GOING BEYOND THE IDEAL APPROXIMATIONS : resistive and anisotropic Ohm law

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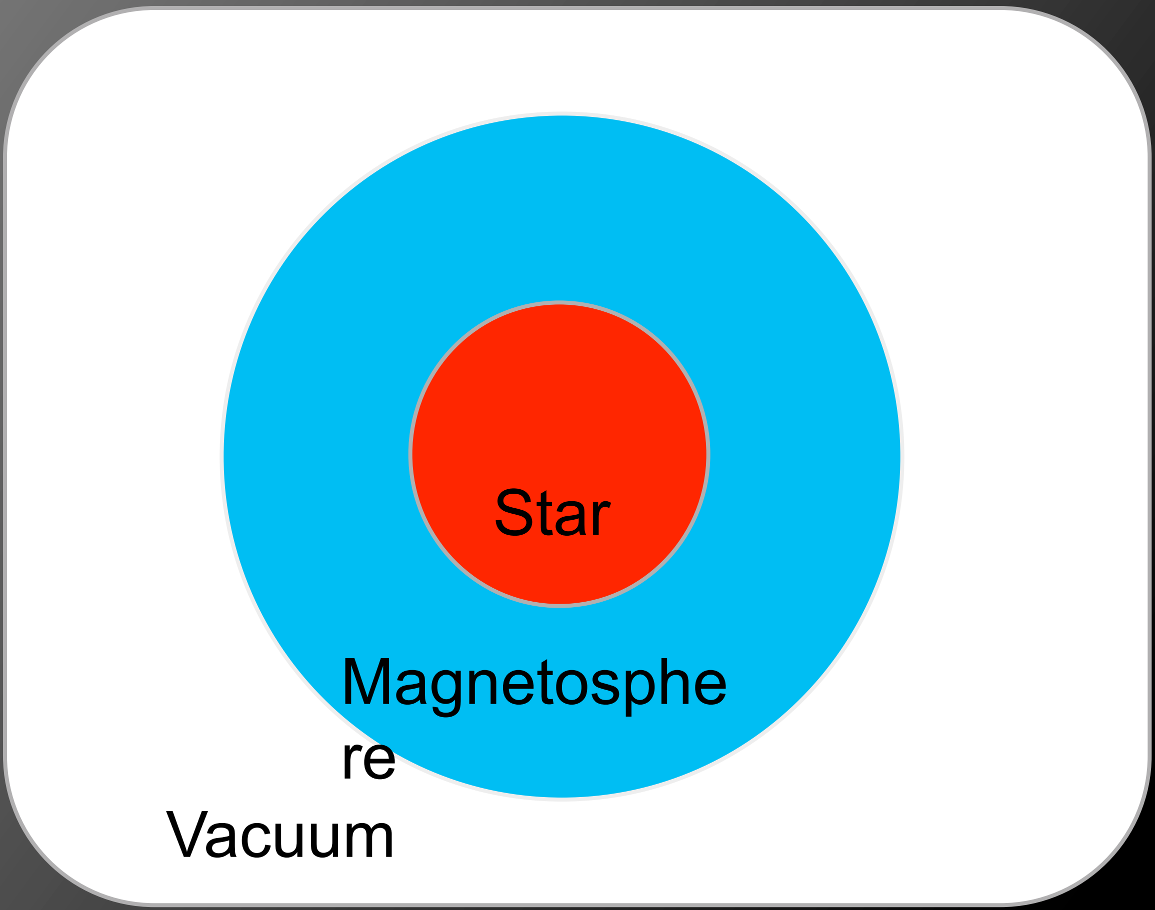
Overview

- Motivation
  - Different regimes of the Maxwell equations
  - Resistive effects and anisotropies
- The system of equations
  - The relativistic MHD equations
  - The generalized Ohm law
  - The ideal MHD and the force-free approximation
- Solving the hyperbolic-relaxation eqs.
  - Approaches to the problem
  - The IMEX Runge-Kutta methods
- Application to the resistive MHD equations
  - The inversion from conserved to primitive fields
  - Numerical tests
  - Pulsars in 3D : matching ideal MHD with vacuum

Motivation

- Different regimes of the Maxwell eqs.
- Resistive effects and anisotropies

Different regimes of the Maxwell eqs (I)



- Star or disk
  Dominated by the fluid

  IDEAL MHD

- Magnetosphere
  Dominated by the EM

  FORCE FREE

- ElectroVacuum
  no sources

  MAXWELL EQS.

Different regimes of the Maxwell eqs. (II)

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -\mathbf{J}$$
$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \cdot \mathbf{E} = q$$

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

σ : conductivity

IDEAL MHD ($\sigma \rightarrow \infty$) ⇒ $\mathbf{E} = - \mathbf{v} \times \mathbf{B}$

FORCE FREE ⇒ $q\, \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$

VACUUM ($\sigma \rightarrow 0$) ⇒ $\mathbf{J} = q = 0$

Resistive effects and anisotropies

• The ideal MHD approximation seems to describe properly many astrophysical systems (stars, disks,…), but
  - they may lead to very distorted field lines→ reconnections
  - anisotropic effects coming from the Hall term

• The force free approximation describe well the magnetospheres of NS and BHs, but
  - they may lead to current sheets → anomalous resistivity

• Is it possible to have different limits/approximations in the same physical system?

The system of equations

- The relativistic MHD equations
- Generalized Ohm law
- Ideal and force-free approximation

The relativistic MHD equations (I)

- the description of a fluid in presence of EM fields is given by:

1) Conservation of mass and total energy and momentum + EOS closure relation

Hydrodynamic equations to describe the fluid

ρ: density, u_a: 4-velocity, ε: internal energy, P: pressure

$$\nabla_a (\rho\, u^a) = 0 \quad , \quad \nabla_a T^{ab} = 0 \quad , \quad P = P(\rho,\varepsilon)$$

$$T_{ab} = [\rho(1+\varepsilon) + P] u_a u_b + P\, g_{ab} + [F_{ac} F^c{}_b - (F_{cd} F^{cd}) g_{ab}/4]$$

The relativistic MHD equations (II)

2) (Extended) Maxwell equations for the EM fields

$\nabla_a (F^{ab} + g^{ab} \Psi) = -I^b + \kappa n^b \Psi$

$\nabla_a (^*F^{ab} + g^{ab} \Phi) = \kappa n^b \Phi$

$\nabla_a I^a = 0$

F^{ab} : Maxwell tensor

I^b : current 4-vector

q : charge, J^a: 3-current

$F^{ab} = n^a E^b - n^b E^a + \varepsilon^{abc} B_c$, $I^a = n^a q + J^a$

3) The coupling between the fluid and the EM fields, which is given by the choice of current J^i .

