Relativistic HD and MHD models for AGN jet propagation and deceleration

Rony Keppens

Centre for Plasma Astrophysics, K.U.Leuven
FOM-Institute for Plasma Physics Rijnhuizen
Astronomical Institute, Utrecht University

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In collaboration with:

- Z. Meliani
Outline

• Introduction:
  ⇒ SRMHD model and AMRVAC software
  ⇒ Fanaroff-Riley classification and HYMORS
  ⇒ contemporary challenges in AGN jet modeling

• Relativistic (M)HD simulations
  ⇒ relativistic HD jet simulations and HYMORS
  ⇒ relativistic (M)HD two-component jet simulations
  ⇒ helically magnetized, relativistic jets

• Outlook
Special Relativity and MHD

- **special relativistic magnetofluids** → flat Minkowski space-time
  ⇒ particle, tensorial energy-momentum conservation, full Maxwell
- **ideal magnetohydrodynamic**: vanishing electric field in comoving frame

\[ E = -v \times B \]

⇒ fix Lorentz frame, use 1 + 3 split (time/space), obtain

\[ \partial_t U + \partial_i F^i = 0 \]

⇒ conserved variables \( U = (D, S, \tau, B) \)

- \( D = \) rest mass density \( \rho \times \Gamma \)
  ⇒ lab number density \( \Gamma n_0 \): volume change by length contraction
  ⇒ conserved variables \( U = (D, S_{tot}, \tau, B) \), primitives \( (\rho, v, p, B) \)
MHD waves

- 7 wavespeeds \textit{entropy}, \pm slow, \pm Alfvén, \pm fast [anisotropic!]
- MHD waves in uniform medium
Relativistic MHD waves I

- in MHD: anisotropic wave behavior in rest frame
  ⇒ phase & group (Friedrich) diagrams for slow, Alfvén, fast

⇒ horizontal $B$, uniform plasma
⇒ $\delta$-perturbation yields group diagram, also Huygens construction
⇒ **Alfvén waves**: circles in phase diagrams, pointlike in Friedrich
Relativistic MHD waves II

- draw phase diagram when source moves at

\[ \mathbf{v} = 0.9 \left[ \sin(\pi/4)\mathbf{e}_x + \cos(\pi/4)\mathbf{e}_z \right] \]

- group speed diagram then fully 3D objects, no more symmetry

⇒ use Huygens construction: slow and fast wave fronts
Relativistic MHD waves III

- when speed $v = 0.9c e_z$ aligned with $B$, still up-down symmetry
  $\Rightarrow$ from Lorentz transform get group diagram

- see *Physics of Plasmas* 15, 102103, 2008
  $\Rightarrow$ knowledge of exact group diagrams: stringest code test!
Adaptive Mesh Refinement & AMRVAC

- extreme contrasts, positive $p, \rho, \tau, v < 1, \Gamma \geq 1$, solenoidal $B$
  $\Rightarrow$ stringent demands on numerics and accuracy: AMR vital

- Special relativistic HD and MHD: ‘modules’ in AMRVAC
  $\Rightarrow$ advection, hydro, MHD, relativistic (M)HD modules
  $\Rightarrow$ different EOS implemented for relativistic modules
  $\Rightarrow$ any-D, explicit grid adaptive framework
  $\Rightarrow$ full MPI octree variant, cartesian/cylindrical/spherical

- shock-capturing schemes (TVDLF/HLL/HLLC/Roe), reconstructions (linear/PPM)
RMHD Orszag-Tang test

- relativistic analogue of 2D MHD Orszag-Tang test
  - double periodic, supersonic relativistic vortex rotation
  - initial field configuration: double island structure

⇒ current sheets form, shock interactions, reconnections
AMR vital: captures small-scale reconnection effects

\[ \text{time} = 6.4 / 0906 \]

\[ \text{min} = 1.000000, \text{max} = 1.704703 \]
• Equation of state in relativistic numerical simulations
  ⇒ mostly assumed constant polytropic index $\gamma$
  ⇒ specific internal energy $e_{th} = p/(\gamma - 1)\rho$

• Relativistically correct ideal gas: effective $\hat{\gamma}(T)$
  ⇒ compare Synge with convenient proxy (no Bessel functions)
MPI-AMRVAC and HPC-Europa2

