# Homework #3, AST 203, Spring 2012

*Due in class (i.e. by 4:20 pm),* **Thursday** *March 8*

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in. Please read `http://www.astro.princeton.edu/ast203/writeup.pdf`

- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.
  Please read `http://www.astro.princeton.edu/ast203/mathtips.pdf`

- A number of useful formulas and quantities are summarized at `http://www.astro.princeton.edu/ast203/formulas.pdf`

- Feel free to work with your classmates on this homework, but your write-up and wording should be **your own**. Answer all questions.

**100 total points**

**1. Greenhouse on Titan** (*30 points*)

This question asks you to calculate temperatures and the implications of those temperatures for Saturn's large moon Titan. Titan (radius 2,576 km) is slightly larger than the planet Mercury and is the only moon in the solar system with a dense atmosphere – its atmospheric surface pressure is 50% greater than Earth's, in fact. Nevertheless, it is far from the Sun...

a) (*10 points*) Saturn orbits 9.6 AU from the Sun, and Titan orbits Saturn. Treat Titan as having, on average, the same distance from the Sun as Saturn. Neglecting the greenhouse effect for now, write down an equation that balances the power (or, energy per time) from the Sun absorbed by Titan with the power radiated away from Titan, treating Titan as a black body with albedo A. Use symbols for luminosity $L$ of the Sun, distance from Sun to Titan, $d$, radius of Titan, $r$, and its temperature $T$. Do not yet fill in any values for your variables.

b) (*10 points*) Take Titan's albedo to be 0.22. Assuming no greenhouse atmosphere, what is the expected black-body temperature (in Kelvin) of Titan from the equation you set up in part a)? Take the solar luminosity to be $3.9 \times 10^{26}$ W.

c) (*2 points*) Titan's atmosphere is mostly nitrogen, like Earth's, but contains nearly 2% methane – a powerful greenhouse gas. Assume Titan's atmosphere provides an ideal greenhouse. We saw in lecture that this would be expected to increase the surface temperature of a world by a factor of $2^{1/4}$ or about 1.2. If Titan's atmosphere were a perfect greenhouse, what would you expect Titan's surface temperature to be?

d) (*2 points*) Earth's average temperature is 288 K. Extreme temperatures at the poles and equator vary about this average by as much as 90 K lower and higher, respectively. Assume that Titan sees a temperature variation (about its surface greenhouse temperature) from its poles to equator proportionally the same as Earth's. What are the minimum and maximum temperature on Titan likely then to be?

e) (*6 points*) Download and print the file "methane phase diagram" (Fig. 1) from the Blackboard site under Course Materials. Titan's surface pressure is 1.5 bar, but on the logarithmic pressure scale of Fig. 1, this can be well approximated as 1.0 bar, written as "1.0E0". Using a pencil and ruler, and referring to the temperature scale on the x-axis, draw a horizontal line on this plot indicating the temperature range that could be experienced on Titan's surface. What phases of methane are possible across this temperature range? On Earth, a meteorological cycle is driven as water moves between solid, liquid, and vapor phases. Is a meteorological cycle, but with methane instead of water, possible on Titan?

**Problems 2-5. A New Denizen of the Solar System**

The discovery of the dwarf planet "Eris" in 2005 threw the astronomical community into a tizzy and made international headlines; it is slightly larger than Pluto (as we'll see) and brought up interesting questions about what the definition of a planet is. Eventually, this resulted in the controversial demotion of Pluto from the 9th planet of the Solar System to just one of a number of dwarf planets.

As a top AST 203 student, you have been named to head a team of astrophysicists to calculate some of properties of Eris. For the next four problems of this homework, you are going to be using the same tools that professional astronomers used to understand the properties of this object. Throughout, assume that Eris is spherical and is observed at opposition (i.e., the Earth lies on the straight line connecting the Sun and Eris).

**2.** (15 points) First, you will determine how far Eris is. In five hours, Eris is observed to move 7.5 arcseconds relative to the background stars as seen from Earth; see `http://www.astro.princeton.edu/ast203/eris.gif`. Because (as we'll see) Eris is much further from the Sun than is the Earth, it is moving quite a bit slower around the Sun than the Earth, so this apparent motion on the sky is essentially entirely *parallax* due to the Earth's motion. First, calculate the speed with which the Earth goes around the Sun, in kilometers/second. Then use this information and the small-angle formula to calculate the distance from the Earth to Eris. Express your result in AU. Compare with the semi-major axis of Pluto's orbit (which you'll need to look up). *Hint: it will help to draw a diagram.*

**3.** (30 points) We're next going to calculate how big Eris is, from its apparent brightness. We'll have to do a few calculations along the way. Eris shines in two ways: from its reflected light from the Sun (which will be mostly visible light), and from its blackbody radiation from absorbed sunlight (which will mostly come out as infrared light); it will be important in the calculations that follow to keep in mind which of the two you're dealing with. The albedo of Eris (i.e., the fraction of the sunlight incident on Eris that is reflected) is very high, about 85%. This suggests that Eris is covered by a layer of shiny ice; spectroscopy tells us that the ice is composed of frozen methane, $CH_4$.

(a) (10 points) We are going to start by deriving an expression for the brightness of Eris which depends on its distance from the Sun $d$, and its radius $r$. First, calculate the amount of sunlight reflected by Eris per unit time (i.e., its luminosity in reflected light); express your answer in terms of $d$, $r$, the albedo $A$, and the luminosity of the Sun $L$ (you will find this similar to problem 1, but with one important difference). Don't plug in numbers yet!

(b) (10 points) We detect only a tiny fraction of this light reflected by Eris. Calculate the brightness, via the inverse square law, of Eris as perceived here on Earth. Again, don't plug in numbers yet; just do algebra; again your answer will involve $d, r, A$, and $L$. (*Hint: With Eris so distant from the Sun, the tiny 1 AU separation between the Earth and the Sun is negligible. So make the approximation that the distances from the Sun to Eris, and from Eris to the Earth, are the same.*)

(c) (10 points) OK, now for some numbers. The measured brightness of Eris is $2.4 \times 10^{-16}$ Joules meters$^{-2}$ second$^{-1}$. We know the distance $d$ to Eris and its albedo $A$, and can look up the luminosity of the Sun $L$. Use this information to solve for the radius $r$ of Eris. (*Hint: be careful to be consistent with your units!*) Compare with the radius of Pluto (1200 km). This is what led to the controversy of what a planet is: if Pluto is considered a planet, then certainly Eris should be as well. We'll write much more about this controversy, and some of the related issues, in the solution set to this homework.

**4.** (10 points) Calculate the angular size of Eris (i.e., the angle the diameter of Eris makes on the sky). Compare this to the resolution of the Hubble Space Telescope (i.e., the smallest things it can see as extended), which is about 40 milli-arcseconds; will you be able to resolve Eris (i.e., will it look like a point of light or a finite-size object in a telescope)?

**5.** (15 points) You go ahead and observe Eris with Hubble, and find that it has a moon orbiting it! Observations with Hubble show that this moon (called Dysnomia) makes an almost circular orbit around Eris with a period of 15.8 Earth days. The semi-major axis of the orbit subtends an angle of $0.53''$ as seen from Earth. Calculate the semi-major axis in kilometers, and calculate the mass of Eris in kilograms. Compare with the mass of Pluto ($1.3 \times 10^{22}$ kg). Is Eris more massive?