The relativistic MHD equations (III)

- 3+1 decomposition (special relativistic)

$$
\begin{aligned}
\partial_t \psi + \nabla \cdot \boldsymbol{E} &= q - \kappa\, \psi\,, \\
\partial_t \phi + \nabla \cdot \boldsymbol{B} &= -\kappa\, \phi\,, \\
\partial_t \boldsymbol{E} - \nabla \times \boldsymbol{B} + \nabla \psi &= -\boldsymbol{J}\,, \\
\partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} + \nabla \phi &= 0\,.
\end{aligned}
$$

$$\partial_t \tau + \nabla \cdot \boldsymbol{F}_\tau = 0\,,$$

$$\partial_t \boldsymbol{S} + \nabla \cdot \mathbf{F}_{\boldsymbol{S}} = 0\,,$$

$$\partial_t q + \nabla \cdot \boldsymbol{J} = 0\,,$$

$$\partial_t D + \nabla \cdot \boldsymbol{F}_D = 0$$

$$
\begin{aligned}
\tau &\equiv \frac{1}{2}(E^2 + B^2) + h\, W^2 - p \\
\boldsymbol{S} &\equiv \boldsymbol{E} \times \boldsymbol{B} + h\, W^2\, \boldsymbol{v}\,.
\end{aligned}
$$

$$\boldsymbol{F}_\tau \equiv \boldsymbol{E} \times \boldsymbol{B} + h\, W^2\, \boldsymbol{v}\,,$$

$$\mathbf{F}_{\boldsymbol{S}} \equiv -\boldsymbol{E}\boldsymbol{E} - \boldsymbol{B}\boldsymbol{B} + hW^2 \boldsymbol{v}\boldsymbol{v} + \left[\frac{1}{2}(E^2 + B^2) + p\right] \mathbf{g}\,.$$

$D = W\rho$

$h = \rho(1+\varepsilon) + p$

$W = (1-v^2)^{-1/2}$

…But, what is J?

The generalized Ohm's law (I)

- The first charge moment of the Boltzmann equation for a two-component fluid (electrons and ions) in the Newtonian case (Goossens)

$$\vec{E} = -(\vec{v}\times\vec{B}) + \frac{\vec{j}}{\sigma} + \frac{1}{en_e}\vec{j}\times\vec{B} - \frac{1}{en_e}\nabla.\mathbf{P_e}$$
$$+\frac{m_e}{e^2 n_e}\left\{\frac{\partial\vec{j}}{\partial t} + \nabla.(\vec{\mathbf{v}}\vec{\mathbf{j}} + \vec{\mathbf{j}}\vec{\mathbf{v}} - \frac{1}{en_e}\vec{\mathbf{j}}\vec{\mathbf{j}})\right\}$$

induction, ideal MHD

Ohmic term, allows for dissipation

electron inertia, negligible

Hall term, introduces anisotropies wrt B

Battery term

The generalized Ohm's law (II)

- Keep not only the induction term, but also the Ohmic and the Hall ones. In the collision-time approximation, in full GR covariant form (Bekenstein)

$$I_a = q\, u_a + \sigma^{ab}\, e_a \qquad \sigma^{ab}=\sigma(g^{ab} + \xi^2 b^a b^b + \xi \varepsilon^{abcd}\, u_c\, b_d)$$

$$\xi = e\tau / m \quad , \quad \sigma = n_e e \xi / (1 + \xi^2\, b^2)$$

written in terms of the charge density and EM fields measured by a observer co-moving with the fluid

$$q = -I_a u^a \ , \ e_a \equiv F_{ab}\, u^b \ , \ b_a \equiv F^*_{ab}\, u^b$$

The generalized Ohm's law (III)

• Neglecting the second and third term, in 3+1 form

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -\mathbf{J} = -q\,\mathbf{v} - \sigma W\,[\mathbf{E} + \mathbf{v} \times \mathbf{B} - (\mathbf{E}\cdot\mathbf{v})\,\mathbf{v}]$$
$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

•There are two important reasons to avoid this form:

- contains electromagnetic waves (v_{max}=c) → more expensive for Newtonian fluids (but consistent limit!)
- contains strong stiff terms (large σ)→ difficult to solve with standard explicit numerical methods

The ideal MHD approximation

• The induction terms is much larger than all the others, formally recovered when $\sigma \rightarrow \infty$

J finite → $\mathbf{E} = - \mathbf{v} \times \mathbf{B}$

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

- the EM waves has been removed ($v_{max}=v_{Alfven}$)
- the evolution of E is not needed → no stiffness

The force free approximation

- From the total energy-momentum conservation and Maxwell equations

$$\nabla_a T^{ab}=0 \quad \rightarrow \quad \nabla_a T^{ab}_{(fluid)} = -\nabla_a T^{ab}_{(em)} = -F^{ab} I_a$$

- if $\rho, P << B^2$ then $\nabla_a T^{ab}_{(fluid)} << F^{ab} I_a \approx 0$

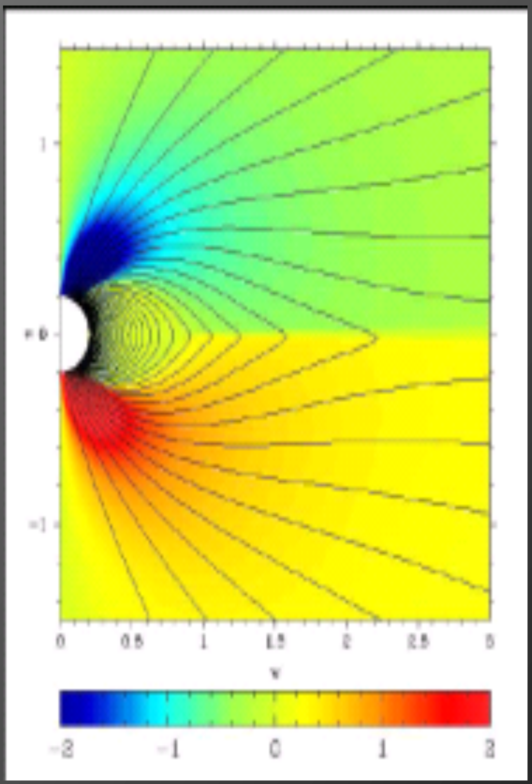
3+1 decomposition $\quad \mathbf{E}\cdot\mathbf{J} = 0 \,,\quad q\,\mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$

$\times\, \mathbf{B} \rightarrow \quad \mathbf{J} = q\, \mathbf{E}\times\mathbf{B}/B^2 + (\mathbf{J}\cdot\mathbf{B})\, \mathbf{B}/B^2$

$\cdot\; \mathbf{B} \rightarrow \quad \mathbf{E}\cdot\mathbf{B} = 0$

$\partial_t(\mathbf{E}\cdot\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E} - \mathbf{B}\cdot\mathbf{J} \qquad \partial_t(\mathbf{E}\cdot\mathbf{B})=0 \rightarrow \mathbf{B}\cdot\mathbf{J}$

Magnetospheres of NS and BHs with force-free (Komissarov, Spitkovski, Gruzinov,...)