- excellent scaling for domain decomposition and full multi-level AMR
  \[ \Rightarrow \text{2D MHD at } \approx 400^2, \ 1000 \ \Delta t \ \text{in less than 5 seconds (includes IO)} \]
  \[ \Rightarrow \text{10 level AMR for RHD with sustained 80\% efficiency on 2000 CPUs!} \]
Fanaroff-Riley 1974 classification

- correlation **radio luminosity** - positions high-low surface brightness

  ⇒ Class I – Class II transition: at well-defined $L_{178\text{Mhz}}$

- FR I: brightest near core, jets in 80 %, **relativistic at parsec scale while diffuse and subrelativistic at kpc**

- FR II: emission in **lobes and hot spots**; narrow, highly relativistic jet

- relation radio appearance - **IGM energy transport/deposition**
HYMORS

- Gopal-Krishna & Wiita 2002: Hybrid Morphology Radio Sources
  - FR I appearance on one side, FR II characteristics
  - FR I/II classification relates to ambient medium differences

⇒ FR II lobe with hotspot to SE, diffuse jet to NW (FR I)
⇒ source yields ‘identical’ launch conditions at each side
AGN jet challenges

- How to decelerate highly energetic (especially FR II) flows?
  ⇒ HYMORS suggest external medium influence, study with axisymmetric HD jet models
- Jet launch models and observations point out transverse stratification
  ⇒ source region controls inner/outer jet properties
  ⇒ FR II/FR I transition: liability to relativistic Rayleigh-Taylor mode
  ⇒ relativistic 2.5D and 3D (M)HD simulations
- Magnetization of jet flows
  ⇒ role of helical B in collimation/propagation
Model parameters

- jet kinetic energy & Lorentz factor $\Gamma$ (order 10-20)
- ratio between jet/IGM inertia (density contrast)
- opening angle: cylindrical/conical models

- external medium stratification: include **density discontinuities**
  $\Rightarrow$ inevitable boundaries separating differing regions of influence
- **special relativistic (magneto)hydro** equations
• Relativistically correct ideal gas: effective $\gamma(T)$
  ⇒ varying polytropic index: affects compression rate, shock strength
• Application: **AGN jets encountering density discontinuity**
  ⇒ simulate jet propagation through layered media
  ⇒ $\Gamma \approx 20$ beam Lorentz factor, $L_{\text{Jet, Kin}} \sim 10^{46}\text{ergs/s}$.
• lower region: lighter medium $\rho_{\text{Low}}/\rho_{\text{b}} = 0.1496$
  ⇒ after 160 light crossings **zoom on jet head before jump encounter**
• preformed jet head: ultrarelativistic state in shocked, swept-up ISM
  ⇒ effect on dynamics as it penetrates denser region
• explored differences between **low-high energy jets**: \(10^{43}\) or \(10^{46}\) ergs/s

⇒ jet beam kinetic luminosity

\[
L_{\text{jet},\text{Kin}} = (\Gamma_b h_b - 1) \rho_b \Gamma_b \pi R_b^2 v_b
\]

⇒ 10 model computations, **varying** \(\Gamma_b = 10 - 20\) and \(\theta = 0 - 1\)

⇒ always CD, with/out \(\rho\) variation, Case II \(10^{46}\) ergs/s at \(t = 900\)

⇒ **FR II jet at first, then dramatic slowdown with FR I appearance**
• high energy jets: need significant contrast to induce FR I transition

\[ \Gamma = 10, \text{ density jump 10-1000}, \text{ IGM stratification effects} \]
• overall findings on jet deceleration
  ⇒ FR II-FR I transition feasible at large density contrast
  ⇒ FR I changeover: relativistic at pc to subrelativistic at kpc
• FR I low energy jets: **Richtmeyer-Meshkov instability** as shock passes CD

⇒ all *high-resolution, grid-adaptive computations, effective resolutions of* $3000 \times 5000$, *4 to 6 refinement levels*

⇒ typical execution times: *4 days on 64 processors*
Internal stratification effects and jet deceleration

- AGN jets **radial stratification: fast inner, slow outer jet**
  - different launch mechanism $\rightarrow$ different rotation
- outer ‘disk’ jet launched magnetocentrifugally
  - Magnetized Accretion-Ejection Structure (MAES)

