The Effects of Anisotropic Transport on Dilute Astrophysical Plasmas

Eliot Quataert (UC Berkeley)

in collaboration with

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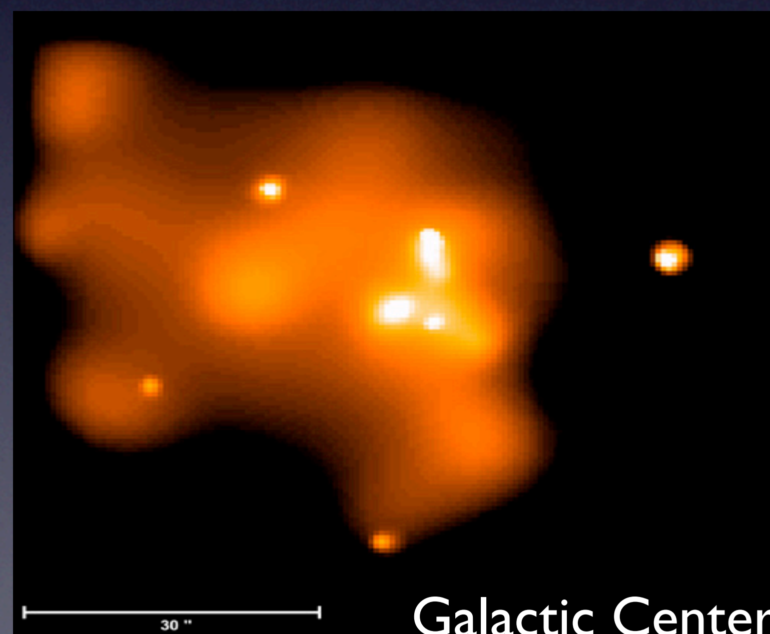
Hydra A w/ Chandra

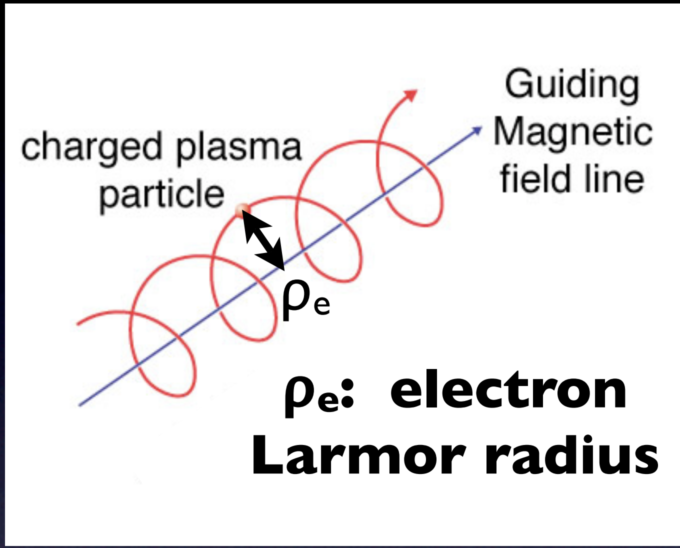


Galactic Center

Anisotropic Transport in Dilute Plasmas

$$\frac{l_e}{\rho_e} \sim 10^{14} \left(\frac{B}{10^{-6}\,\mathrm{G}}\right) \left(\frac{n}{0.01\,\mathrm{cm}^{-3}}\right)^{-1} \left(\frac{T}{3\,\mathrm{keV}}\right)^{-3/2}$$

l_e: electron mean free path
ρ_e: electron Larmor radius
#s scaled for galaxy clusters



$l_e >> \rho_e$ ⇒ heat transport is anisotropic, primarily along B

ion momentum transport is also anisotropic

Theme: anisotropic heat & momentum transport are crucial for the thermodynamics & dynamics of weakly magnetized dilute plasmas

Overview

- Anisotropic Heat Transport
  - Convection induced by Anisotropic Thermal Conduction
    - Particularly important for the intracluster plasma in galaxy clusters
- Anisotropic Momentum Transport
  - The MRI in a dilute plasma is strongly modified by anisotropic ion momentum transport; similar physics $\Rightarrow$ efficient electron heating
    - Important for hot accretion flows onto compact objects
- Focus on physics, not astrophysical implications - happy to discuss the latter

Hydrodynamic Convection

- Schwarzschild criterion for convection: **$ds/dz < 0$**
- Motions slow & adiabatic: **pressure equil, s ~ const**

  solar interior: t_{sound} ~ hr << $t_{buoyancy}$ ~ month << $t_{diffusion}$ ~ 10^4 yr

low entropy (s)

gravity

high s

$$\rho_i = \rho_{bg}$$
$$s_i = s_{bg}$$
$$p_i = p_{bg}$$

Hydrodynamic Convection

- Schwarzschild criterion for convection: **$ds/dz < 0$**
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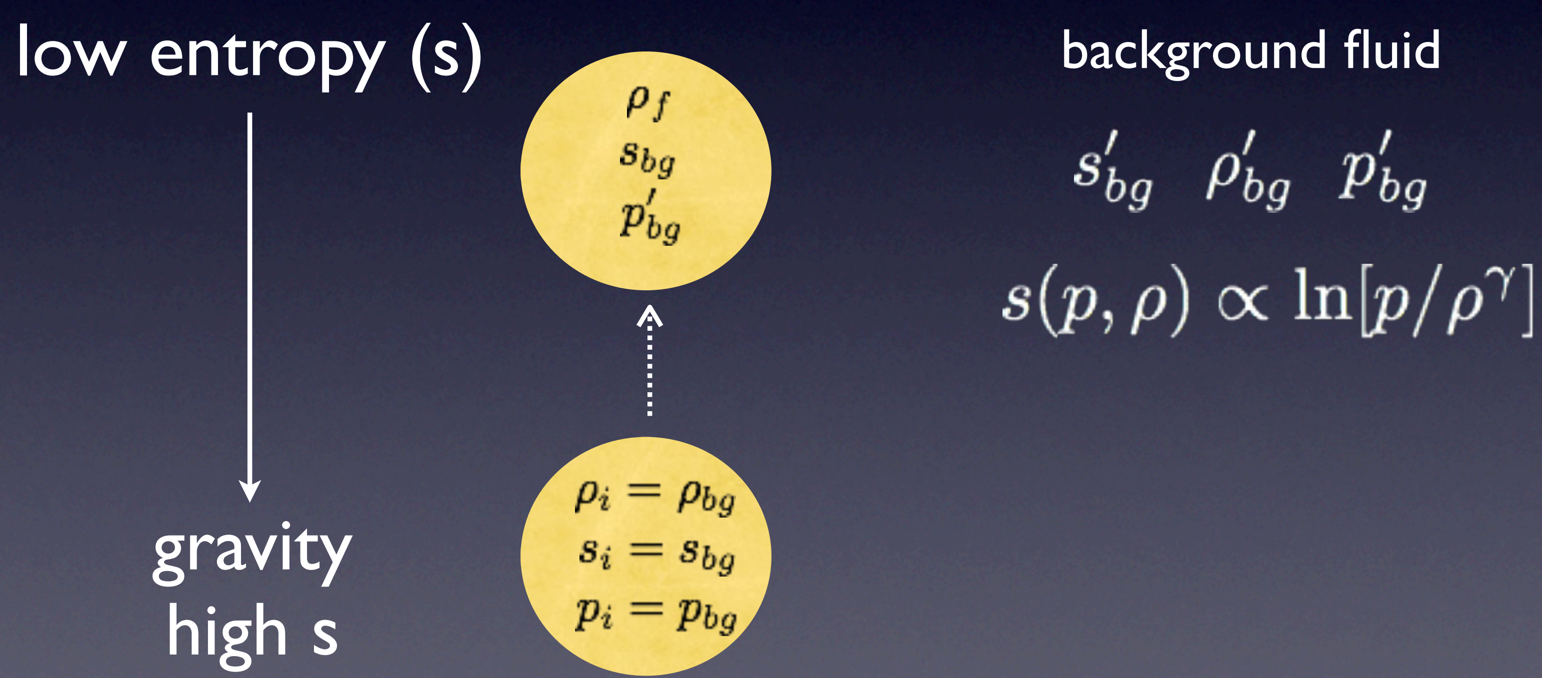
  solar interior: $t_{sound} \sim$ hr << $t_{buoyancy} \sim$ month << $t_{diffusion} \sim 10^4$ yr



