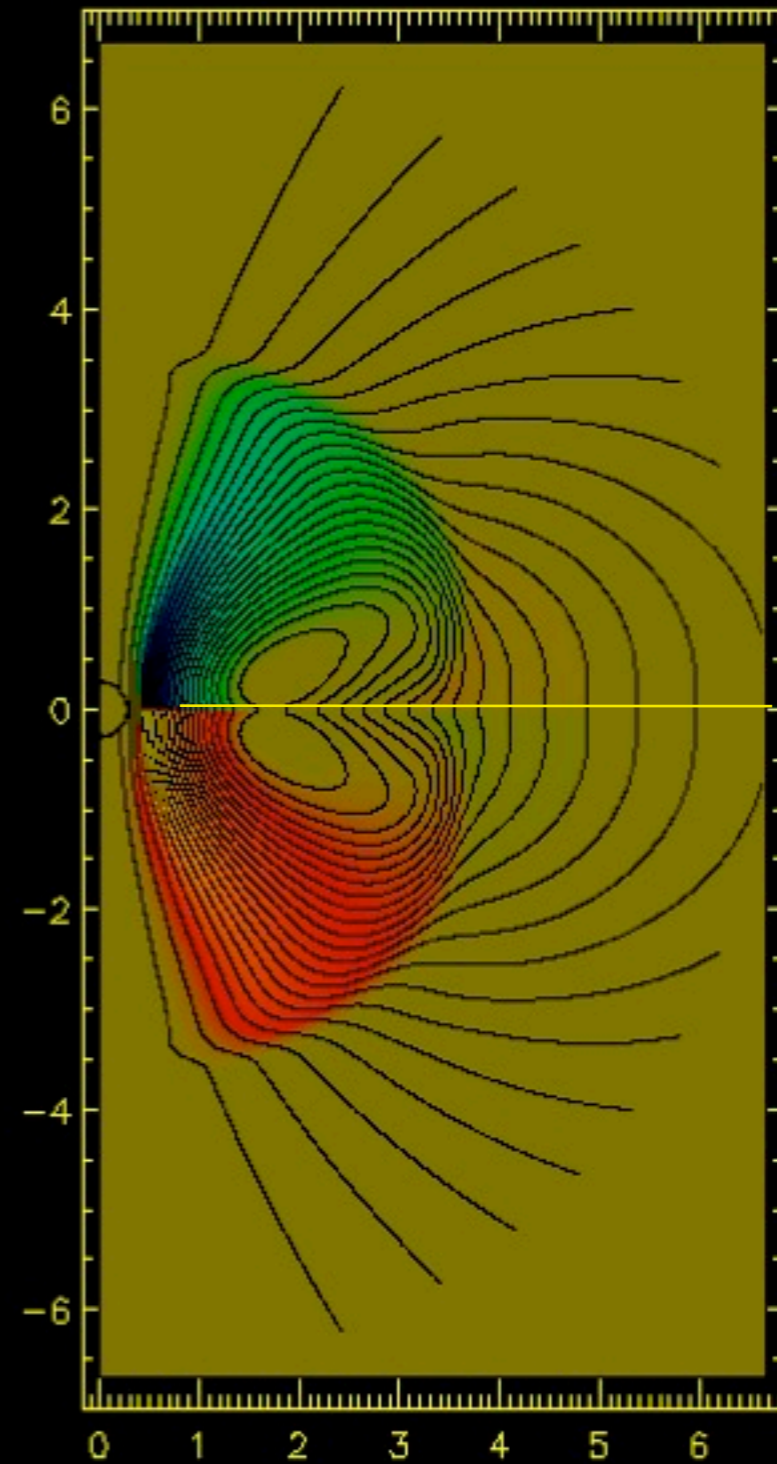
Force-free zoology: disks

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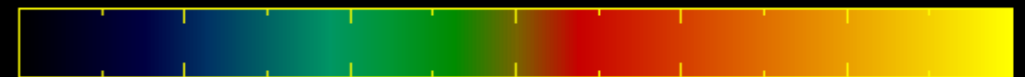
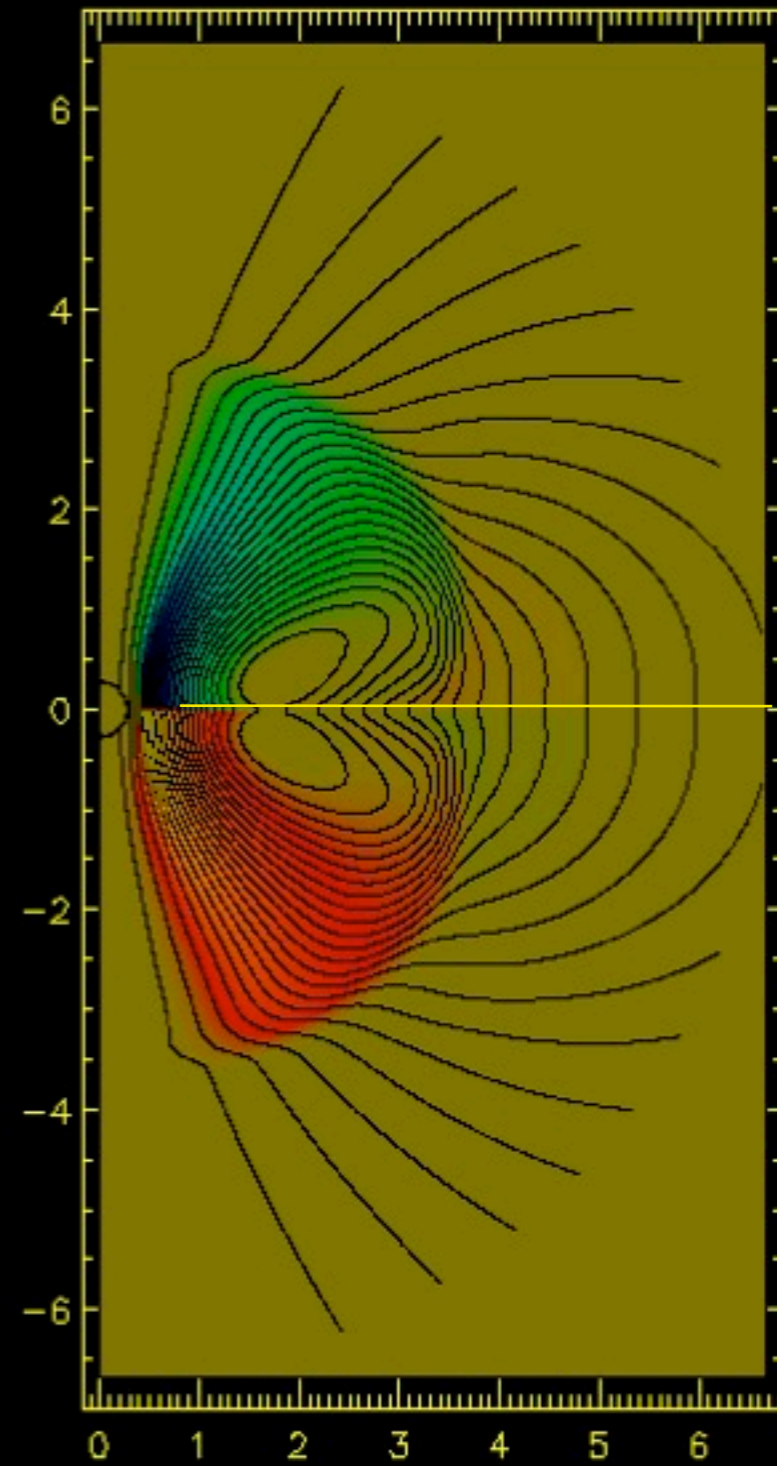
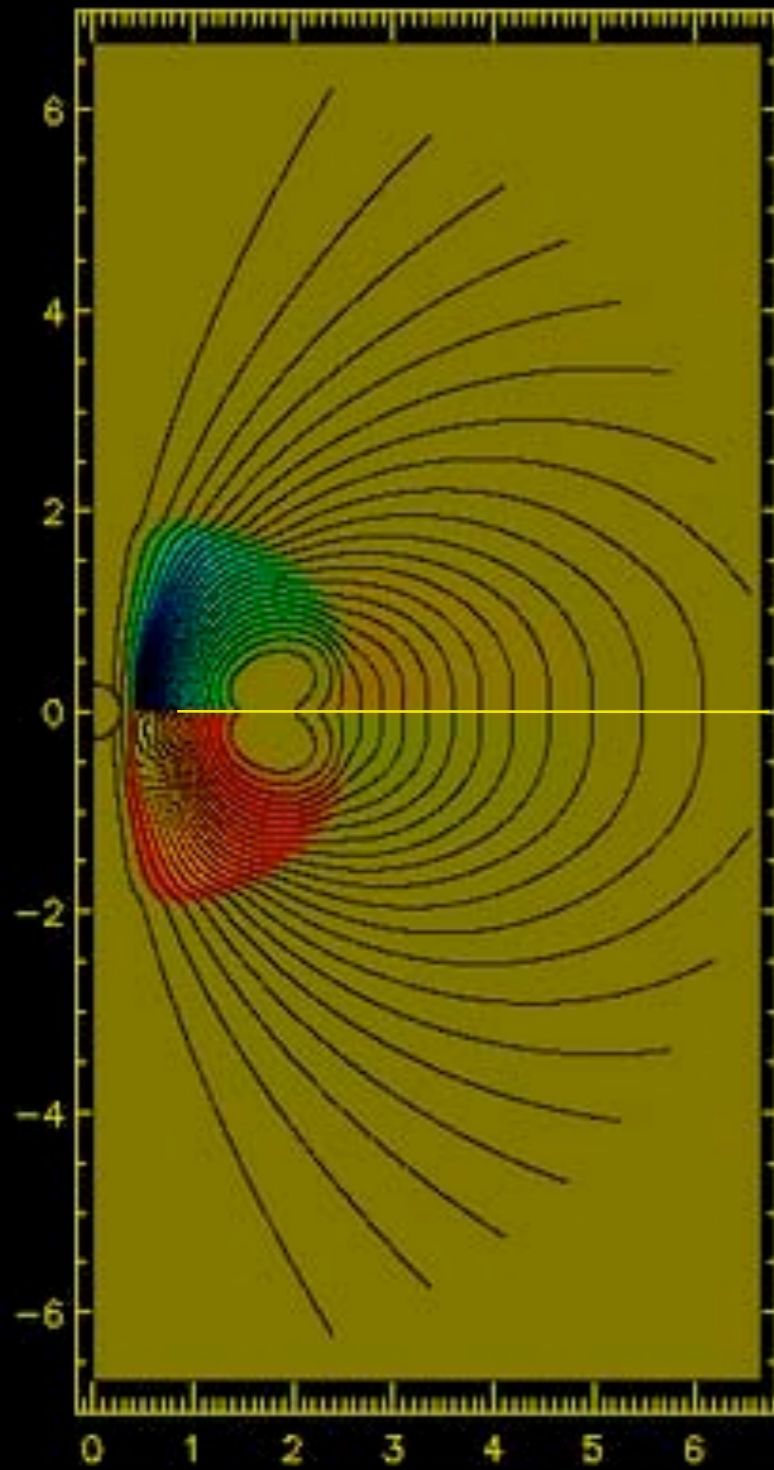
Force-free zoology: disks

Accretion disk corona



Force-free zoology: disks

Accretion disk corona



summary so far

Current sheets form from smooth initial conditions universally.

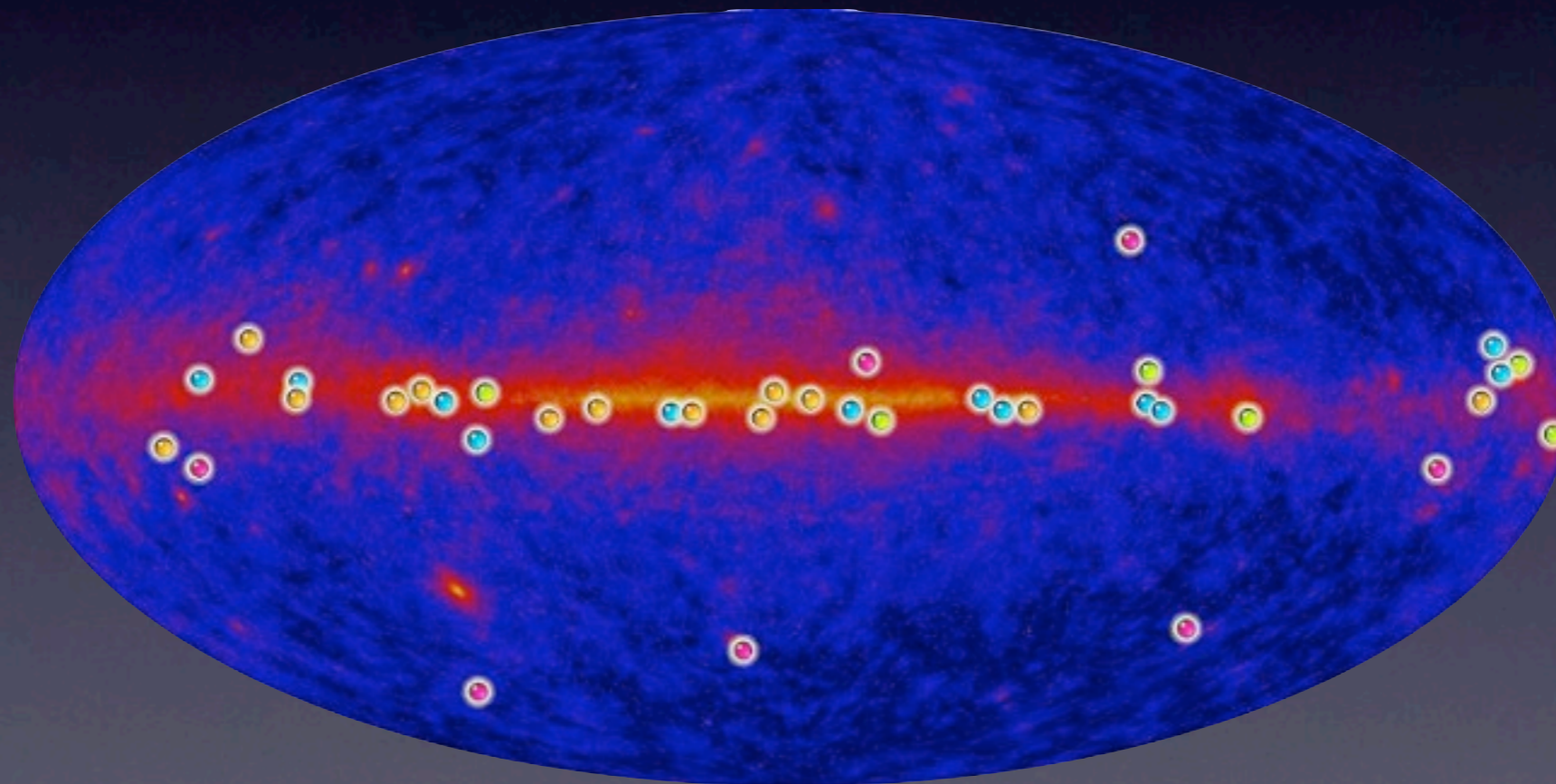
They are integral part of the evolution, and delineate distinct regions.

Can we observe them?

FF solutions of pulsars produce geometrical shape of the magnetosphere.

Is there any observational signature of the FF magnetosphere?