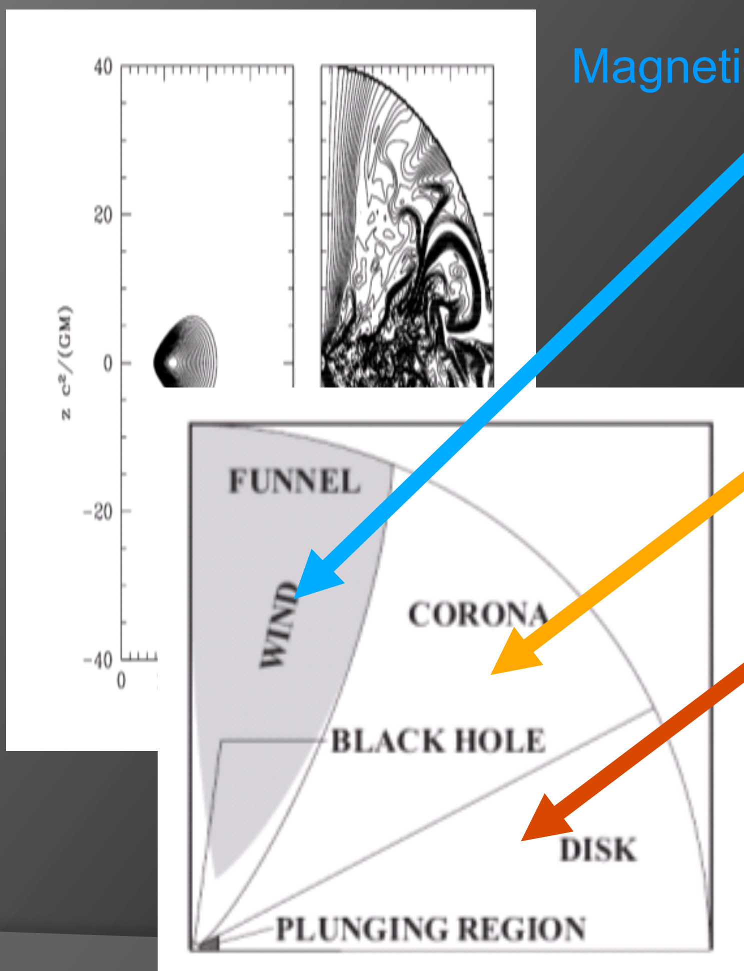
- Current sheet at the equator and instabilities when $B^2 - E^2 < 0$
$\rightarrow$ inertia effects are not neglegible
$\rightarrow$ dissipation processes restore E=B

- Let us consider $\mathbf{B} \cdot \mathbf{J} = \sigma_{//} (\mathbf{E} \cdot \mathbf{B})$, and add $\sigma_{\perp}$

$$\mathbf{J} = [\, q\, \mathbf{E} \mathrm{x} \mathbf{B} + \sigma_{//} \, (\mathbf{E} \cdot \mathbf{B})\, \mathbf{B}\,] \,/\, \mathbf{B}^2 + \sigma_{\perp} \, E_{\perp}$$

$\rightarrow$ $\partial_t(\mathbf{E} \cdot \mathbf{B}) = \ldots - \sigma_{//}(\mathbf{E},\mathbf{B})\, (\mathbf{E} \cdot \mathbf{B})$
implies $E \cdot B = 0$ when $\sigma_{//} \rightarrow \infty$
$\rightarrow$ $\sigma_{\perp} E_{\perp}$ can restore $B^2 > E^2$
$\rightarrow$ similar to generalized Ohm law

Force-free with ideal MHD BH+disk (McKinney & Gammie)



Magnetically dominated (effectively force-free)
$B^2 >> P$

$P \sim B^2$

Matter dominated
$P >> B^2$

the dependence on the Ohm law seems to diminish as $\rho, P << B^2$

Summarizing…

- A complete description of the different regions may be necessary to study magnetized fluid, but it is difficult to match solutions of different limits of the MHD equations

- The equations may lead to very distorted fields, where the limits are not valid anymore and there are significant dissipative effects inside the star or in the current sheets

- Naïve approach : evolve the full Maxwell equations with a generic current prescription in the three domains with no approximations, just changing the effective conductivity. The simplest example is to go from ideal MHD ($\sigma \rightarrow \infty$) to vacuum ($\sigma = 0$).

...Resistive MHD

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -q\, \mathbf{v} - \sigma W \left[E + \mathbf{v} \times \mathbf{B} - (\mathbf{E}\cdot\mathbf{v})\, \mathbf{v} \right]$$
$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

ε (= $1/\sigma$) : relaxation time

$\varepsilon \to 0 \rightarrow R(U) = 0$

Hyperbolic-relaxation equation (STIFF)

difficult to evolve with standard numerical methods

Solving the hyperbolic-relaxation eqs.

- Approaches to the problem
- The IMEX Runge-Kutta methods

Approaches to the problem

- SOLUTION 1 : let us consider a simple case discretized with an explicit scheme

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \iff \partial_t u = a\, \partial_x u - u / \varepsilon$$

$$(a=0) : \; u^{n+1} - u^n = -\Delta t\, u^n / \varepsilon \rightarrow \; u^{n+1} = u^n \,(1 - \Delta t/\varepsilon)$$

amplification factor $C^n = | u^{n+1}/u^n | < 1$ for stability

- CFL stability condition: $\Delta t < \Delta x / a$
- Stiff stability condition with explicit method: $\Delta t < 2\varepsilon$

if $\Delta t \sim \varepsilon = 1/\sigma \sim 10^{-6}$ → computationally VERY expensive

Approaches to the problem

- SOLUTION 2 : solving the full equation implicitly

- Let us consider an implicit method

$$(a=0): \quad u^{n+1} - u^n = -\Delta t\, u^{n+1} / \varepsilon \;\rightarrow\; u^{n+1} = u^n / (\,1 + \Delta t/\varepsilon\,)$$

- Stiff stability condition with implicit method: $\Delta t > 0$
- But… it is expensive/complicated with non-vanishing F(U) containing partial derivatives

Approaches to the problem

- SOLUTION 3 : the equilibrium system
  - expand the solution around $\varepsilon \to 0$

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \quad \Leftrightarrow \quad \partial_{tt} B - \Delta \mathbf{B} = [-\partial_t \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})] / \varepsilon$$

$$\mathbf{U} = \mathbf{U}_0 + \varepsilon\, \mathbf{U}_1 + O(\varepsilon^2) \qquad \mathbf{B} = \mathbf{B}_0 + \varepsilon\, \mathbf{B}_1 + O(\varepsilon^2)$$

$\Downarrow$

$O(\varepsilon^0)$: IDEAL MHD $\qquad \partial_t \mathbf{B_0} - \nabla \times (\mathbf{v} \times \mathbf{B_0}) = 0$

$O(\varepsilon^1)$: $\qquad \partial_t \mathbf{B_1} - \nabla \times (\mathbf{v} \times \mathbf{B_1}) = -(\partial_{tt} \mathbf{B}_0 - \Delta \mathbf{B}_0)$

- hierarchy of solutions : compute $\mathbf{B}_0$, then $\mathbf{B}_1$,… but it is only valid close to $\varepsilon \to 0$

Approaches to the problem

- SOLUTION 4 : Strang Splitting

$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$ ⇔ $\partial_t \mathbf{U} = S(\Delta t/2) ¤ T(\Delta t) ¤ S(\Delta t/2)\, \mathbf{U}$

U^* : $U^* = U^n + (\Delta t/2)\, R(U^n) /\varepsilon$

U^{**} : $U^{**} = U^* + \Delta t\, F(U^*)$

U^{n+1}: $U^{n+1} = U^{**} + (\Delta t/2)\, R(U^{**})/\varepsilon$

- The source step can be solved exactly with the analytical solution (Komissarov 2007)… but it does not work for general Ohm law and have problems with strong stiff terms in the presence of shocks

Approaches to the problem

- SOLUTION 5 : discontinuous Galerkin methods

- There are high order schemes (3-5th order) which can deal with the stiff terms (Dumbser & Zanotti 2009)… but they are complicated and expensive

The IMEX Runge Kutta methods

- treat implicitly the stiff part and explicitly the non-stiff
  IMplicit-EXplicit methods (Pareschi & Russo 05)

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

$$U^{(i)} = U^n + \Delta t \, \Sigma \, \underline{a}_{ij} \, F(U^{(j)}) + \Delta t \, \Sigma \, a_{ij} \, R(U^{(j)}) / \varepsilon$$

$$U^{n+1} = U^n + \Delta t \, \Sigma \, \underline{\omega}_i \, F(U^{(i)}) + \Delta t \, \Sigma \, \omega_i \, R(U^{(i)}) / \varepsilon$$

$$\begin{array}{c|cccc} \underline{c}_1 & 0 & 0 & 0 & \cdots \\ \underline{c}_2 & \underline{a}_{12} & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \underline{c}_n & \underline{a}_{1n} & \underline{a}_{2n} & \cdots & 0 \\ \hline & \underline{\omega}_1 & \underline{\omega}_2 & \cdots & \underline{\omega}_n \end{array}$$