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low entropy (s)

gravity
high s

background fluid

$$s'_{bg} \quad \rho'_{bg} \quad p'_{bg}$$

$$s(p,\rho) \propto \ln[p/\rho^\gamma]$$

$$\text{if } \mathrm{ds/dz} < 0 \rightarrow \rho_{\mathrm{f}} < \rho'_{\mathrm{bg}}$$

convectively unstable

The Magnetothermal Instability (MTI)

Balbus 2000, 2001; Parrish & Stone 2005, 2007; Quataert 2008

cold

g

hot

$$\rho_i = \rho_{bg}$$
$$T_i = T_{bg}$$
$$p_i = p_{bg}$$

weak B-field

no dynamical effect;
only channels heat flow

The Magnetothermal Instability (MTI)

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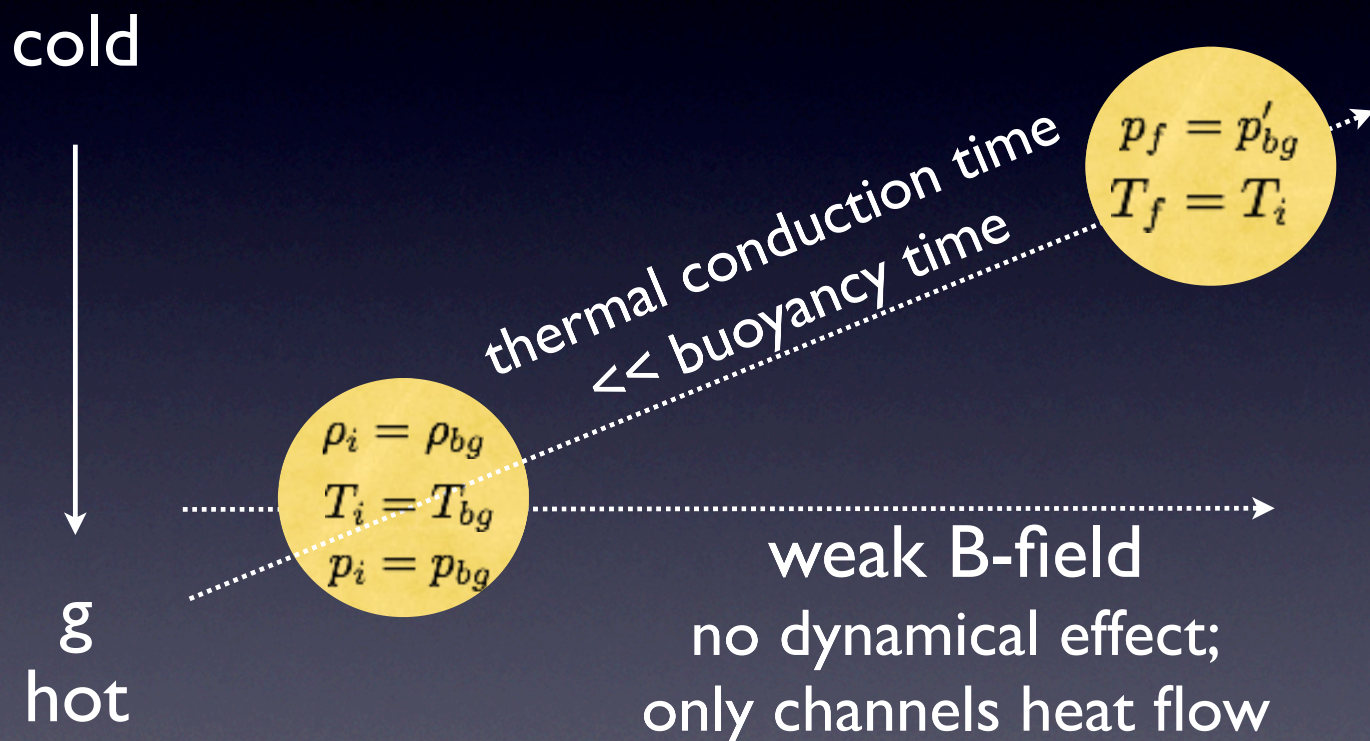

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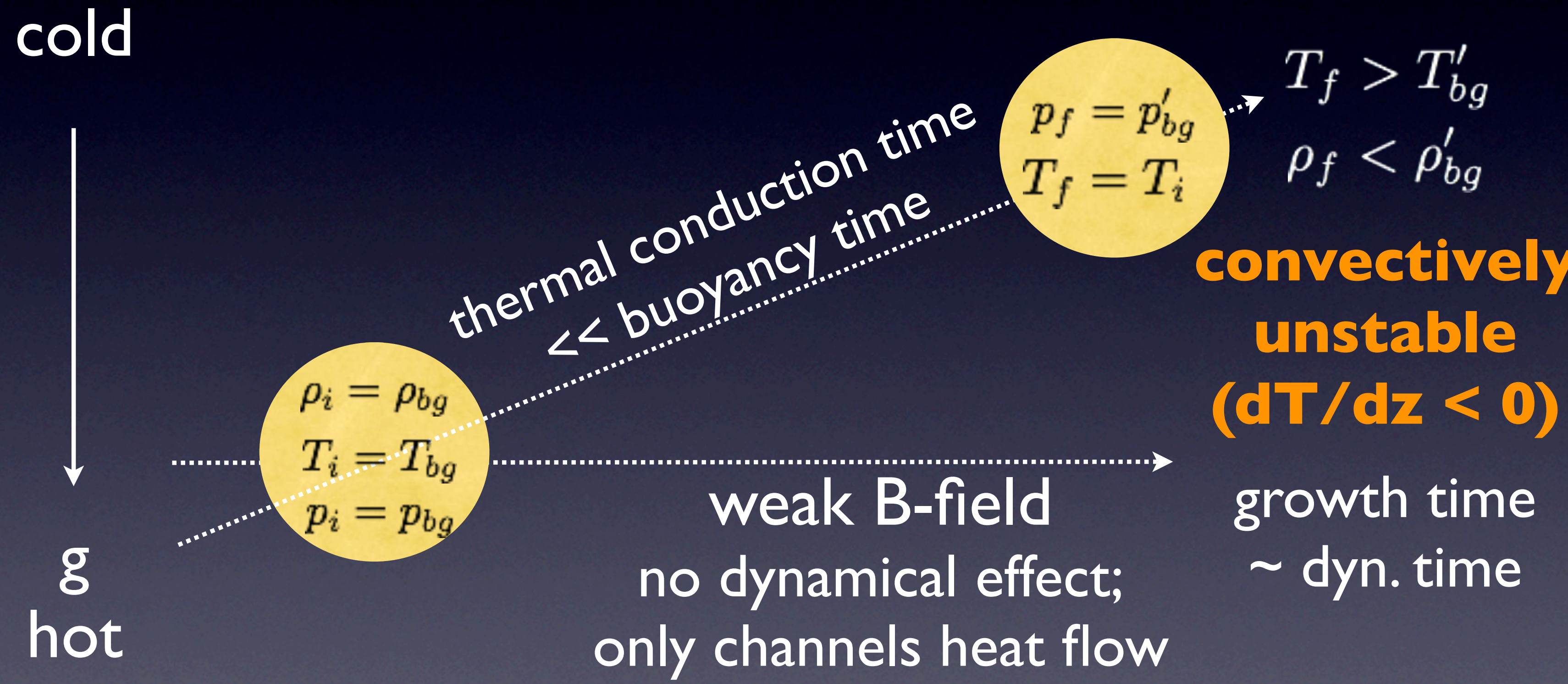
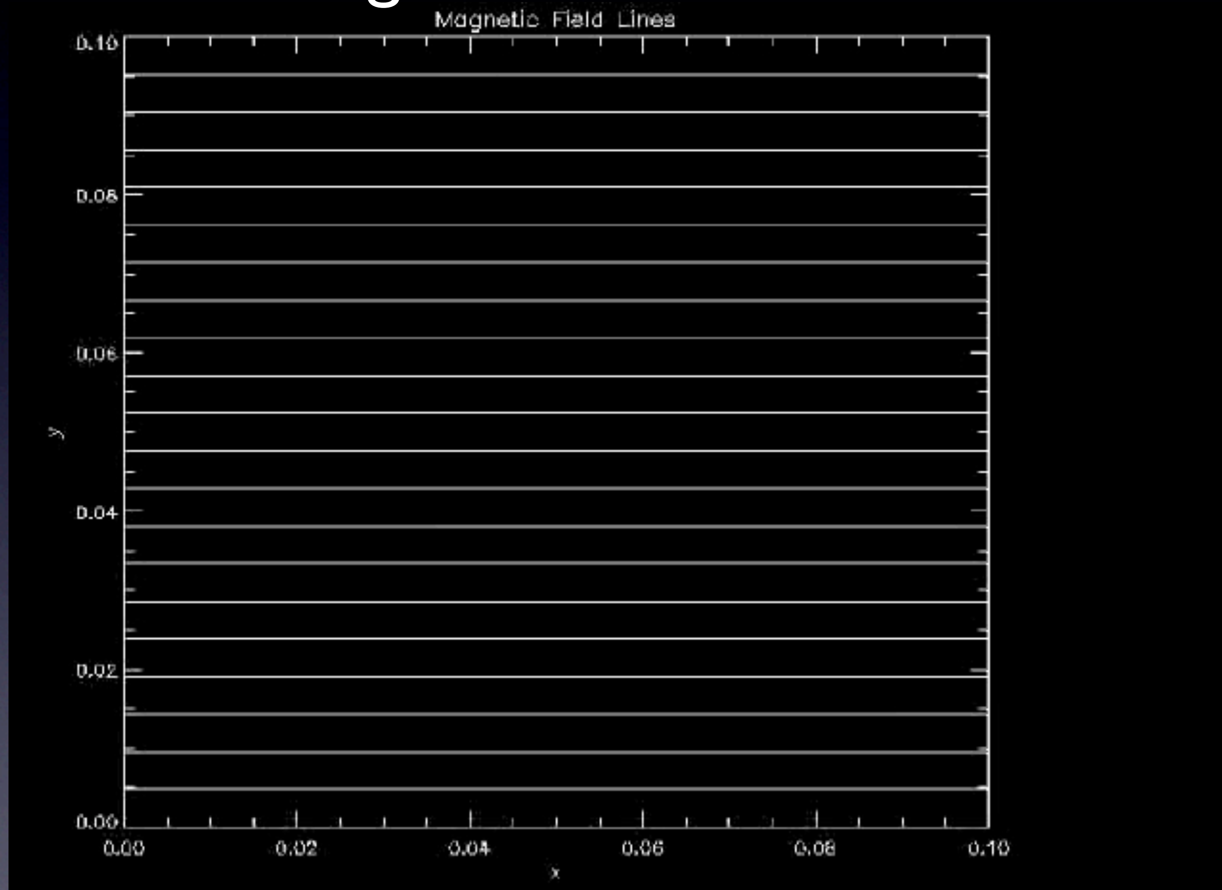