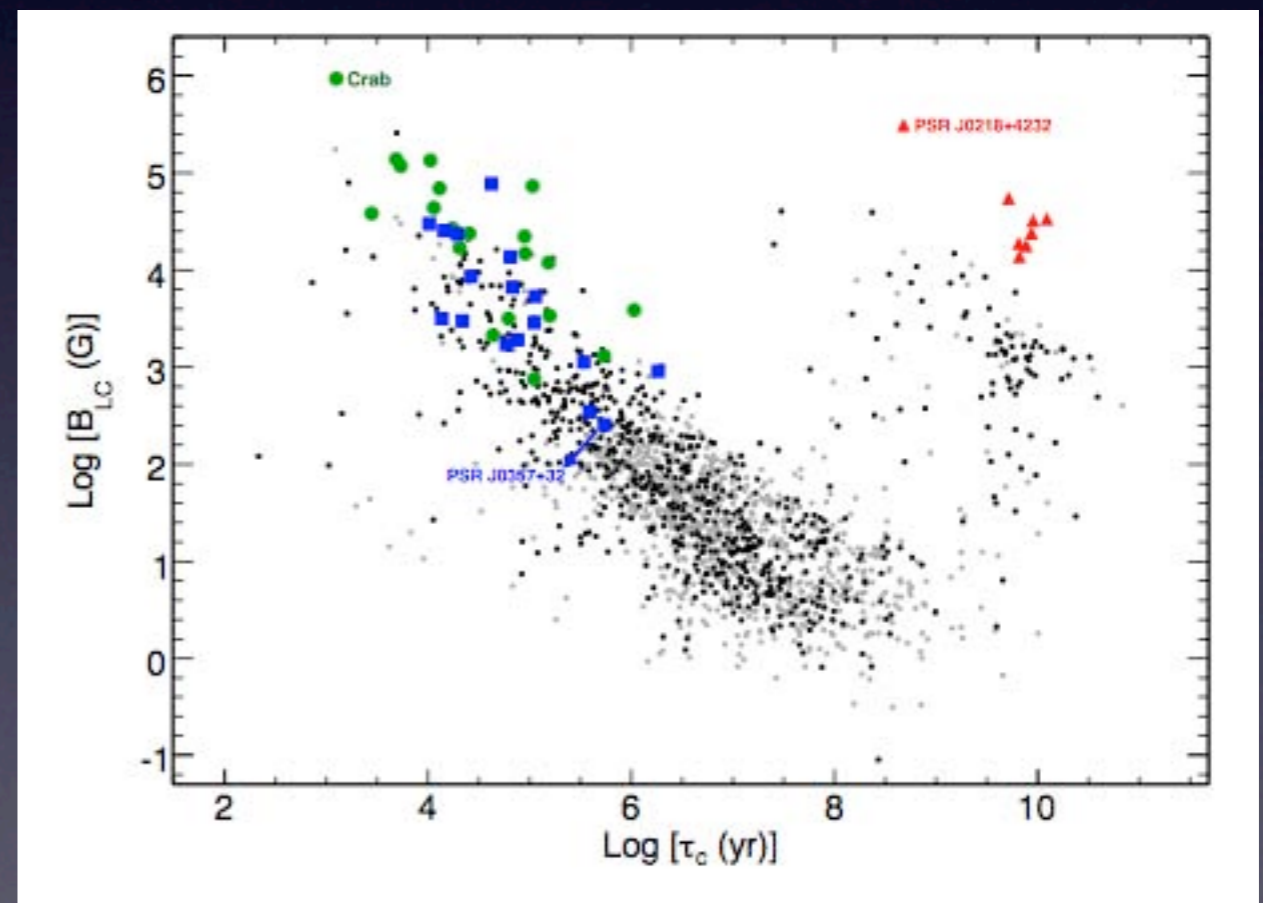
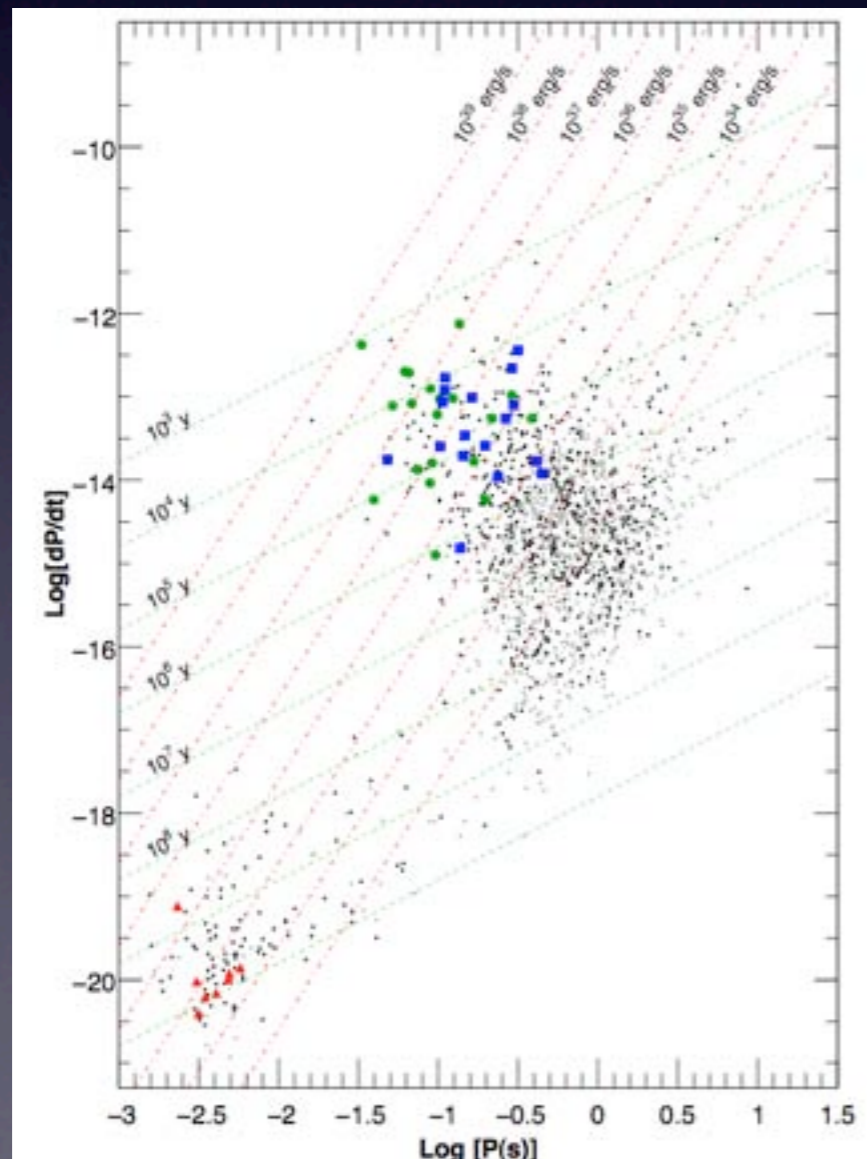
Gamma-ray emission from pulsars



Fermi Pulsar Detections

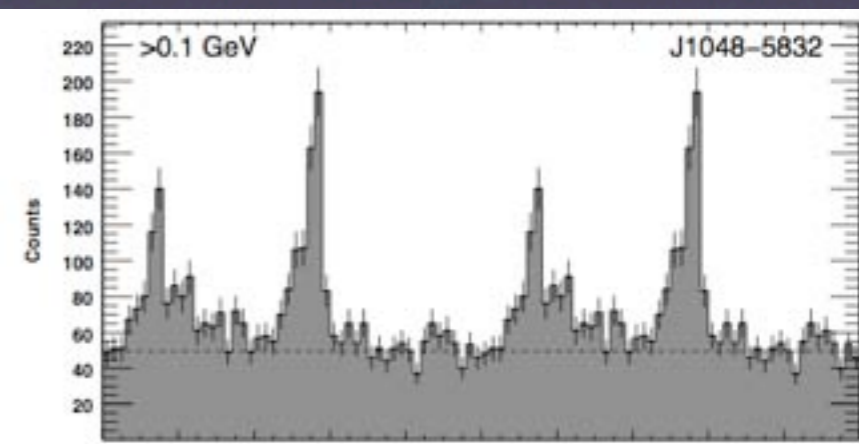
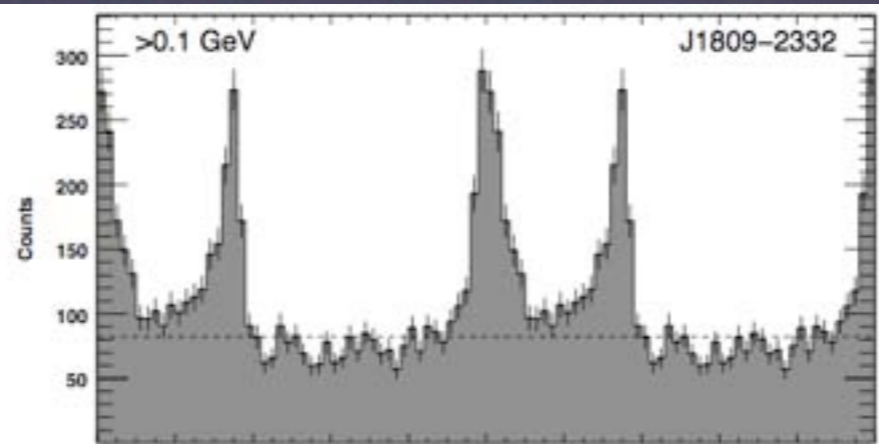
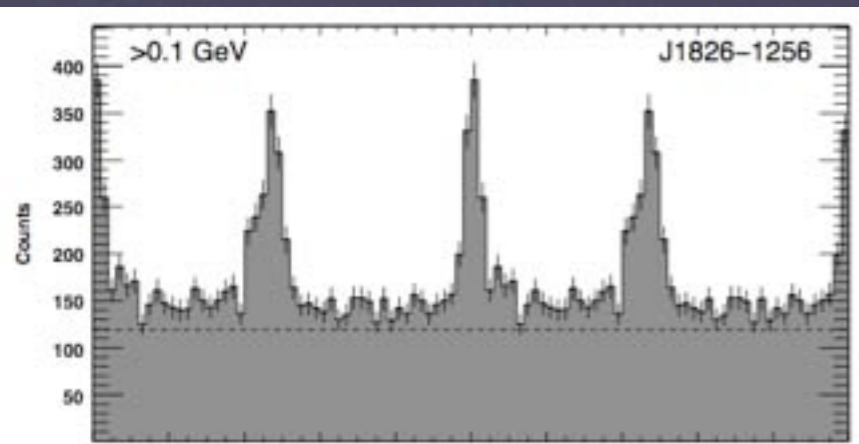
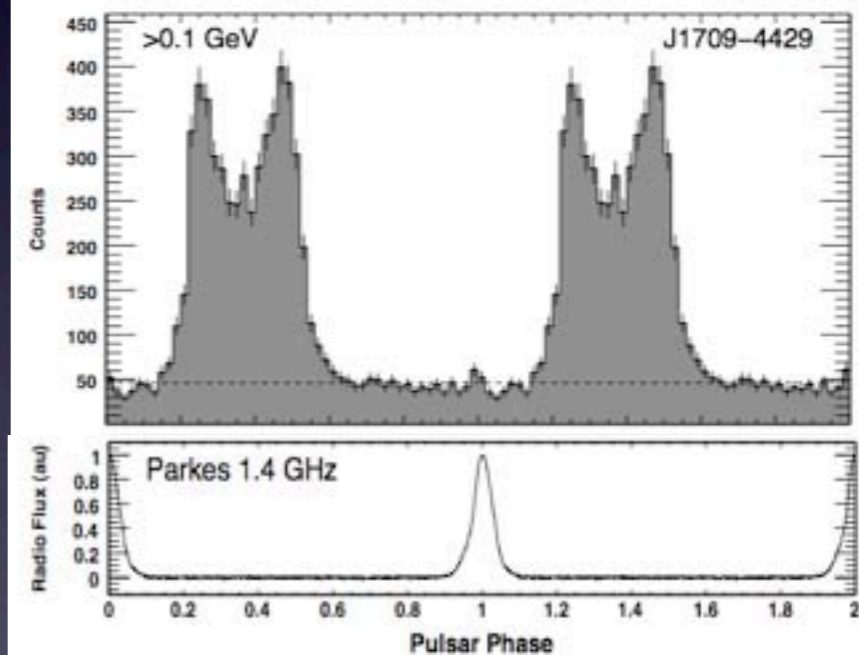
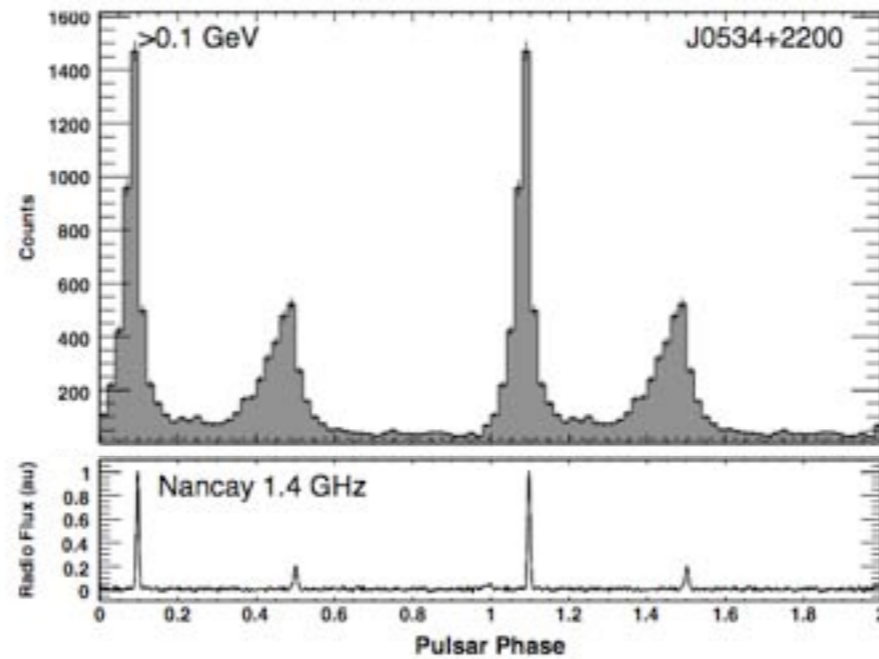
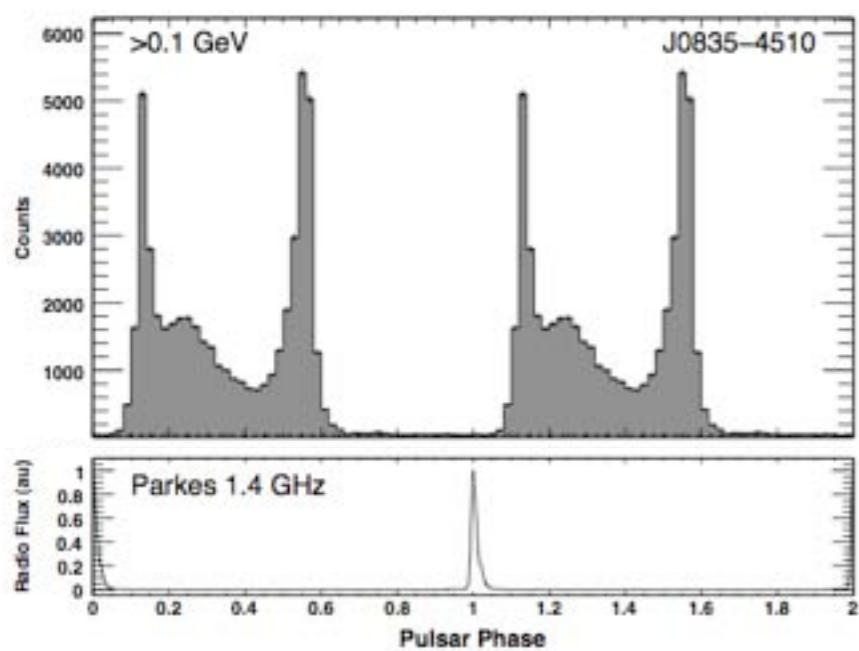
- New pulsars discovered in a blind search
- Millisecond radio pulsars
- Young radio pulsars
- Pulsars seen by Compton Observatory EGRET instrument

Gamma-ray emission from pulsars



High B at light cylinder required

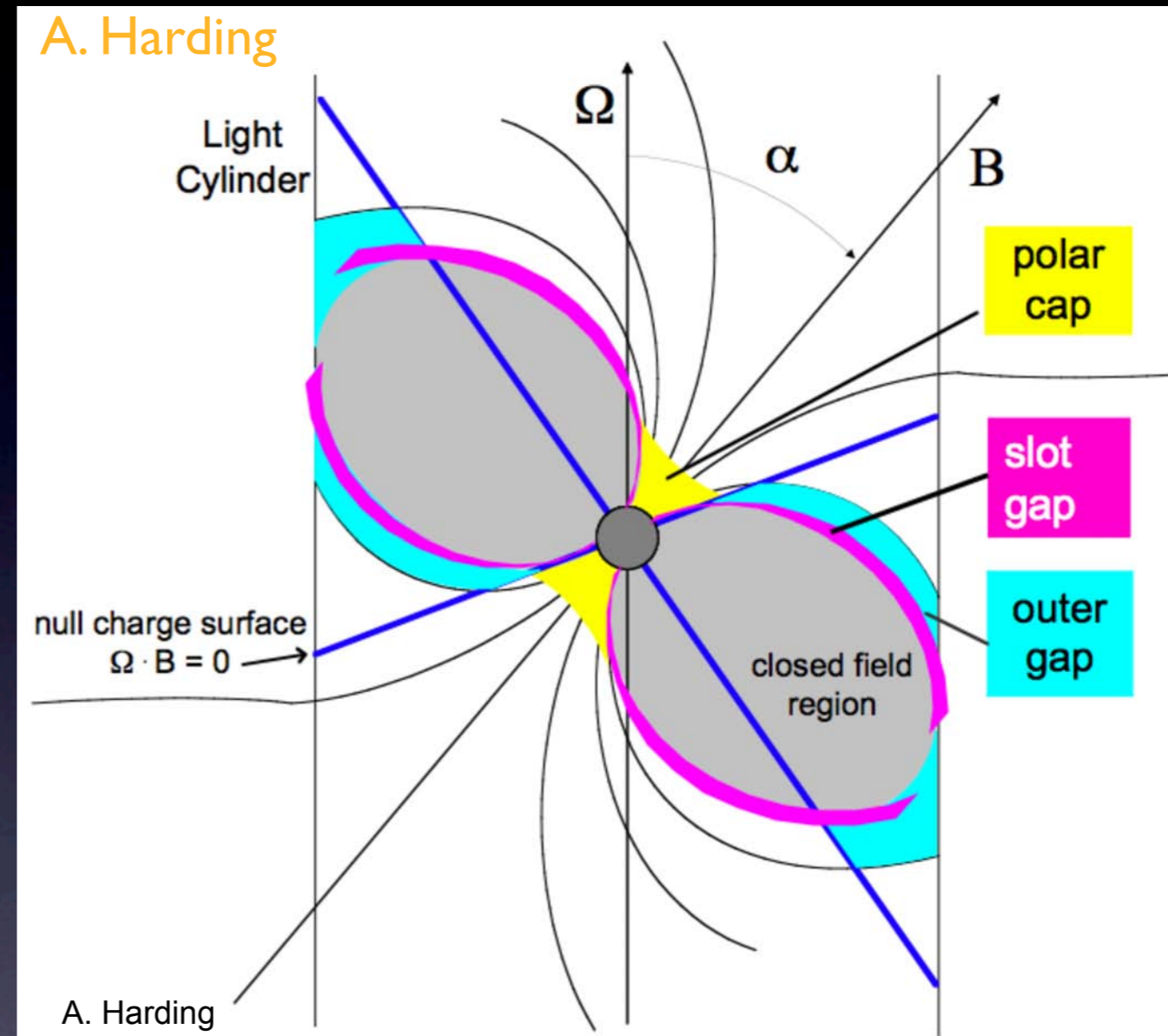
Gamma-ray emission from pulsars



What emits?

Emission process less complicated than in the radio: curvature, IC, or synchrotron.

