

Homework #2, AST 203, Spring 2012

Due in class (i.e. by 4:20 pm), Tuesday February 28

*General grading rules: One point off per question (e.g., 1a or 1b) for egregiously ignoring the admonition to set the context of your solution. Thus take the point off if relevant symbols aren't defined, if important steps of explanation are missing, etc. If the answer is written down without *any* context whatsoever (e.g., for 1a, writing "164.85 years", and nothing else), take off 1/3 of the points. One point off per question for inappropriately high precision (which usually means more than 2 significant figures in this homework). However, no points off for calculating a result to full precision, and rounding appropriately at the last step. No more than two points per problem for overly high precision. Three points off for each arithmetic or algebra error. Further calculations correctly done based on this erroneous value should be given full credit. However, if the resulting answer is completely ludicrous (e.g., 10^{-30} seconds for the time to travel to the nearest star, 50 stars in the visible universe), and no mention is made that the value seems wrong, take three points further off. Answers differing slightly from the solutions given here because of slightly different rounding (e.g., off in the second decimal point for results that should be given to two significant figures) get full credit. One point off per question for not being explicit about the units, or for not expressing the final result in the units requested. Two points off for the right numerical answer with the wrong units. Leaving out the units in intermediate steps should be pointed out, but no points taken off. Specific instructions for each problem take precedence over the above. In each question, one cannot get less than zero points, or more than the total number the question is worth.*

100 total points

1. Neptune and Pluto 20 points total

The planet Neptune, the most distant gas giant from the Sun, orbits with a semimajor axis $a = 30.066$ AU and an eccentricity $e = 0.01$. Pluto, the next large world out from the Sun (though much smaller than Neptune) orbits with $a = 39.48$ AU and $e = 0.250$.

- a. To correct number of significant figures given the precision of the data in this problem, how many years does it take Neptune to orbit the Sun? (4 points)

Solution: This is a simple application of Kepler's Third Law, P (years) = $a^{3/2}$ (AU). For $a = 30.066$ AU, this gives (yes, I used a calculator) 164.85 years. I give the answer to five significant figures, the same number as I have for the semi-major axis.

Take 1 point off for fewer or more significant figures. However, any answer close to 164.8 years gets credit; calculators can actually differ in the exact value in the fifth decimal place. 1 point for recognizing that Kepler's Third Law is relevant, and going no further.

- b. How many years does it take Pluto to orbit the Sun? (4 points)

Solution: The exact same calculation for Pluto gives 248.1 years, to four significant figures.

Take 1 point off for fewer or more significant figures. However, any answer close to

248 years gets credit; calculators can actually differ in the exact value in the fourth decimal place.

- c. Take the ratio of the two orbital periods you calculated in parts (a) and (b). You'll see that it is very close to the ratio of two small integers; which integers are these? Thus the two planets regularly come close to one another, in the same part of their orbits, which allows them to have a maximum gravitational influence on each other's orbits. This is an example of an orbital resonance (other examples in the solar system can be found among the moons of Jupiter, and between the moons and various features of the rings of Saturn). (4 points)

Solution: The ratio of orbital times is $248/164.8 = 1.505$ to four significant figures. This is quite close (within 0.3%) to a ratio of 3:2. That is, every time Pluto makes two orbits around the Sun, Neptune makes three orbits.

Full credit for calculating the ratio and recognizing that this is 3:2. Stating the ratio is 1.5 and not saying 3:2 gets two points off. Taking the difference in the result from 1.5 seriously, and working out the ratio as some enormous pair of numbers (e.g., 301 to 200) gets three points off. These resonances are actually quite common in the Solar System. It turns out that many of the gaps in Saturn's rings are due to resonances with the various moons of Saturn, and more complicated resonances explain some of the stunning detailed features seen in those rings. See for example the pictures taken by the Cassini spacecraft at:

<http://saturn.jpl.nasa.gov/multimedia/images/images.cfm?categoryID=5>.

- d. What is the aphelion distance of Neptune's orbit? Express your answer in AU. (3 points)

Solution: The eccentricity of the orbit is very close to zero, so we're not surprised that the aphelion distance, $a(1 + e) = 30.4$ AU, is very close to the semi-major axis. Here, the number of significant figures is subtle. You might think that the eccentricity is known to only a single significant figure, so that the aphelion should be given to the same significance. In fact, what counts in this calculation is the quantity $1 + e$, which has three significant figures.

