



PULSAR WINDS – COSMIC PEVATRONS

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The Eponymous Crab Nebula a prototype for all Pulsar Wind Nebulae and (a lot of) AGN and GRB physics – magnetar version: UHECR?



Powered by relativistic outflow electrons-positrons + ions (?) O-X Energy extracted electromagnetically from rotating neutron star Radiation: Synchroton (TeV: IC, hadronic emission not yet seen)



Radio

D: 2 kpc Size: 2x1 ellipsoid, radio – 6pc x 3 pc optical – 4 pc x 2 pc X-ray (10 kev): 0.6 pc x 1 pc synch: ε up to 100 MeV X + g-rays measure current energy input from the pulsar

 $L_{X}+L_{g} = 10^{38} \text{ erg/s}$

 $L_{spindown} = 5 \times 10^{38} \text{ erg/s}$



Main X-ray Source = Torus; Polar Pinch(es) along rotation axis; created by backflow B in nebula, not by hoop stress in wind

The Crab is Not Alone - Nebula of B0540 is Similar



B0540 Nebula also shows Torus, perhaps Jet



Chandra Images, Gotthelf and Wang 2000 B0540 too distant (55 kpc) for detail Vela Pulsar (T ~ 10^4 yr)has X-ray nebula with structure like Crab,

perhaps distorted by proper motion induced ram pressure (Chandra)



A Soft Shelled Crab Chandra Image of P=0^s.067 pulsar in 3C58 SNR of 1181 Sne? - T = 815 yr ($P/2\dot{P} = 3335$)



A Hard Shelled Crab Chandra Image of P=0^s.0714 pulsar (ASCA) in G11.2-0.3 SNR of SNe in 386 AD - T = 1615 yr ($P/2\dot{P} = 24,000$ yr)



$$\dot{\Omega} = -K\Omega^{n} \Longrightarrow$$
$$T = \frac{P}{(n-1)\dot{P}} \left[1 - \left(\frac{P_{i}}{P}\right)^{n-1} \right]$$

"theory": n = 3 Obs: 2<n<3 Crab: $P_i \approx P/2$ Vela: $P_i << P$ (T_{SNR}~10⁴ yr)) 3C58, G11.2: $P_i \approx P$ (3C58: 87% G11.2: 97%))

G320.4/PSR1509-58(T_{spin} ~1250 yr) from Chandra



Crab cools radiatively at high energy: t(flow) ~ t(rad) in X-ray, gamma ray

Nebula smaller at high energy: Pulsar emits particles shrinkage not universal: e.g., 3C58 has R_X ~ R_{radio}: B field weaker – environment? + somewhat weaker pulsar

		Spin-Down		
Name	Period	Luminosity	Voltage	
	(msec)	(erg/s)	(Volts)	
Crab	33	5.0E+38	4.0E+16	
3C58	67	6.5E+37	1.4E+16	
1509-58	110	1.8E+37	7.4E+15	
Vela	89	1.2E+37	6.0E+15	
G11.2	71	7.7E+36	4.8E+15	

Pulsars: Pulsed Radio (and IR, optical, X, gamma) Sources



FIG. 1.—A series of individual pulses from the pulsar B0943 + 10 (center panel) along with their average (bottom panel) and energy (left panel) as a function of pulse number and longitude (360° longitude corresponds to 1 stellar rotation). Note the drifting subpulses, the alternate pulse modulation, and the single average profile.

1.6 msec < P < 9 sec dP/dt > 0

Pulsar Radio Emission: Lighthouse Beam



Model: Rotating Neutron Star (1 msec < P < 10 sec) Unipolar Inductor Polar Beam: Electron-Positron Pairs, Charge Neutralized Electron Current, Ion return current in "walls" (flipped in opposite geometry) - Ultrarelativistic Outflow

Polar models have voltage across B~10¹² Gauss of F \approx W²m/c² > 10¹³ V. Polar current I ~ W²m/c ~ 10¹² Amps and created by acceleration through potential DF << F (space charge limited beam): DF > 10¹² V: pair creation Rotating magnet sends out EM Poynting flux + particle flux, spins down (dP/dt > 0) ("Wind") Energy deposited in surrounding interstellar medium, creates nebula USE NEBULAE TO CHARACTERIZE OUTFLOW