- generic mechanism for jet launch
  - magnetic torque brakes disk matter azimuthally
  - magnetic torque spins up jet matter
  - mass source for jet: disk
  - $\mathbf{B}$ collimates, accelerates
  - **Jet formation animation & Escaping accretion**

- accretor can be very different
  - YSO, compact object, AGN
Two-component jet model

- close to central engine: GR mechanisms launch additional inner jet
  - efficient extraction AM from inner disk + black hole
    (Blandford-Znajek mechanism)
  - fast rotating inner jet, introduce radially layered jet
  - inner $\Gamma \sim 30$, outer $\Gamma \sim 3$

- perform 2.5D runs in cross-section
  - variation along jet axis ignored
  - both HD and MHD runs
  - explore differences in effective inertia

- repeat in full 3D HD
  - confirm & complement 2.5D scenario
• vary relative contribution inner jet to total $L_{Jet,Kin} \sim 10^{46}\text{ergs/s}$

$\Rightarrow$ discovery new relativistic, centrifugal Rayleigh-Taylor mode

$\Rightarrow$ FR I versus FR II related to launch efficiency inner jet!
• novel relativistically enhanced Rayleigh-Taylor mode
  ⇒ approximate dispersion relation
  ⇒ insert spatio-temporal dependence \( \exp(\lambda t - k |\zeta|) \) with displacement \( \zeta \)
  \[
  \lambda^2 \propto k \left[ (\Gamma^2 \rho h + B_z^2)_{\text{in}} - (\Gamma^2 \rho h + B_z^2)_{\text{out}} \right]
  \]
• Stability: effective inertia outer jet > effective inertia inner jet
  ⇒ works for both HD and MHD relativistic jets
  ⇒ purely poloidal \( B \) effect incorporated
  ⇒ No classical counterpart (relativistic flow essential)!
  ⇒ \( \Gamma^2 h \) effect with \( h \) specific enthalpy
  ⇒ relativistic EOS crucial: cold/hot outer/inner jet
• stable versus unstable jets: design initial conditions with varying contribution of inner/outer jet to total kinetic energy flux
  ⇒ criterion predicts cases A, C, D stable; B1, B2 unstable
  ⇒ evolution of inner jet mean Lorentz factor
  
  \[\gamma_{\text{mean}} \text{ vs. } t \text{ (year)}\]
  
  The evolution of mean Lorentz factor
  
  Case A
  Case B1
  Case B2
  Case C
  Case D

  ⇒ efficient AM redistribution and enhanced inner/outer jet mixing when mode develops
• can quantify jet de-collimation due to mode development

⇒ non-axisymmetric mode development ultimately responsible

⇒ relativistic RT mode decelerates inner, decollimates total jet

• FR II/FR I transition thereby related to central engine

⇒ depends on distribution kinetic energy over two-component jet
Preview: 3D two-component scenarios

- set up cylindrical, two-component jet models
  ⇒ assume periodic segment, ignore jet opening angle
  ⇒ visualization: exploit Paraview (www.paraview.org)
- 3D case liable to RT mode versus 3D case stable to RT mode
Summary two-component jet evolutions in 3D

- despite additional Kelvin-Helmholtz modes with axial variation
  ⇒ main evolution driven by newly discovered RT mode
- further work: analyse observational consequences
  ⇒ synthetic radio maps!
  ⇒ role of helical magnetic fields (to do . . . )
  ⇒ combined internal/external stratification effects (to do . . . )
Helically magnetized jets

- Axisymmetric **helical field configurations**
  - ⇒ again 2.5D, density contrast 1/10: light jet
  - ⇒ inlet profile of $\Gamma$ and $\mu = \frac{R_j B_\varphi}{R B_Z}$

- average $\bar{\Gamma} \approx 7$, $\beta_I = 0.3$ and $\sigma = 0.006$
  - ⇒ kinetic energy dominated, near equipartition jets
- both helical field and rotation within jet!
• follow jet to 147 light crossing times of $R_j$: $p_{\text{mag}}$ top, $\rho$ down