The MTI

magnetic field lines

cold



g

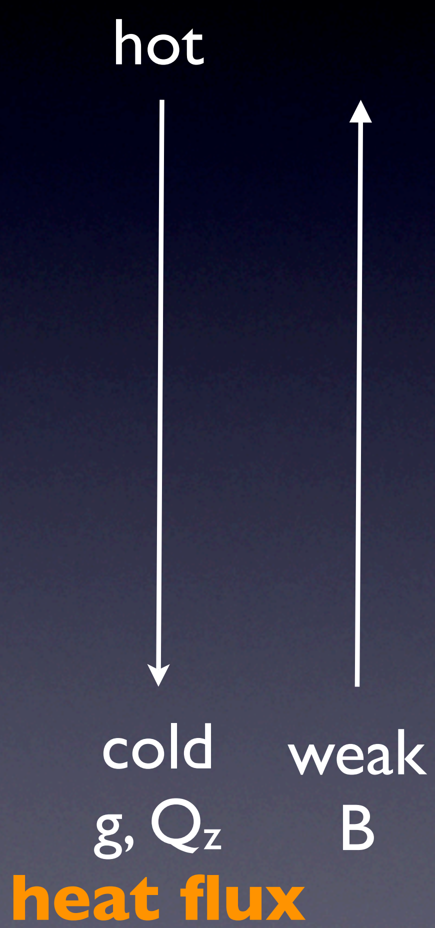
hot

2D simulation from Ian Parrish (Athena)

instability saturates by amplifying & rearranging the magnetic field

The Heat Flux-Driven Buoyancy Instability (HBI)

Quataert 2008; Parrish & Quataert 2008



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hot

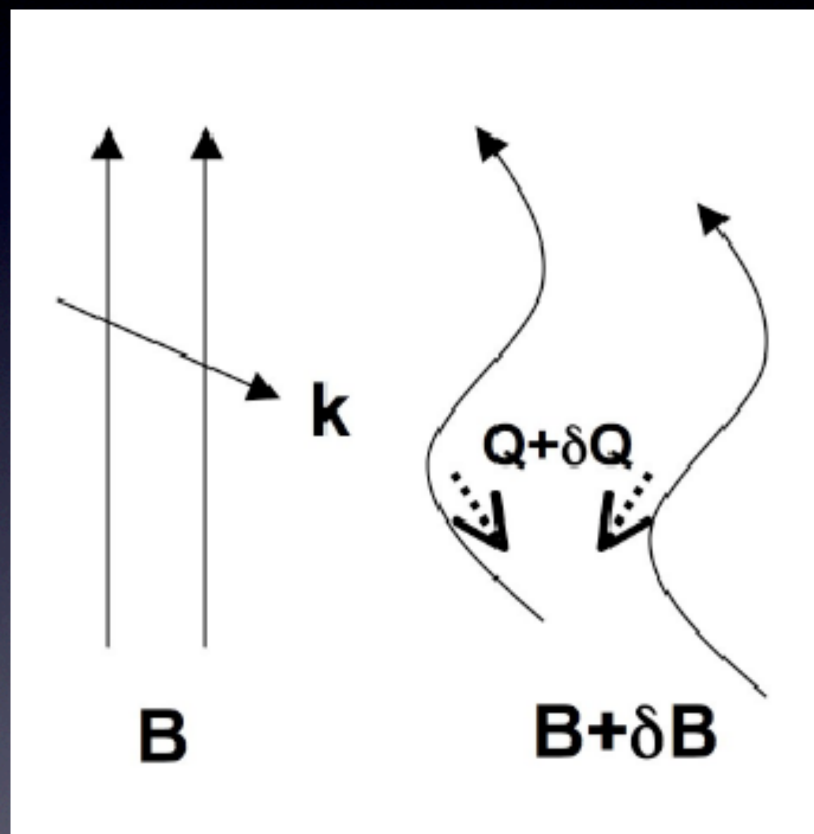
cold

g, Q_z

heat flux

weak

B



converging & diverging heat flux ⇒ conductive heating & cooling

for **dT/dz > 0** upwardly displaced fluid heats up & rises, bends field more, ...