- Need acceleration of particles
- Depending on how much plasma is in the magnetosphere, postulate emission regions, where E field is not shorted out: gap models
- Trace emission in field geometry, usually assumed to be rotating vacuum dipole
- Remarkably successful in fitting the light curves and spectra

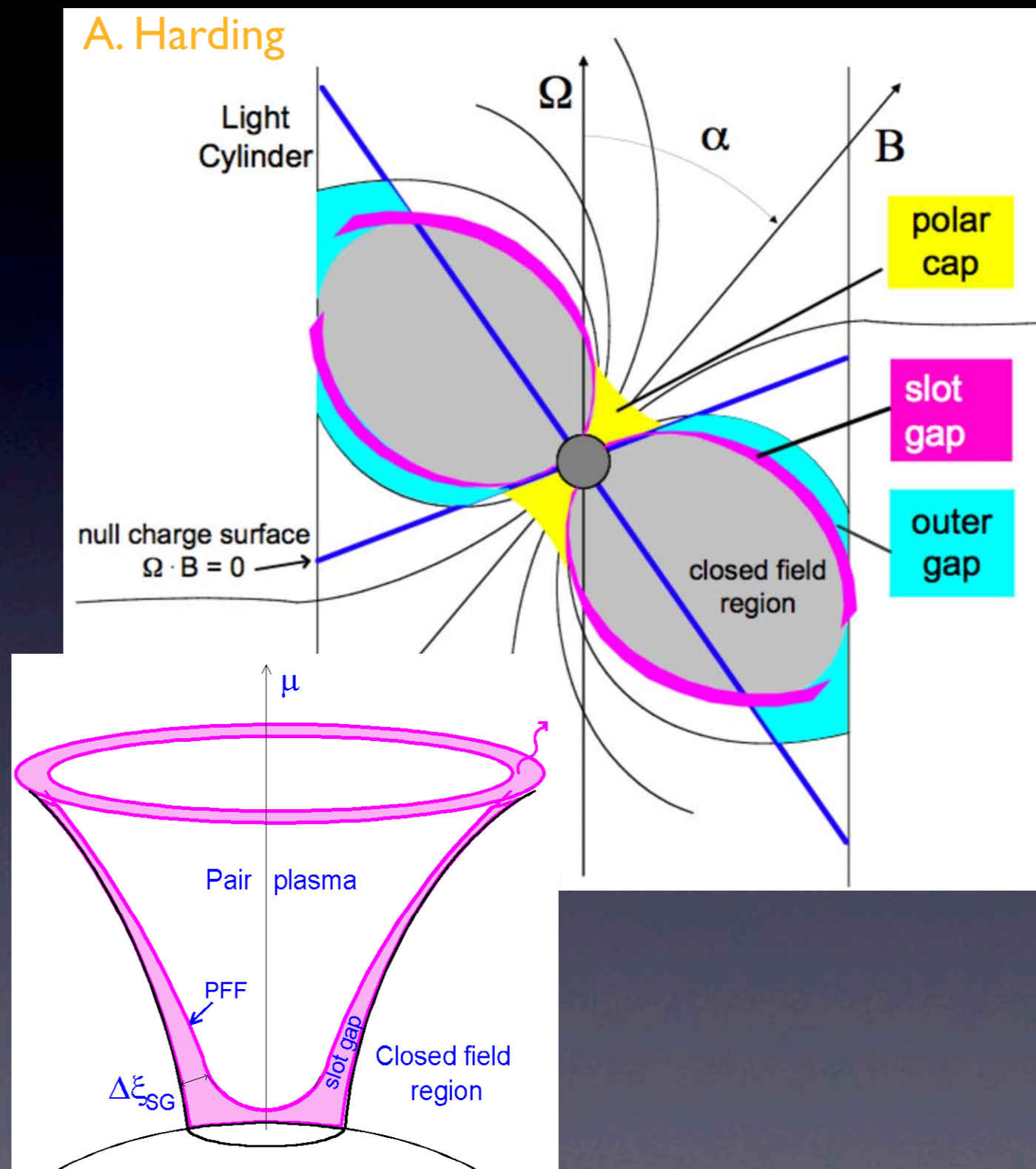


Geometry is crucial to the formation of light curves

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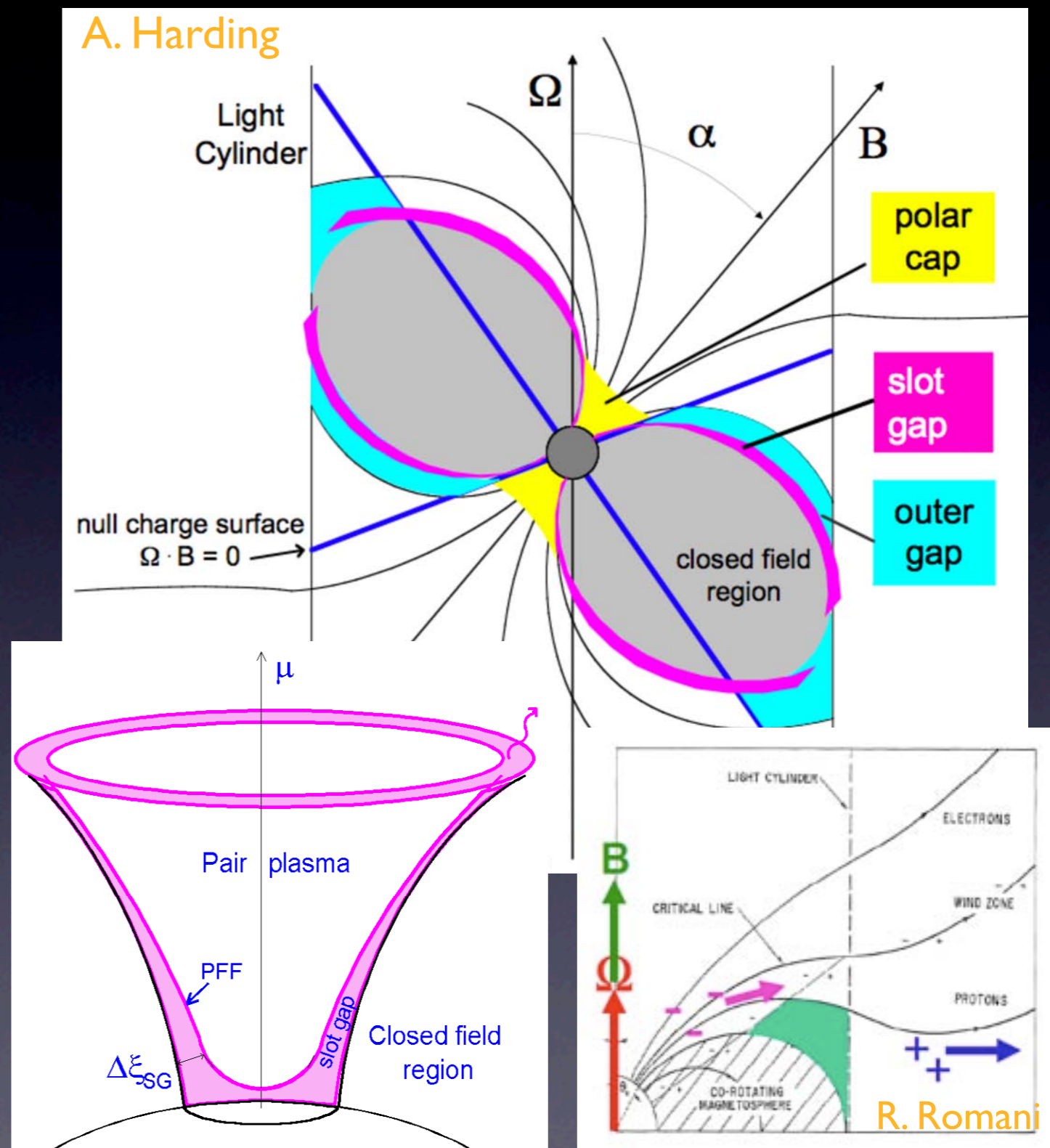


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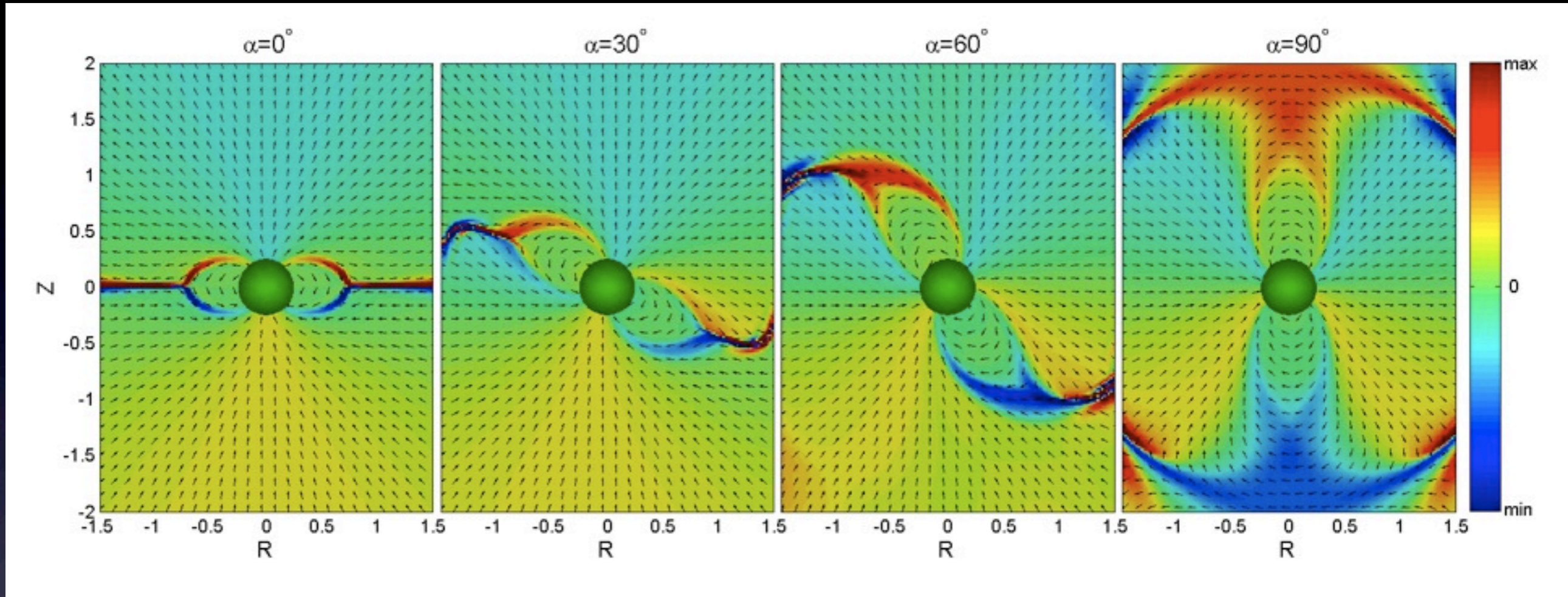
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Geometry is crucial to the formation of light curves

Oblique rotator: force-free



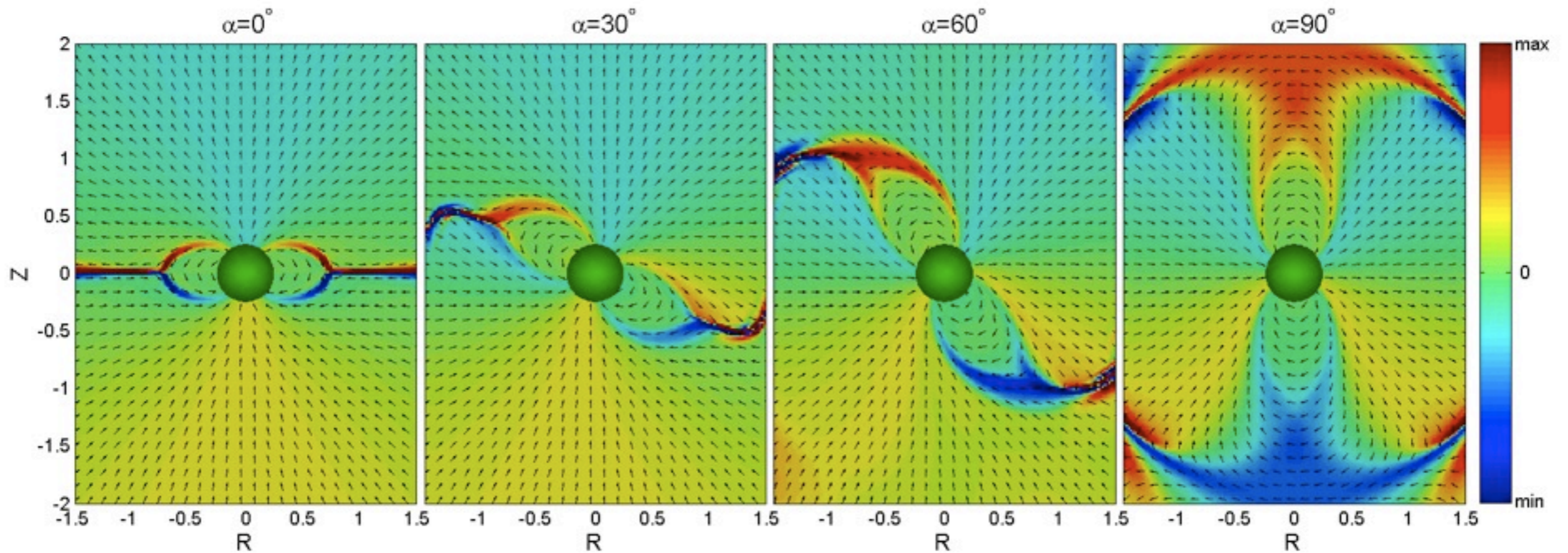
X. Bai & A. S. arXiv:
0910.5041

Distribution of current in the magnetosphere

Force-free field provides a
more realistic magnetic
geometry

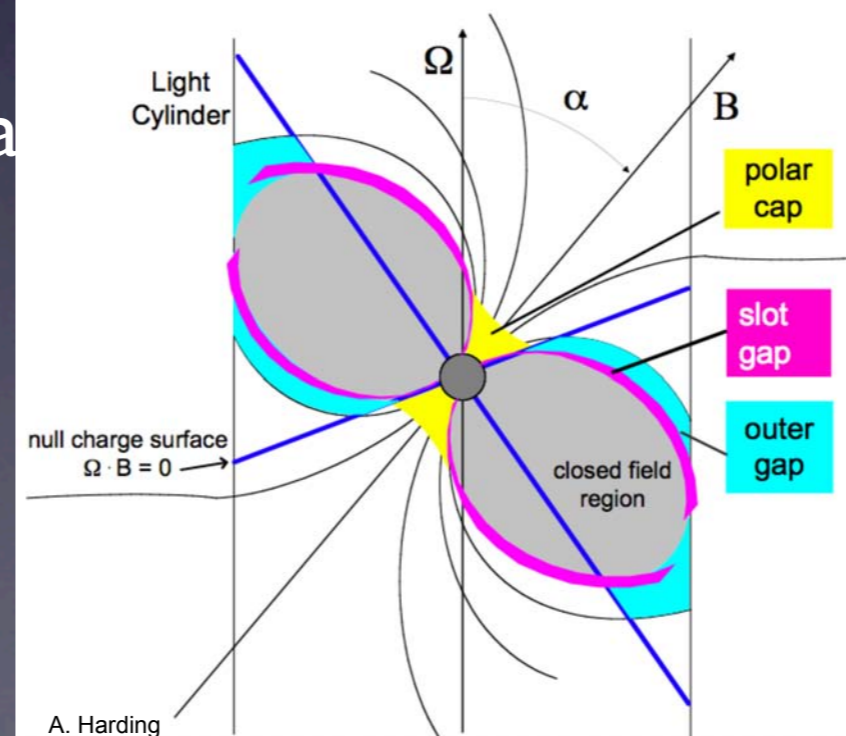
Tempting to
associate gaps
with currents.
Can we?

Oblique rotator: force-free



Distribution of current in the ma

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X. Bai & A. S. arXiv: 0910.5041

Tempting to associate gaps with currents. Can we?

Light curve calculation

1. Pick field (static dipole, retarded dipole [Deutch], force-free)
2. Find the polar cap (field lines touching LC, or all closed?)
3. Decide which field lines emit
4. Assume uniform emissivity (with cuts in radius)
5. Trace field lines emitting photons along field line
6. Add aberration and time of flight effect
7. Bin photons on the sky -- > sky map + light curves
8. Repeat

Geometry is crucial to the formation of light curves: affects aberration and definition of polar cap.

