Homework #4, AST 203, Spring 2012

Due in class (i.e., by 4:20 pm), Thursday April 5

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in.

- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.

- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

100 total points

1. Transits of extrasolar planets (30 points)
NASA mission called "Kepler" was launched in 2009 to observe the transits of extrasolar planets. During a transit, a planet goes in front of its host star as seen from our vantage point on Earth. This blocks a little bit of light from the star, so an accurate measurement of the change in brightness of the star will show a temporary dimming. The main goal of the Kepler mission is to detect Earth-size planets in the habitable zone around stars like our Sun. In this problem, we will calculate the magnitude of the effect that the Kepler mission observes. Consider a planet with the radius of the Earth on a circular orbit with radius of 1 AU about a star that has parameters like our Sun. The orbital plane of this planet lies along our line of sight.

a) How long does a transit last (in hours)? (*10 points*)

   Hint: the Earth is very far from this planetary system.

b) By how many percent does the brightness of the host star diminish in the middle of the transit? (*10 points*)

c) Consider now the planet the size of Jupiter, orbiting at 0.5 AU from the same star. By how many percent does the brightness of the host star diminish in the middle of the transit by this planet? How long does this transit last? (*10 points*)
   Hint: the Earth is still very far from this planetary system.

2. White Dwarfs and Neutron Stars (30 points)
White dwarfs and neutrons stars are "compact objects" – stars whose large mass (near a solar mass) is concentrated in a small volume. Such stars possess exotic properties that allow us to probe the behavior of matter at extreme densities. Consider a white dwarf (WD) with the mass of 1 $M_\odot$ and radius 1 $R_{\rm Earth}$, and a neutron star (NS) with a mass of $1.4 M_\odot$ and radius 10 km.

a) Find how much mass is contained in one teaspoon ($1 cm^3$) of material from WD and NS. Express the answer in metric tons. (*7 points*)
Hint: the material can be taken to be at the mean density of these stars

b) Calculate the mass density of a neutron and compare it to the mean density of NS. A neutron can be considered as a sphere of radius 1 femto-meter (10^{-15} m). (*6 points*)

c) Calculate the mean distance between atoms in WD. Express your result in Ångstroms (1 Ångstrom = 10^{-10} meters). You may approximate the white dwarf to be made entirely of carbon atoms (12 times the mass of hydrogen). Compare with the typical sizes of atoms under normal conditions, $\sim$ 1 Ångstrom. *Hint: We are looking for an approximate answer here.* (*7 points*)

d) We can estimate the rate of rotation of compact objects by knowing that they are the end product of contraction of rotating main sequence stars after they exhaust their nuclear fuel. Each piece of gas in a star contracts in such a way that the product of its distance to the rotation axis times the velocity of rotation about the axis is a constant. (*Hint: we really mean the velocity of rotation, not angular velocity here*). This is known as conservation of "angular momentum," and is the same phenomenon that causes a spinning ice skater to turn faster when she raises her hands. Consider a point on the equator of a the core of the star before the collapse. The radius of the core is half the radius of the Sun and its rotation period is 30 days (like it is for the Sun). Now imagine that this point contracts with the core and ends up on the surface of a neutron star. Find the expected rotation period of such a neutron star (in milliseconds). (*10 points*)

3. Neutrinos Coursing Through Us (25 points total).
In 1987, the astronomical world was electrified with the news of a supernova exploding in the Large Magellanic Cloud, a dwarf galaxy companion to the Milky Way, at a distance of 150,000 light years. It was the nearest supernova to have gone off in 400 years, and was studied in great detail. Its luminosity was enormous; the explosion released as much visible light energy in a few weeks as the Sun will emit in its entire lifetime of 10^{10} years (note that this luminosity is substantially greater than the kinetic energy you calculated in Problem 1c). It was easily visible to the naked eye from the Southern hemisphere. However, theories of the mechanisms taking place in the supernovae predict that the visible light represents only 1% of the total energy of the supernova; there is 100 times more energy emitted in the form of neutrinos, in a blast lasting only a few seconds! When the supernova was discovered, the late John Bahcall of Princeton and his colleagues did the calculation that follows, asking the question whether any of the neutrinos emitted from the supernova should have been detected here on Earth.

a. (7 points) From the information given, calculate the total amount of energy emitted by the supernova in neutrinos. Express your answer in Joules.

b. (5 points) Each neutrino has an energy of roughly 1.5×10^{-12} Joules. Calculate how many neutrinos are emitted by the supernova. (This is an easy calculation, but will give you a very large number!).

c. (8 points) Scientists have built a number of detectors of neutrinos (in order to look for neutrinos from the Sun, as we discussed in class). One of the largest is called Kamiokande in Japan. In 1987, it consisted of a cube of water roughly 10 meters on a side (it has since been expanded). Calculate how many neutrinos from the supernova would have passed through the cube of water. *Hint: the inverse square law holds for neutrinos just as it does for light. Consider the area of the detector, and remember that the neutrinos are sent out in all directions from the supernova; at the time that the neutrinos hit Kamiokande, they are spread out over an enormous sphere centered on the supernova.*

d. (5 points) Neutrinos are ghostly particles; they are very difficult (but not completely impossible) to detect. A typical neutrino can pass through the entire Earth without anything happening to them. However, it turns out that roughly 1 in 10^{15} (i.e., one in a quadrillion!) of the neutrinos passing through Kamiokande will interact with the water there, and thereby be detected. Calculate how many neutrinos should have been detected by Kamiokande.

The neutrinos detected by Kamiokande in 1987 serve as a dramatic confirmation of the theories that claimed that most of the supernova energy comes out in the form of neutrinos. These remain the only neutrinos ever seen from an astronomical source more distant than our own Sun.

4. True or False? (15 points)

Determine if the following statements are true or false, and give the reasoning to support your conclusion in a short paragraph.

a) If the Sun had been born as a high-mass star some 4.5 billion years ago, rather than as a low-mass star, the planet Jupiter would probably have Earth-like conditions today, while the Earth would be hot like Venus. (*5 points*)

b) An open cluster that contains many O stars is unlikely to also have lots of white dwarfs. (*5 points*)

c) If a $3M_\odot$ main-sequence star is in a binary with a 2.5 $M_\odot$ red giant, the red giant must have been more massive than $3M_\odot$ when it was a main-sequence star. (*5 points*)