One point off for simply stating that e is tiny, so the aphelion distance is identical to the semi-major axis. One point off for rounding to a single significant figure. No points off for writing 2, 3, 4 or 5 significant figures (30.367); the correct number of significant figures in this problem is a quite subtle one.

- e. What are the perihelion and aphelion distances of Pluto's orbit? Is Pluto always farther from the Sun than Neptune? (5 points)

Solution: The perihelion is $a(1 - e) = 39.48 * (1 - 0.250)$ AU = 29.61 AU, and the aphelion, similarly, is $a(1 + e) = 49.35$ AU. The perihelion distance of Pluto is less than the aphelion distance of Neptune, so indeed, Pluto is sometimes a bit closer to the Sun than is Neptune. It only gets a little inside Neptune's orbit, and it turns out that it was last inside Neptune's orbit from 1979 to 1999.

Two points each for the aphelion and perihelion calculation, and one point for an answer to the question about whether Pluto ever comes closer to the Sun than Neptune.

2. Satellites *30 points total*

In this problem we will talk about properties of satellite orbits.

- a) A satellite in geosynchronous orbit (GEO) orbits the Earth once every day. A satellite in geostationary orbit (GSO) is a satellite in a circular GEO orbit in Earth's equatorial plane. Therefore, from the point of view of an observer on Earth's surface, a satellite in GSO seems always to "hover" in the same point in the sky. For example, the satellites used for satellite TV are in GSO so that satellite dishes can be stationary and need not track their motion through the sky. Take a look; you'll notice all satellite dishes on people's houses point towards the Equator, i.e., South. How far above Earth's equator (i.e., above the Earth's *surface*) is a satellite in GSO? Express your answer in kilometers, and in Earth radii. (*10 points*).

Solution: This is another application of Kepler's Third Law; we have a period (24 hours) and want to find a radius of the orbit. However, note here that this is *not* an orbit around the Sun, and so Kepler's Third Law in its original form is not valid. Rather, we can use Newton's Form of Kepler's Third Law:

$$a^3 = \frac{GM_{\oplus}P^2}{4\pi^2}.$$

In doing this calculation, we need to work in MKS units throughout. $M_{\oplus} = 6 \times 10^{24}$ kg is the mass of the Earth, $G = \frac{2}{3} \times 10^{-10} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$ is Newton's constant, the period is one day, which I'll approximate as 90,000 seconds. Thus:

$$a^3 = \frac{2/3 \times 10^{-10} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1} \times 6 \times 10^{24} \text{ kg} \times 8 \times 10^9 \text{ sec}^2}{40},$$

where I've approximated $\pi^2 \approx 10$. Gathering the terms together, I find:

$$a^3 = 8 \times 10^{22} \text{ m}^3$$

So far, no calculator. How am I going to take the cube root without a calculator? Well, I know that $8 \times 10^{22} = 80 \times 10^{21}$, and I know that the cube root of 10^{21} is 10^7 . I know that $4^3 = 64$ and $5^3 = 125$, so the cube root of 80 is a shade over 4. So to a single significant figure, I find

$$a = 4 \times 10^7 \text{ m} = 40,000 \text{ km}.$$

Are we done? Well, we were asked for the distance from the Earth's *surface*, whereas what we've calculated is from the Earth's *center*. So we need to subtract from this the radius of the Earth, 6400 km, leaving roughly 34,000 km. And we're asked to express this number in Earth radii; dividing by 6400 km gives a bit over 5 Earth radii.

If I redo the calculation, using precise values for the Earth's mass, G , and π (and using a calculator), I find a distance of 35,900 km above the Earth's surface, about 5.63 Earth radii.

Another possible way to do the problem would be to recognize that Kepler's Third Law can be written in the form:

$$a^3 = P^2/M,$$

with a in AU, P in years, and M in solar masses. This has the advantage of skipping the need to look up the values of G and π , but would require the additional work of converting all our numbers to those (in this case inconvenient) units.