$\Rightarrow$ significant magnetic pressure within beam and backflow regions
- magnetic field: helicity throughout the jet beam
  ⇒ changes at internal cross-shocks
  ⇒ localized mainly toroidal field within vortical backflows
beam cross-shocks: increased helical field pinches flow downstream
⇒ matter reaccelerates up to next cross-shock
⇒ deceleration relativistic jet with equipartition $B$: extreme lengths
- detailed variation of field quantities at jet head
  \[\Rightarrow\] significant 2D effects compared to related 1D Riemann problems
explored transition $\tilde{\Gamma} = 1.15 \rightarrow 7$
\[\Rightarrow\text{non-relativistic: strong toroidal field in cocoon}\]
- quantified propagation characteristics
  ⇒ varied field inlet topology and external medium

⇒ propagation characterized by $\Gamma > \bar{\Gamma}$
• power maps give **indication of sites of synchrotron emission**

⇒ total radiation emitted is \( \propto v^2 \Gamma^2 B^2 \sin^2 \psi \)

⇒ varies significantly from toroidal to poloidal field cases

⇒ simultaneous plots of pressure/temperature at right

**Ref2: time=217.91**
Outlook

- **In summary:** realistic relativistic (M)HD models
  - external medium influence HYMORS: one-sided FR II to FR I
  - radial jet stratification: FR II versus FR I
  - helical $B$ jets: magnetic reacceleration across cross-shocks

- **Related References:**
As a sequel to **Principles of Magnetohydrodynamics** (Goedbloed & Poedts, CUP 2004), I can warmly recommend ...
Quadtree-Octree AMR

- example with 2D domain covered by $8 = 4 \times 2$ base level grid blocks
  $\Rightarrow$ hierarchically nested AMR levels, fixed factor 2 refinement

- Space-filling Morton (Z-order) curve for $N_{\text{block}} = 17$ grid blocks
  $\Rightarrow$ load-balancing: $N_p$ CPUs each $N_{\text{block}}/N_p$ ‘adjacent’ blocks
  $\Rightarrow N_p = 4$: $P_0 : 1 \rightarrow 5$, $P_1 : 6 \rightarrow 9$, $P_2 : 10 \rightarrow 13$, $P_3 : 14 \rightarrow 17$
  $\Rightarrow$ after every timestep: full grid-tree re-evaluated
AMR criteria

- automated block-based regridding procedure: 3 steps
  - consider all blocks at level $1 < l < l_{\text{max}}$
  - quantify local error $\mathcal{E}_i$ at each gridpoint $x_i$ in a grid block
  - if ANY point has $\mathcal{E}_i > Tol^l$ refine block (and ensure nesting)
  - if ALL points have $\mathcal{E}_i \leq f_{\text{Tol}}^l Tol^l$ coarsen block

- involves (user) parameters:
  - error tolerance per level $Tol^l$
  - coarsen fraction $f_{\text{Tol}}^l$ per level
Error estimation

- choice between 3 different local error $\mathcal{E}_i$ estimators
  - Richardson-based: quantify error at $t^{n+1}$, use $w^{n-1}, w^n$
  - local comparison between $w^{n-1}, w^n$
  - Löhner (& FLASH3) estimator: use $w^n$, normalized 2nd derivatives
- all estimators use user-selection of (conserved or auxiliary) variables

\[
\mathcal{E}_i = \sum_{i,w} \sigma_{i,w} \mathcal{E}_{i,w}^{\text{Rel}}
\]

- local relative variable errors $\mathcal{E}_{i,w}^{\text{Rel}}$, weights obey $\sum_{i,w} \sigma_{i,w} = 1$
- all error estimators augmented with user-coded (de)refinement

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Richardson estimator

- Richardson procedure: compute 2 future solutions $w^{n+1}$, 3 time levels
  ⇒ start from $w^{n-1}$, coarsen to $2\Delta x$, integrate with $2\Delta t$
  ⇒ **Coarsened-Integrated** solution $w^{CI}$
  ⇒ start from $w^n$, integrate with $\Delta t$, coarsen to $2\Delta x$
  ⇒ **Integrated-Coarsened** solution $w^{IC}$
  ⇒ local relative variable errors $\mathcal{E}_{iw}^{Rel}$ from

\[
\mathcal{E}_{iw}^{Rel} = \frac{|w_{iw}^{CI} - w_{iw}^{IC}|}{\sum_{iw} \sigma_{iw} |w_{iw}^{IC}|}
\]

