

## Midterm Exam, AST 203, Spring 2012

Thursday, March 15, 3:00-4:20 PM

*General grading rules for calculational problems: 4 points off for each arithmetic or algebraic error. Two points off per problem for excess significant figures in a final result (i.e., more than 2 significant figures). In this part of the exam, there is no requirement to write in full sentences. Explanations should be clear, but if the context is unambiguous, no points off for undefined symbols. However, take off two points per problem if a student gives absolutely no context for their calculation. Not giving units or not giving results in the units asked for, is 2 points off. Giving the correct number with incorrect units is 4 points off. No points off for not indicating units in intermediate calculations. In no case do the results of one part depend on the answer to another, so errors should not propagate from one part to another.*

*In many of the problems, there is an easy way and a hard way to do it; full credit for doing problems the hard way (and getting the right answer).*

The total exam is worth 200 points.

### Calculational Problems

**1. White dwarf** (50 points) A hot white dwarf star has the same luminosity as does the Sun. It can be seen with a moderate-size telescope; it is  $10^{16}$  times less bright than is the Sun. From its spectrum, its surface temperature is measured to be a blazing 60,000 K. Scaling from what you know about the Sun:

**a.** (25 points) How far away is the white dwarf? Express your answer in light years.

**Answer:** The inverse square law relates brightness  $b$ , luminosity  $L$ , and distance  $r$ :

$$b = \frac{L}{4\pi r^2} \quad (1)$$

We can write such an equation down separately for the Sun and the white dwarf. If we take the ratio of the two, realizing that the luminosities of the two are similar, we find:

$$\left(\frac{r_{WD}}{r_{\odot}}\right)^2 = \frac{b_{\odot}}{b_{WD}} = 10^{16} \quad (2)$$

Taking the square root of both sides, and solving for  $r_{WD}$ :

$$r_{WD} = 10^8 r_{\odot} = 10^8 \text{ AU} \quad (3)$$

There are 60,000 AU in a light year, so we find:

$$r_{WD} = 1700 \text{ ly.} \quad (4)$$

*7 points for writing down the inverse square law and no more. Another 7 points for understanding that you need to write the inverse square law for each of the Sun and the WD, and take their ratio and no more. Full credit for laboriously calculating the brightness*

of the Sun, dividing by  $10^{16}$ , and solving for the distance to the white dwarf, if done correctly. Full credit does not require going through the intermediate steps as above; e.g., if the student sees immediately that the brightness ratio is the inverse distance ratio squared...

**b.** (25 points) What is the radius of the white dwarf? Express your answer in kilometers. You should assume that the white dwarf and the Sun both radiate like a blackbody.

**Answer:** We know that the Sun and the white dwarf have the same luminosity. We also know that for black bodies, luminosities are related to radius and temperature:

$$L = 4\pi R^2 \sigma T^4 \quad (5)$$

Writing down such an equation for both the WD and the Sun, and taking their ratio:

$$1 = \left(\frac{R_{WD}}{R_{\odot}}\right)^2 \left(\frac{T_{WD}}{T_{\odot}}\right)^4 \quad (6)$$

The temperature ratio we know; the WD is 10 times hotter than the Sun. Taking the square root of both sides, and solving for  $r_{WD}$  gives:

$$R_{WD} = R_{\odot} \times 10^{-2} = 7 \times 10^5 \text{ km}/100 = 7000 \text{ km} \quad (7)$$

*Full credit for those who do this the hard way (and get the right answer), by plugging in numbers for the luminosity of the Sun and  $\sigma$ . As above, 7 points for writing down the relation between  $L$ ,  $T$ , and  $r$ , and 7 more points for realizing that the way to go is to take a ratio.*

**2. Two Planets** (70 points) Consider two planets in circular orbits around a main sequence B star. The star has the same radius as the Sun, with four times the mass of the Sun, and twice the surface temperature. The first of the planets, Romulus, has an orbital radius of 1 AU, while the other, Vulcan, has an orbital radius of 4 AU. *Hint: In these questions, you may scale from what you know about the Earth's orbit.*

**a.** (20 points) What is the orbital period of each of these planets? Express your result in years.

**Solution:** Here, you can take the hint given seriously. A planet orbiting a one solar mass star at a distance of 1 AU has an orbital period of one year, as we know from Earth's orbit. Kepler's Third Law says that the square of the period is inversely proportional to the mass of the parent star, or

$$P^2(\text{years}) = a^3(\text{AU})/M(\text{solar masses}).$$

For Romulus,  $a = 1$  AU, and  $M = 4$ , so  $P = 1/2$  year.