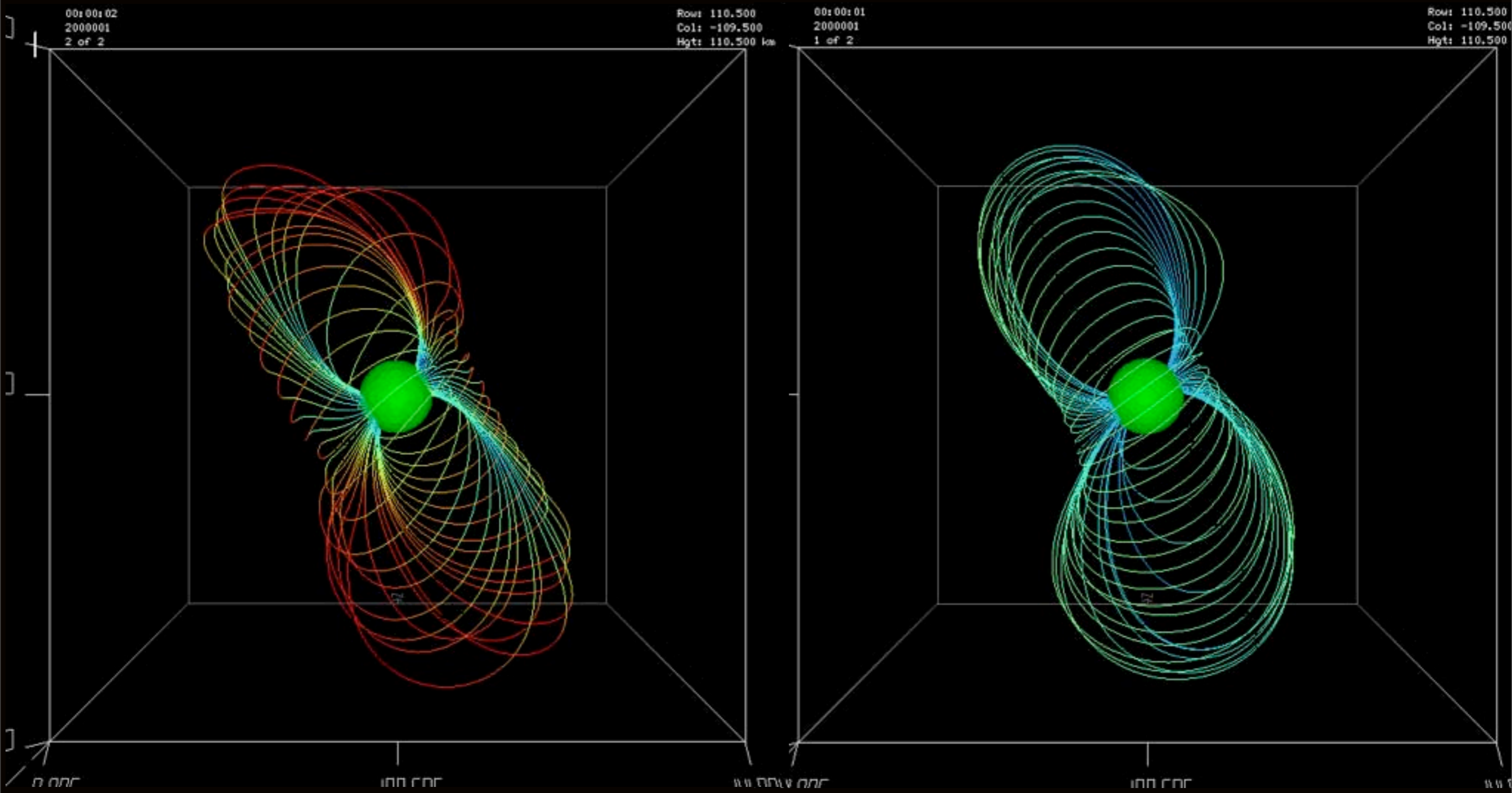
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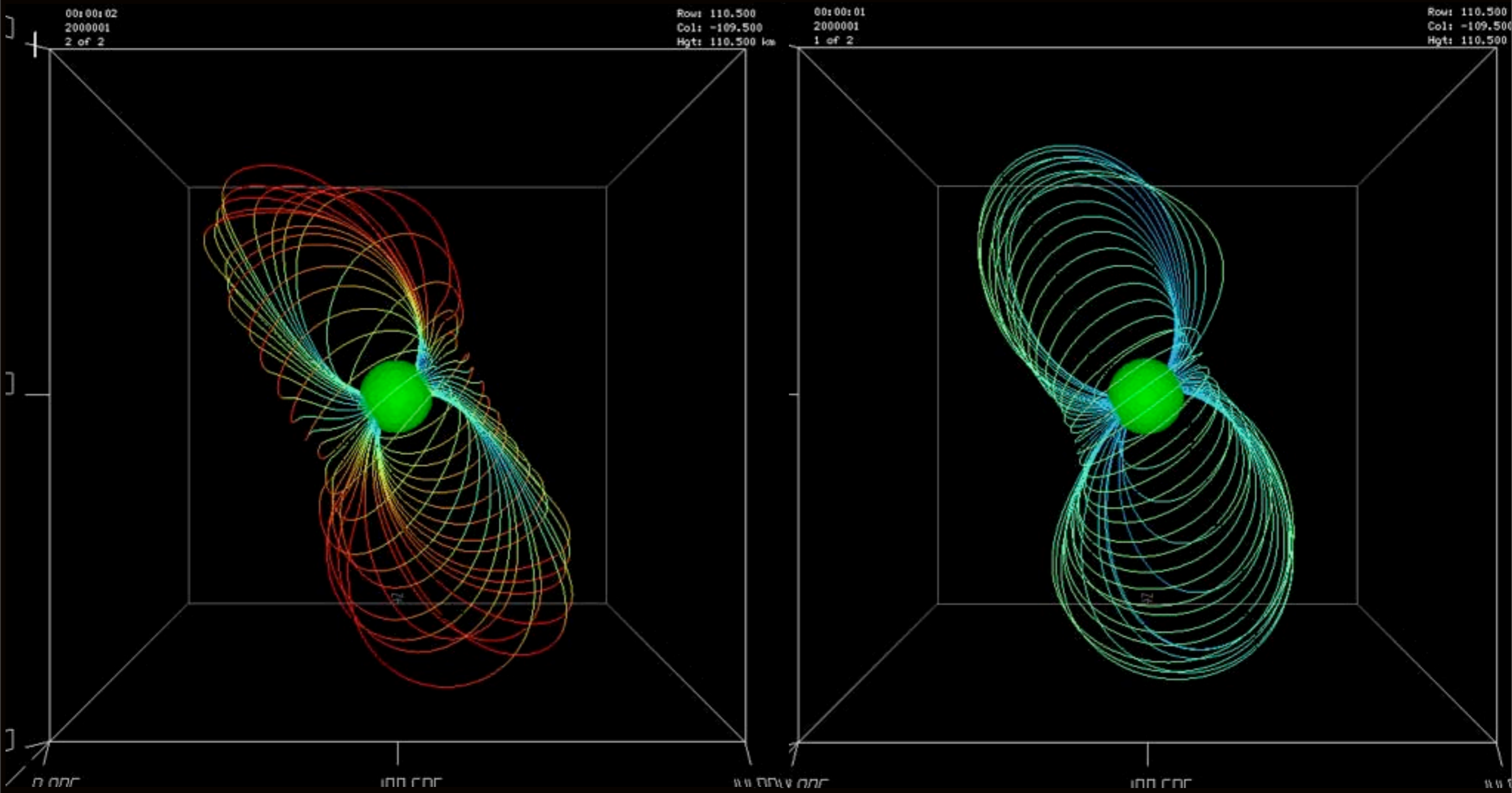
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Force-free vs Vacuum: Last Closed Lines

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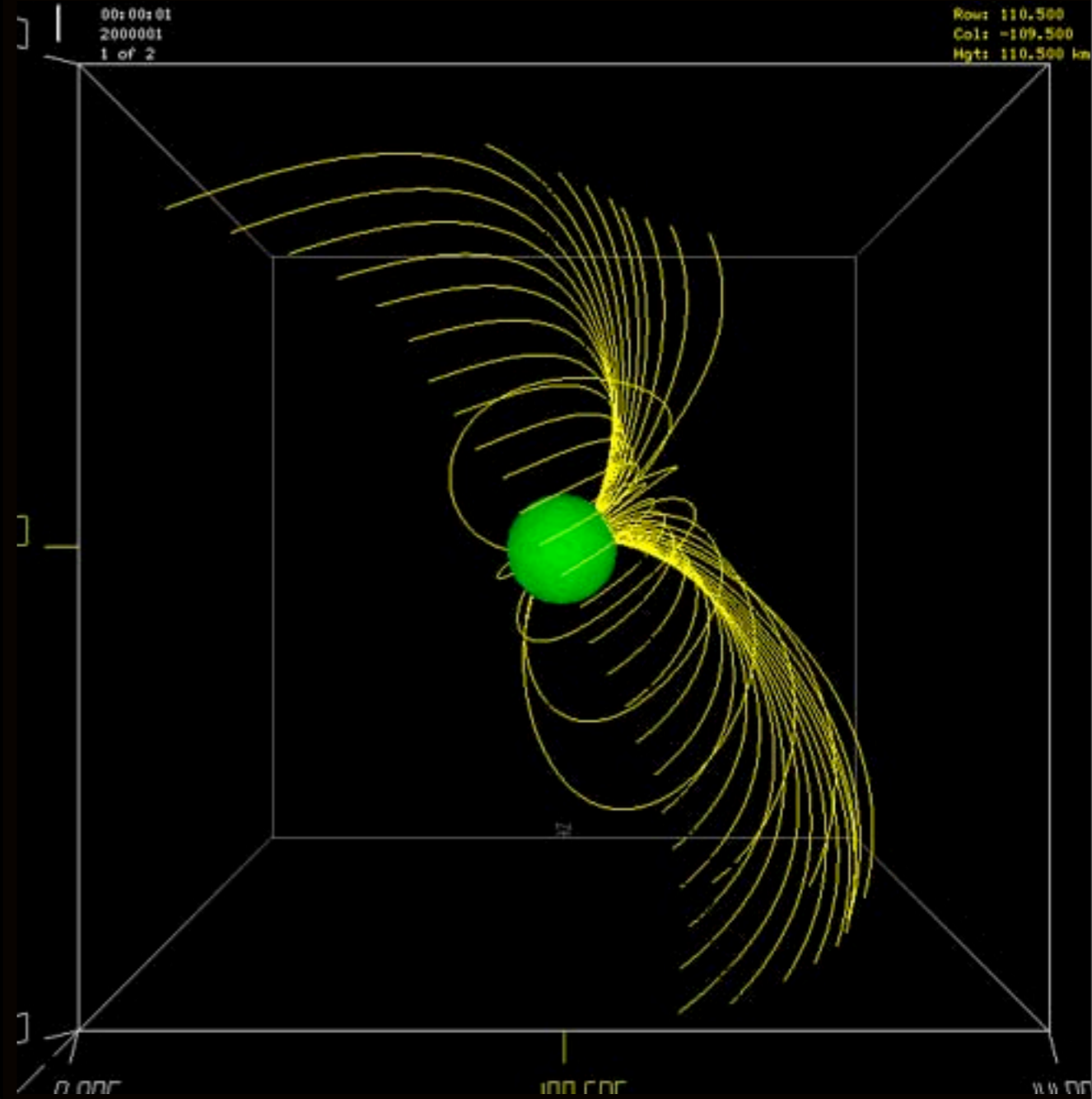
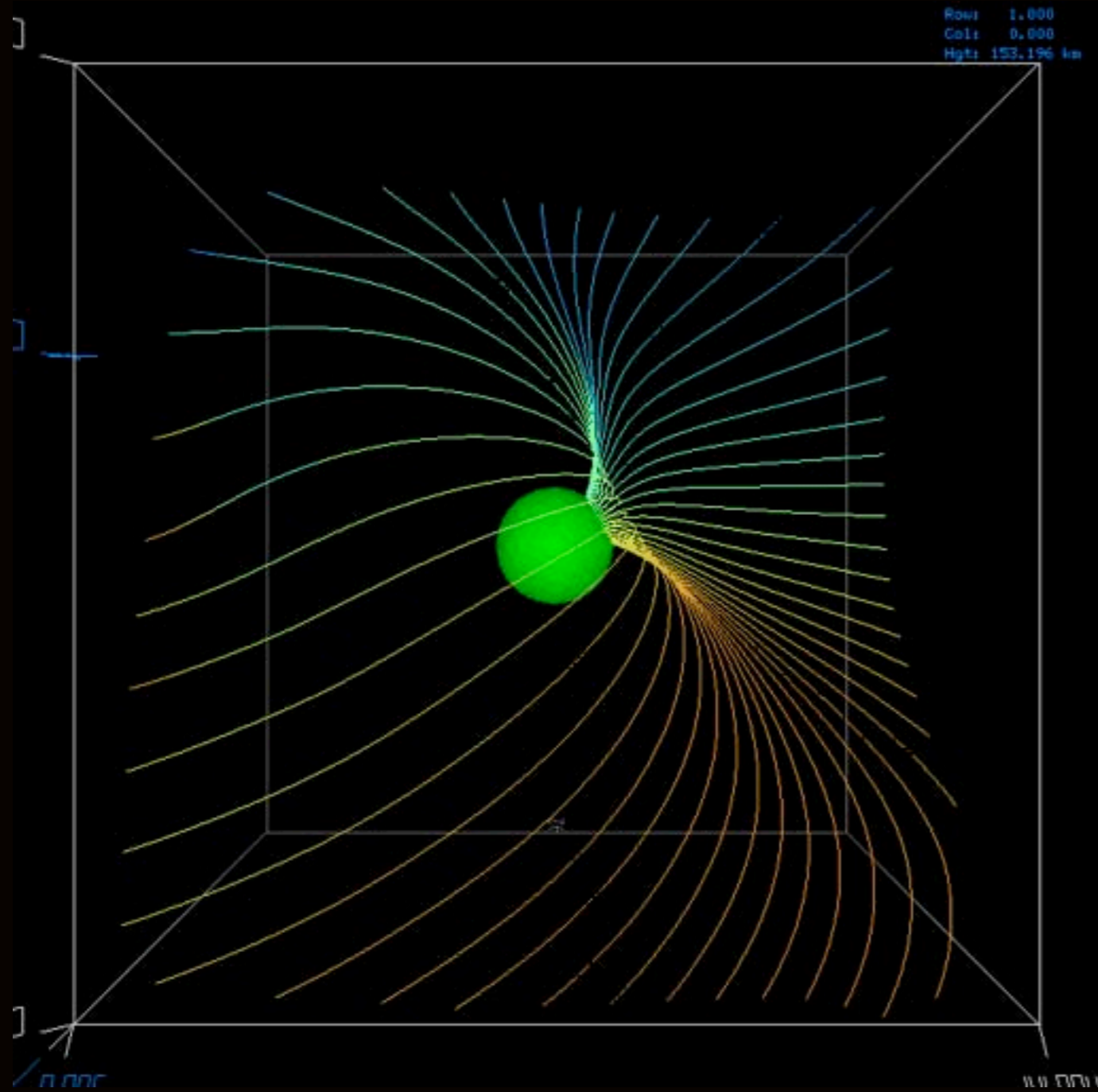


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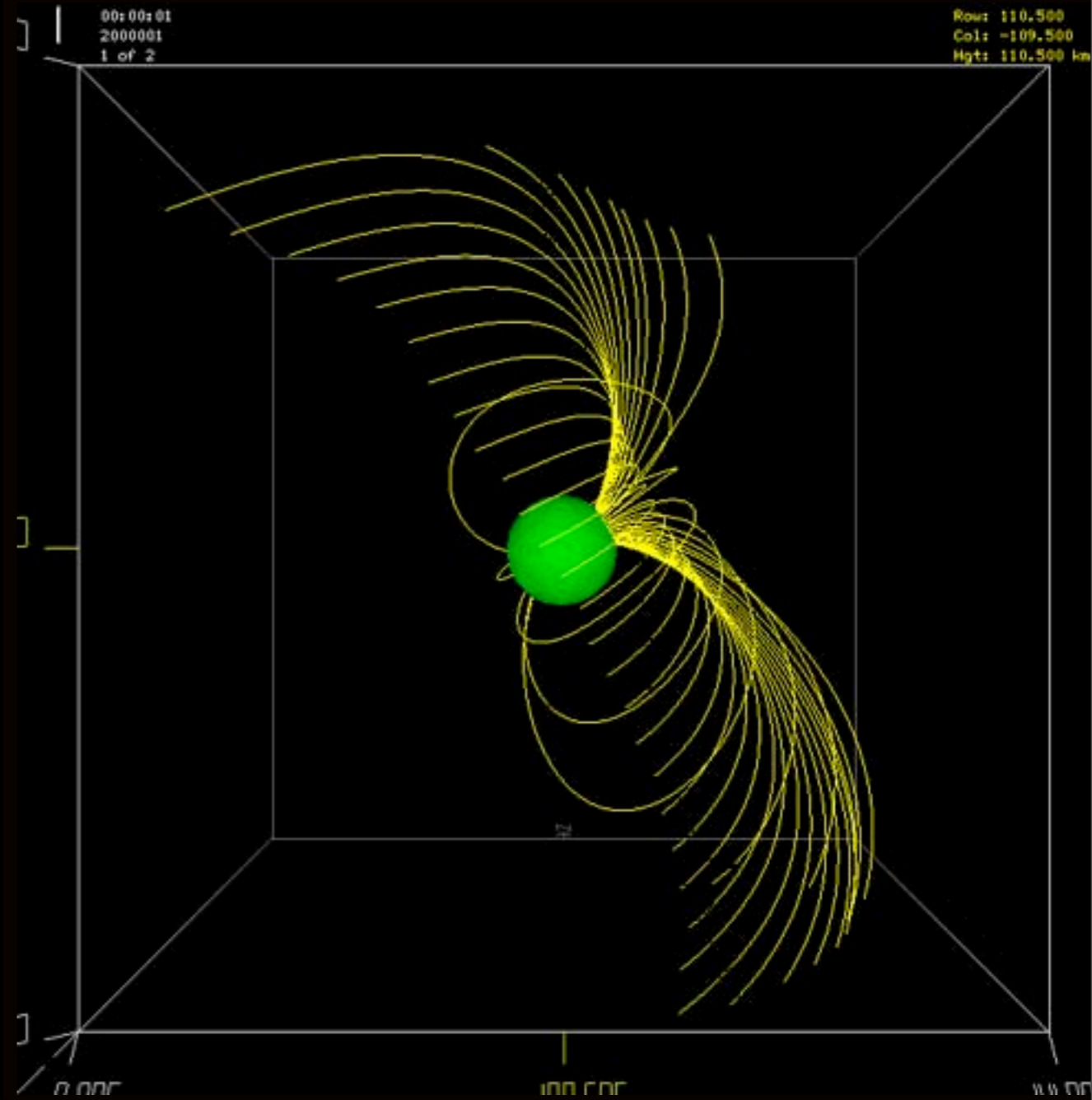
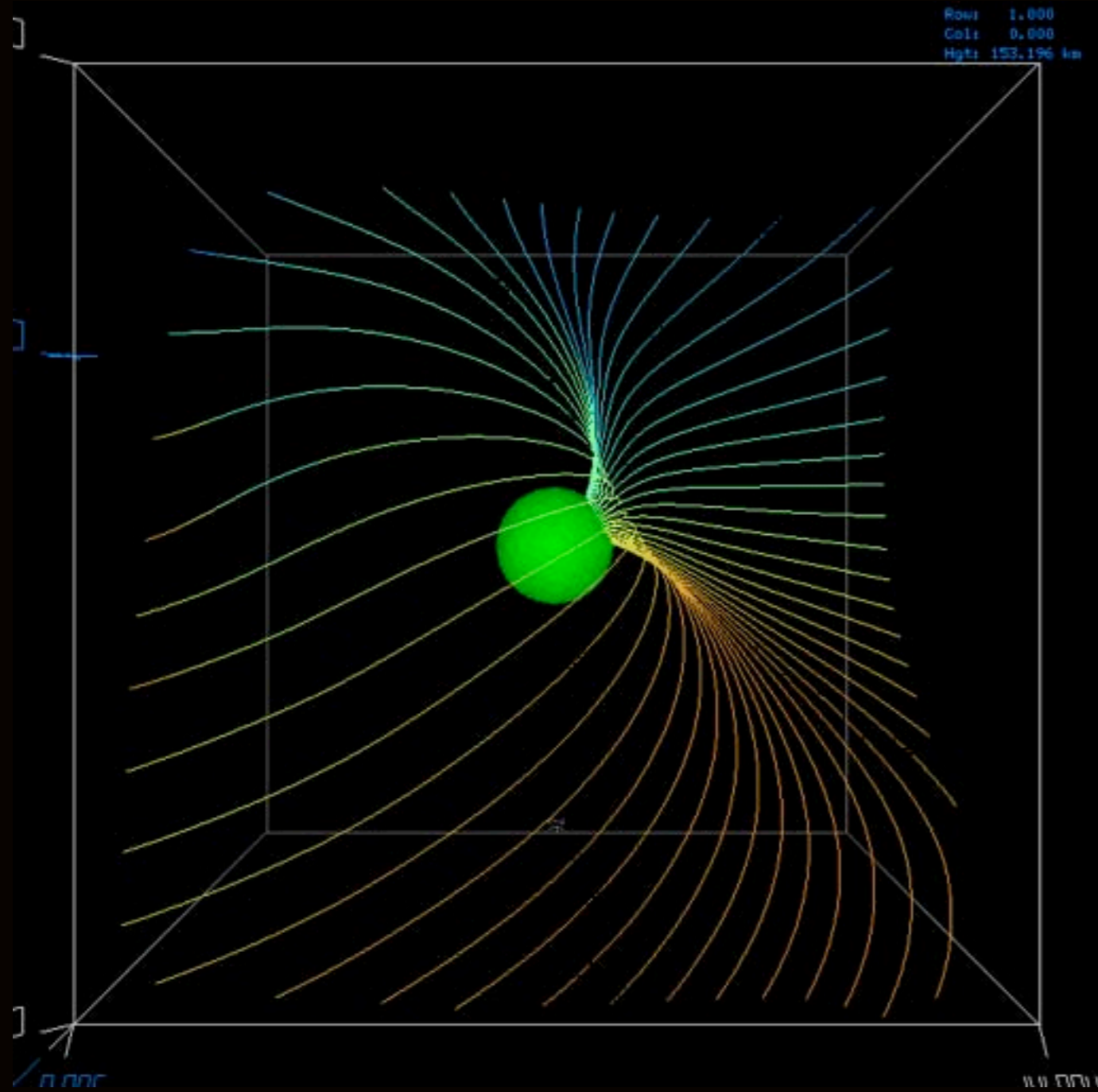