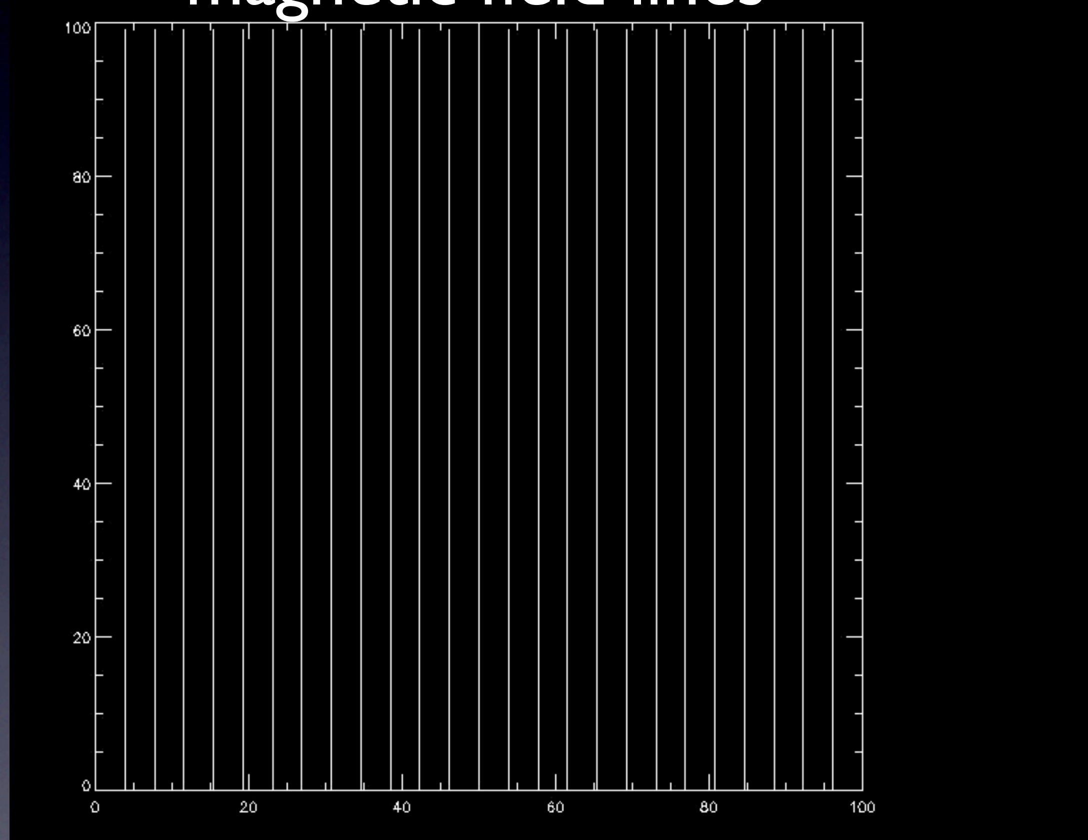
convectively unstable

The HBI

hot

cold
g, Q_z
heat flux

magnetic field lines

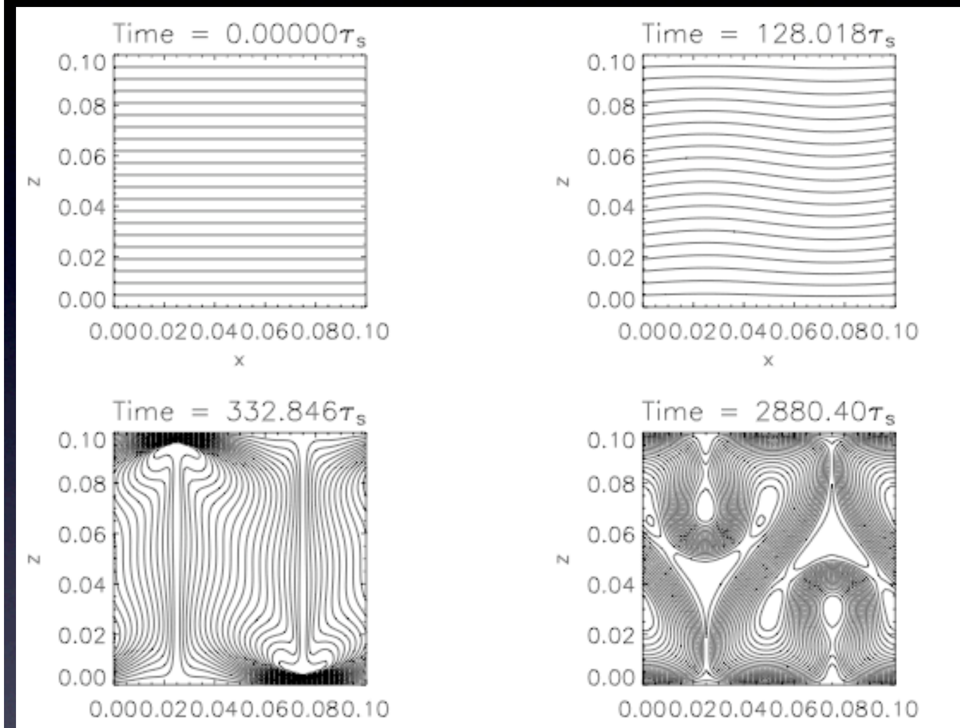


Parrish & Quataert 2008

HBI saturates by amplifying & rearranging the magnetic field

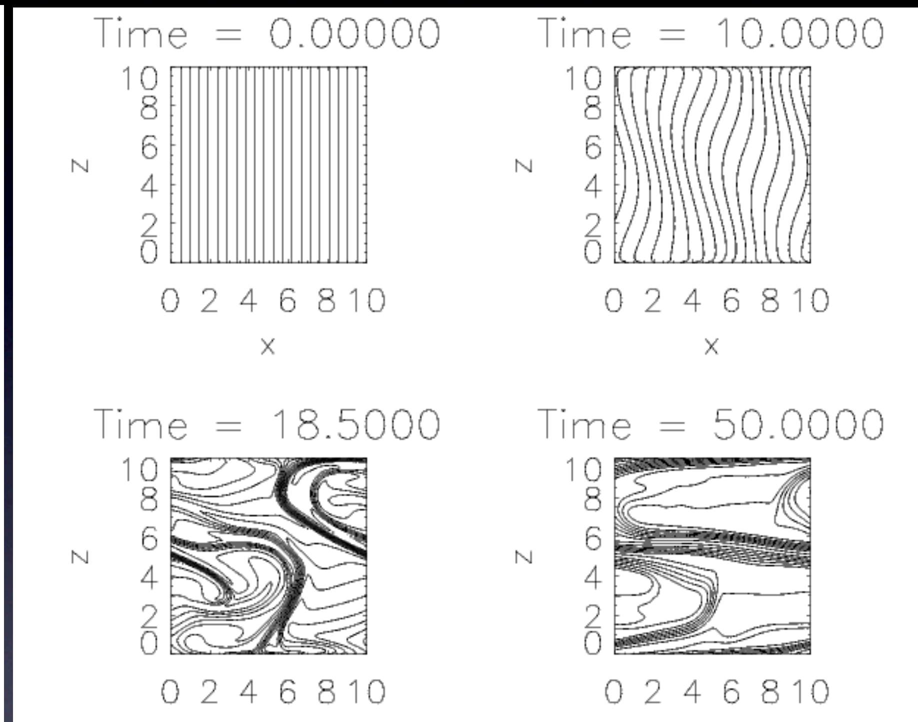
Buoyancy Instabilities in Magnetized Plasmas

MTI (dT/dz < 0)



Parrish & Stone 2005

HBI (dT/dz > 0)

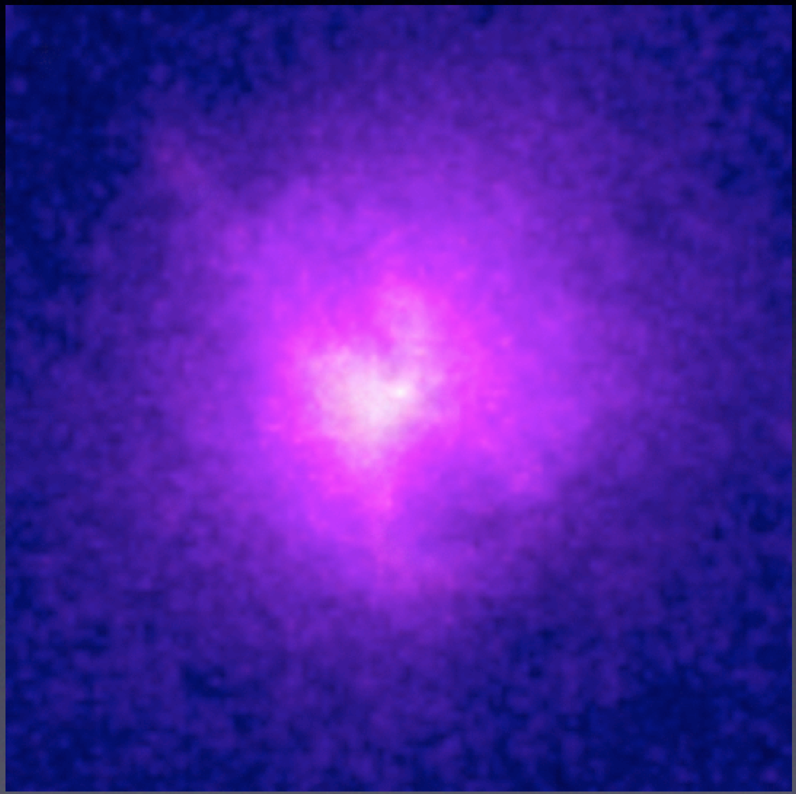


Parrish & Quataert 2008

a weakly magnetized plasma w/ anisotropic heat transport is always buoyantly unstable, independent of dT/dz

Instabilities suppressed by 1. strong B (β < 1; e.g., solar corona) or 2. isotropic heat transport >> anisotropic heat transport (e.g., solar interior)