One can do this the difficult way, from Newton's form of Kepler's Third Law; if you do it correctly, you get full credit.

Planet Vulcan then has a period:

$$P(\text{years}) = \sqrt{a^3/M} = \sqrt{4^3/4} = \sqrt{16} = 4.$$

The second planet has a period of four years.

*4 points off for each arithmetic mistake. 5 points for writing down Newton's form of Kepler's Third Law (or at least  $P^2 \propto a^3/M$ ), and going no further. Writing down the simpler form,  $P^2 = a^3$  alone is worth only 2 points. 5 points off for the calculation ignoring the fact that the B star has four times the mass of the Sun (you then get an answer of 1 and 8 years, not 0.5 and 4). Full credit for doing this the hard way (i.e., not scaling from the Earth's orbit, but rather plugging in MKS numbers into Newton's form of the equation), if done correctly.*

**b.** (20 points) What is the equilibrium surface temperature of each of these planets?

You may ignore the effects of albedo and greenhouse effect on each planet.

**Solution:** The relation between the temperature of a planet, and the distance to the parent star, its size, and its temperature, is:

$$T_{\text{planet}} = T_{\text{star}} \sqrt{\frac{r_{\text{star}}}{2d}}.$$

We ignored albedo ( $A=0$ ) and the greenhouse effect in this formula. Thus for a planet at a given distance from its parent star, the temperature is proportional to the temperature of the star. In this problem, we're comparing two planets around the same star, so  $T_{\text{star}}$  and  $R_{\text{star}}$  are the same. The problem states that we can scale from the Earth, for which the cheatsheet gives a temperature of 270K. The parent star here is a factor of 2 warmer, so for the planet at 1 AU, the temperature is  $2 \times 270\text{K} = 540\text{K}$ . For the second one, the only thing different is the distance; 4 times further away. The temperature is therefore a factor of  $\sqrt{4} = 2$  cooler, or 270 K.

*Four points off for arithmetic or algebra errors. Full credit for doing this using the form of the formula that uses the luminosity of the parent star. Four points for writing down the relationship for the planetary temperature in any of its relevant forms, and going no further. 12 points for calculating the temperature of one of the planets, and going no further.*

**c.** (15 points) Astronomers on Romulus measure the parallax of a distant star to be 1 arcsecond. Vulcan astronomers measure the parallax of the same star; what value do they find?

**Solution.** Parallax of a distant star is linearly proportional to the baseline used for the measurement. For a baseline of 1AU, the formula is  $p(\text{arcsec}) = 1/d(\text{pc})$ . For a baseline of 4AU, the parallax is going to be 4 times larger, or 4 arcseconds.

*6 points for writing the 1AU parallax formula and going no further.*

**d.** (15 points) Angular size of Vulcan as seen from Romulus at the point of closest approach between the planets is 10 arcseconds. What is the angular size of Vulcan when it is farthest from Romulus in its orbit?

**Solution.** The angular size of an object is given by its diameter divided by the distance to the object (small angle formula). At the point of closest approach, Romulus and Vulcan are  $4\text{ AU} - 1\text{ AU} = 3\text{ AU}$  apart. At the farthest point, they are separated by  $4\text{ AU} + 1\text{ AU} = 5\text{ AU}$ . The angular size of Romulus is then  $10'' \times 3\text{ AU}/5\text{ AU} = 6''$ .

*4 points for small angle formula and going no further.*

### Concept Questions

Please answer the following question, using coherent English sentences. Feel free to use equations, calculations, and/or diagrams as part of your explanation. To answer the

question in full will take at least three or four paragraphs, but feel free to write more if you need to. This is your opportunity to show us what you have learned, so aim for complete explanations.

**3. (45 points)** Describe the Aristotelian worldview, emphasizing its description of the physical nature of, and laws that govern the motions of, heavenly bodies relative to those on Earth. Explain how this differs from our current scientific understanding of the Universe, and explain the importance of the discoveries of Galileo Galilei, Nicolas Copernicus, Johannes Kepler, Isaac Newton and Cecilia Payne-Gaposchkin in overturning the Aristotelian worldview.