Explicit RK

DIRK

$$\begin{array}{c|cccc} c_1 & a_{11} & 0 & 0 & \cdots \\ c_2 & a_{12} & a_{22} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c_n & a_{1n} & a_{2n} & \cdots & a_{nn} \\ \hline & \omega_1 & \omega_2 & \cdots & \omega_n \end{array}$$

Butcher Tableau

The IMEX Runge Kutta methods

- Let us consider a simple IMEX RK as an example

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon$$

IMEX-Midpoint(1,2,2)

0	0	0		0	0	0
1/2	1/2	0		1/2	0	1/2
	0	1			0	1

$$\mathbf{U}^1 = \mathbf{U}^n$$

$$\mathbf{U}^2 = \mathbf{U}^n + \Delta t\, F(\mathbf{U}^1)/2 + \Delta t\, R(\mathbf{U}^2)/(2\,\varepsilon)$$

$$U^{n+1} = U^n + \Delta t\, F(U^2) + \Delta t\, R(U^2) / \varepsilon$$

- only the stiff part has to be inverted
- high order convergence in time (usually 3 order)
- strong theoretical background (it has to work!)

Application to the Maxwell eqs.

- Inverting explicitly the stiff part
- Numerical tests
- Pulsars in 3D: matching ideal MHD and vacuum

(CP,Lehner,Reula,Rezzolla 09)

Inverting explicitly the stiff part (I)

- only the evolution of the electric field has stiff terms

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -q\,\mathbf{v} - \sigma W [\,\mathbf{v} \times \mathbf{B} + \mathbf{E} - (\mathbf{E}\cdot\mathbf{v})\,\mathbf{v}]$$

- use standard TVD explicit RK scheme for the other fields and apply the IMEX only to E

$$\partial_t \mathbf{U} = F(\mathbf{U}) + R(\mathbf{U}) / \varepsilon \longrightarrow \begin{array}{l} F(\mathbf{E}) = \nabla \times \mathbf{B} - q\,\mathbf{v} \\ R(\mathbf{E}) = -W[\mathbf{v} \times \mathbf{B} + \mathbf{E} - (\mathbf{E}\cdot\mathbf{v})\,\mathbf{v}] \\ \varepsilon = 1/\sigma \end{array}$$

Inverting explicitly the stiff part (II)

Example:

$$\mathbf{U}^1 = \mathbf{U}^n$$

$$\mathbf{U}^2 = \mathbf{U}^n + \Delta t\, F(\mathbf{U}^1)\,/2 + \Delta t\, R(\mathbf{U}^2)\,/(2\,\varepsilon)$$

$$U^{n+1} = U^n + \Delta t\, F(U^2) + \Delta t\, R(U^2)\,/\,\varepsilon$$

- compute the explicit part, partial evolution for **E**

$$\mathbf{E}^* = \mathbf{E}^n + \Delta t\, F(\mathbf{E}^1)\,/2$$

- invert explicitly the implicit part, since $R(E) = \mathcal{A}\, E$

$$\mathbf{E}^2 = M(\mathbf{v}, \mathbf{B})\,[\,\mathbf{E}^* - \Delta t\, W\,(\mathbf{v} \times \mathbf{B})\,/\,(2\,\varepsilon)\,]$$

- compute $F(E^2)$ and $R(E^2)$ to update E^{n+1}

Inverting explicitly the stiff part (III)

- the conserved variables (D,τ,S^i,E^i,B^i,q) are evolved by using HRSC methods for conservation laws
- the primitive variables $(\rho,\varepsilon,P,v^i,E^i,B^i,q)$ are needed to compute the rhs of the evolution equations

  * with the IMEX, only the explicit part of E^i is evolved
  * the implicit part can be solved explicitly, but depends on the unknown velocity

-The transformation from conserved to primitive variables is non-linear and has to be solved numerically in general
  * with the IMEX $E^i=f(..,v^i)$ so the implicit evolution and the inversion from conserved to primitive has to be done at the same time (4-dimensional system)

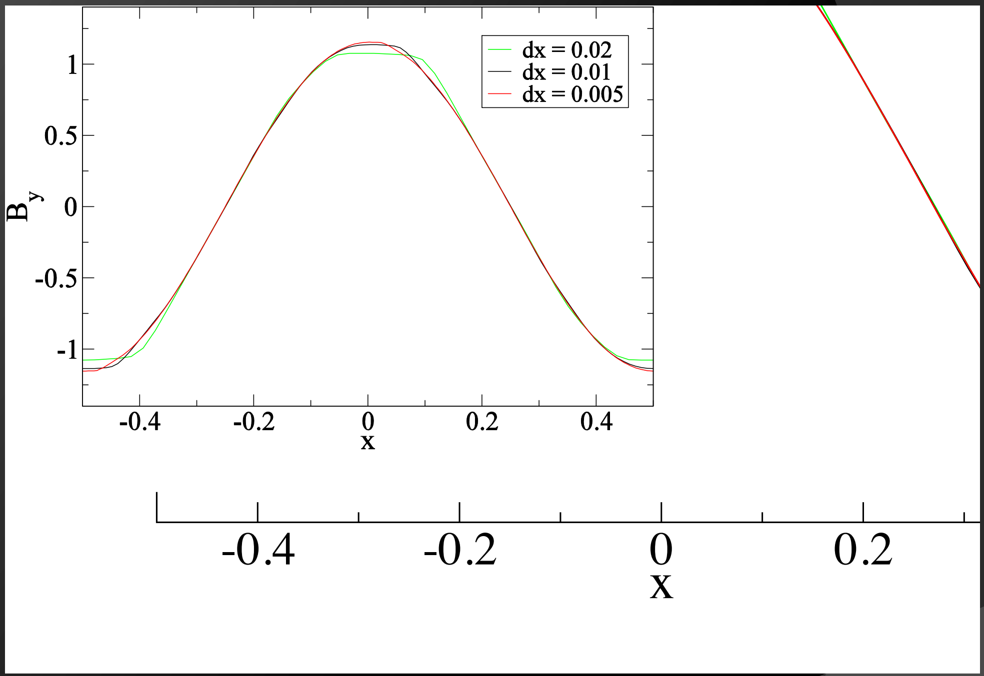
Test 1: Alfven wave (del Zanna 2007)

- Testing the high conductivity limit (ideal MHD)

$B_y = B_o \cos(x - v_A t)$
$B_z = B_o \sin(x - v_A t)$
$v_y = -v_A B_y/B_o$
$v_z = -v_A B_z/B_o$

Alfven speed v_A

$P = \rho = 1$, $v_A = 1/2$
conductivity $\sigma = 10^6$



Solution after one period
(periodic boundary conditions)

Test 2: current sheet (Komissarov 2007)

