

These are useful quantities and facts that you can use in preparing your homework; this list will also be made available in the exams. You will not understand all of this at the beginning of the course, but by the end, you should be familiar with all of it. If there are quantities that you would like to see added to this list, please let Professor Spitkovsky (anatoly@astro.princeton.edu) know. The appendices to Bennett et al are also a good source of such numbers and formulas. All the numbers here are approximate.

Note that most of the quantities below are given in MKS (meters-kilograms-seconds) units; we will freely switch between those and cgs (centimeters-grams-seconds) in the class.

### Useful Numbers and Formulas

- 1 day  $\approx 1 \times 10^5$  sec
- 1 month  $\approx 3 \times 10^6$  sec
- 1 year  $\approx 3 \times 10^7$  sec
- 1 Astronomical Unit  $\approx 1.5 \times 10^{11}$  m
- 1 light year  $\approx 1 \times 10^{16}$  m  $\approx 60,000$  AU
- 1 parsec  $\approx 3$  light years  $\approx 200,000$  AU
- $0^\circ$  Centigrade corresponds to 273 Kelvins
- Radius of Earth  $\approx 6.4 \times 10^6$  m
- Radius of Moon  $\approx 1.7 \times 10^6$  m
- Radius of Sun  $\approx 7 \times 10^8$  m
- Mass of Earth  $\approx 6 \times 10^{24}$  kg
- Mass of Sun  $\approx 2 \times 10^{30}$  kg
- Surface temperature of Sun  $\approx 6000$  K
- Radius of Moon's orbit around the Earth  $\approx 4 \times 10^8$  m
- Luminosity of Sun  $\approx 4 \times 10^{26}$  Joules/sec
- Newton's constant  $G \approx 2/3 \times 10^{-10}$  m<sup>3</sup> s<sup>-2</sup> kg<sup>-1</sup>
- Planck's constant  $h \approx 2/3 \times 10^{-33}$  Joule sec
- Speed of light  $c \approx 3 \times 10^8$  m/sec
- 1 Joule = 1 kilogram meter<sup>2</sup>/second<sup>2</sup> (a unit of energy)
- Stefan-Boltzmann Constant  $\sigma \approx 5.6 \times 10^{-8}$  W/m<sup>2</sup>/K<sup>4</sup>.
- Mass of proton  $\approx$  Mass of neutron  $\approx$  Mass of Hydrogen atom  $\approx 1/6 \times 10^{-26}$  kg
- 1 radian  $\approx 60$  degrees  $\approx 200,000$  arcsec. A full circle covers 360 degrees, or roughly 6 radians.
- $\pi \approx 3$ ;  $\pi^2 \approx 10$
- The angle subtended by an object in radians is given by its diameter, divided by its distance (the small-angle formula).
- The circumference of a circle of radius  $r$  is  $2\pi r$ ; its area is  $\pi r^2$ .
- The surface area of a sphere of radius  $r$  is  $4\pi r^2$ ; its volume is  $\frac{4}{3}\pi r^3$ .
- The closest and farthest approach of a planet from its parent star in an orbit with semi-major axis  $a$  and eccentricity  $e$  are  $a(1 - e)$  and  $a(1 + e)$ , respectively.
- The acceleration required to keep an object in a circle of radius  $r$  at uniform speed  $v$  is  $a = v^2/r = \omega^2 r$ , where  $\omega = 2\pi/P$ , and  $P$  is the period.

