Homework #6, AST 203, Spring 2012

Due in class (i.e., by 4:20 pm), Thursday May 3 (last lecture of the course)

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in.
- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.
- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

100 total points

1. Rocket ships (10 points)
Two spaceships, each measuring 300 m in its own rest frame, pass by each other traveling in opposite directions. Instruments on board spaceship A determine that the front of spaceship A requires 1 microsecond to traverse the full length of B. What is the relative velocity v of the two spaceships (in units of the speed of light)?

2. A Hitchhiker's Challenge (30 points)

> "A full set of rules [of Brockian Ultra Cricket, as played in the higher dimensions] is so massively complicated that the only time they were all bound together in a single volume they underwent gravitational collapse and became a Black Hole."
> *Chapter 17 of Life, the Universe and Everything, the third volume of the Hitchhiker's Guide to the Galaxy series (1982, Douglas Adams)*

A quote like that above is crying out for a calculation. In this problem, we will answer Adams' challenge, and determine just how complicated these rules actually are.

An object will collapse into a black hole when its radius is equal to the radius of a black hole of the same mass; under these conditions, the escape speed at its surface is the speed of light (which is in fact the defining characteristic of a black hole!). We can rephrase the above to say that an object will collapse into a black hole when its *density* is equal to the density of a black hole of the same mass.

a. Derive an expression for the density of a black hole of mass M. Treat the volume of the black hole as the volume of a sphere of radius given by the Schwarzschild radius. As the mass of a black hole gets larger, does the density grow or shrink? (5 points)

b. Determine the density of the paper making up the Cricket rule book, in units of kilograms per cubic meter. Standard paper has a surface density of 75 grams per square meter, and a thickness of 0.1 millimeters. (5 points)

c. Calculate the mass (in solar masses), and radius (in AU) of the black hole with density equal to that of paper. (10 points)

d. How many pages long is the Brockian Ultra Cricket rule book? Assume the pages are standard size ($8.5'' \times 11''$). For calculational simplicity, treat the book as spherical (a common approximation in this kind of problem). What if the rule book were even longer than you have just calculated? Would it still collapse into a black hole? (10 points)

3. Galaxy Rotation Curves and Dark Matter (*35 points*)
We will examine galaxy rotation curves and show that they imply the existence of dark matter.

a) Recall that the orbital period P is given by $P^2 = 4\pi^2 a^3/GM$. Write down an expression that relates the orbital period and the orbital velocity for a circular orbit, and then write down an expression that relates the orbital velocity with the mass enclosed within R. (*Hint: we've done this several times before*) (*5 points*)

b) The Sun is 8,000 parsecs from the center of the Milky Way, and its orbital velocity is 220 km/s. Use your expression from a) to determine roughly how much mass is contained in a sphere around the center of the Milky Way with a radius equal to 8,000 parsecs? (*5 points*)

c) Assume that the Milky Way is made up of only luminous matter (stars) and that the Sun is at the edge of the galaxy (not quite true, but close). What would you predict the orbital velocity to be for a star 30,000 parsecs from the center? 100,000 parsecs? (*10 points*)

d) *Observations* show that galaxy rotation curves are flat: stars move at the same orbital velocity no matter how far they are from the center. How much mass is *actually* contained within a sphere of radius 30,000 parsecs? 100,000 parsecs? Take the orbital velocity at these radii to be the same as the orbital velocity of the Sun. (*10 points*)

e) What do you conclude from all of this about the contents of our galaxy? (*5 points*)

4. Alternative Universe (*25 points*)
Imagine Alan Sandage was right, and the Universe had a Hubble Constant of 50 km/s/Mpc. Calculate each of the following, and compare with the values for the real Universe:

a) You observe a galaxy with the hydrogen 656.3 nm line redshifted to a value of 926 nm. Calculate the redshift for this galaxy, and determine the distance to the galaxy in the "Alan" Universe and the real Universe. (*5 points*)

b) Calculate the age of the universe in the "Alan" Universe and compare with the real value. (*5 points*)

c) How many galaxies are there in the observable Universe? Here you can assume that the average distance between galaxies is comparable to the distance between the Milky Way and Andromeda (2.5 mega light-years). Do this calculation for both Universes. (*10 points*)

d) Finally, using your answer to (c), calculate the total number of stars in the observable universe. Here you can assume that most galaxies are comparable to the Milky Way. (*5 points*)