Protostellar disks: gas/grain dynamics (Part I)

AST 541

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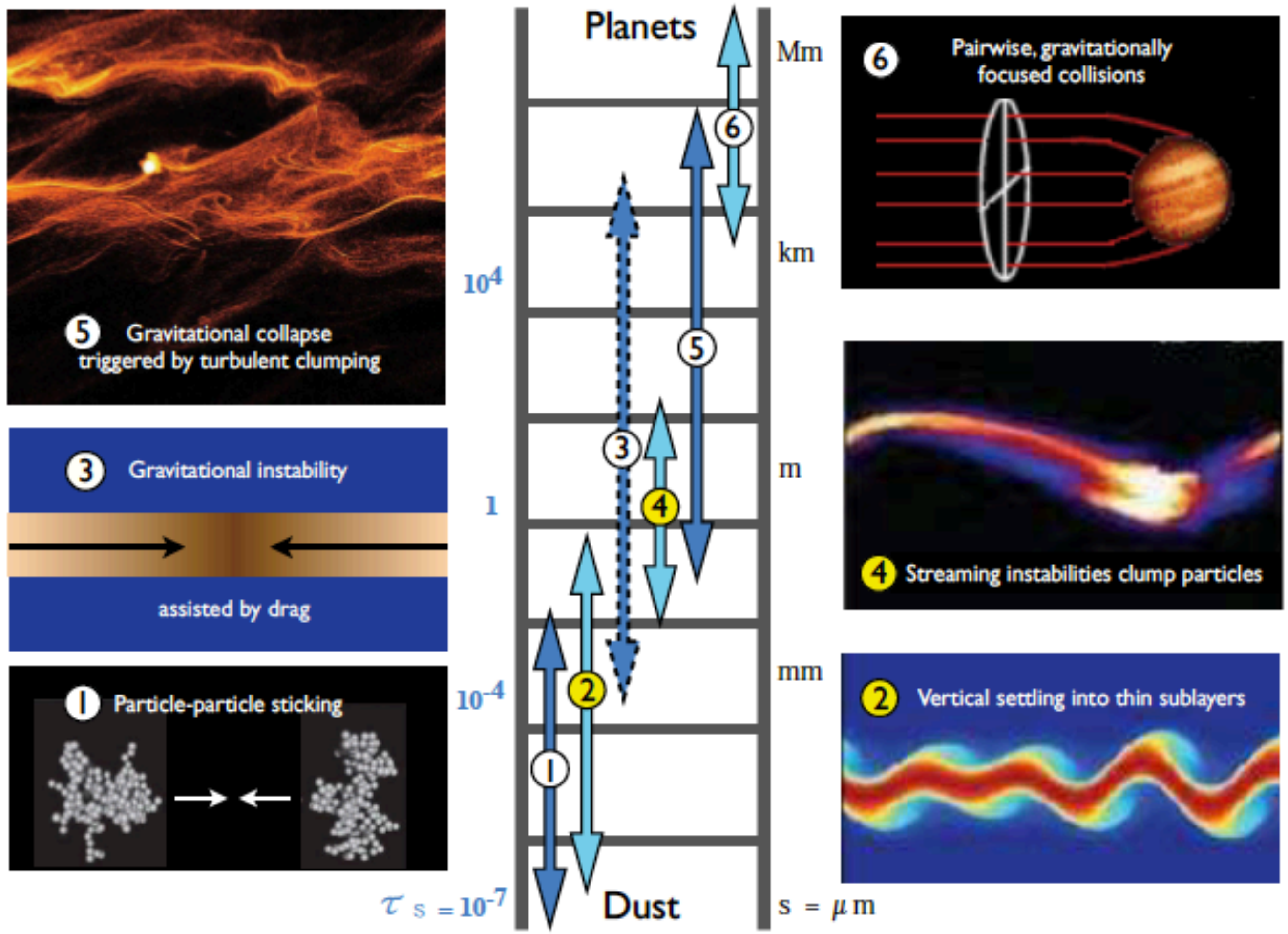
Motivation: planet formation

As we know, planets form from dust in protoplanetary disks. It requires growth in size by many orders of magnitude, and different mechanisms operate in different size regimes.