- Testing the low conductivity limit

P=cte, ρ=cte

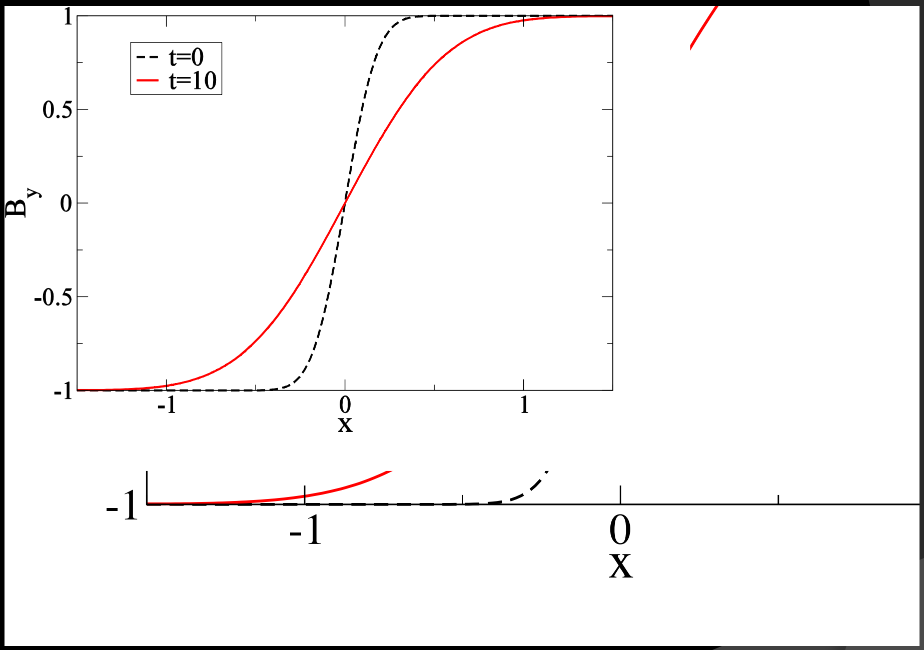
E = v = 0

B= $(0, B_y(x,t), 0)$

$\partial_t B_y - (1/\sigma)\, \partial_{xx} B_y = 0$

$B_y = B_o \, \mathrm{erf}[(\sigma/(4\,\xi))^{1/2}]$

with $\xi = t/x^2$



Solution at t=10 with σ=100

Test 3: shock tube problem

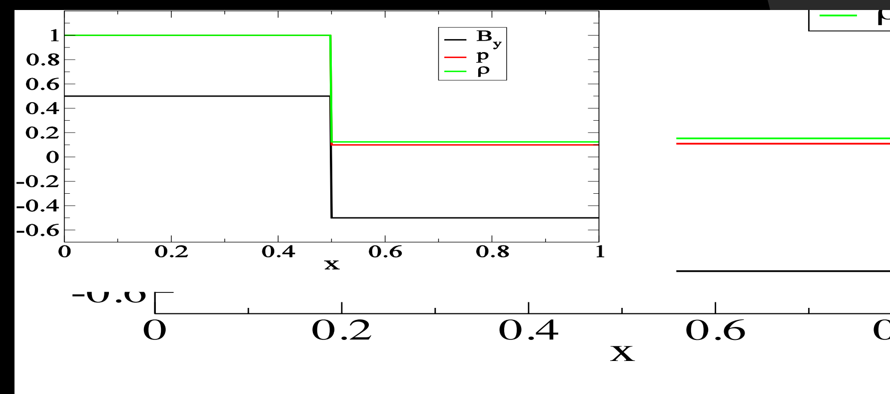
- Testing the resistive MHD with shocks

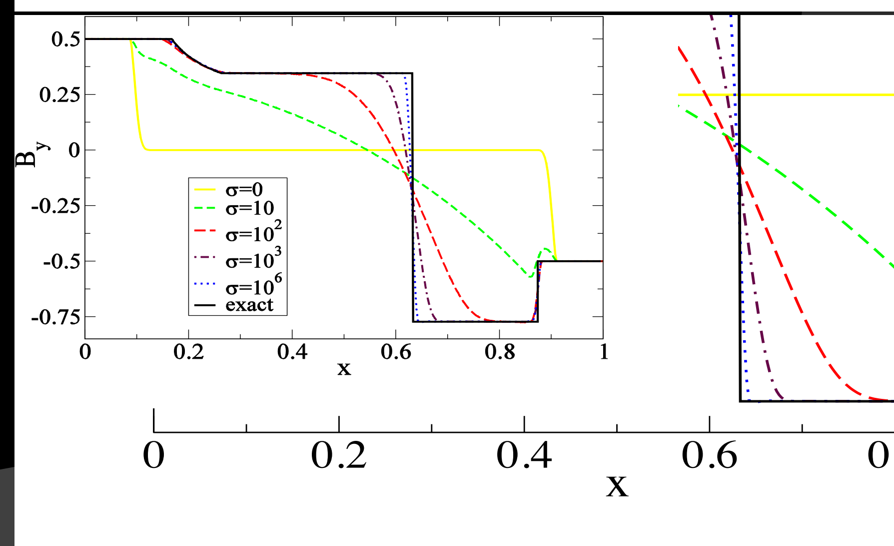
Left state

$(\rho^L, p^L, B_y^L) = (1, 1, 1/2)$

Right state

$(\rho^R, p^R, B_y^R) = (1/8, 0.1, -1/2)$





Solution at t=0.4

Test 4: cylindrical explosion

- Testing the resistive MHD with shocks in 2D

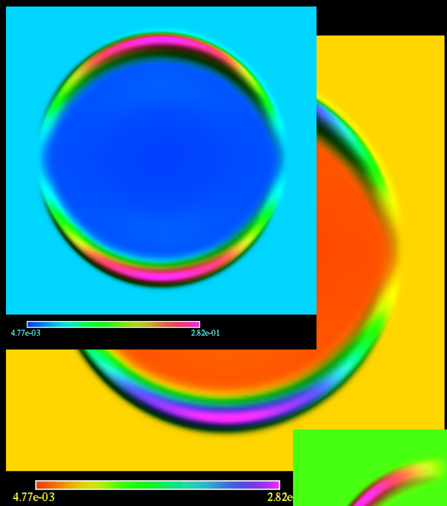
$r<0.8 \quad p=1,\ \rho = 0.01$

$r>1.0 \quad p= \rho = 0.001$

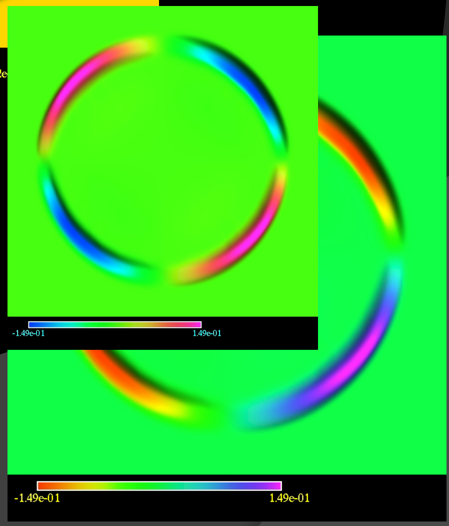
$B = (0.05, 0, 0)$

$E = q = 0$

Solution at t=4



B_x

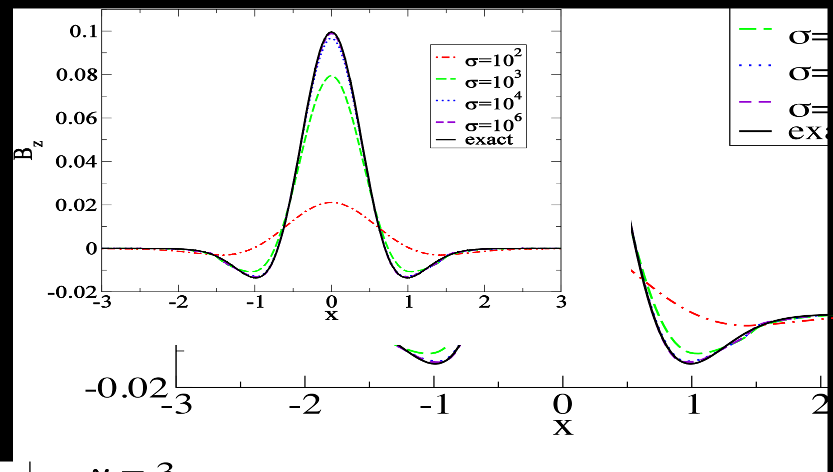


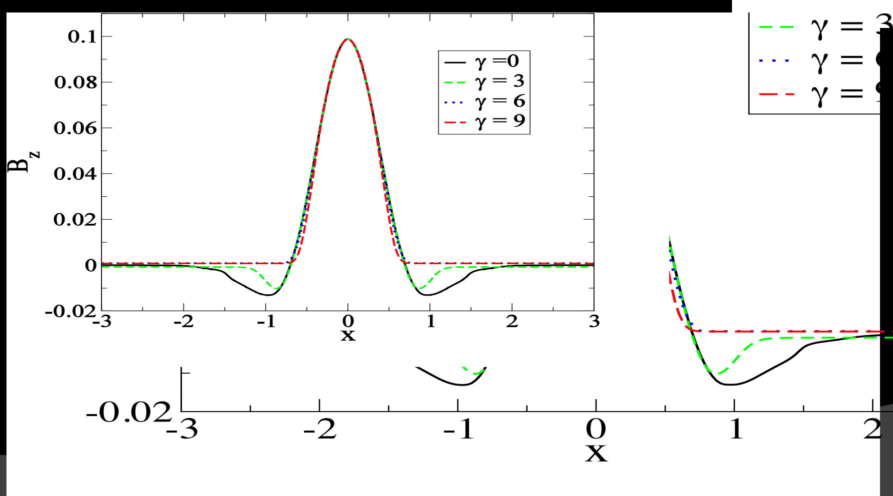
B_y

Test 5: cylindrical star

- Testing the resistive MHD in toy model stars

$\rho = \rho_0 \exp[-(r/r_o)^2]$
$v_\varphi = \rho\, \Omega$
$B_z = 2\, B_o[1 - (r/r_o)^2]$
E,q from ideal MHD





$\sigma = \sigma_0\, D^\gamma$ D=ρW

Neutron Stars in 3D (I)

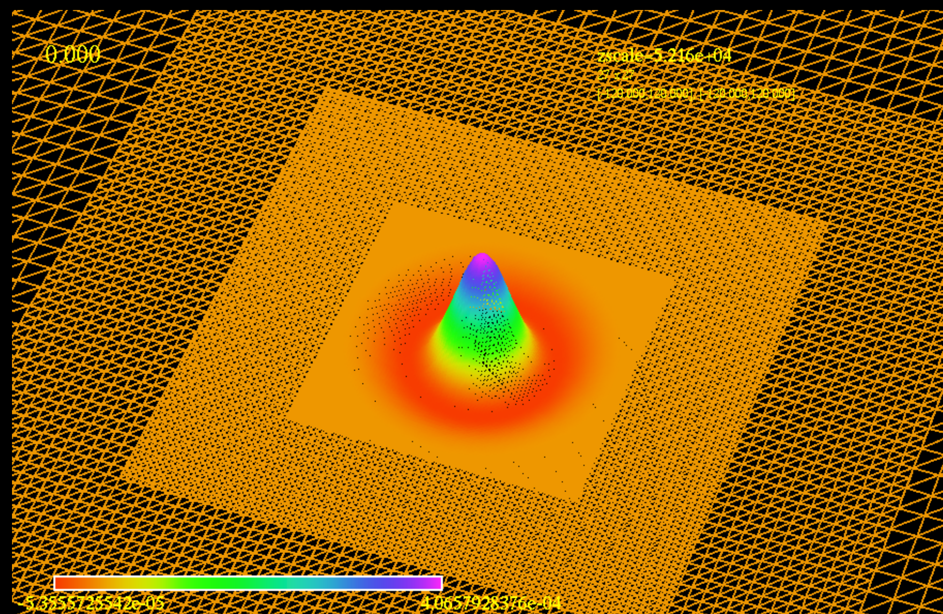
-Very compact objects → needs General Relativity

$$
\begin{aligned}
\partial_t(\sqrt{\gamma}\,B^i) + \partial_k[-\beta^k\sqrt{\gamma}\,B^i + \alpha\epsilon^{ikj}\sqrt{\gamma}\,E_j\,] &= \\