Hot Plasma in Galaxy Clusters



$L_x \sim 10^{43\text{-}46}$ erg s^{-1}

$n \sim 10^{-4}\text{-}1$ cm^{-3}

$T \sim 1\text{-}15$ keV

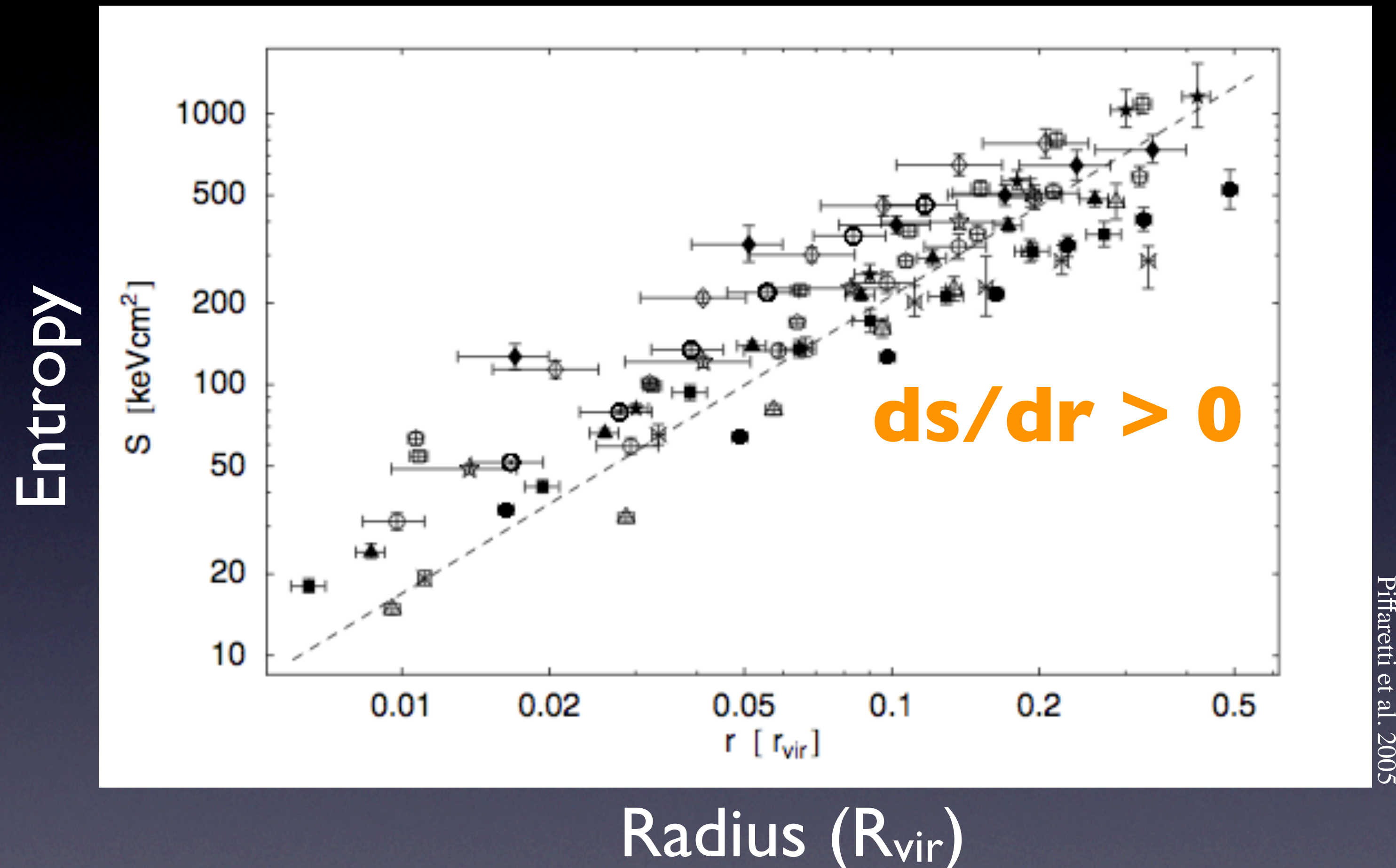
$M_{gas} \sim 10^{13\text{-}14}\ M_\odot$

large electron mean free path:

$$\ell_e \simeq 2 \left(\frac{T}{3\,\mathrm{keV}}\right)^2 \left(\frac{n}{0.01\,\mathrm{cm}^{-3}}\right)^{-1} \mathrm{kpc}$$

→ thermal conduction important

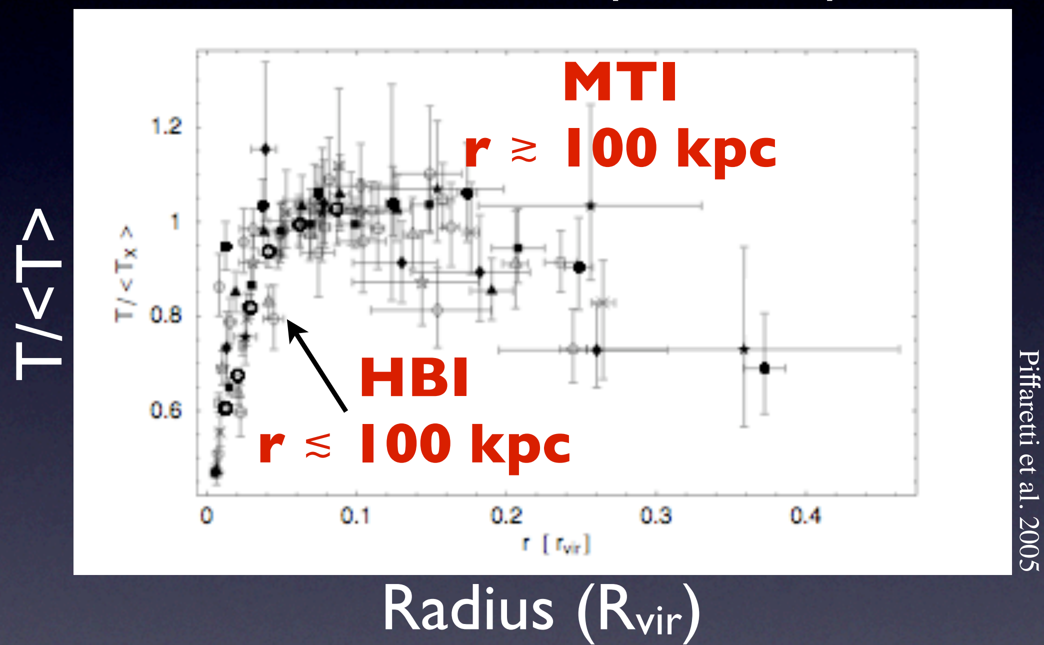
Cluster Entropy Profiles

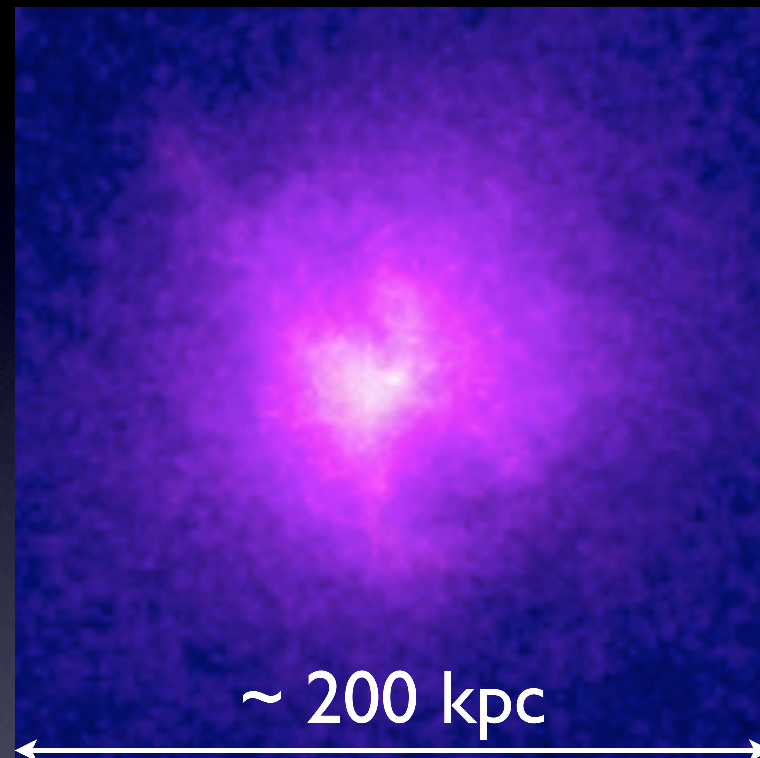


Schwarzschild criterion → clusters are buoyantly **stable**

The MTI & HBI in Clusters

cool core cluster temperature profile





The Entire Cluster is 'Convectively' Unstable in MHD, driven by anisotropic thermal conduction

Important implications for the thermal evolution of clusters, cluster bi-modality, cluster B-fields, ...