2 points for writing down some form of Kepler's Third Law and going no further. 3 points off for forgetting to subtract the radius of the Earth in the last step. 6 points off for using the Sun's mass rather than the Earth's mass in the calculation. If you do this, you get a radius about 60 times larger than we've calculated above.

Calculations to a single significant figure, or self-consistent calculations to more significant figures, get full credit. But take a point off if the result is given to many significant figures, and the input numbers used (e.g., mass of the Earth, length of a day) are to significantly fewer significant figures.

- b) Space station MIR traveled 3.6 billion kilometers during its life. It's circular orbit was 200 km above the surface of the Earth. How many years was it in orbit? (10 points)

Solution

The radius of the orbit was $R = 6400 \text{ km} + 200 \text{ km} = 6600 \text{ km}$. The circumference of this circular orbit is $2\pi R = 4.2 \times 10^4 \text{ km}$. So, the station has done 8.6×10^4 orbits. Now, how long does each orbit take? This is the inverse of the problem we've done above in a). We know the semi-major axis of the orbit (the radius for a circular orbit), and can use Kepler's law applied to the Earth to find the period:

$$P = \left(\frac{4\pi^2}{GM_{\oplus}} \right)^{1/2} a^{3/2} \approx 5400\text{s} = 90\text{min}$$

So, if every orbit takes roughly 90 minutes, 8.6×10^4 orbits take $4.6 \times 10^8 \text{ s}$, or 15 years, rounding off to the needed precision. We could have also done this calculation by scaling the Kepler's law that we found for the geostationary satellite in part a). We show how to do a similar scaling in part c) below.

3 points off for not adding the Earth's radius to get the radius of the orbit. Answers close to 15 years are also accepted.

- c) How many times did MIR circle the Earth per day (i.e., 24 hours)? Can you put a satellite into such an orbit that it circles the Earth 20 times per day? (10 points)

Solution

We found that the period of each orbit was close to 90 minutes. Therefore, it made $24 \times 60/90 = 16$ turns per day. Ok, what would an orbit of 20 revolutions per day look like? Let's find the radius of this orbit. The period of this orbit is shorter by 16/20 from 90 minutes for Mir (or 72 minutes). We can find its orbital radius the hard way by using the full Newton's version of Kepler's law again. Or, we can remember that Kepler's law holds for both orbits around the Earth, so we can find the ratio of both expressions. Namely:

$$\left(\frac{P_{\text{new sat}}}{P_{\text{Mir}}} \right)^2 = \left(\frac{a_{\text{new sat}}}{a_{\text{Mir}}} \right)^3,$$

$$a_{\text{new sat}} = a_{\text{Mir}} \left(\frac{P_{\text{new sat}}}{P_{\text{Mir}}} \right)^{2/3} = a_{\text{Mir}} \left(\frac{16}{20} \right)^{2/3} = 0.86 a_{\text{Mir}} = 5700 \text{ km}.$$

This would have been a nice circular orbit, except for the fact that it is 700 km underground...

the number of revolutions is worth 4 points, 4 points off for not realizing something is wrong with the orbit in question.

3. Emission of Light 10 points total

This problem concerns the luminosities, sizes, and temperatures of stars and planets. You may assume that all radiate as blackbodies.

- (a) (*5 points*) Venus has an average surface temperature of about 700 K, while the Sun's surface temperature is about 6000 K. Venus' radius is roughly 6050 km, and the radius of the Sun is roughly 700,000 km. What is the ratio of luminosity of the Sun to that of Venus? At what wavelength does Venus' emitted radiation peak? Express your answer in microns.

Solution: The luminosity of a blackbody of surface temperature T and radius R is proportional to R^2T^4 , by the Stefan-Boltzmann Law. So the ratio of the luminosities is:

$$\frac{L_{Sun}}{L_{Venus}} = \left(\frac{R_{Sun}}{R_{Venus}} \right)^2 \left(\frac{T_{Sun}}{T_{Venus}} \right)^4.$$

Note that I first take the ratio of radii, then square (and similarly for temperature), as that will be much easier arithmetically. See the mathtips at the beginning of the course. Plugging in numbers, we get:

$$\frac{L_{Sun}}{L_{Venus}} = \left(\frac{7 \times 10^5 \text{ km}}{6 \times 10^3 \text{ km}} \right)^2 \left(\frac{6000 \text{ K}}{700 \text{ K}} \right)^4.$$

Here I rounded 6050 km to 6000 km, as all numbers I had in the problem were to one significant figure. Doing the calculation (and yes, I used my calculator), I get

$$\frac{L_{Sun}}{L_{Venus}} = 7 \times 10^7.$$

Wow, the Sun is almost 100 million times more luminous than is Venus!