Necessity of Magnetospheric Plasma
Star = good conductor
Electric field vanishes **inside** star in co rotating frame:
(works in GR, co-rotating frame not inertial)

$$E + \frac{1}{c}(\Omega \times r) \times B = 0, r < R$$

$$v_E = c \frac{E \times B}{B^2} = -\frac{[(\Omega \times r) \times B] \times B}{B^2} = \Omega \times r$$
Co-rotation electric
field (inertial frame) $V = V_E + V_{\parallel} b, b = \frac{B}{B}$
Non-zero E requires charge density:

$$\nabla \cdot E = 4\pi\eta_R, \quad \eta_R = -\frac{\Omega \cdot B}{2\pi c} + \dots$$
Parallel electric field reduced to small
compared to vacuum Vacuum

$$\frac{\eta_R}{e} = 7 \times 10^{10} \frac{B_{12}}{P} \text{ cm}^3$$

$$D = \int_0^{r_R} dr_L E_{co} = \frac{\mu}{r_{LC}^2} = \sqrt{\frac{E_R}{c}} \approx 10^{13} \sqrt{\frac{P_{15}}{P^3}} \text{ Volts (Crab: 10^{16.5} V,...)}$$
Goldreich-Julian density

$$I = \int_0^{r_R} dr_L r_C \eta_R = c\Phi = 10^{12} \sqrt{\frac{P_{15}}{P^3}} \text{ Amps}$$

$$B_f = F/r$$

$$\dot{E} = I\Phi = c\Phi^2 = \dot{E}_R \text{ (without details of angles)}$$

Nebulae require Number Loss Rate >> Goldreich-Julian rate

$$\dot{N}_{\pm} \gg \frac{c\Phi}{e} \approx 10^{34} \frac{\Phi}{10^{16.6} V} s^{-1}, \quad \Phi = \sqrt{\frac{\dot{E}_R}{c}} = 3.9 \times 10^{16} \frac{\dot{P} / 10^{-12.35}}{(P / 33 \,\text{msec})^3} \text{ Volts,}$$
$$I = c\Phi = 3.9 \times 10^{16} \frac{\dot{P} / 10^{-12.35}}{(P / 33 \,\text{msec})^3} \text{ Amp, } \dot{E}_R = I\Phi = c\Phi^2 = \frac{\Omega^4 \mu^2}{c^3} \text{ (~ dipole to LC)}$$

Feeding Nebulae needs particle outflow > > cF/e: large multiplicity

Composition = electron-positron plasma Termination Shock (TS) located at $R_{TS} = F/(4pP_{neb})^{1/2} >> R_L$



STerSI makes termination shock flow "turbulent":



B from Camus et al 2009 pressure

If turbulence cascades to short wavelength, fast 2nd order Fermi acceleration creates radio emitting spectrum (Fermi 1948, Kardashev 1962 – magnetic eddies; Stawarz 2008 (eddies, waves) $Q_{inject} \propto E^{-(1+p)}, p = \frac{\tau_{accel}}{\tau_{esc}} \sim 0.5$ plausible: diffusion in space, energy from same eddies/waves, v_{eddv/wave}/c ~ 1

Follow the Mass Loss: From Whence all the Pairs?

Pulsar Wind Nebulae: Nebular Synchrotron requires particle injection >> Goldreich-Julian current PAIR PROBLEM

X-Rays:current injection rate (compact, strong B nebulae - Crab, G54,...) measured rates ~ existing (starvation) gap rates $k_{\pm} \leq 10^4$ pairs/GJ

Radio measures injection rate averaged over nebular histories, $\langle k_{+} \rangle > 10^{6}$



Low s = B²/8pm_±c²n_±G_w at termination \rightarrow G_w = eF/2m_±c²k_± =

S₀

PWN Name	k _±	G _w	F _{init} (PV)	Age (yr)
Crab	> 10 ⁶	5 x 10 ⁴	100	955
3C58	> 10 ^{5.7}	3 x 10 ⁴	15	2100
B1509	> 10 ^{5.3}	1 x 10 ⁴	121	1570
Kes 75	> 10 ⁵	7 x 10 ⁴	22	650

From one zone evolutionary model of observed spectrum <u>including</u> radio (with Bucciantini, Amato) – injection spectrum <u>convex</u>, $g^{-1.5}$ (radio) $\rightarrow g^{-2.3}$ (X)