Force-free vs Vacuum: Last Open Lines

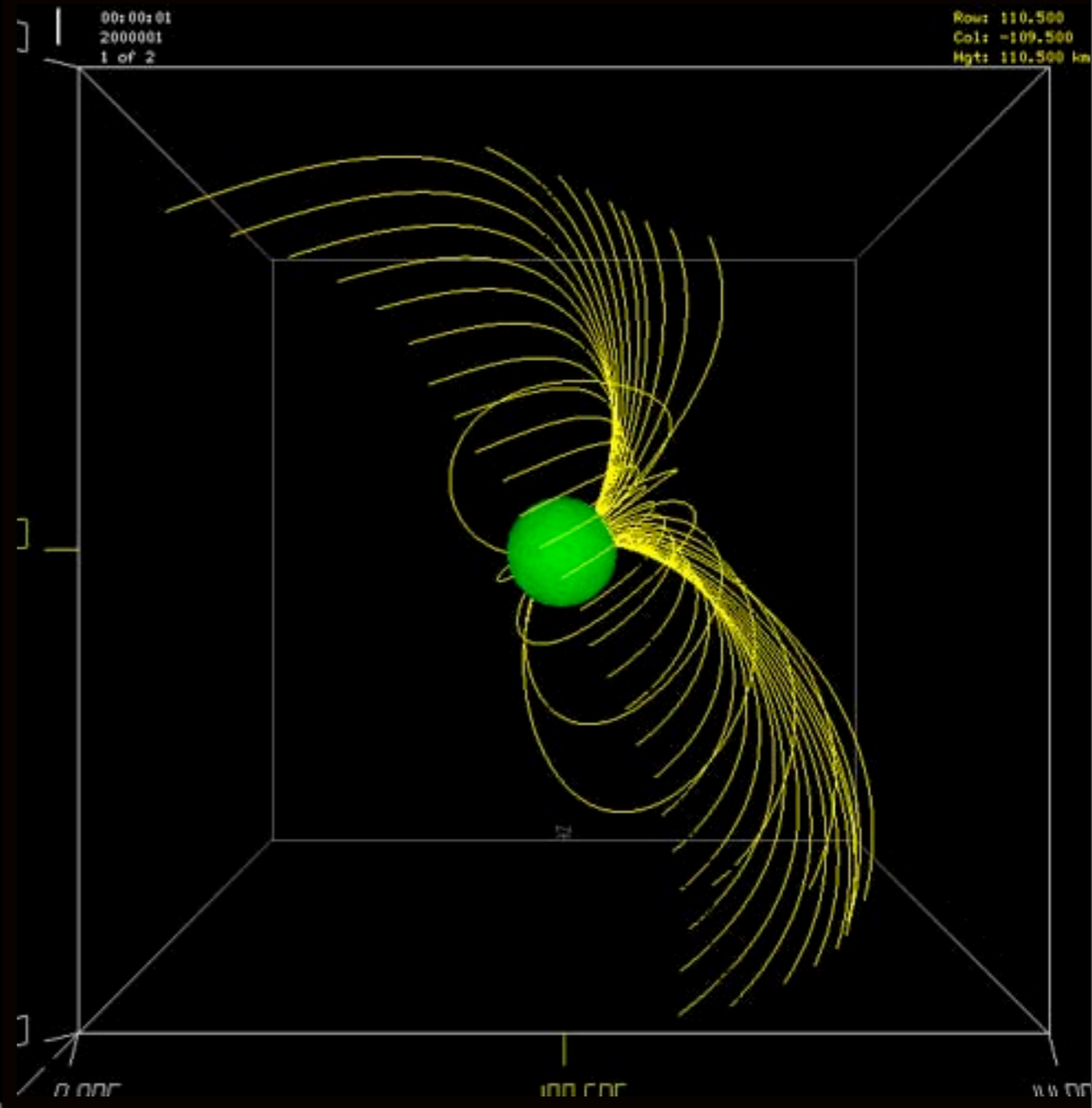
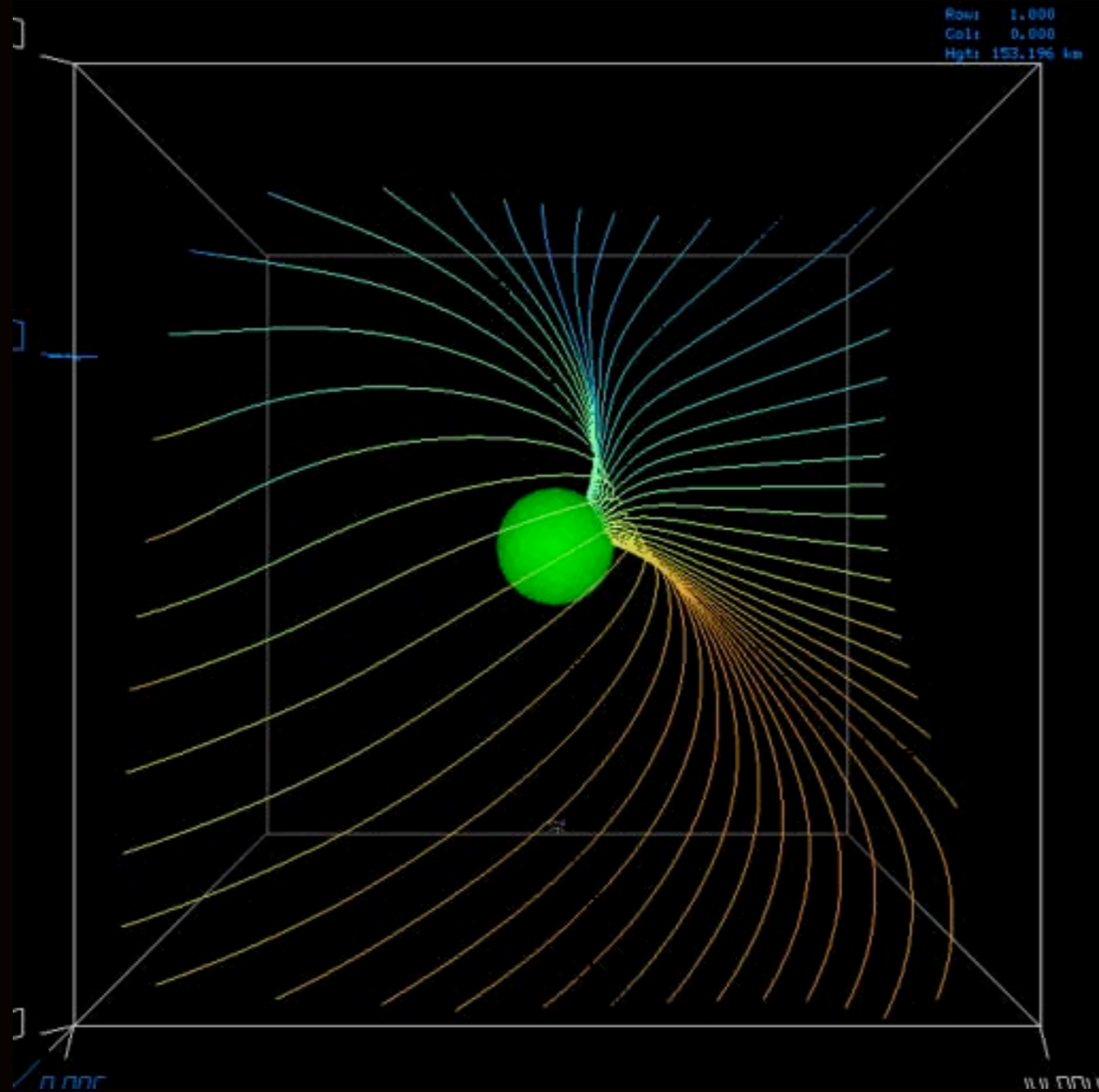
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Force-free vs Vacuum: Last Open Lines

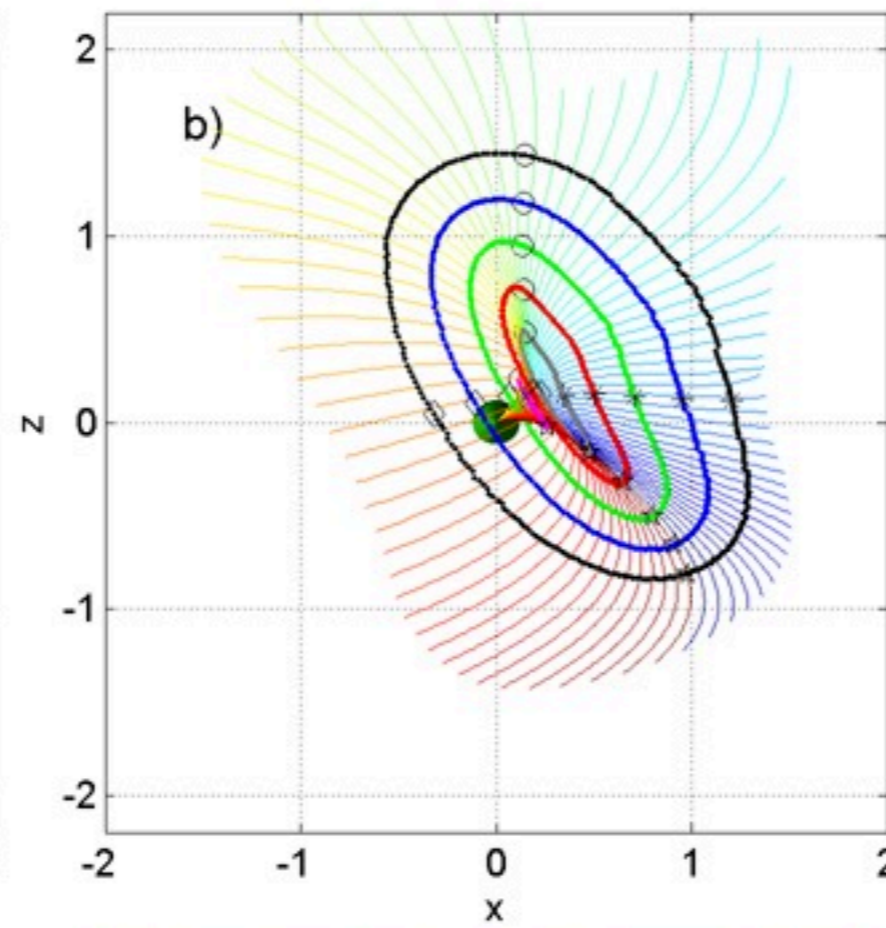
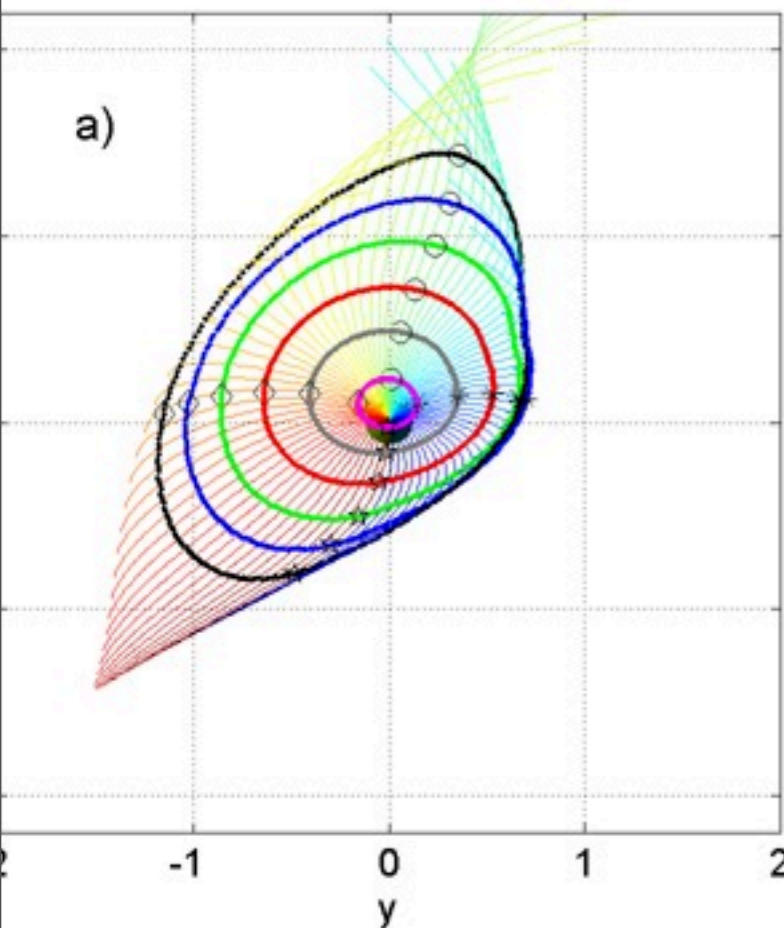