(Parrish et al. 2009, 2010; Bogdanavic et al. 2009; Ruszkowski & Oh 2010)

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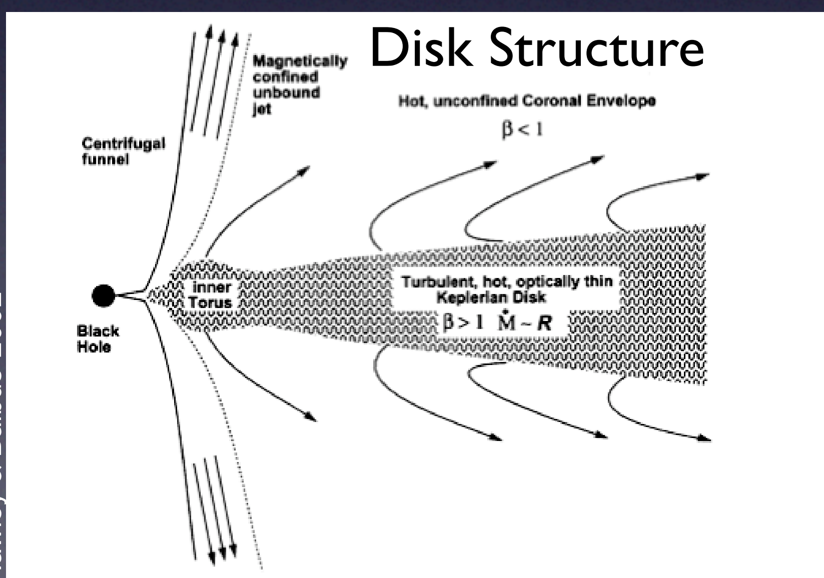
Radiatively Inefficient Accretion Flows

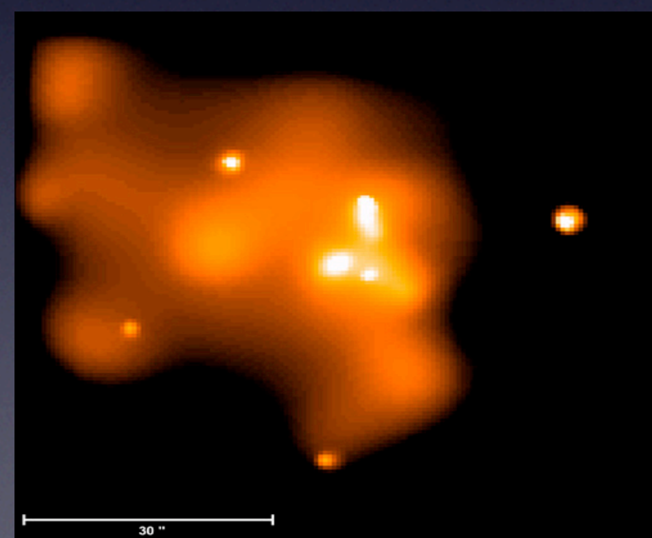
- At low densities (accretion rates), cooling is inefficient
- grav. energy ⇒ turbulence (MRI) ⇒ thermal energy: not radiated

$$L \equiv \eta \dot{M} c^2 : \quad \eta \lesssim 0.1$$

- kT ~ GMm_p/R (virial): $T_p \sim 10^{11\text{-}12}$ K > $T_e \sim 10^{10\text{-}11}$ K near BH
- **collisionless plasma**: e-p equil. time > inflow time for $\dot{M} \lesssim \alpha^2 \dot{M}_{Edd}$



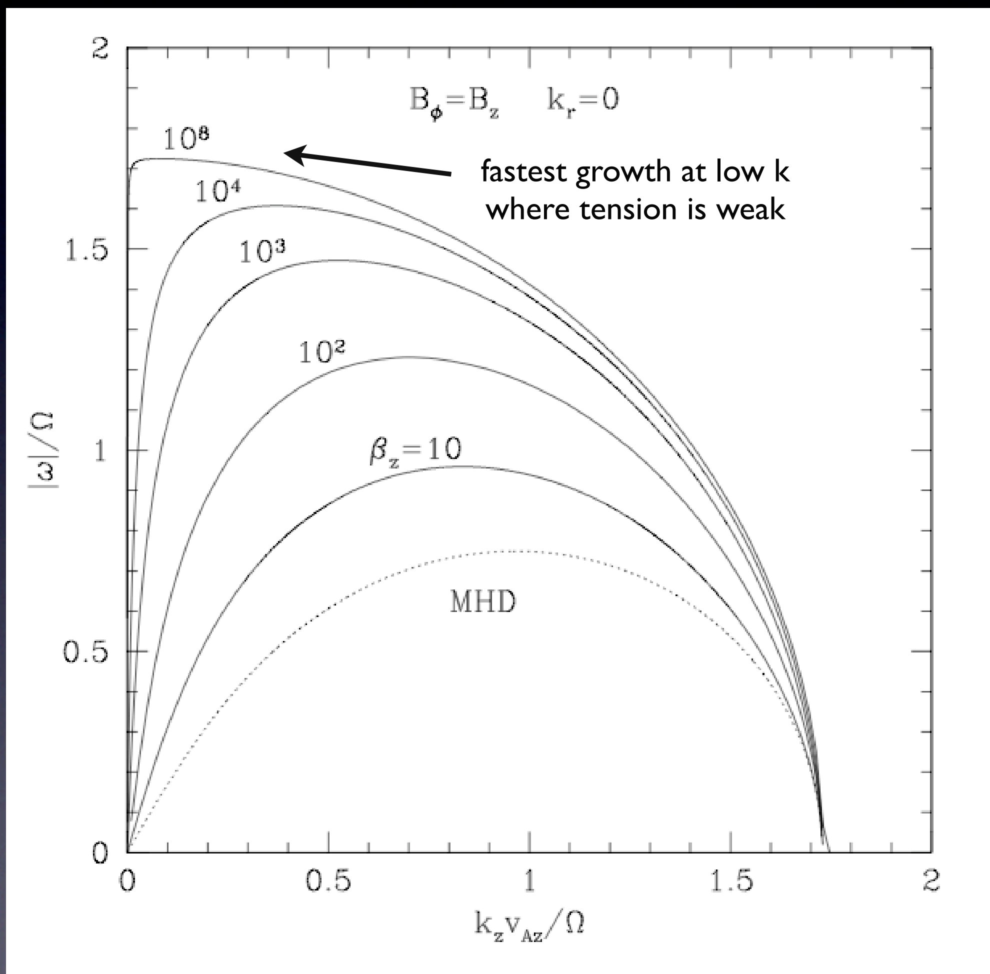
Hawley & Balbus 2002



Relevant to low luminosity compact objects such as Sgr A* i.e., Galactic Center

The Linear MRI in Kinetic Theory

Growth Rate



Quataert, Dorland, Hammett 2002; also Sharma et al. 2003; Balbus 2004

angular momentum transport via free-streaming along field lines (viscosity!), in addition to magnetic stresses

anisotropic viscosity is **destabilizing**, unlike isotropic viscosity

Nonlinear Evolution Simulated using “Kinetic-MHD”

- Large-scale dynamics of collisionless plasmas: expand Vlasov eqn using “slow timescale” and “large lengthscale” assumptions of MHD (Kulsrud 1983)
- Particles efficiently transport heat and momentum along B-field lines

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla\right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B}\right),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_\parallel - p_\perp\right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

Evolution of the Pressure Tensor

$$\frac{\partial f}{\partial t} + \left(v_{\|}\hat{\mathbf{b}} + \mathbf{v}_E\right) \cdot \nabla f + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{v}_E}{Dt} - \mu\hat{\mathbf{b}} \cdot \nabla B + \frac{e}{m}(E_{\|} + F_{g\|}/e)\right) \frac{\partial f}{\partial v_{\|}} = C\left(f\right),$$

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$$\rho B \frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = -\nabla \cdot (\hat{\mathbf{b}} q_{\perp}) - q_{\perp} \nabla \cdot \hat{\mathbf{b}}$$