Low s ($G_w \rightarrow s_0$) in <u>unconfined</u> wind requires magnetic dissipation somewhere

Ideal MHD, poloidal field lines almost radial:

acceleration parallel to velocity, inertial force for change of speed proportional to longitudinal mass mg³:

$$\rho c\beta \frac{\partial}{\partial r} (\gamma c\beta) = \rho c^2 \left(\beta \frac{\partial \gamma}{\partial r} + \gamma \frac{\partial \beta}{\partial r}\right) = \rho c^2 \gamma^3 \frac{\partial \beta}{\partial r} \sim -\frac{\partial}{\partial r} \frac{B^2}{8\pi} = \frac{B^2}{4\pi r}$$

Magnetic Spring > Inertia: $1 > \frac{\rho c^2 \gamma^3 (\partial \beta / \partial r)}{B^2 / 4\pi r} = \gamma^2 \frac{4\pi \rho c^2 \gamma}{B^2} r \frac{\partial \beta}{\partial r} \approx M_F^2 \Rightarrow$

Unconfined Relativistic MHD winds accelerate to

$$M_F \approx 1 \left(\text{not } \sigma = \frac{\sigma_0}{\gamma} = 1 \right), \Rightarrow \gamma_\infty \approx \sigma_0^{1/3}$$
$$(\text{not } \gamma_\infty \approx \sigma_0)$$

Observations (models) require stronger, non-radiative (equatorial) acceleration for $r >> R_F \sim 10^2 R_{LightCyl}$

Current Sheet: Return Current Particle Beams with Pair Content

Inner wind magnetically dominated (s >> 1) – has generic return current sheet

Circuit is open – net electrons (protons) on central path, protons & positrons (electrons) on auroral path –current "closes" in "earth" = external nebula (interstellar medium) through eddy currents



Reconnection flow supports return current: pairs from wind into layer - electrons flow down, contribute to the return current - rest is particles (protons in geometry shown) attracted up from the surface by precipitating charge - current outflow in equatorial sheet (plasmoids) = ions from surface + positrons from reconnection singular region



Relativistic MHD Simulation of Aligned Rotator – dissipation = numerical resistivity - limit cycle reconnection in current sheet (Bucciantini et al 2006)

O Regions recede relativistically -Recurrence time ~ rotation period

Observed PSR = oblique rotators Equatorial Current Sheet \longrightarrow Frozen-in Transmission Line Inner Wind: Magnetically Striped



Force Free Simulation of i=60° Rotator (Spitkovsky)



) i=60° - topology = aligned rotator (Bai and Spitkovsky) Equatorial cross-section



Current Sheet Separating Stripes (from Bogovalov's analytic model)



Meridional cross-section

 $\dot{E}_R = -I\Omega\dot{\Omega} = k \frac{\mu^2 \Omega^2}{\sigma^3} (1 + \sin^2 i), \ k = 1 \pm 0.1 \qquad i = \angle(\mu, \Omega)$

Stripe Dissipation Kills B_{f.} restores aligned rotator thick CS

If wrinkled current dissipates, striped field dissipates, magnetic energy coverts to flow kinetic energy, "heat" & radiation, perhaps strong waves - partition?



From Coroniti 1990

Sheet spacing: v/W – cold between sheets

Sheet separation = R_L , wavelength = $2G_{\omega}$ RL TS at many R_L (10⁹ R_L for Crab) Ideal MHD: Intersheet S >> 1 conserved Current sheets' dissipation:

- I) Anomalous resistivity forms, plasma in sheets heats, current channels widen, merge at r << R_{TS}
- Reconnection (also needs resistivity/ inertia) collapses field onto sheets, energy goes to hot islands, sheets spread and merge, devour field upstream of TS
- Mode conversion sheet converts to relativistically strong EM waves in flow frame

Plasma pressure in sheets causes wind acceleration as G_{ω} drops

Maximum dissipation

Lows

Heating: sheet expansion in wave frame = wind flow rest frame spreads at speed $v_s < c$; sheets expand, merge, S \longrightarrow "0" sheet separation in wave frame: $I/2 = pG_{wind}R_I$,

Merger time in PWN frame: $T_{m} = \Gamma_{wind} \left(\frac{\lambda/2}{v_{s}} \right) = \pi \Gamma_{wind}^{2} \frac{R_{L}}{v_{s}}$