The Wein Law tells us the wavelength at which the emission from Venus peaks, namely:

$$\lambda_{peak} = \frac{0.29 \text{ cm}}{T} = \frac{0.29 \text{ cm}}{700} = 4 \times 10^{-4} \text{ cm} = 4 \text{ microns}.$$

This is light in the infrared part of the spectrum.

Our eyes are insensitive to infrared light, but Venus is easy to see with the naked eye (indeed, it is the brilliant "star" visible soon after sunset in the West on clear evenings). What we're seeing is not the light *emitted* by the planet, but rather the light *reflected* from the Sun. It turns out that the total luminosity of the two are roughly comparable, as we'll learn in another few lectures.

3 points for the luminosity ratio calculation (full credit if they calculate the inverse ratio, consistently), and 2 points for the peak wavelength calculation.

- (b) (5 points) Consider a red giant star that has a diameter eighty times that of the Sun, but a surface temperature of only 3,000 K. What is the ratio of its luminosity to that of the Sun? At what wavelength does the red giant's emitted radiation peak? What color is it? (Hint: if you conclude its color is blue, check your work.)

Solution: This is very similar to the previous calculation. We write:

$$\frac{L_{Red\ Giant}}{L_{Sun}} = \left(\frac{R_{Red\ Giant}}{R_{Sun}}\right)^2 \left(\frac{T_{Red\ Giant}}{T_{Sun}}\right)^4.$$

We are told that the ratio of the diameters (and therefore the radii) is 80. The Sun has a surface temperature of 6000 K, and therefore the ratio of temperatures is 1/2. Thus:

$$\frac{L_{Red\ Giant}}{L_{Sun}} = (80)^2 \times \left(\frac{1}{2}\right)^4.$$

$2^4 = 16$, so this becomes:

$$\frac{L_{Red\ Giant}}{L_{Sun}} = \frac{8 \times 8 \times 100}{16} = 400,$$

without having to touch a calculator. By the Wein Law, the peak wavelength is

$$\lambda_{peak} = \frac{2.9\text{ cm}}{3000} = 1\text{ microns}.$$

This is in the near-infrared part of the spectrum. Our eyes are not sensitive to light of wavelength one micron, but there is substantially more light coming out at red wavelengths than blue wavelengths, so the object appears red (as its name implies!). *Two points for the luminosity ratio, two points for the peak wavelength, and one point for the color.*

4. Radioactive Decay 10 points

You will need to refer to the periodic table, e.g. in Appendix D of the text, to answer this problem.

Uranium-238 (^{238}U), an isotope of uranium that cannot be used to make a nuclear explosion, can absorb a neutron in a nuclear reactor. Which isotope does it then become? The resulting nucleus then undergoes beta decay with a half-life of 24 minutes. In the beta decay, one of the neutrons in the nucleus of the atom decays into a proton and an electron, and the electron is emitted from the nucleus. What type of atom results? This element is also unstable, and undergoes another beta decay with a half-life of 2.4 days. What type of atom results from this second beta decay? The resulting isotope is one that can be used in an atomic weapon. This is why nuclear reactors provide one potential route to atomic weapons. For all of the above, give the the symbol of the resulting isotope and its atomic mass.

Solution: The first neutron absorption doesn't change the number of protons (so the nucleus is still Uranium), but its atomic mass has gone up by one, so we have ^{239}U .

With the emission of an electron, an extra proton now resides in the nucleus, so it moves up one space on the periodic table, namely element 93, Neptunium (^{239}Np). Note that

the atomic mass doesn't change. A second beta decay does the same; the atomic mass is unchanged, but we're now at element 94, and thus we have plutonium, ^{239}Pu .

4 points for the first isotope, 3 points for each of the subsequent ones. Two points off any of these for getting the atomic mass wrong.