**Answer:** The Aristotelian worldview had several characteristics:

- The Earth lay at the center of the Universe (*3 points*)
- Earthbound objects were made of four basic elements: Earth, Water, Air, and Fire (*3 points*)
- The natural motion of the elements was towards their “proper place”: for Earth and Water, it was towards the center of the earth, while the natural motion of Air and Fire was away from the center of the Earth. (*3 points*)
- The material of the heavens, or Celestial Spheres, was made of a fifth element, different from that found on Earth, namely quintessence. (*3 points*)
- The natural motion of celestial objects was in circles; indeed, the Celestial Spheres rotated around the Earth. (*3 points*)

Indeed, for this class, the most important aspect of the Aristotelian worldview is that the substance that objects in the heavens were made of, and the physical laws that governed those objects, were different from those on Earth. *Up to 7 points for making this general statement, in the absence of the details described above. Give up to five extra points for further discussion of either Aristotelian philosophy or the Ptolemaic model, perhaps including diagrams, and explaining the nature of epicycles, etc.*

Perhaps the most profoundly revolutionary aspect of the Scientific Revolution, as exemplified by the work of Copernicus, Newton, Galileo, and Payne-Gaposchkin, was the understanding that the substance of heavenly bodies, and the laws that govern their motions, are the *same* as those on Earth. (*8 points*) Let’s take the work of each of these great scientists in turn:

Nicolas Copernicus showed that motions in the Solar System are much more easily explained if instead of placing the Earth in the center, the Sun lies in the center of the Solar System. (*5 points*) This contradicts one of the Aristotelian notions, i.e., that all heavenly objects move in circles around the Earth. *3 more points for more detailed descriptions of the Copernican picture and its advantages.*

Galileo Galilei was the first to use a telescope to study the sky. His discoveries were in strong support of Copernicus’ ideas, and served to further undermine the Aristotelian notions of the heavenly objects as perfect:

- He saw mountains on the Moon, very much like those on Earth. (*3 points*)

- He saw spots on the Sun, demonstrating that it was not unblemished. (*3 points*)
- He saw that Venus went through phases, easily explained (and predicted!) in the heliocentric view, and not expected in the geocentric view. (*3 points*)
- He saw that Jupiter had a set of four moons orbiting around it, showing directly that heavenly bodies can exhibit circular motions around objects other than the Earth. (*3 points*)

Johannes Kepler used the detailed astronomical observations of the motions of planets to empirically obtain his laws of planetary motion: he understood that the planets move on ellipses (3 points) and the period of the planet squared is proportional to its semi-major axis cubed (4 points). (2 points for law of equal areas per time).

Isaac Newton developed his laws of motion, and his laws of gravity, showing explicitly that the laws that govern motions on Earth and in the heavens are the same *Up to 5 points*. In particular, he showed that the acceleration that pulls an apple to the ground, is due to the same force of gravity that keeps the Moon “falling” in its orbit around the Earth. (*4 points*). *Up to 5 more points for a coherent description of Newton’s Laws, and/or its explanation of Kepler’s Laws of planetary motion.*

Finally, Cecilia Payne-Gaposchkin studied the spectra of stars, and used the (then new-fangled) notions of quantum mechanics to infer the chemical composition of stars. (*4 points*) The important point here was that stars are made up of the same elements that are found on Earth, going against one of the basic Aristotelian tenets. (*4 points*) In particular, the Sun, and stars in general, are composed of 73% Hydrogen and 25% Helium, leaving 2% for all the rest of the elements in the periodic table. (*4 points*). *Up to 4 more points for further discussion of spectral line formation in stars, how the lines which appear in a star depend on the surface temperature of a star, and that Payne-Gaposchkin sorted out what the OBAFGKM sequence actually means.*

**4. Multiple choice.** This is a multiple choice problem. Write your answer in the exam book (e.g., 5.1 – A), NOT on the exam text. No explanation is necessary. *35 total points*

Answers: 5 points for each correct answer. No partial credit, except 4.3, 4.4 and 4.6

4.1 D (5 points);

4.2 D (5 points);

4.3 Three letters: B (2 points),C (2 points),E (1 point). If others are also selected, -2 points each for each wrong answer.

4.4 B (5 points). Subtract 1 point for each selected wrong answer;

4.5 C (5 points);

4.6 D (5 points). Subtract 1 point for each selected wrong answer;

4.7 C (5 points);

4.1 (5 points) The spectral sequence sorts stars according to

- A) mass.
- B) core temperature.
- C) luminosity.
- D) surface temperature.
- E) radius.