Flow time from star to TS in nebula frame at $r = R_{TS}$: $T_{TS} = R_{TS}/c$ Sheet merger occurs before wind terminates only if $T_m < T_{TS}$:

$$\Gamma_{wind} < \left(\frac{R_{TS}}{\pi R_L} \frac{v_s}{c}\right)^{\nu_z} = (Crab) \ 3.2 \times 10^4 \left(\frac{v_s}{c}\right)^{\nu_z}$$

ow s wind at TS $\Rightarrow \dot{E}_R = \dot{N}_{\pm} m_{\pm} c^2 (\Gamma_{wind} - 1) \Rightarrow \Gamma_{wind}^{(max)} = \frac{\dot{E}_R}{2\dot{N}_{\pm} m_{\pm} c^2} = 3 \times 10^3 \frac{10^{41} s^{-1}}{\dot{N}_{\pm}} (Crab \ radio)$
Sheet dissipation upstream of TS may work if

$$v_{s} / c > 0.01 \left(\frac{10^{41} s^{-1}}{\dot{N}_{\pm}} \right)^{-1}; v_{s} / c < 1 \Leftrightarrow \dot{N}_{\pm} > 10^{40} s^{-1}$$

"fast" sheet dissipation if "slow", dense wind: $G_{wind} << 10^6 \dot{N}_{\pm} >> 10^{38} s^{-1}$

Simplified Sheet Structure





Dynamics of plasma inside thin sheets as if each sheet is unmagnetized; intersheet medium is high S MHD $(B^2 \gg 4\pi\rho_0 c^2)$ - <u>sheet current = runaway beam</u>

Two Symmetric Sheet Instability



Growth Rate 2 symmetric sheets = purely growing in proper frame

Wave vector parallel to $B = 2\pi / k_{\parallel}$

Alfvenic magnetic ripple at each sheet $\langle \delta B_x(y) \rangle \propto \exp \left[i \left(k_{\parallel} y - \omega t \right) \right]$

Intersheet plasma MHD - sheets couple through Alfven waves modified by inhomogeneity

j₀ x dB_x force compresses each sheet's <u>surface</u> <u>density</u> into filaments parallel to j₀ Surface current filaments reinforce dB_x currents flow in unmagnetized sheets' cores *Weibel instability in flatland*

Proper Growth Rate $(v_A=b_A, v_{beam}=cb_b)$ $\Gamma_{2sheet} = \frac{2c}{\lambda} \beta_A (\beta_b \beta_A k_{\parallel} \lambda / 2)^{2/3} \left(\frac{\lambda / 2}{H}\right)^{1/3}, \text{ use } k_{\parallel} \lambda / 2 \sim 1$ $\omega_A = k_{\parallel} c \beta_A$ 0.8 $z_0 = \frac{\beta_A^2 \beta_b^2}{k_{\scriptscriptstyle \parallel} H}$ $\frac{1}{\omega_{\rm A} \, z_0^{1/3}}$ 0.6 0.4 0.2 0.01 0.1 10 $k_{\mu}x_{0}z_{0}^{1/3}$

 $G_{2sheet}T_{flow} >> 1$,

Sustained Weibel turbulence inside current sheets in wind

Weibel scatters particles

Weibel in pairs, colliding shells (shock simulations)



Magnetic Energy Density - Peak and Downstream





Test Particle in a Strong Weibel Mediated Shock

Large 2D shock PIC simulation Labeled plasma particles show scattering

[x,y,z]=c/ω_ρ

Current carriers scattering nonresonant, $t_{scat} \propto \pi^2 \Rightarrow \rho \upsilon \nu \alpha \omega \alpha \psi \beta \epsilon \alpha \mu \sigma$ G_{beam} may be as high as qF Alternate model – currents are in main body of sheet plasma, not very relativistic, dissipation = internal Instabilities of Sheets: Collisionless Tearing, Drift Kink (stronger for pairs)