asymptotic split-monopole is ideal for caustic formation

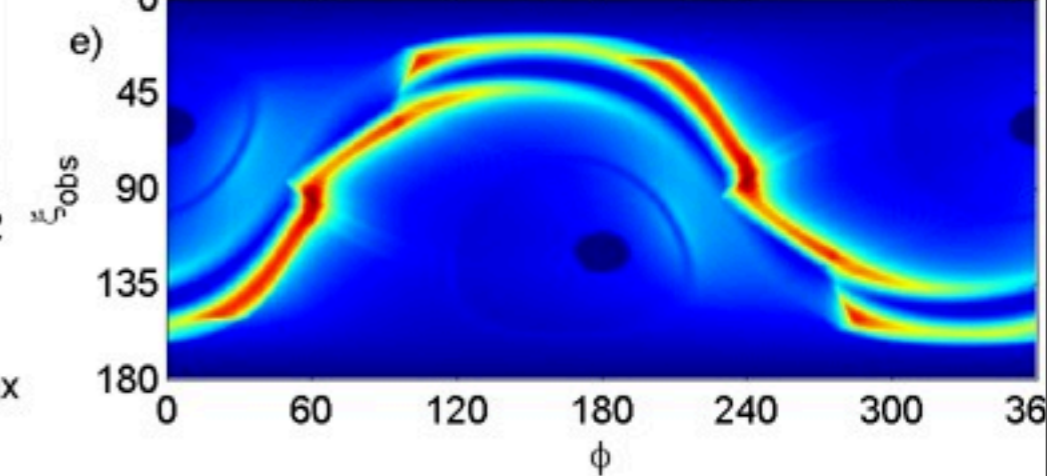
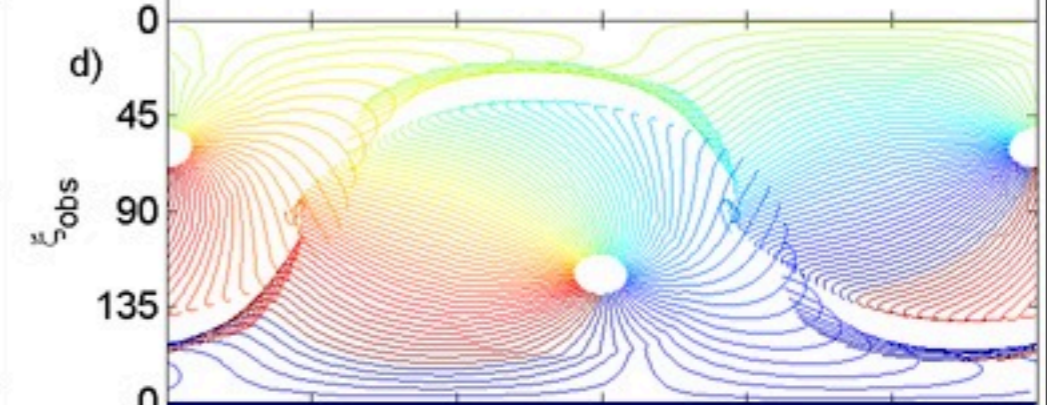
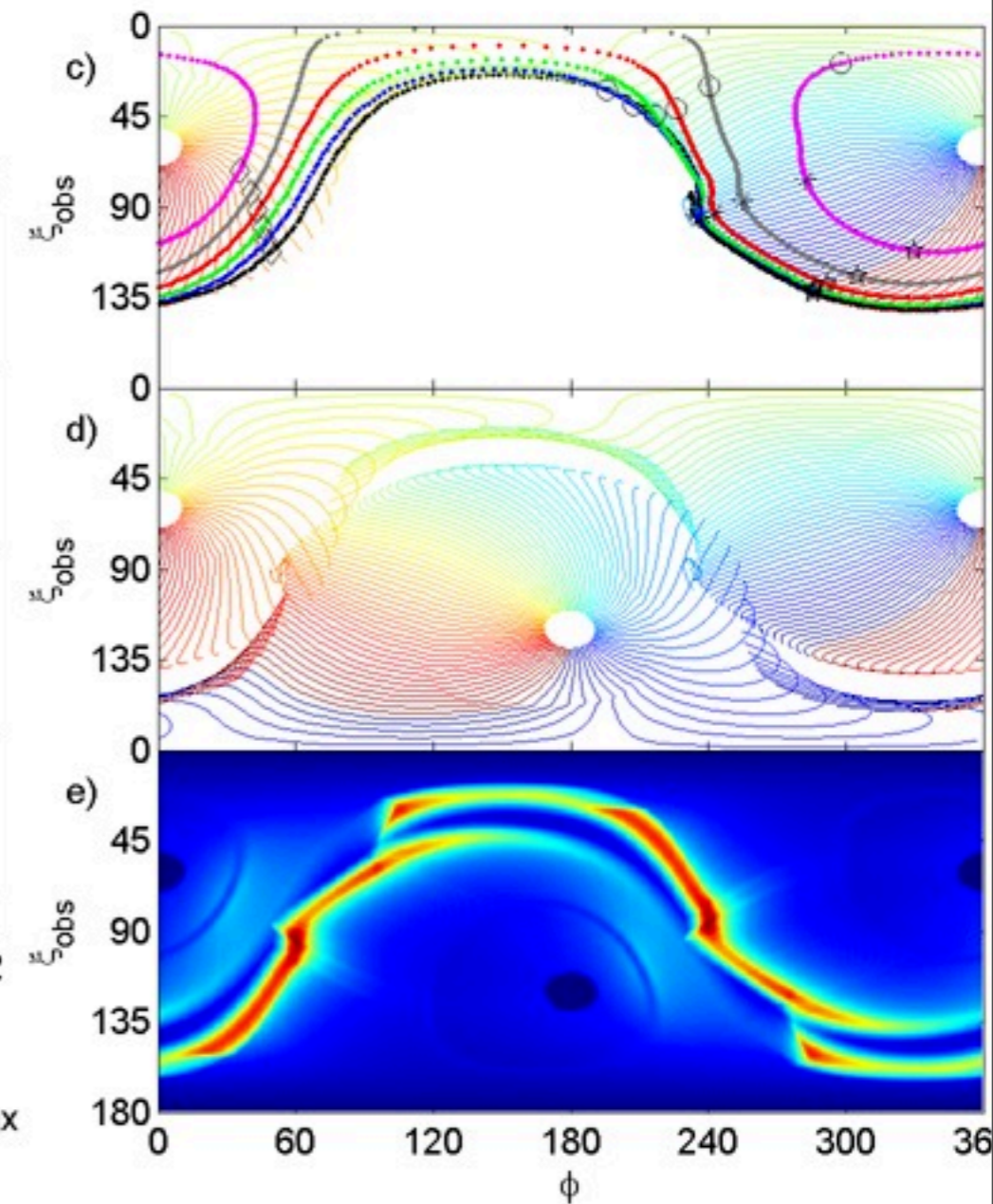
Force-free sky map

AG-F60-90-90

a)-b): spatial plot; e): sky map intensity
c)-d): projection to sky map;



Color scale for e):



Force-free field, 60 degree inclination, flux tube starting at 0.9 of the polar cap radius.

“Sky map stagnation”

Force-free from different flux tubes

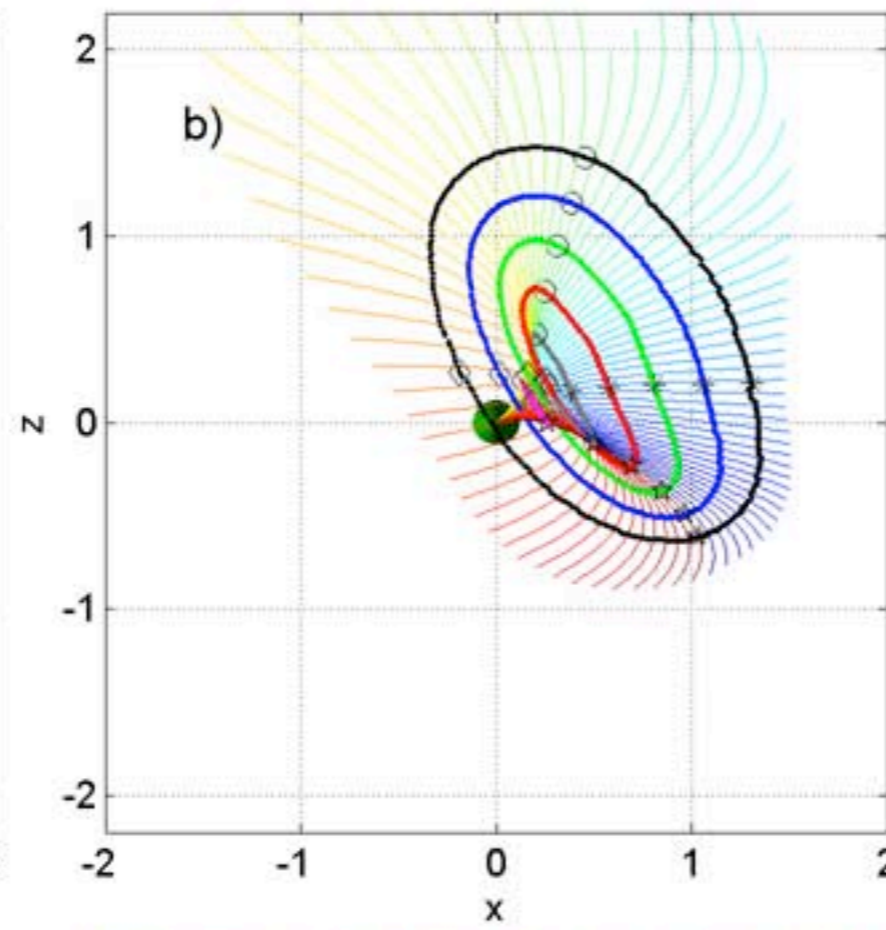
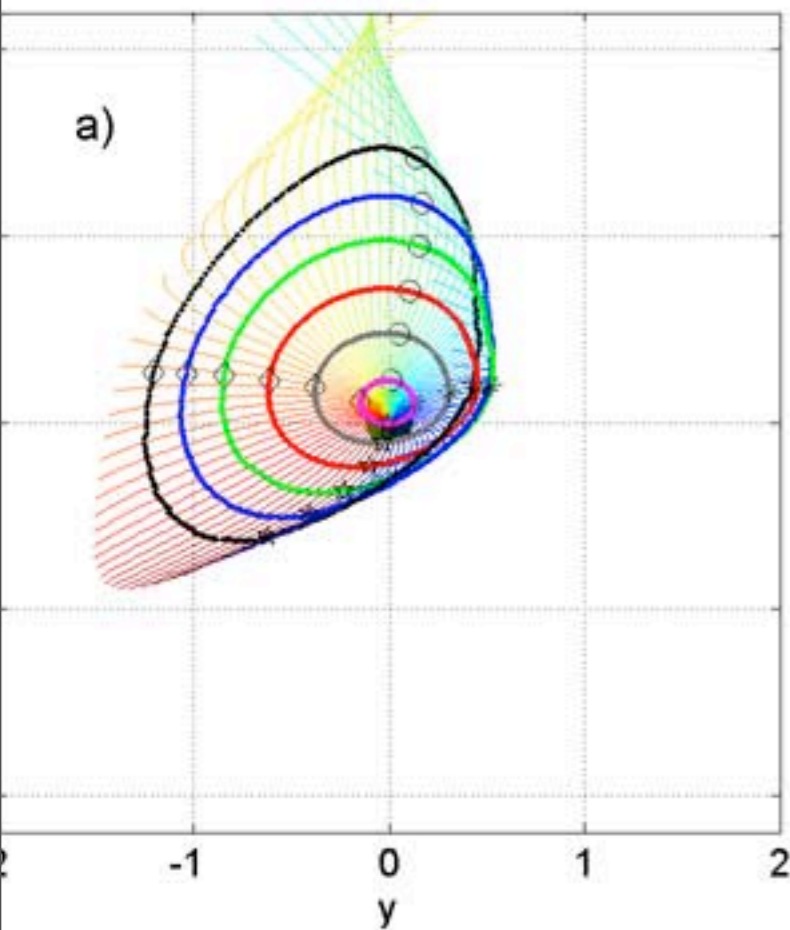
Emissions from two poles merge at some flux tubes: what's special about them?

Bai & A. S. [arXiv:0910.5041](https://arxiv.org/abs/0910.5041)

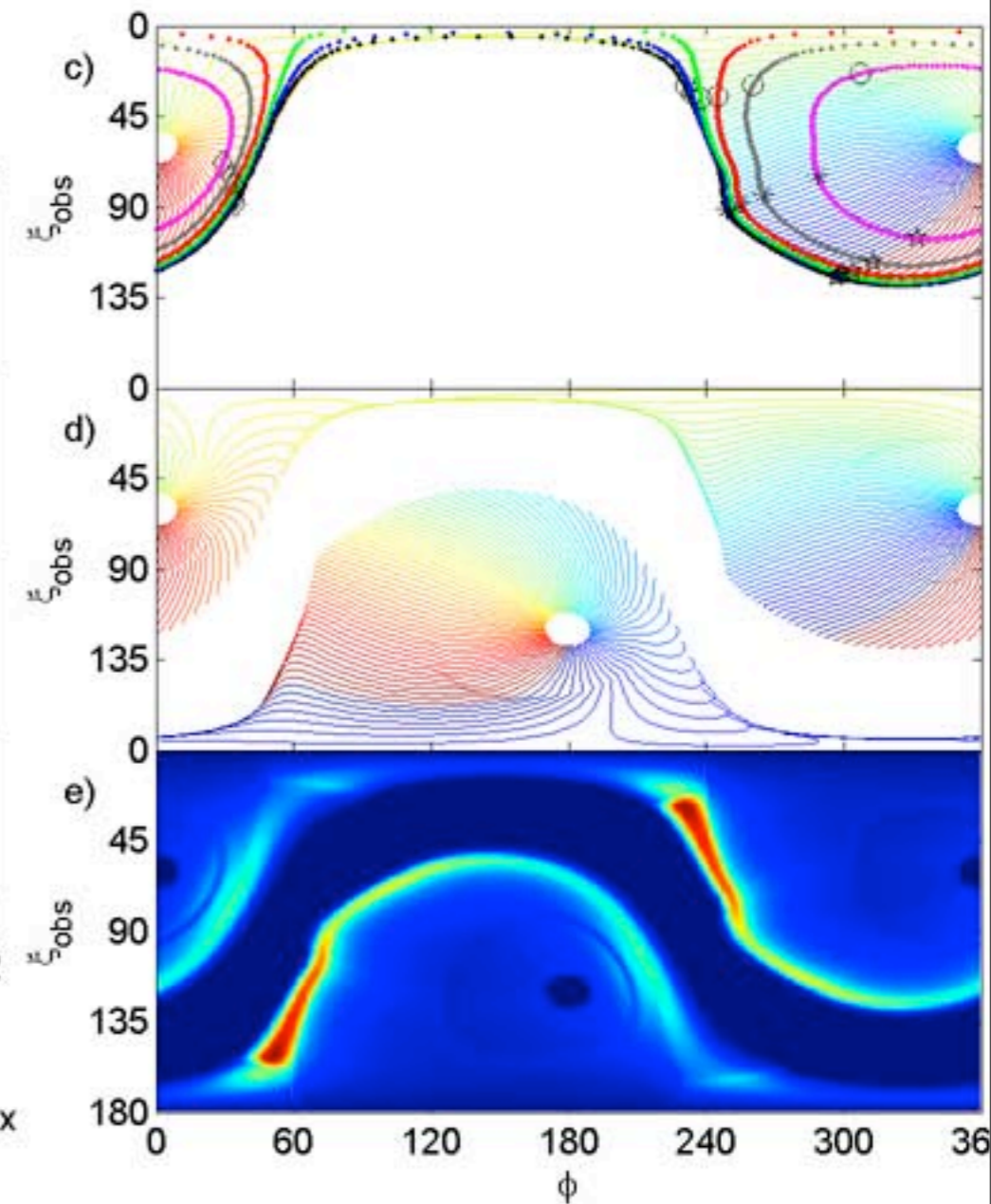
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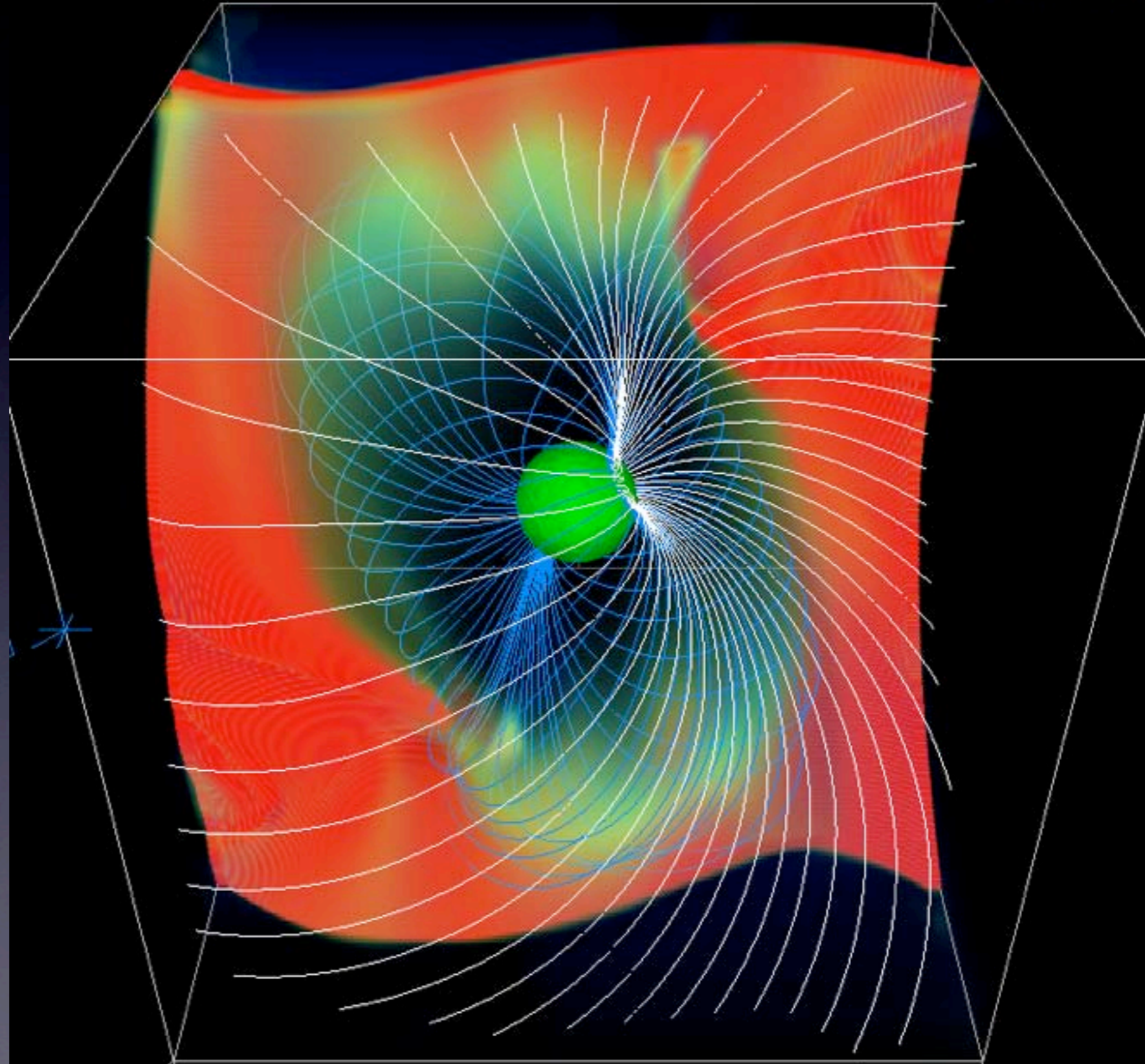
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Bai & A. S. arXiv:0910.5041