- integrator: use first order D-unsplit scheme, only unsplit source terms
Local comparison

- Local comparison: employ 2 time levels $w^{n-1}, w^n$
  \[ \Rightarrow \text{local relative variable errors } E_{iw}^{Rel} \text{ from} \]
  \[ E_{iw}^{Rel} = \frac{|w_{iw}^{n-1} - w_{iw}^n|}{|w_{iw}^{n-1}|} \]

- Richardson or Local may need added user-set buffer zone to refine
  \[ \Rightarrow \text{additional user parameters } n_{buff} \]
Löhner estimator I

- Löhner (1987) as adjusted in PARAMESH & FLASH3
  \[ \Rightarrow \text{instantaneous } w^n, \text{ quantifies normalized 2nd derivatives} \]
  \[ \Rightarrow \text{local relative variable errors } \mathcal{E}_{iw}^{\text{Rel}} \text{ from} \]
  \[ \mathcal{E}_{iw}^{\text{Rel}} = \sqrt{\frac{N_{iw}}{\max (D_{iw}, \epsilon)}} \]
  \[ \Rightarrow \text{numerator } N_{iw} = \sum_{i_1} \sum_{i_2} \left[ \Delta_{i_1} (\Delta_{i_2} w_{iw}) \right]^2, \text{denominator} \]
  \[ D_{iw} = \sum_{i_1} \sum_{i_2} \left[ | L_{i_1} w_{iw} | + | R_{i_1} w_{iw} | + f^l S_{i_2}(S_{i_1} | w_{iw} |) \right]^2 \]
  \[ \Rightarrow \text{discrete central, left and right shifts } \Delta_i, L_i, R_i, \text{ sum operator } S_i \text{ for direction } i \]
Löhner estimator II

- estimator quantifies a weighted 2nd derivative in grid point \( i \) as in

\[
\left\{ \frac{\sum_{i_1} \sum_{i_2} \left( \Delta x_{i_1} \Delta x_{i_2} \left( \frac{\partial^2 w}{\partial x_{i_1} \partial x_{i_2}} \right) \right)^2}{\sum_{i_1} \sum_{i_2} \left[ | \Delta x_{i_1} \frac{\partial w}{\partial x_{i_1}} |_{i-1} + | \Delta x_{i_1} \frac{\partial w}{\partial x_{i_1}} |_{i+1} + f^l | \bar{w} | \right]^2} \right\}^{\frac{1}{2}}
\]

- (level dependent) 'wavefilter' parameter \( f^l \), order \( 10^{-2} \)
  \( \Rightarrow \) can also use logarithm for (positive) variables

- Note: tolerance \( Tol^l \) order 0.1, smaller for Richardson or Local
Special Relativity I

- 4D flat space-time, with $c$ as maximal propagation speed
  - four-vector $X = (ct, x)^T$ squared length invariant
    \[ X \cdot X = -c^2 t^2 + x_1^2 + x_2^2 + x_3^2 \]
  - Minkowski metric $g_{\alpha\beta} = g^{\alpha\beta} = \text{diag} \ (-1, 1, 1, 1)$
  - contra- & covariant components $X^\alpha = g^{\alpha\beta} X_\beta$ reverse $X^0 = -X_0$
- particle wordline: ideal clock for proper time $\tau$
  - four-velocity $U = dX/d\tau$, components
    \[
    U^\alpha = \begin{pmatrix}
    c \\
    \frac{dt}{d\tau} \\
    \frac{dx_i}{dt} \\
    \frac{dt}{d\tau}
    \end{pmatrix} = (c\Gamma, \Gamma v)^T \tag{1}
    \]
  - spatial three-velocity $v$ in prechosen Lorentzian reference frame
  - Lorentz factor $\Gamma = \frac{1}{\sqrt{1-v^2/c^2}}$
Special Relativity II