**Answer D.**

4.2 (5 points) A telescope on the Earth has an angular resolution of  $4 \times 10^{-2}$  arcseconds. The maximum distance to a star that it can measure using the parallax method is

- A) 0.5 parsecs.
- B) 2.5 parsecs.
- C) 5 parsecs.
- D) 50 parsecs.
- E) 500 parsecs.

**Answer:** D. The distance to a star in parsecs is 1 over the parallax angle. The resolution of the telescope determines the minimum total angular shift that the telescope can measure from July to January. This is twice the parallax angle. So, for parallax angle of  $2 \times 10^{-2}$  arcsecs, we get  $d = 1/(2 \times 10^{-2}\text{arcsec}) = 50\text{pc}$ .

4.3 (5 points) Select all that apply: a spaceship on a geostationary orbit

- A) must orbit close to the Sun.
- B) completes 365 orbits per year.
- C) is moving slower than a spaceship in low Earth orbit.
- D) is moving faster than a spaceship in low Earth orbit.
- E) always appears above a fixed point on the Earth.

**Answer:** B, C, E. If the satellite is always over a fixed point on the Earth (E), it is making one orbit every 24 hours, or 365 orbits per year (B). From HW 2 we remember that it was on a fairly high orbit (35 000 km). By Kepler's laws, the speed of the object on a lower orbit is faster than on the higher orbit (C).

4.4 (5 points) Which statements about the H-R diagram are true (select all that apply)?

- A) Stellar mass is increasing from the lower left to the upper right of the H-R diagram.
- B) Hydrogen-fusing stars are clustered along a diagonal on the H-R diagram.
- C) Luminosity of red stars is always smaller than the luminosity of blue stars.
- D) Stars with the largest radii on H-R diagram are located at the top left tip of the main sequence.
- E) Stars with the smallest radii on H-R diagram are located at the top right corner of the diagram.

**Answer:** B is the answer. Explanation why other answers are wrong:

- A) Stellar mass is increasing from the lower RIGHT to the upper LEFT.
- C) This is true only on the main sequence, and is not true of the whole H-R diagram
- D) Stars with the largest radii on H-R diagram are located in the top right corner – the supergiants.
- E) Smallest radii are in the bottom left corner.

4.5 (5 points) Approximately, what basic composition are all stars born with?

- A) half hydrogen, half helium, no more than 2 percent heavier elements
- B) 90 percent hydrogen, 10 percent helium, no more than 1 percent heavier elements
- C) three-quarters hydrogen, one-quarter helium, no more than 2 percent heavier elements
- D) one-quarter hydrogen, three-quarters helium, no more than 2 percent heavier elements
- E) 98 percent hydrogen, 2 percent helium

**Answer:** C.

4.6 (5 points) The half-life of Uranium-238 is roughly 4 billion years. Select all that apply:

- A) if you start with 1 kg block of pure Uranium-238 it will weigh 0.5kg after 4 billion years.
- B) based on its half-life this isotope cannot be used for radioactive dating of rocks on the Earth.
- C) after one half-life the remaining Uranium-238 nuclei will decay in 2 billion years.
- D) in 4 billion years half of the nuclei of Uranium-238 will remain.
- E) number of nuclei of Uranium-238 that decay per second does not change with time.

**Answer:** D. Explanation why other answers are wrong:

A) Upon decay, Uranium-238 will transform into another element (Thorium), and most of the mass of Uranium will not disappear, it will be left as Thorium. Only a tiny fraction of the mass will be released as energy. So, the block will not lose half the mass.

B) Half-life of 4 billion years is very good for dating ancient rocks, as the age of the Earth is close to 4 billion years. If the half life were much shorter, not much Uranium-238 would have been left in the rocks, so it would be difficult to tell their age.

D) The half life does not change with time, so the remaining U-238 will still have a half-life of 4 billion years.

E) Number of decays is decreasing with time as the sample is getting smaller.

4.7 (5 points) When the Sun and planets were created, there was no element Poodlonium. Labradorium atoms decay to form Poodlonium with a half-life of 800 million years. If you find a rock that is one quarter Labradorium, and three quarters Poodlonium, how old is the rock?

- A) 400 million years.
- B) 800 million years.
- C) 1.6 billion years.
- D) 3.2 billion years.
- E) 6.4 billion years.

**Answer:** C. After one decay, half of the Labradorium remains, after 2 decays, one quarter.