&-\sqrt{\gamma}\,B^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\,\gamma^{ij}\partial_j\phi \\
\partial_t(\sqrt{\gamma}\,E^i) + \partial_k[-\beta^k\sqrt{\gamma}\,E^i - \alpha\epsilon^{ikj}\sqrt{\gamma}\,B_j\,] &= \\
&-\sqrt{\gamma}\,E^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\,\gamma^{ij}\partial_j\Psi - 4\pi\alpha\sqrt{\gamma}\,J^i \\
\partial_t\phi + \partial_k[-\beta^k\phi + \alpha B^k\,] &= \\
&-\phi\,(\partial_k\beta^k) + B^k(\partial_k\alpha) - \alpha\Gamma^i_{ki}B^k - \alpha\kappa\phi \\
\partial_t\Psi + \partial_k[-\beta^k\Psi + \alpha E^k\,] &= \\
&-\Psi\,(\partial_k\beta^k) + E^k(\partial_k\alpha) - \alpha\Gamma^i_{ki}E^k + 4\pi\alpha q - \alpha\kappa\Psi \\
\partial_t(\sqrt{\gamma}\,q) + \partial_k[-\beta^k\sqrt{\gamma}\,q + \alpha\sqrt{\gamma}\,J^k\,] &= 0 \\
\partial_t(\sqrt{\gamma}\,D) + \partial_k[\sqrt{\gamma}\,D\,(\alpha v^k - \beta^k)\,] &= 0 \\
\partial_t(\sqrt{\gamma}\,\tau) + \partial_k[\sqrt{\gamma}\,(\alpha S^k - \beta^k\tau)\,] &= \sqrt{\gamma}\,[\alpha S^{ij}K_{ij} - S^j\partial_j\alpha\,] \\
\partial_t(\sqrt{\gamma}\,S_i) + \partial_k[\sqrt{\gamma}\,(\alpha S^k{}_i - \beta^k S_i)\,] &= \sqrt{\gamma}\,[\alpha\Gamma^j_{ik}S^k{}_j + S_j\,\partial_i\beta^j - \tau\partial_i\alpha\,]
\end{aligned}
$$

Neutron stars in 3D (II)

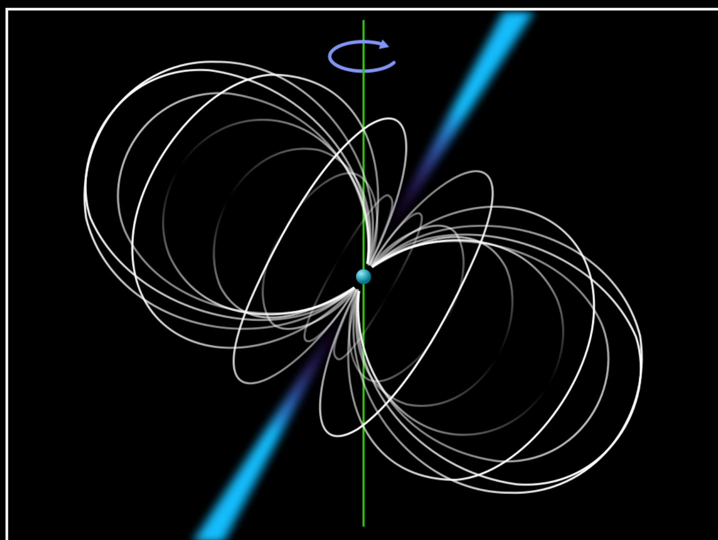
- IMEX scheme implemented in the had infrastructure (Lehner talk), which provides parallelization & AMR