Relativistic Harris-Hoh Equilibrium instead of unidirectional charge neutralized beam

$$\begin{split} \boldsymbol{B} &= B_0 \tanh{(z/\lambda)} \hat{\boldsymbol{x}}, \\ f_s &= \frac{n_0 \cosh^{-2}{(z/\lambda)}}{4\pi m^2 c T K_2 (mc^2/T)} \exp\left[\frac{-\gamma_s (\varepsilon - \beta_s m c u_y)}{T}\right] \\ &+ \frac{n_{\text{bg}}}{4\pi m^2 c T_{\text{bg}} K_2 (mc^2/T_{\text{bg}})} \exp\left(-\frac{\varepsilon}{T_{\text{bg}}}\right), \end{split}$$

Counterstreaming electrons/positrons in channel drives kinking perpendicular to B

$$\tau_c = \frac{\lambda}{c} = \omega_c^{-1} = \frac{m_{\pm}c^2\Gamma_w}{e\Phi} \frac{r}{2\pi R_l}P$$



Zenitiani & Hoshino initial value PIC (current stops at late time, not true for PSR sheet) 1011 12/14/09 6:52 PM

Anomalous Resistivity in Sheets & Sheet Merging (beam model)

 $\langle (\delta B)^2 \rangle \neq 0 \Rightarrow$ scattering of beam particles ("collisions")

$$v_{c} = \left\langle \left(\delta \omega_{c} \right)^{2} \right\rangle \tau_{ac} = \Gamma(\Gamma \tau_{ac}) = K_{c} \Gamma, K_{c} \ge 1$$

Conductivity inside sheet :
$$\sigma_{\text{beam}} = \frac{\omega_{p,\text{beam}}^2}{4\pi v_c}$$
,

$$v_{\rm m} = \frac{c^2}{4\pi\sigma_{\rm beam}} = \frac{1}{3}cH\alpha_{\rm beam} \left(\frac{H}{\lambda}\right)^{2/3},$$
$$\lambda = 2\pi\Gamma_{\rm wind}R_{\rm L} = \text{ stripe wavelength, } D_{\rm Bohm} = \frac{1}{3}cH \text{ since } H \approx r_{\rm Larmon}$$

Sheet Heating: Non-MHD electric field $E_{beam} = J_{beam} / \sigma_{beam}$ entropy not conserved,

$$\Gamma_{\text{wind}}^{2} r \frac{d}{dr} \left(\frac{H}{\Gamma_{\text{wind}}} \right) + \frac{\Gamma_{\text{wind}}}{3} \frac{dH}{dr} = \alpha_{\text{beam}} \left(\frac{4H}{\lambda} \right)^{2/3}$$

Heating accelerates the wind

Sheet Heating: Non-MHD electric field $E_{beam} = J_{beam} / \sigma_{beam}$ entropy not conserved:

$$H = \frac{2T}{eB}, \quad \Gamma_{\text{wind}}^2 r \frac{d}{dr} \left(\frac{H}{\Gamma_{\text{wind}}}\right) + \frac{\Gamma_{\text{wind}}}{3} \frac{dH}{dr} = \alpha_{\text{beam}} \left(\frac{4H}{\lambda}\right)^{2/3}$$

Energy Conservation:

$$R_{L} \frac{d\Gamma_{wind}}{dr} = \frac{\alpha_{beam}}{2\pi\Gamma_{wind}^{2}} \frac{\dot{E}}{\dot{M}c^{2}} \left(\frac{4H}{\lambda}\right)^{2/3}$$

Similarity Solution:

$$\Gamma_{\text{wind}} = \left(\frac{7}{6\pi} \frac{\dot{E}_{R}}{\dot{M}c^{2}}\right)^{1/7} \alpha_{\text{beam}}^{2/7} \left(\frac{r}{R_{L}}\right)^{3/7}, \quad \frac{4H}{2\pi\Gamma_{\text{wind}}R_{L}} = \left(\frac{36\pi^{2}}{49} \alpha_{\text{beam}} \frac{\dot{M}c^{2}}{\dot{E}_{R}}\right)^{3/7} \left(\frac{r}{R_{L}}\right)^{3/7}$$

Current sheet merger complete, striped B field ~ gone when $4H=2\pi\Gamma_{wind}R_L$ at $r=R_{merge}$.

$$R_{merge} = \frac{49}{36\pi^{2}\alpha_{beam}} \left(\frac{\dot{E}_{R}}{\dot{M}c^{2}}\right)^{2} R_{L} = (Crab) \frac{5 \times 10^{8}}{\alpha_{beam}} \frac{10^{40} \, s^{-1}}{\dot{N}_{\pm}} R_{L} < R_{shock} \approx 10^{9} R_{L}$$