5. Dating a lunar meteorite 10 points

You have been asked to determine the age of a strange meteorite that, from its appearance, you hypothesize used to be part of a volcanic lava flow on the Moon. You are using the potassium-argon ($^{40}\text{K}/^{40}\text{Ar}$) method, and (in the simplified form in which we've discussed it) you know that the half-life for this decay is 1.3 billion years. Your measurement gives $^{40}\text{K}/^{40}\text{Ar} = 1/7$. How old is the meteorite?

Solution: As we learned in class, the Argon is created by the radioactive decay of the Potassium. So if there is one part Potassium to 7 parts Argon, there was originally 8 parts Potassium, and no Argon whatsoever. Let us make a chart of what the relative amounts of Potassium and Argon there are after a certain number of half-lives, where we recognize that the Potassium and Argon have to add up to 8 parts at any time, and the amount of Potassium drops by a factor of two each half-life (by definition!):

Half-lives	Parts Potassium	Parts Argon
0	8	0
1	4	4
2	2	6
3	1	7

That's the answer we're looking for; after 3 half-lives, we expect the ratio of Potassium to Argon we observe. Three half-lives is 4 billion years, so the sample is 4 billion years old.

Two points off for giving the table, without any explanation whatsoever.

6. Cratering on the Moon 20 points

This problem asks you to work with a lunar photograph returned by one of the Apollo missions to estimate the age of a surface on the lunar far side.

The photo you will be working with is posted on the Blackboard site for the course under Course Materials, as "Apollo photo AS16-4136" – indicating that this photograph (recorded on film) was obtained by the Apollo 16 orbiter. The orbiter was at an altitude of 119 km when the photo was taken. What is important for us is the 10 km scale bar reproduced with the photo. The text (from a NASA special publication) that accompanies the photo is interesting, but not directly germane to this problem.

- a) (*5 points*) AS16-4136 is called figure 103 in the page reproduced and available on Blackboard. Print out a copy of the page, including the photo. Note the 10 km scale bar. Using a centimeter ruler, and noting the direction north indicated at the opposite corner of the photo, determine the east-west and north-south extent of the photo, in km. Then determine the total number of square kilometers captured by the photo. Report your results to two significant figures only.

Solution: From the printout we measure the size of the picture to be 25 by 55 km, or surface area of 1400 km^2 , to two significant figures.

- b) (*5 points*) Using the 10 km scale bar, determine the diameter on your hardcopy that corresponds to a crater 4 km in diameter. Survey the photo and circle with a pen or pencil every crater 4 km or larger that you see. Circles may overlap. Note that some craters are sharp and others may be very degraded. Do your best; crater counting, especially with a small image such as this, has subjectivity to it. Count any partial craters that are more than 50% in the photo. Include this sheet, with the circles, in the homework that you hand in. How many craters greater than 4 km in diameter do you count?

Solution: We count 6 craters that are larger than 4km in diameter in an area of 1400 km^2 . Estimates may range between 5 and 9 craters, depending on how people count the degraded craters.

- c) (*5 points*) Now refer to the lunar cratering histogram, also posted on the blackboard site under course materials. Note that its y-axis is in units of number of craters bigger than 4 km across (per million square kilometers). Using ratios and your results from steps 1 and 2 of this problem, convert your results these units.

Solution: 6 craters / 1400 km^2 translates to 4100 craters per million square kilometers. At the high end of 9 craters, this translates to 6100 craters/million square kilometers.

- d) (*5 points*) To 2 significant figures, in billions of years, how old is the lunar terrain photographed in AS16-4136? Do we have evidence that there was life present on Earth at this time, or does this surface predate the origin of life on Earth (at least as far as we can tell from the evidence available)?

If you want to see exactly where this image is situated on the lunar far side, you may look for the number 103 on the lunar far-side map in figure 96 of NASA SP-362, available at <http://history.nasa.gov/SP-362/ch5.1.htm> .

Solution: We get the number of craters translating to the Uplands point on the cratering diagram to be The correct number is about 4.2 Gyrs (Gyr –giga-year– is a billion years). Answers between 4 Gyr and 4.4 Gyrs are ok.

Life is thought to be about 3.5 Gyrs old on the Earth based on the fossil record, so the lunar surface predates the emergence of life on Earth.