- minimal changes
- fixed background, easily full GR
- HLLE flux formulae
- PPM reconstruction
- ideal gas EOS, being generalized



Neutron stars in 3D (III)

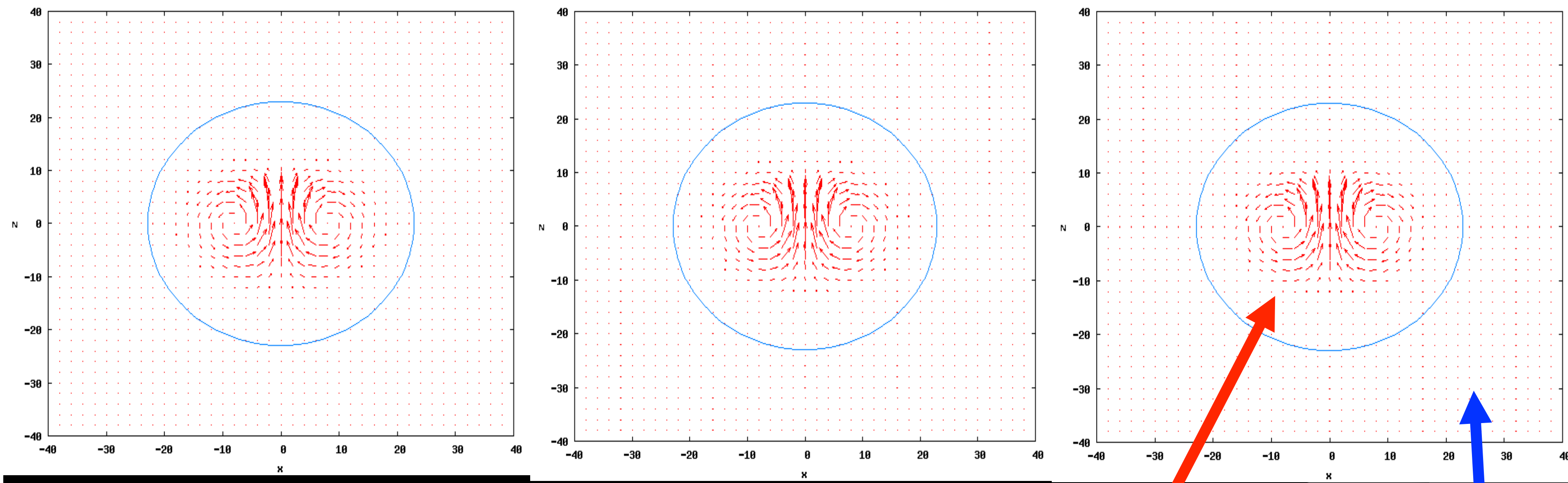
•Rotating neutron star with a poloidal magnetic field



-Full 3D simulations!!
(no symmetries)
- Aligned/disaligned cases
- Ideal MHD at the star
- Vacuum at the magnetosphere

Neutron stars in 3D (IV)

- magnetic moment aligned with spin

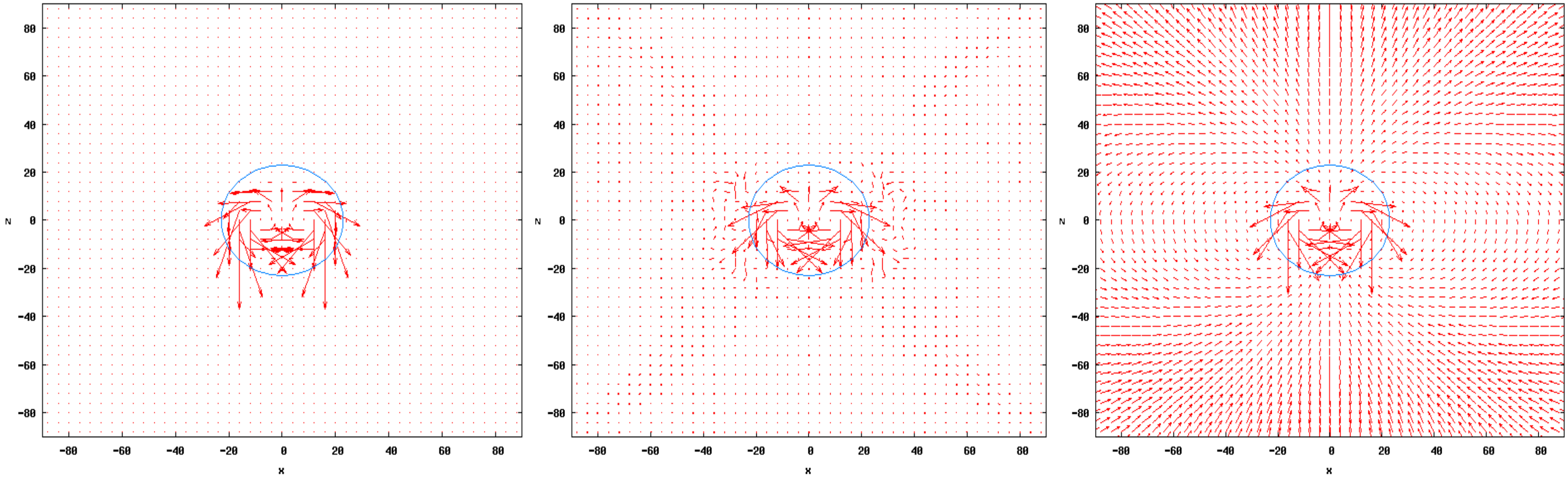


$t=0$ after 2 periods $\sigma = \sigma_0 = 10^6$ $\sigma = \sigma_0\, \rho^2$

$\sigma = 10^6$ $\sigma \approx 0$

Neutron Stars in 3D (V)