$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_{||}B^2}{\rho^3}\right) = -\nabla \cdot (\hat{\mathbf{b}} q_{||}) + 2q_{\perp} \nabla \cdot \hat{\mathbf{b}},$$

adiabatic invariance of $\mu \sim v_{\perp}^2/B \sim T_{\perp}/B$

Evolution of the Pressure Tensor

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$$q \simeq \frac{n v_{th}}{|k_{\|}|}\nabla_{\|}T$$

strong parallel heat conduction (both e & p)

Pressure Anisotropy

$$\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\parallel \text{ as B } \uparrow$$

Pressure Anisotropy

- MRI ⇒ Amplification of B

$$\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\parallel \text{ as B } \uparrow$$

- a background pressure anisotropy ($p_\perp > p_{||}$) can stabilize the MRI

$$\text{uniform plasma DR}: \;\; \omega^2 = k_\parallel^2 \left[v_A^2 + \frac{(p_\perp - p_\parallel)}{\rho} \right]$$

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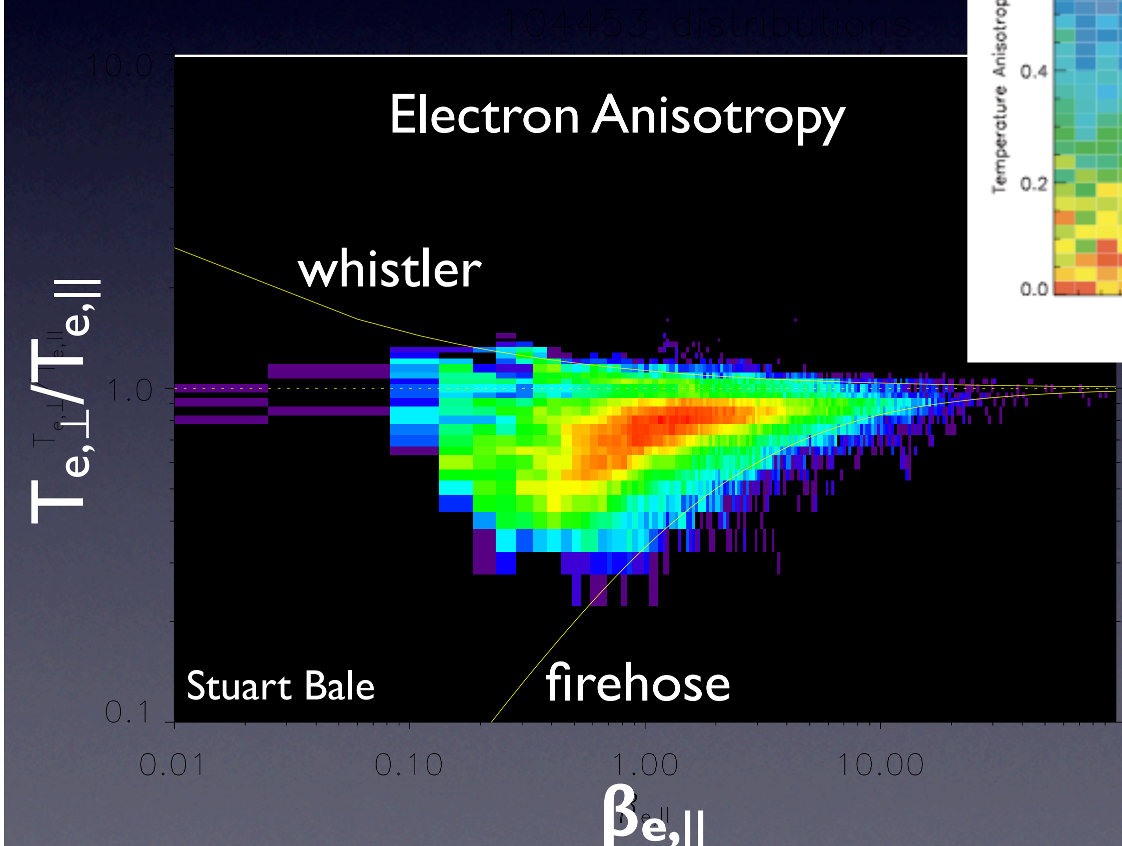
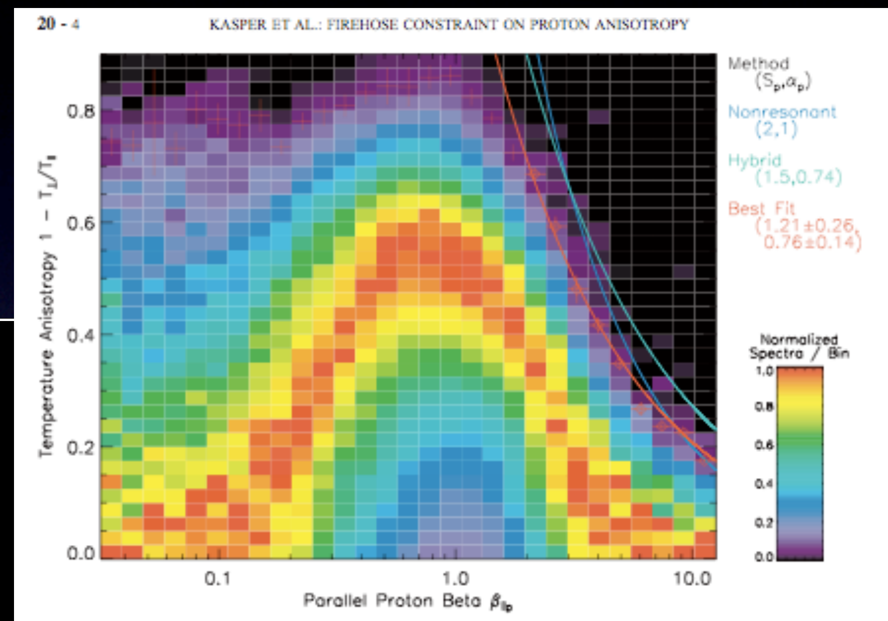
- But .... $T_\perp \neq T_{||}$ unstable to small-scale (Larmor radius) instabilities that act to isotropize the pressure tensor (velocity space instabilities)
  - e.g., mirror, firehose, ion cyclotron, electron whistler

Velocity-Space Instabilities in the Solar Wind





proton || anisotropy limited by firehose

Kasper et al. 2002

Pressure Anisotropy

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  - e.g., mirror, firehose, ion cyclotron, electron whistler
- Use subgrid model to account for this physics in Shearing Box Sims

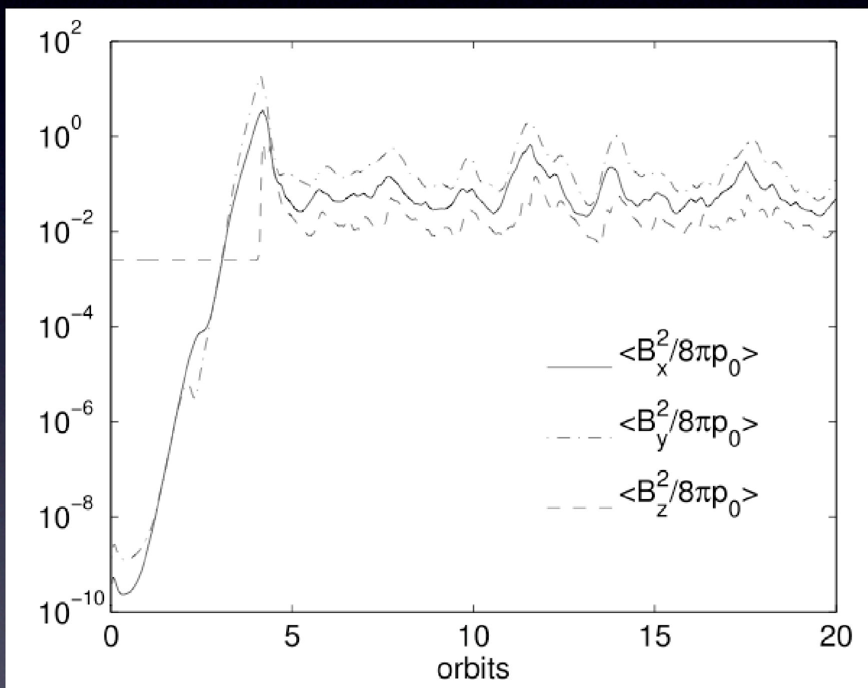
$$\frac{\partial p_\perp}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)[p_\perp - p_\parallel]$$

$$\frac{\partial p_\parallel}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)[p_\parallel - p_\perp]$$