Association with the current sheet

Row: 1.000
Col: 0.000
Hgt: 153.196 km

Color -> current

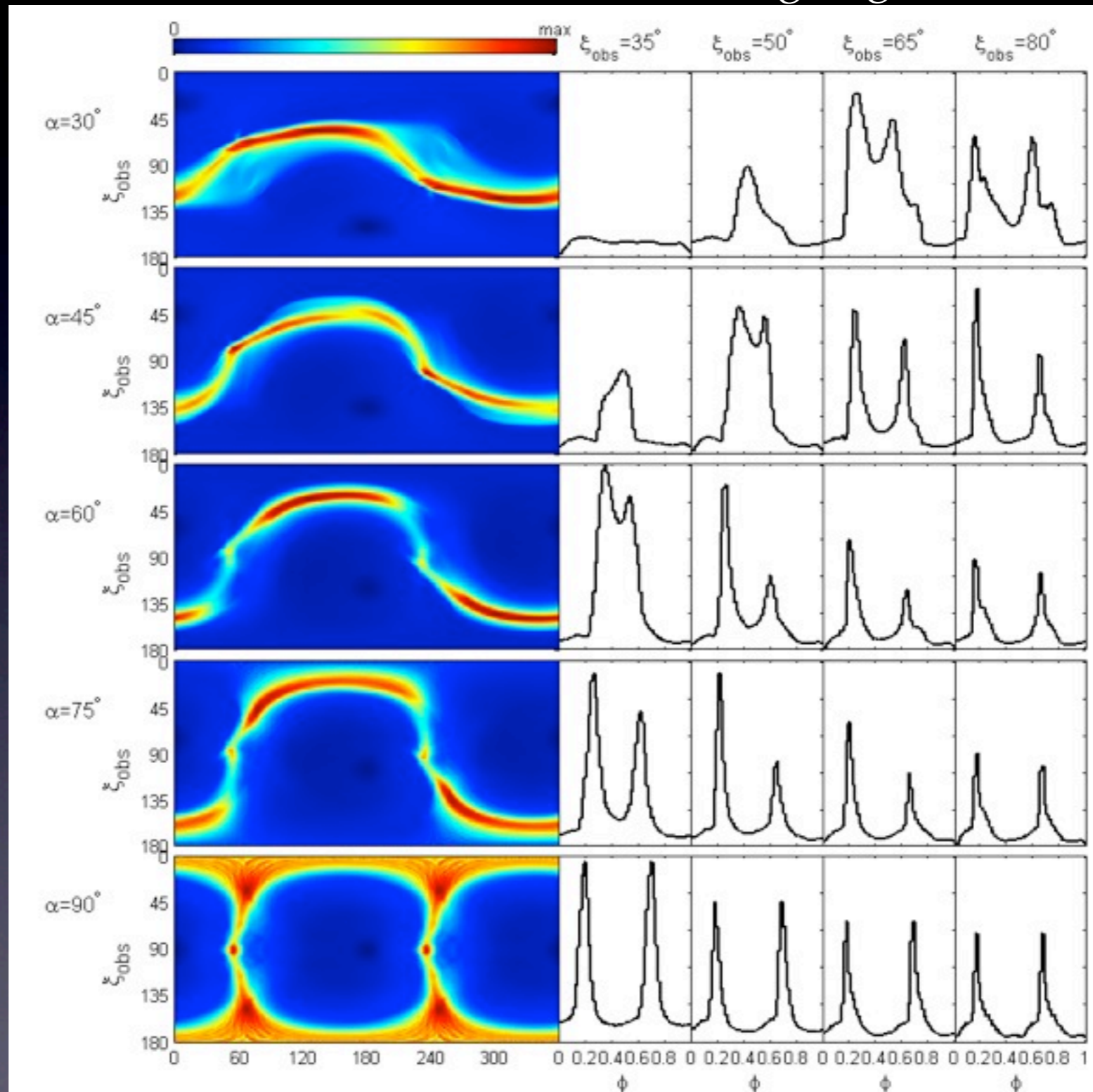


Field lines that produce best force-free caustics seem to “hug” the current sheet at and beyond the LC.

Force-free gallery

Viewing angle

Inclination angle



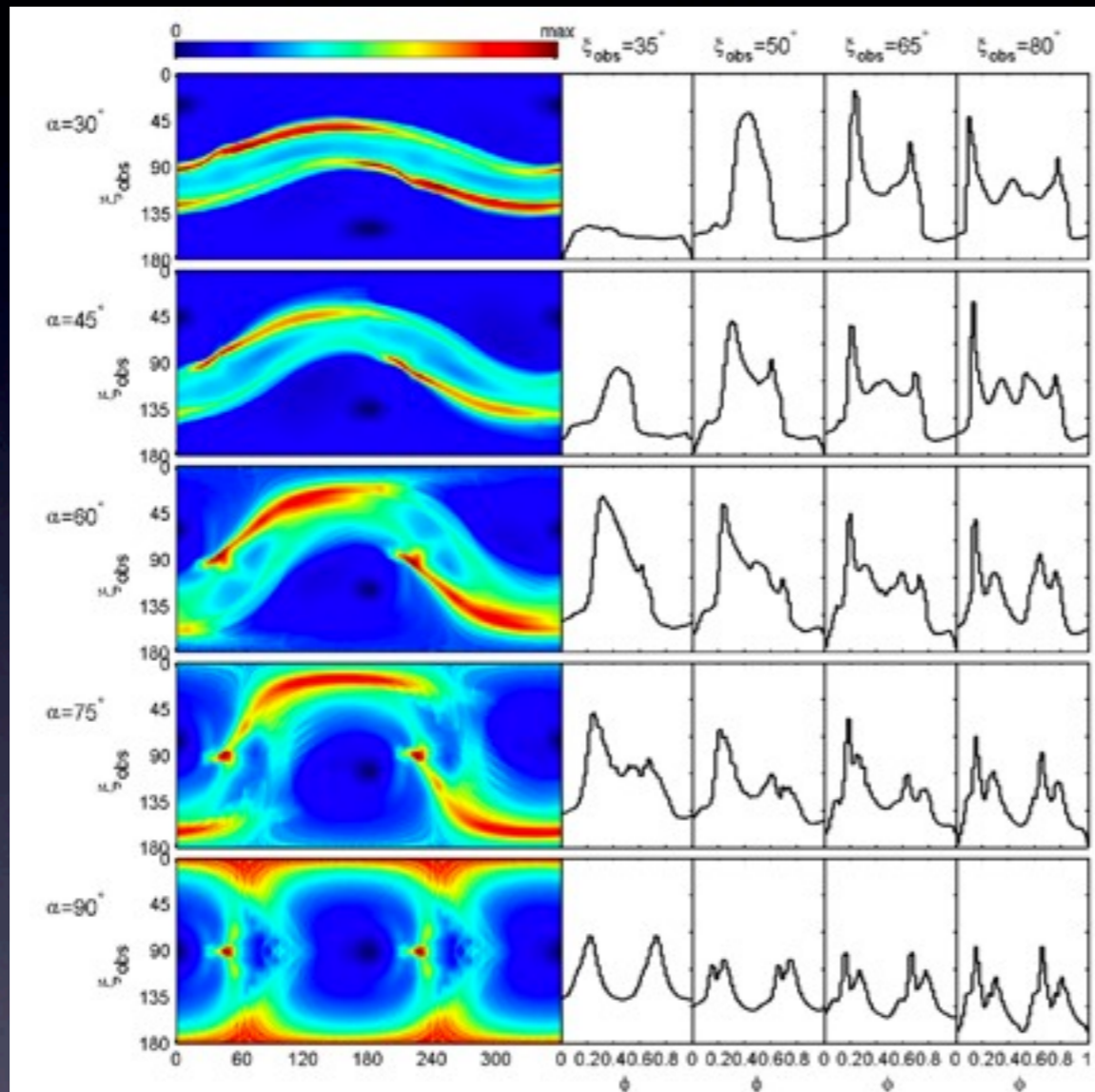
Double peak profiles very common.

Bai & A. S. arXiv:0910.5041

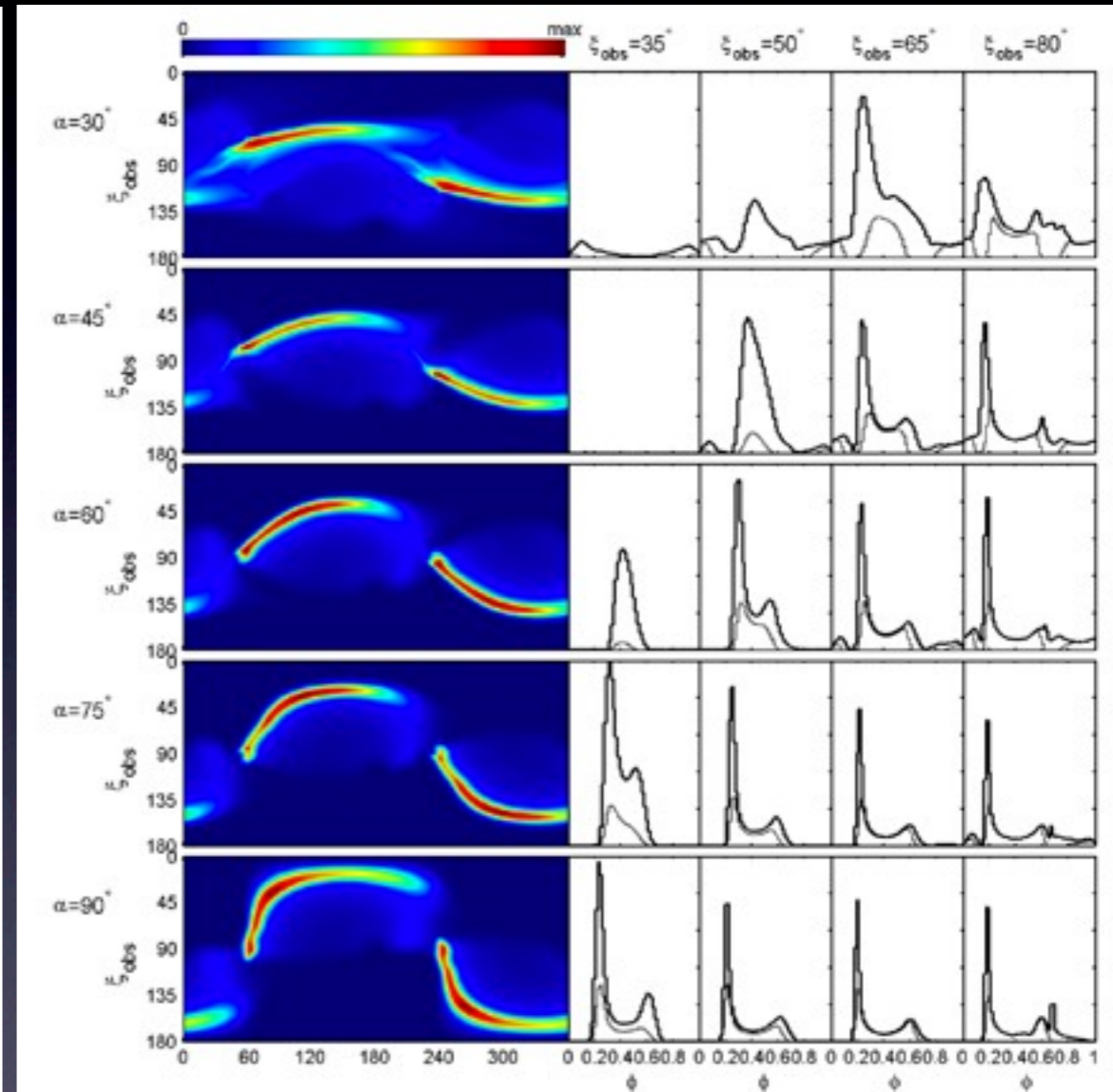
Force-free gallery: TPC and OG

Viewing angle

Inclination angle



SG/TPC with FF



OG with FF

SG/TPC and OG with FF field do not produce double peaks!

Bai & A. S. arXiv:0910.5041

Gamma-rays from pulsars: summary

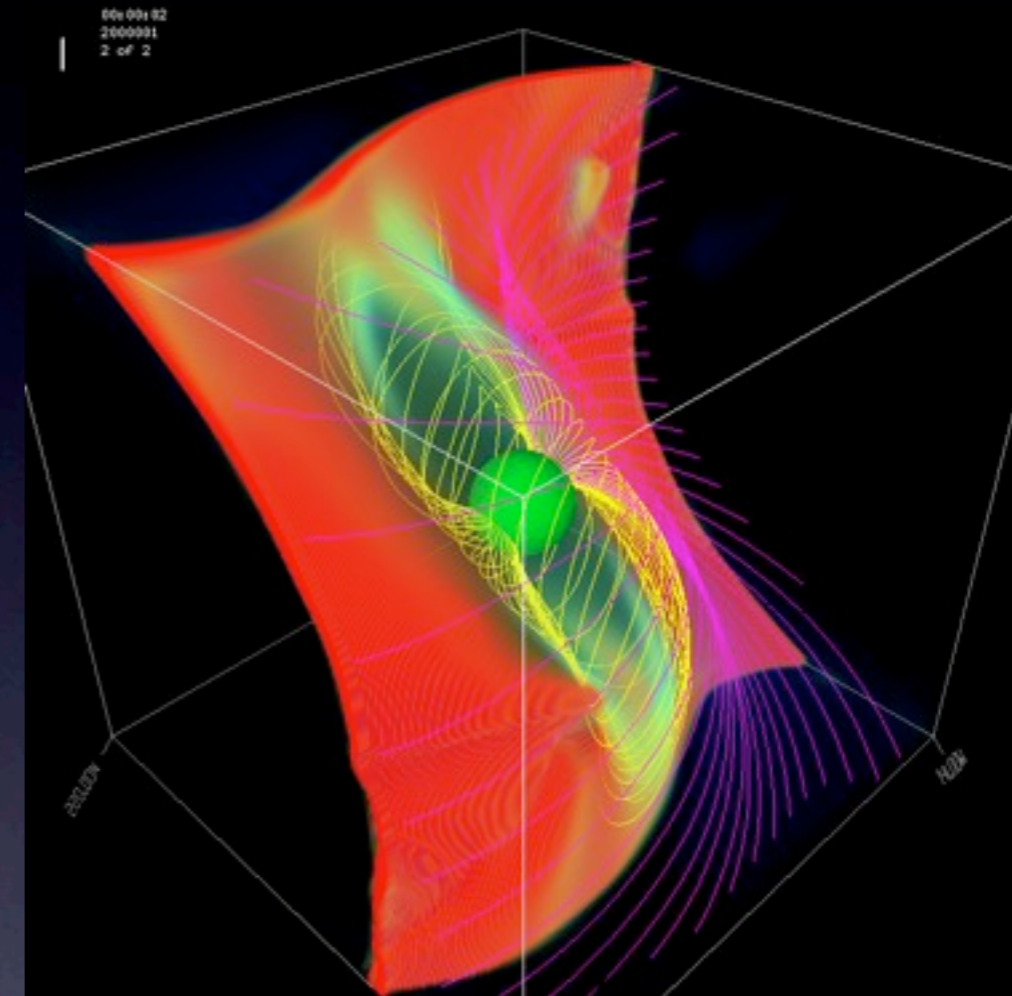
Pulsar gamma-ray emission is coming from the outer magnetosphere.

Two well-established models for the location of emission in magnetosphere exist: SG & OG. Both rely on the vacuum field. The physical basis for existence of these accelerating regions and their extents is very uncertain, but they fit the data!

More realistic field, force-free magnetosphere, can produce double peaks. However, neither SG nor OG locations work for FF. The best fit is from emission near the current sheet at and beyond the LC.

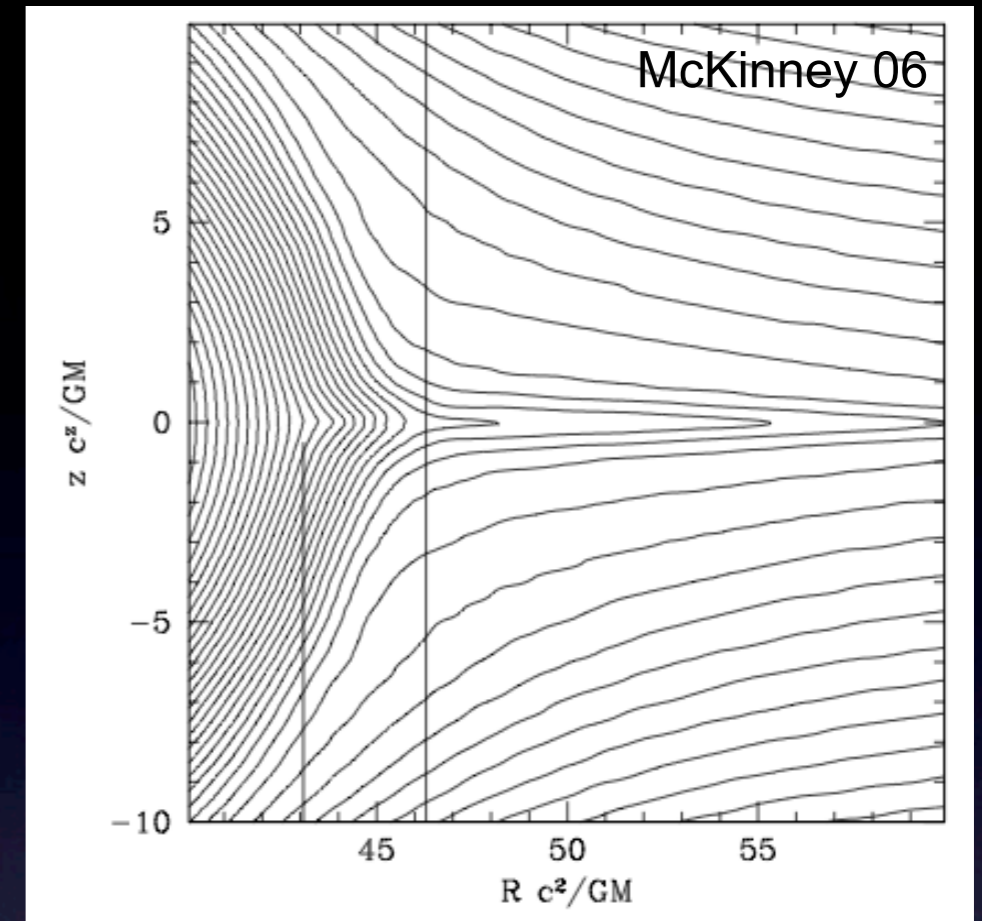
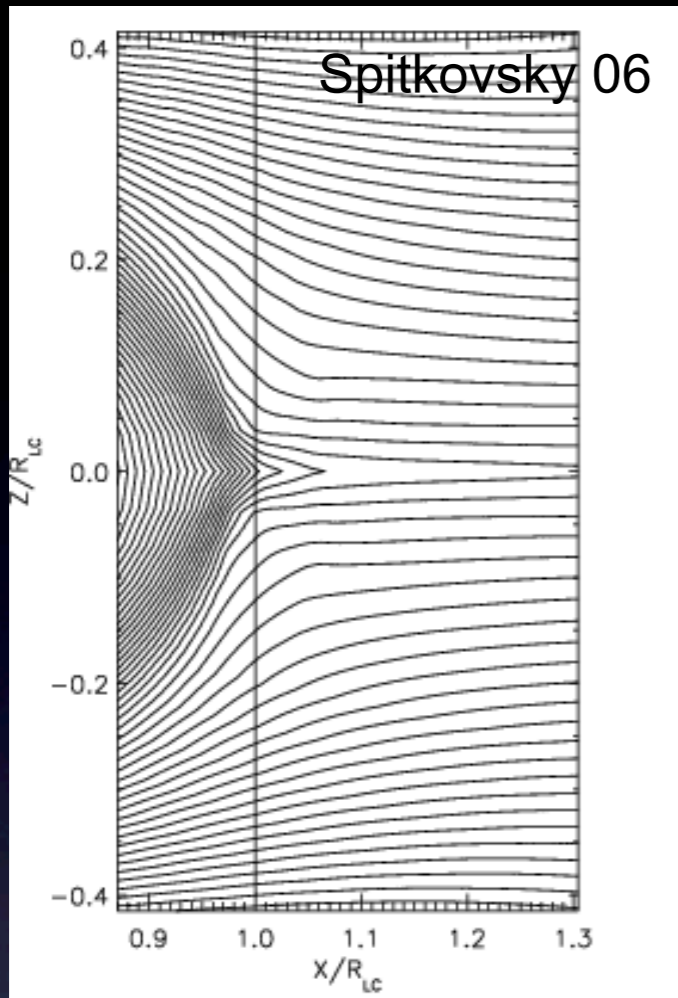
Caustics in FF due to split-monopolar asymptotics. Theory of emission from current sheet is not well developed at all, and much more theoretical work has to be put in. Large L_γ makes sense w/ cur sheet.