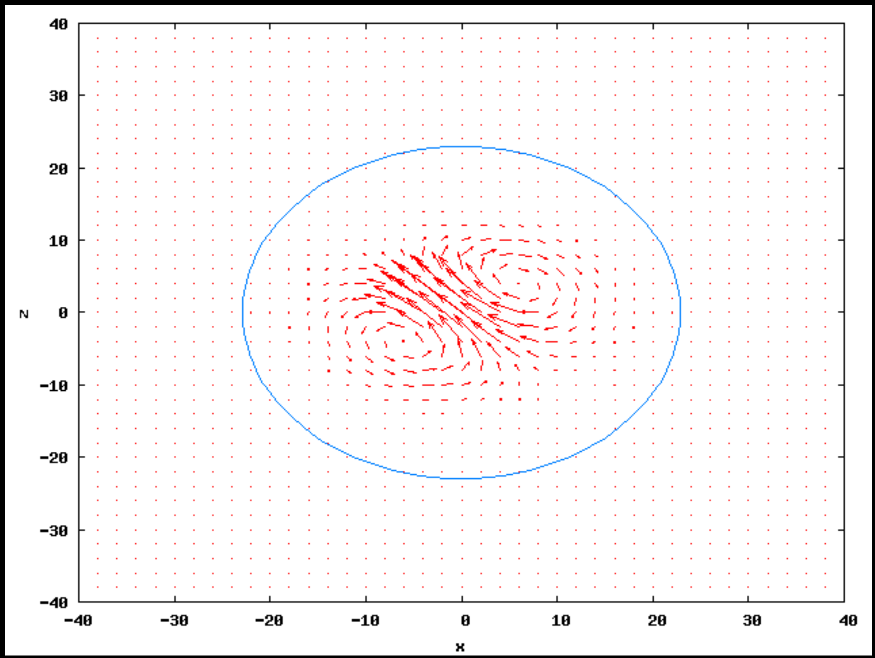
- plot $r^2 B$ to show the outer region

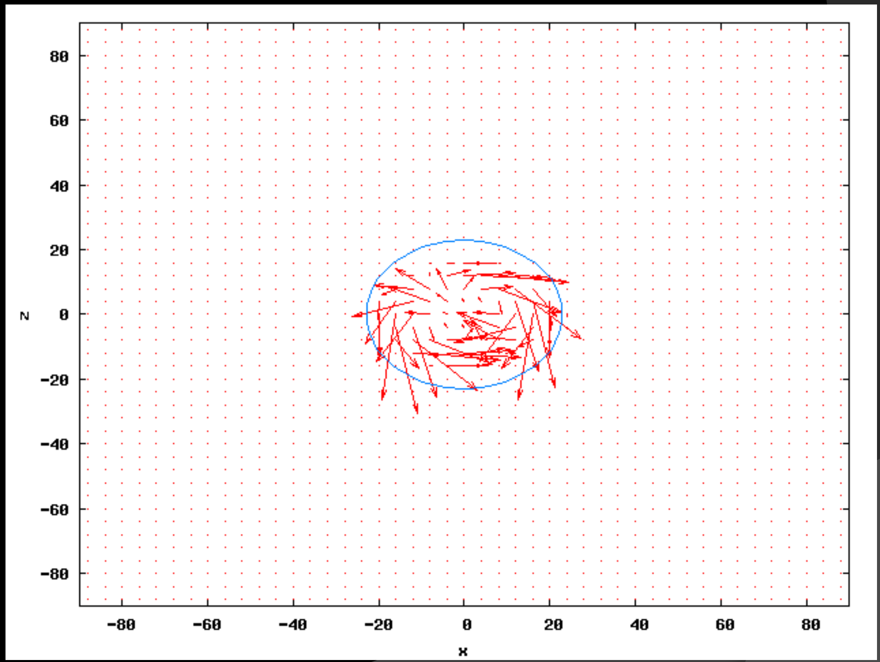


t =0 after 2 periods $\sigma = \sigma_0 = 10^6$ $\sigma = \sigma_0 \rho^2$

Neutron Stars in 3D (VI)

- Magnetic moment misaligned 45^o wrt spin



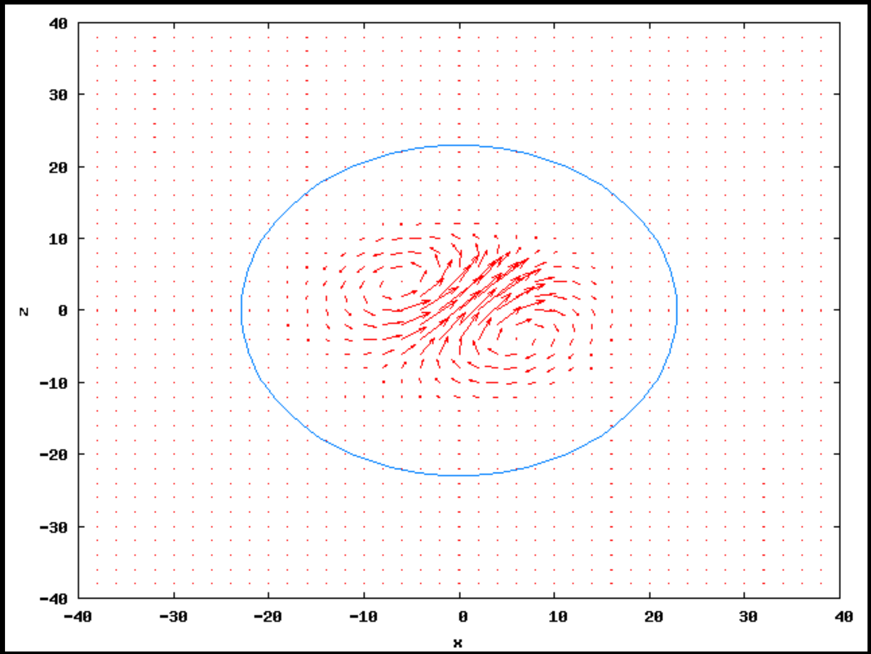


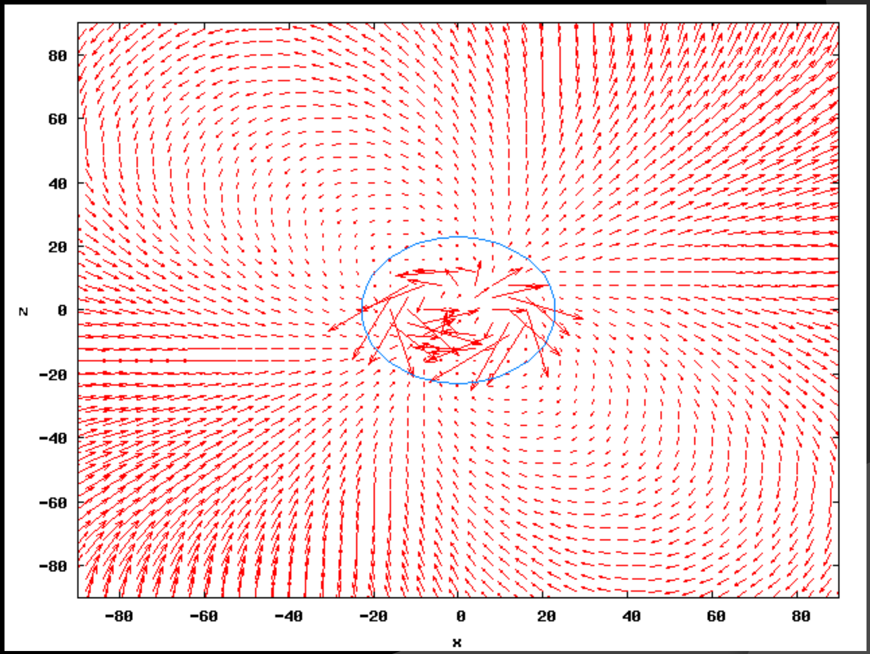
t=0 B r^2 B

Neutron star in 3D (VII)

- Magnetic moment misaligned 45° wrt spin

$$\sigma = \sigma_0 \rho^2$$





t=1.5P B

r^2 B

Summary

- the IMEX Runge-Kutta allows to solve easily hyperbolic-relaxation eqs. where the stiff terms have no partial derivatives
- in particular, the resistive-anisotropic MHD equations in different regimes
  - modify only on the RK (add DIRK) [simple!]
  - add extra-memory only for E [cheap!]
  - change your con2prim/solve implicit eq. via Newton-Raphson [straight!]
- the limit of ideal MHD and electrovacuum can be recovered easily, force free on the way: preliminary studies of a pulsar surrounded by electrovacuum