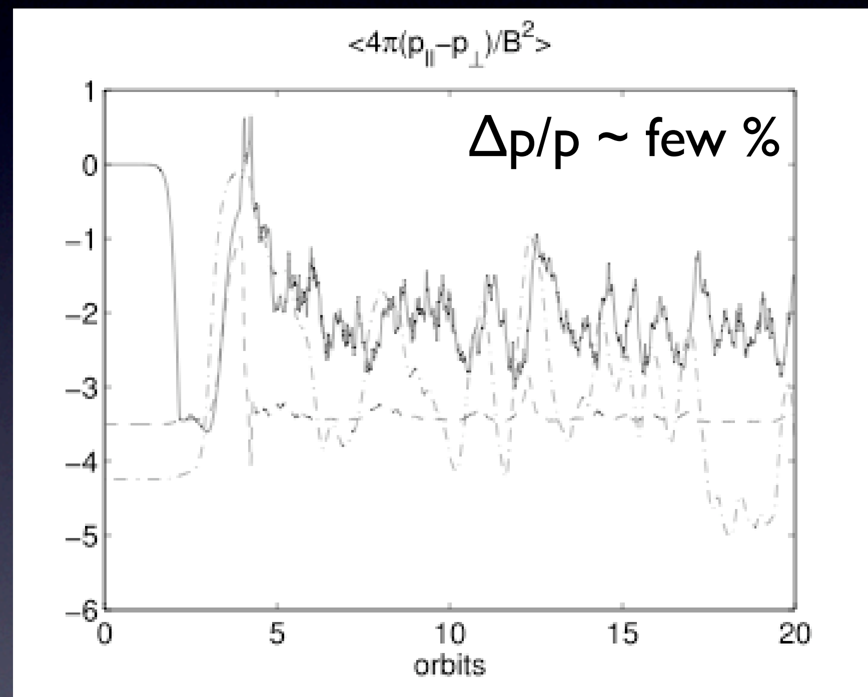
Shearing Box Sims of the Kinetic MRI

(w/ pitch angle scattering & mean B_z)

magnetic energy



volume averaged pressure anisotropy



Sharma et al. 2006

$$\text{Stress}: W_{r\phi} = \rho \delta V_r \delta V_\phi - \frac{\delta B_r \delta B_\phi}{4\pi}\left[1 - \frac{4\pi(p_{\|} - p_{\perp})}{B^2}\right]$$

Anisotropic Stress ~ Maxwell Stress

Energetics in the (Kinetic) Shearing Box

Shear

Anisotropic Pressure

Turbulence (MRI)

Direct "Viscous" Heating on Resolved Scales
(~ 1/2 of Grav. Pot. Energy)

Grid-scale Loss of Magnetic & Kinetic Energy (~ 1/2 of GPE)

Collisionless Damping of Fluctuations (small)

$$q^{+} \propto \frac{d\Omega}{d\ln r}(p_{\parallel} - p_{\perp})\frac{\delta B_r \delta B_\phi}{B^2}$$

- Heating ~ Shear*Stress $$q^+ \propto \frac{d\Omega}{d\ln r}(p_\parallel - p_\perp)\frac{\delta B_r \delta B_\phi}{B^2}$$

- Collisional Plasma
  - Coulomb Collisions set $\Delta p \rightarrow q^+ \sim m^{1/2} T^{5/2}$
  - Primarily Ion Heating

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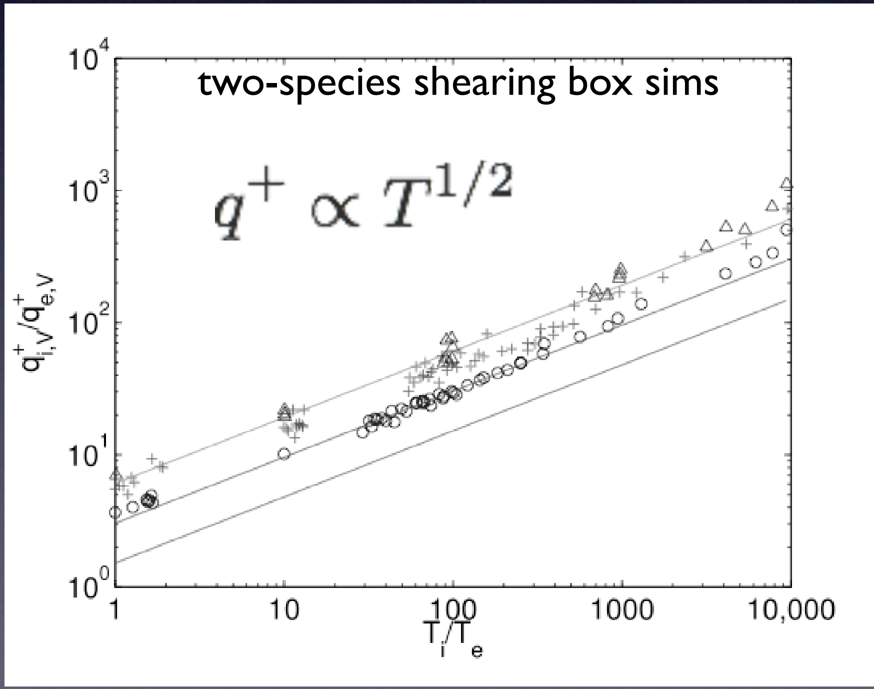
- Collisional Plasma
  - Coulomb Collisions set Δp → $q^+ \sim m^{1/2}\, T^{5/2}$
  - Primarily Ion Heating
- Collisionless Plasma
  - Microinstabilities Regulate Δp
  - *Significant Electron Heating*

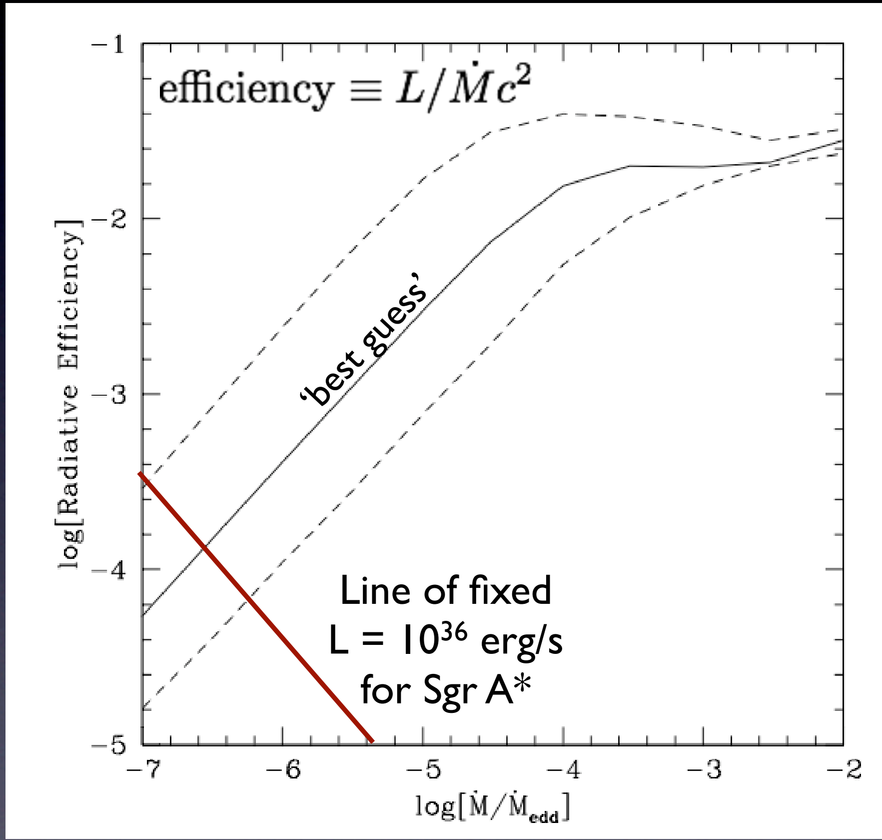


Sharma et al. 2007

Astrophysical Implications

Use $q^+ \sim T^{1/2}$ in 1D models
to determine $T_e(r)$ & radiation

Sgr A*: Predicted $\dot{M} \sim 10^{-8} M_\odot\,\mathrm{yr}^{-1}$
consistent w/ Faraday Rotation &
measured T_e from VLBI



First-principles estimate of luminosity
and efficiency for RIAFs

Summary

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