

# Homework #1, AST 203, Spring 2012

*Due in class (i.e. by 4:20 pm), Thursday February 16*

- To receive full credit, you must give the correct answer *and* show that you understand it. This requires writing your explanations in full, complete English sentences, clearly labeling all figures and graphs, showing us how you did the arithmetic, and being explicit about the units of all numbers given. All relevant mathematical variables should be explicitly defined. And please use your best handwriting; if we can't read it, we can't give you credit for it! Please staple together the sheets of paper you hand in. Please read <http://www.astro.princeton.edu/ast203/writeup.pdf>
- Most of the calculations in this course involve numbers that are only approximately known. The result of such a calculation should reflect this imprecision. In particular, it is *wrong* to simply write down all the digits that your calculator spits out. Your final answer should have the same number of significant figures as the least precise number going into your calculation. In many (but not all!) cases, it's best to do the problems without a calculator.  
Please read <http://www.astro.princeton.edu/ast203/mathtips.pdf>
- A number of useful formulas and quantities are summarized at <http://www.astro.princeton.edu/ast203/formulas.pdf>
- Feel free to work with your classmates on this homework, but your write-up and wording should be your own. Answer all questions.

## **100 total points**

### **1. Scientific notation review**      *10 total points*

Write the following in proper scientific notation, giving the proper number of significant figures. Full sentences are not required here.

- a) Four hundred seventy million (*2 points*)
- b) Seven trillion, two hundred seventy-one billion (*2 points*)
- c) one two hundredth (*2 points*)
- d) The speed of light is  $3.0 \times 10^5$  km/sec. What is the speed of light in units of cm/sec? (*2 points*)
- e) The number  $\pi \times 13$  (*2 points*)

### **2. Looking out in space and back in time**      *20 total points*

Because the speed of light is finite (not infinitely fast), looking at anything means seeing the thing as it was some time in the past. This usually doesn't matter in daily life on Earth, but it is often important in astronomy and spacecraft engineering.

Take the speed of light to be  $3.0 \times 10^5$  km/sec. Take one astronomical unit (AU) to be 150 million kilometers.

- a) To correct precision, how far back in time are we looking when we look at the Sun from the Earth (give your answer in minutes); (*3 points*)
- b) How far back in time are we seeing the star Betelgeuse, which is 643 light years away? Give your answer in years. (*3 points*)
- c) Suppose you were trying to have a conversation with an astronaut on the Moon. The Moon is 384,400 km from Earth. If you sent a radio signal to an astronaut on the Moon, and she replied immediately, how long would be the gap between when you sent the signal and you received your answer? (*5 points*)

Now imagine that you were an engineer for the Mars Exploration Rover that is driving around on the surface of Mars. Suppose you need to send it an emergency transmission to prevent it from driving into a ditch. Mars is on a roughly circular orbit of radius 1.5 AU around the Sun. The Earth is also on a circular orbit, of radius 1.0 AU, of course. (For these two planets at the level of precision asked for in this problem, the approximation of their orbits as circles is a good one.) About how long would it take your message to reach the rover (in minutes), when:

- d) Mars is in opposition, i.e. when the Earth lies on the line between Mars and the Sun? (*5 points*)
- e) Mars is farthest from Earth in the two planets' orbits? (*4 points*)

### 3. The density of air and the Galaxy *40 points total*

A cubic meter of air at sea level contains approximately  $3 \times 10^{25}$  molecules. To a fair approximation, all the molecules are  $\text{N}_2$ , i.e. a pair of nitrogen atoms bound together into a diatomic molecule. (Note that the part of the air that you need to breathe, i.e., the oxygen ( $\text{O}_2$ ) molecules, comprises only about 21% of the air by volume; we ignore this  $\text{O}_2$  fraction here and pretend that the whole atmosphere is  $\text{N}_2$ .) Each N atom has a mass 14 times that of hydrogen (H).

- a) Calculate the mass density (kilograms per cubic meter) of air on Earth. Do this calculation without a calculator, show your work, and use scientific notation. In this and in all parts of this problem, give the final answer using the appropriate amount of precision. You will need the mass of a hydrogen atom (to a good approximation the same as the mass of a proton,  $1.6 \times 10^{-27}$  kg). (*10 points*)
- b) To a fair approximation, the total volume of Earth's atmosphere can be treated as lying within 8 km of the surface of the planet. (Of course the atmosphere extends much higher, but it becomes exponentially less dense with increasing height; in fact about 60% of the atmosphere lies within 8 km of the surface.) The Earth is a sphere of radius 6,400 km. Using the results of a), calculate the mass of the Earth's atmosphere

in kilograms. You may use a calculator if you wish, but give answers to the proper amount of precision in scientific notation, and show your work. *Hint: the volume of a thin spherical shell is the surface area of the sphere times the thickness of the shell. (10 points)*

- c) How many tons of air are over your head when you walk outside? (*Hint: you may find it helpful to estimate or measure the circumference of your head.*) Do you feel the weight of this air? (10 points)
- d) The Milky Way Galaxy contains about  $10^{11}$  stars with mass similar to that of the Sun, and has a radius of roughly 50,000 light years. Approximating the Milky Way Galaxy as a sphere, compute its mean density in kilograms per cubic meter, and compare with (i.e., take the ratio with) the density you calculated for Earth's air. Do this calculation without a calculator, show your work, and use scientific notation. Do you find the result surprising? (10 points)

#### 4. Kepler's third law      20 total points

Kepler's third law, or the "harmonic law," says that the square of the orbital period  $P$  of a body orbiting the Sun is proportional to the cube of the body's semimajor axis  $a$ . If  $P$  is measured in years and  $a$  is measured in AU, the proportionality constant is  $1 \text{ year}^2/\text{AU}^3$ , and in these units Kepler's third law is:  $P^2 = a^3$ . Consider the following data for three objects orbiting our Sun. In a-c), how long (in years) does it take each of these bodies to complete one orbit? You may use a calculator for these problems.

- a) Venus has a semimajor axis of 0.72333 AU; (4 points)
- b) The asteroid Ceres has a semimajor axis of 2.766 AU; (4 points)
- c) Jupiter's orbit around the Sun has a semi-major axis 5.20 times larger than that of the Earth. (4 points)
- d) Using the result of c), what is Jupiter's orbital velocity, in km/s? Assume it is on a perfectly circular orbit. (8 points)

#### 5. Order of Magnitude Estimate (10 total points)

Most of the homework problems in AST203 will be self-contained, meaning that you will be given all the information needed to solve the problem in the problem text. In the real world, however, we often encounter problems, which are not well defined, where not all the information is given, and some assumptions have to be made to get an answer. To make a first pass at such problems it is very useful to make an educated rough estimate of the unknowns, often to the nearest power of 10, and proceed with a quick calculation to get a "ballpark" estimate of the answer. Such "order of magnitude" (OOM) estimates are very common in science, and can be extremely useful in everyday life as well. A good example of an OOM estimate is the classic Fermi problem (named after Enrico Fermi, who was fond of such calculations): "How many piano tuners are there in Chicago?" You can read its solution on Wikipedia: [http://en.wikipedia.org/wiki/Fermi\\_problem](http://en.wikipedia.org/wiki/Fermi_problem) .

The problem for you is this: *Estimate how many roses are bought on Valentine's day in the US.* Please show your assumptions and estimates. The precise number you get is less important than your reasoning behind it, explaining how you estimated it.