Large $B@LC \rightarrow$ reconnection.
Phase-resolved spectra from Fermi will be crucial!

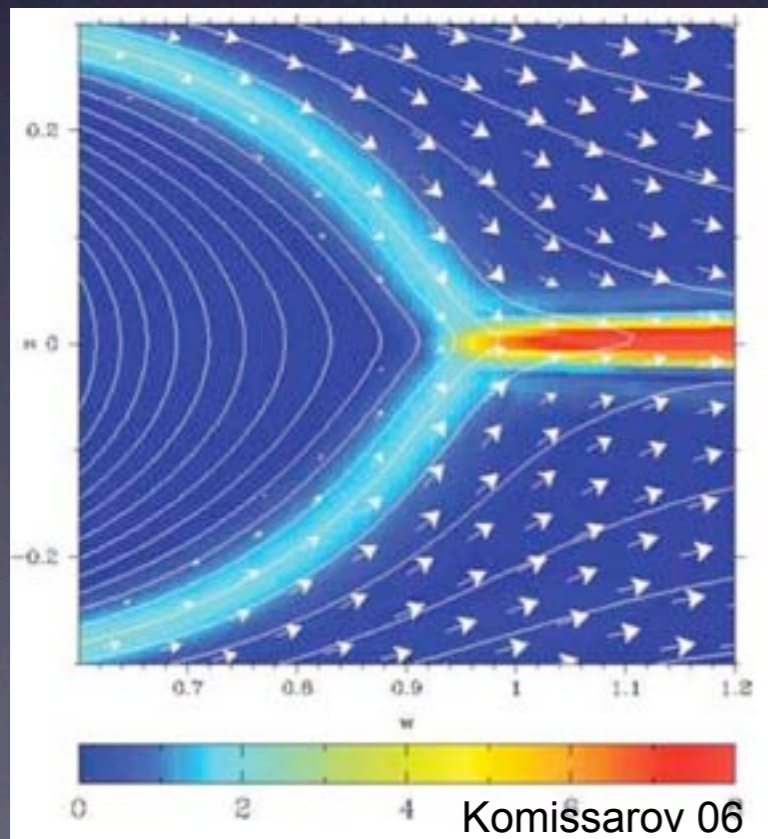


e.g., Lyubarsky 96,
Kirk et al 02,
Petri 09

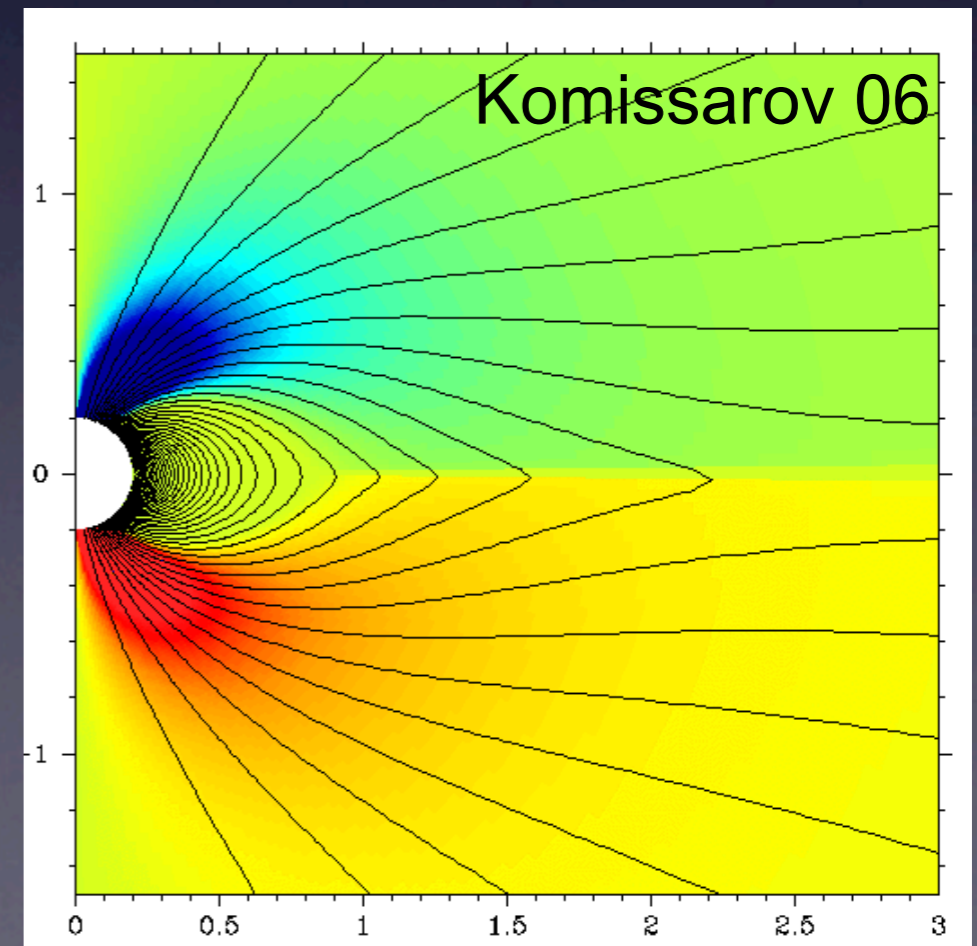
Open issues



Now more than ever
want to resolve the
current sheet!



Solutions are
sensitive to
resistivity
prescription
and code
diffusivity



Resistive FF: Strong Field ED

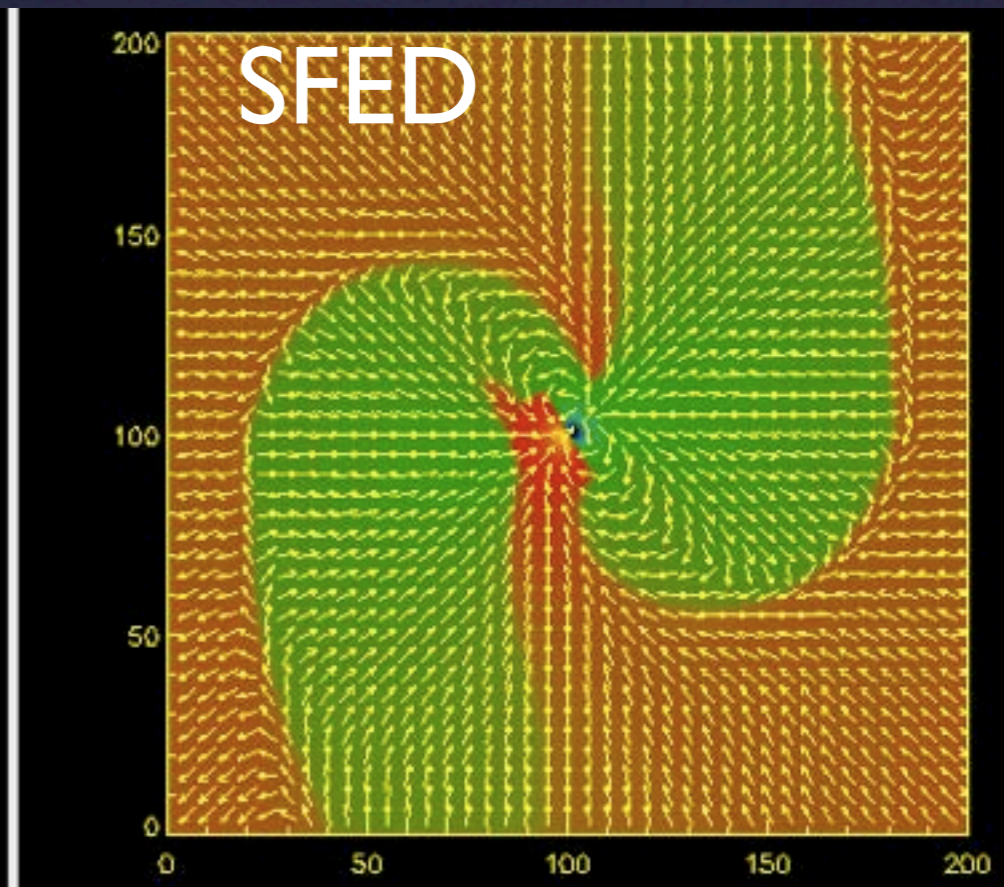
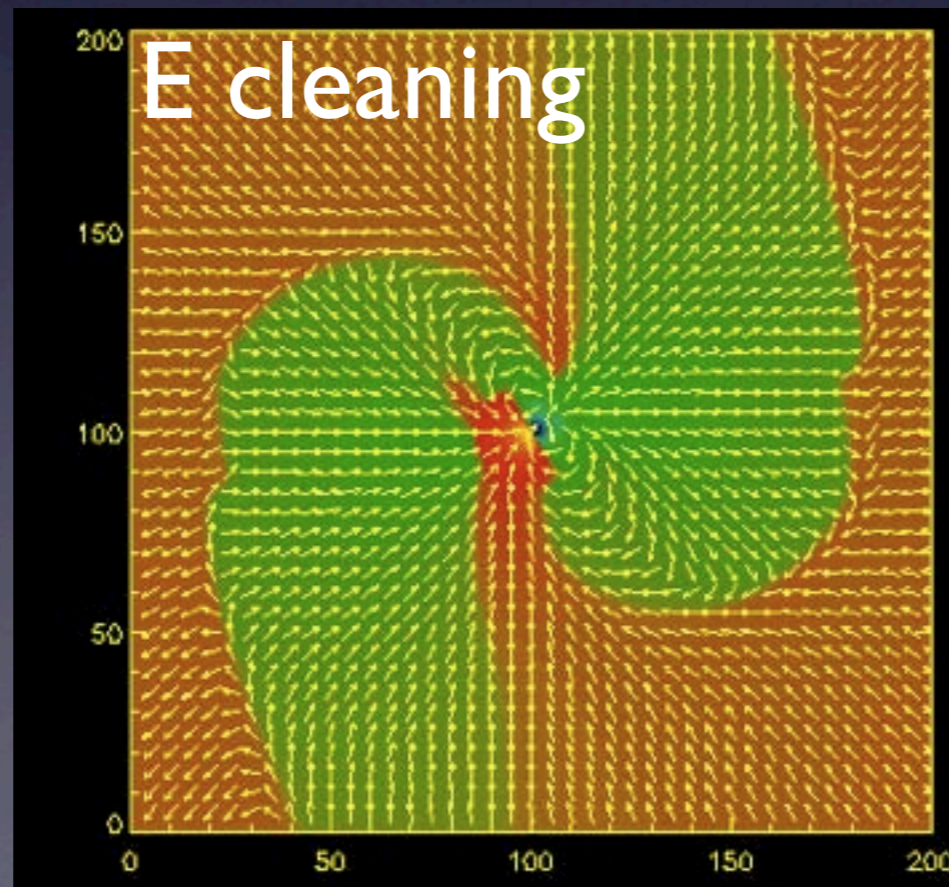
Gruzinov (07,08): in the frame where $\mathbf{E} \parallel \mathbf{B}$ and charge density $\rho=0$, current is $\parallel \mathbf{B}$ and equal to $\sigma \mathbf{E}$.

$$\mathbf{j} = \frac{\rho \mathbf{E} \times \mathbf{B} + (\rho^2 + \gamma^2 \sigma^2 E_0^2)^{1/2} (B_0 \mathbf{B} + E_0 \mathbf{E})}{B^2 + E_0^2},$$

$$B_0^2 - E_0^2 \equiv \mathbf{B}^2 - \mathbf{E}^2, \quad B_0 E_0 \equiv \mathbf{E} \cdot \mathbf{B}, \quad E_0 \geq 0, \quad \gamma^2 \equiv \frac{B^2 + E_0^2}{B_0^2 + E_0^2},$$

$$B_0^2 = \frac{B^2 - E^2 + \sqrt{(B^2 - E^2)^2 + 4(\mathbf{E} \cdot \mathbf{B})^2}}{2} + 0,$$

$$E_0 = \sqrt{B_0^2 - B^2 + E^2}, \quad B_0 = \text{sign}(\mathbf{E} \cdot \mathbf{B}) \sqrt{B_0^2}.$$



Open issues

Numerical:

Treating current sheets: approach from RMHD, or from FF?

Smarter resistivities? (Gruzinov's renormalization)

Explicit-implicit schemes?

What are the test problems?

Do we know what resistive FF equations are? (Gruzinov's SFE)

Origin of the current in FF: can this current always be provided?

Reconnection physics: what happens inside the current sheet that can lead to radiation? What's the spectrum?

Is the current sheet stable physically? Is time-dependence important?

Conclusions

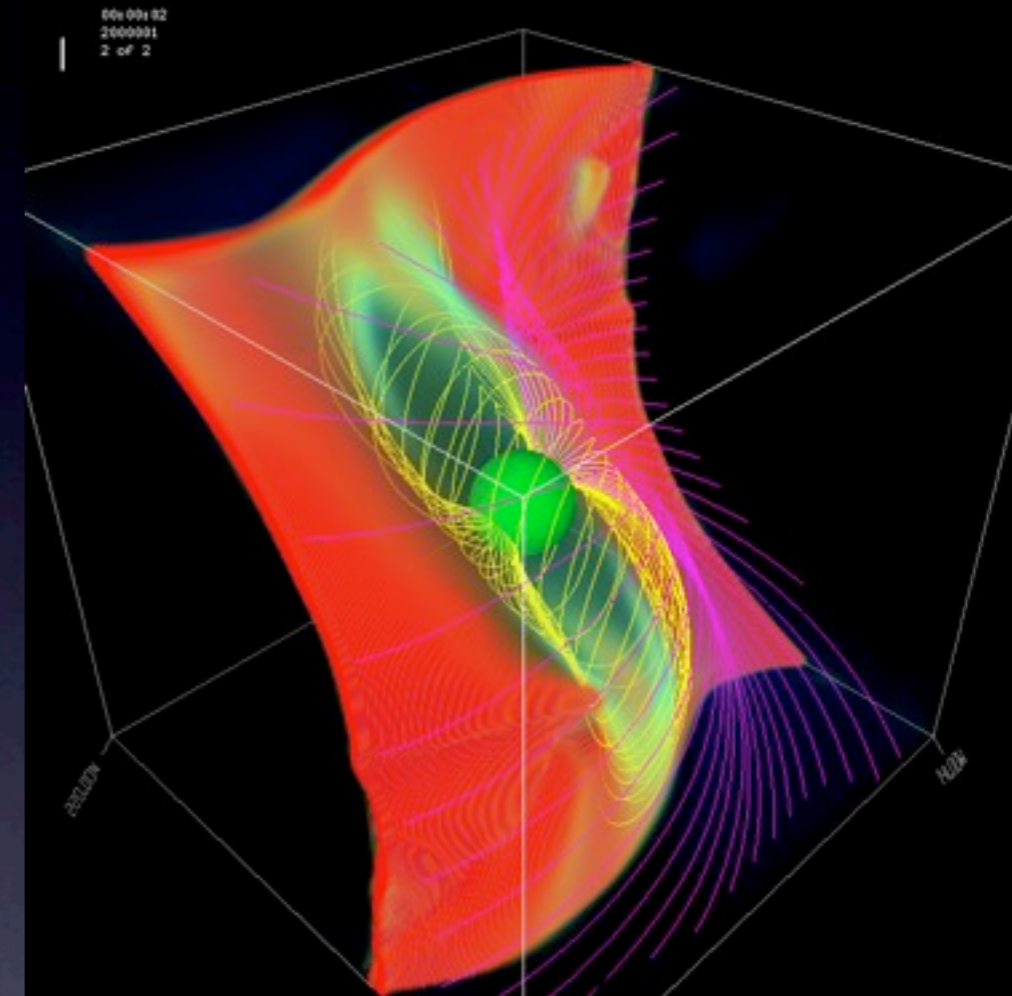
Magnetically-dominated environments now can be modeled numerically in 3D -- force-free method

Applications to pulsars, magnetars and disks allow to find the shape of the magnetosphere, and spin down law and energy loss distribution in angle for oblique rotators

Time-dependent magnetospheres open a new realm for understanding rich pulsar phenomenology (e.g. drifting subpulses)

Current sheets form spontaneously in magnetically-dominated flows. Physics of relativistic reconnection is not understood and needs attention.

Pulsar gamma-ray emission can now be understood both on geometric and physical grounds as the emission from the outer magnetosphere / current sheet. This region has to be understood in much more detail.



**PULSAR
MAGNETOSPHERE:**

The
**40-Year-Old
Virgin** **UNRATED**



BETTER LATE THAN NEVER