 $\alpha_{beam} = 3K_c \beta_A (k_{\parallel} \lambda \beta_{beam} \beta_A)^{2/3} \sim 1(?) = \text{main "wiggle" parameter: } K_c \sim 1? \text{ PIC sims for process}$ $\dot{N} > 10^{40} \text{ s}^{-1} \text{ really needed for feeding radio emission?}$

Radiation from Wind

Beam model has Relativistically hot current sheets: proper temperature ~ g_{beam}m_{beam}c² large

$$\frac{T}{m_{beam}}c^{2} \approx 10^{9} \frac{m_{\pm}}{m_{beam}} \left(\frac{\dot{E}_{R}}{10^{38.7} \text{ erg/s}}\right)^{7/13} \left(\frac{\dot{N}_{\pm}}{10^{41} \text{ s}^{-1}}\right)^{6/13} \left(\frac{R_{L}}{r}\right)^{10/13} \gg 1$$

if $m_{beam} = m_{\pm}$

Synchrotron emission (observer frame):

$$\frac{L_{\text{wind}}^{(\text{synch})}(>r)}{\dot{E}_{R}} \approx 2 \times 10^{-3} \left(\frac{\dot{E}_{R}}{10^{38.7} \text{ erg/s}}\right)^{0.45} \left(\frac{\dot{N}_{\pm}}{10^{41} \text{ s}^{-1}}\right)^{1.44} \left(\frac{m_{\pm}}{m_{\text{beam}}}\right)^{2} \left(\frac{R_{\text{min}}}{r}\right)^{-1.92},$$

$$\hbar\omega_{synch} \approx 40 \frac{33 \text{msec}}{P} \left(\frac{\dot{E}_{R}}{10^{38.7} \text{erg/s}}\right)^{1.04} \left(\frac{\dot{N}_{\pm}}{10^{41} \text{s}^{-1}}\right)^{0.46} \left(\frac{m_{\pm}}{m_{beam}}\right)^{3} \left(\frac{R_{L}}{r}\right)^{1.54} \text{TeV}$$

Spectrum calculations: in progress (add shells of relativistic thermal synch) high energy from inner wind (B_fenclosing sheets large)

Optically thin - yes, except perhaps at highest energy (gg opacity unknown)

Emission from r ~ R_{merge} in optical-UV - unpulsed emission, also faint, B_f small

Tev, GeV emission might be pulsed (inner wind), emission regions can be smaller than rG_{wind}², therefore radiation in phase with sheet? - alternate to SG, OG magnetospheric beamed emission (old idea, recently worked on by Petri and Kirk); there are upper limits on TeV pulsed emission that may challenge model (or allow detection of wind emission - even unpulsed flux might be at energies where nebular flux weakens.)

be modeled by sheet emission (with "knobs"))

old ion beam Idea generalized to all possibilities (e⁻, e⁺, ions – depends on PSR geometry, coupling of current flow to * surface, Y line

Conclusions

Current sheets in striped high S winds decay due to anomalous resistance, sheets in striped pulsar winds don't survive to termination shock - requires large mass loading (observed in Crab, others)

Termination Shock in macro-turbulent medium: shock accel (X-rays) coupled to Fermi II accel of radio emitting pairs?

Inner wind ($r < R_{merge}$) creates synchrotron emission from sheets. Bolometric luminosity small fraction of total energy loss.

Unpulsed? Pulsed? Detectable now or in near future?

Application to jets? Confined (by disk wind?) jets can see $G_w \rightarrow s_0$ without magnetic dissipation \Rightarrow (?) emission from shocks

Lab: sheet spacing = v/W << 1 m - v=1 km/s needs W > 10⁴ s⁻¹ cold: c_s << v (ion, electron; bucky ball pair plasma?) - termination shock? r_L << spacing, transonic inner flow: B >> kiloGauss - less if supersonic



Profile of Magnetic Field in a Current Sheet - Harris Model

$$B = B_0 \tanh\left(\frac{x}{H}\right)$$
$$2H = \text{Larmor radius} = \frac{T}{qB},$$
$$T = \text{beam "temperature"}$$



Beams flow in central unmagnetized channel