- inertial frames Lorentz transform $X' = L_{\alpha}^{\alpha'} X$
  $\Rightarrow$ lost simultaneity, length contracts, time dilates
- proper density: $\rho = m_0 n_0$ with $n_0$ rest frame number density
  $\Rightarrow$ lab frame ‘density’ $D = \Gamma \rho$: volume change by length contraction
- Particle conservation is $\partial_{\alpha} (\rho U^\alpha) = 0$ or
  \[
  \frac{\partial D}{\partial t} + \nabla \cdot (Dv) = 0
  \]
- stress-energy tensor:
  \[
  \begin{pmatrix}
  T^{00} & T^{0i} \\
  T^{i0} & T^{ij}
  \end{pmatrix}
  = \begin{pmatrix}
  \text{energy density} & \text{energy flux} \\
  \text{momentum flux} & \text{stresses}
  \end{pmatrix}
  \]
Special Relativity III

- gas stress-energy contribution from expression in rest frame:
  \[
  \begin{pmatrix}
  \rho c^2 + \rho \epsilon \\
  \text{rest mass + internal energy} \\
  0 \\
  \hline
  0 \\
  \end{pmatrix}
  \]

  \[
  \Rightarrow \text{to lab frame by inverse Lorentz } T_{\alpha \beta} = \mathbf{L}_{\alpha'}^{-1, \alpha} \mathbf{L}_{\beta'}^{-1, \beta} T_{\alpha' \beta'}
  \]

  \[
  \begin{pmatrix}
  T^{00} & T^{0i} \\
  T^{i0} & T^{ij} \\
  \end{pmatrix}
  =
  \begin{pmatrix}
  \tau_g + Dc^2 & \frac{S_g}{c} \\
  \frac{S_g}{c} & \frac{S_g v}{c^2} + pl \\
  \end{pmatrix}
  \]

  \[
  \Rightarrow S_g = (\rho c^2 + \rho \epsilon + p) \Gamma^2 v \quad \text{and} \quad \tau_g + Dc^2 = (\rho c^2 + \rho \epsilon + p) \Gamma^2 - p
  \]
Special Relativity IV

- when also allowing for electromagnetic fields: EM stress-energy

\[
T_{\alpha\beta}^{em} = \begin{pmatrix}
\frac{B^2}{2\mu_0} + \epsilon_0 \frac{E^2}{2} & S_{em} \\
\frac{S_{em}}{c} & \left(\frac{B^2}{2\mu_0} + \epsilon_0 \frac{E^2}{2}\right) I - \epsilon_0 EE - \frac{BB}{\mu_0}
\end{pmatrix}
\]

- EM energy flux is Poynting flux \( S_{em} = \frac{E \times B}{\mu_0} \)
- use \( E = -v \times B \): perfect conductivity

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• energy-momentum conservation

\[ \partial_\beta \left( T^{\alpha\beta} + T_{\text{em}}^{\alpha\beta} \right) = 0 \]

• introduce energy density minus rest mass and total energy flux from

\[ \tau = \tau_g + \frac{B^2}{2\mu_0} + \epsilon_0 \frac{B^2 v^2 - (v \cdot B)^2}{2} \]

\[ S_{\text{tot}} = S_g + S_{\text{em}} \]

⇒ temporal part gives

\[ \frac{\partial \tau}{\partial t} + \nabla \cdot \left( (\tau + p_{\text{tot}}) v - (v \cdot B) \frac{B}{\mu_0} \right) = 0 \]

⇒ spatial part:

\[ \frac{\partial S_{\text{tot}}}{\partial t} + \nabla \cdot \left( S_{\text{tot}} v + p_{\text{tot}} c^2 I - \frac{c^2}{\mu_0} \frac{B B}{\Gamma^2} - \frac{1}{\mu_0} (v \cdot B) v B \right) = 0 \]
Special Relativity VI

- total pressure \( p_{tot} = p + \frac{(v \cdot B)^2}{2} + \frac{B^2}{2 \Gamma^2} \)

- close system with homogeneous Maxwell equations:
  \[ \nabla \cdot B = 0 \]
  \[ \frac{\partial B}{\partial t} - \nabla \times (v \times B) = 0 \]
  \( \Rightarrow \) together with equation of state
  \[ \rho \epsilon = \frac{p}{\gamma - 1} \]

- Summary: ideal relativistic MHD
  \( \Rightarrow \) fix Lorentz frame, use 1 + 3 split (time/space), obtain
  \[ \partial_t U + \partial_i F^i = 0 \]
  \( \Rightarrow \) conserved variables \( U = (D, S_{tot}, \tau, B) \)
  \( \Rightarrow \) primitive variables \( (\rho, v, p, B) \)