- The amplitude of the gravitational force between two objects of mass  $M$  and  $m$  separated by a distance  $r$  is  $GMm/r^2$ .
- Kepler's Third Law states that for orbits around a body of mass  $M$ , the period squared is proportional to the semi-major axis of the orbit cubed, divided by the mass  $M$ . If period is measured in years, the semi-major axis in AU, and the mass in solar masses, the constant of proportionality is 1. In physical units, Newton's form of Kepler's Third Law is  $P^2 = \frac{4\pi^2 a^3}{GM}$ .
- The kinetic energy of a body of mass  $m$  moving at speed  $v$  is  $\frac{1}{2}mv^2$ .
- The energy per unit time emitted by a blackbody of surface area  $A$  and temperature  $T$  is equal to  $\sigma AT^4$ .
- The blackbody spectrum of an object of temperature  $T$  peaks at a wavelength  $\lambda \approx \frac{2.9}{T}$  millimeter, if  $T$  is measured in Kelvin.
- The kinetic energy of each molecule in a gas of temperature  $T$  is  $\frac{3}{2}kT$ , where  $k = 1.4 \times 10^{-23}$  Joules/Kelvin is the Boltzmann Constant.
- The equilibrium temperature of a planet of albedo  $A$  with no greenhouse effect a distance  $d$  away from a star of surface temperature  $T_{star}$  and radius  $r_{star}$  is  $T_{planet} = (1 - A)^{1/4} T_{star} (r_{star}/2d)^{1/2} = ((1 - A)S/4\sigma)^{1/4}$ , where  $S$  is the brightness of the star as seen from the planet. For a star with the radius and surface temperature of the Sun, this gives a temperature of 270 K at  $d = 1$  AU for  $A = 0$ . With an infrared-absorbing one-layer atmosphere, these expressions are increased by the factor  $2^{1/4}$ .
- The wavelength  $\lambda$  and frequency  $\nu$  of a photon are related as  $\lambda\nu = c$ , where  $c$  is the speed of light. The energy of a photon is its frequency times the Planck Constant  $h$ .
- The brightness  $b$  of a distant object is proportional to its luminosity  $L$  times the inverse square of its distance  $r$  to us:  $b = \frac{L}{4\pi r^2}$ .
- The parallax of a star in arcseconds due to the orbit of the Earth around the Sun is the inverse of its distance in parsecs.

- Energy  $E$  and rest mass  $m$  are equivalent:  $E = mc^2$ .
- The luminosity of a star on the Main Sequence is proportional to its mass to the fourth power.
- Light of wavelength  $\lambda_0$  emitted by an object moving towards or away from us at speed  $v$  has its wavelength shifted to  $\lambda$  such that:  $(\lambda - \lambda_0)/\lambda_0 = \pm v/c$  (the Doppler shift); the positive sign is for an object moving away, and the negative sign for an object coming towards us.
- The Sun is 25,000 light years from the center of the Milky Way, and makes a full orbit once every  $2.5 \times 10^8$  years. The mass of the Milky Way within the Sun's orbit is  $10^{11}$  solar masses.
- Best angular resolution of a telescope of diameter  $D$  observing at wavelength  $\lambda$  is  $2 \times 10^5 \lambda/D$  arcseconds. Here,  $\lambda$  and  $D$  are in meters.
- Hubble's Law: The recession velocity of a galaxy is equal to its distance times the Hubble Constant  $H$ .
- Hubble Constant  $\approx 70$  km/s/Mpc
- The age of the universe since the Big Bang is roughly the inverse of the Hubble Constant. This gives roughly 14 billion years.

- The redshift  $z$  of a galaxy is  $(\lambda - \lambda_0)/\lambda_0$ , where  $\lambda$  is the observed wavelength of a spectral line at Earth, while  $\lambda_0$  is the wavelength at which the light in the line was emitted at the source. For small  $z \ll 1$ , galaxy's recession velocity is  $z = v/c$ .
- The critical density of the universe is  $3H_0^2/8\pi G$ , or roughly  $10^{-26}$  kg/m<sup>3</sup>.  $\Omega$  is defined as the ratio of the true density of the universe to the critical density. The present universe has  $\Omega_{matter} = 0.27$  and  $\Omega_{cosmological\ constant} = 0.73$ .
- Time dilation: An observer moving by me at speed  $v$  will age  $\sqrt{1 - v^2/c^2}$  years, for every year I age, according to me.  $c$  is the speed of light. Lengths are contracted by the same factor. At small speeds, this dilation factor is approximated as  $(1 - \frac{1}{2}v^2/c^2)$ , while at speeds close to the speed of light, it is approximately  $\sqrt{2}(1 - v/c)$ .
- The Schwarzschild radius of a black hole of mass  $M$  is  $2GM/c^2$ . For one solar mass, this corresponds to a radius of 3 km.
- In special relativity, the distance between two events in space-time is:  
 $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$ .
- The Einstein Field Equations of General Relativity are:  $R_{\mu\nu} - 1/2g_{\mu\nu}R = 8\pi T_{\mu\nu}$ . You need not understand the details of this, but the left-hand-side involves the curvature of spacetime, and the right-hand-side involves the mass, energy, and pressure at each point in space and time.