Size of block	Formation mechanism
Dust (< cm) → "Rocks"(meter)	Dusty particles collide and stick together due to molecular forces
"Rocks"(meter) → Planetesimals (10 km)	??????
Planetesimals (10 km) → Planets (>1000 km)	Blocks collide and merge due to gravitational attraction
PROFIT!	

Formation of planetesimals is not straightforward. To understand it, we need to carefully consider collective gravitational effects of particles and interaction between dust and gas.

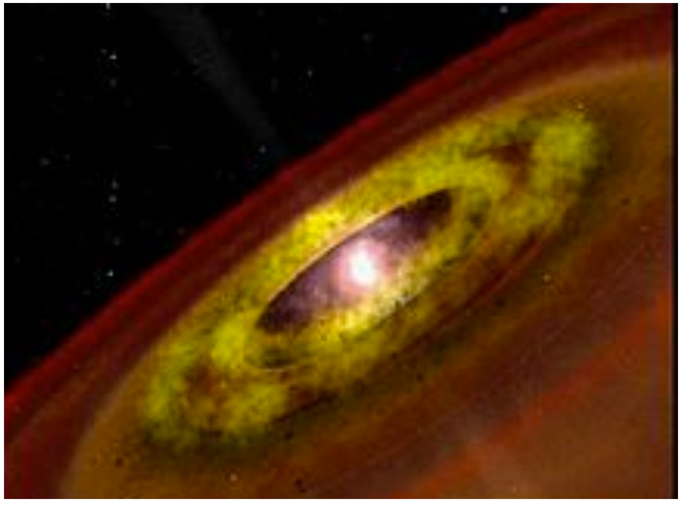
Motivation: planet formation



(Chiang & Youdin 2010)

Motivation: disks around young stars

- Thanks to recent developments in infrared astronomy, we can now directly study protoplanetary disks around T Tauri stars, Herbig Ae/Be stars and even Brown Dwarfs.
- The task of the theory is to model the evolution of dust and gas in the disk, compute spectra and compare them with the observations.



Aerodynamic drag laws

Spherical particle of radius a, moving through the gas with velocity v, experiences an aerodynamic drag force:

$$F_D = -\frac{1}{2} C_D \pi a^2 \rho v^2$$

$$C_D = \frac{8}{3}\frac{\bar{v}}{v}, \quad a < \frac{9}{4}\lambda \quad \text{(Epstein)} \quad \text{ind. molecules;}$$

$$C_D = 24 Re^{-1}, \quad Re < 1 \quad \text{(Stokes)} \quad \sim v;$$

$$C_D = 24 Re^{-0.6}, \quad 1 < Re < 800;$$

$$C_D = 0.44, \quad Re > 800 \quad \sim v^2.$$

Radial drift: basic relations

Due to pressure gradient azimuthal velocity of gas deviates from Keplerian velocity. This difference is small (typically <1%), but important.

$$\frac{v_{\varphi,gas}^2}{r} = \frac{GM_\star}{r^2} + \frac{1}{\rho}\frac{dP}{d\rho},$$

$$P = P_0 \left(\frac{r}{r_0}\right)^{-n}, \quad P_0 = \rho_0 c_s^2,$$

$$v_{\varphi,gas} = v_K (1-\eta)^{1/2}, \quad \eta = n\frac{c_s}{v_K^2}.$$

Radial drift: equations of motion

Let's define friction time scale as

$$t_{fr} = \frac{mv}{|F_D|}$$

Equations of motion for the dust particle are

$$\frac{dv_r}{dt} = \frac{v_\varphi^2}{r} - \Omega_K^2 r - \frac{1}{t_{fr}}(v_r - v_{r,gas}),$$

$$\frac{d}{dt}(rv_\varphi) = -\frac{r}{t_{fr}}(v_\varphi - v_{\varphi,gas}).$$

Radial drift: final result

After some approximations and algebra, we obtain the formula for radial drift:

$$\frac{v_r}{v_K} = \frac{-\eta}{\tau_{fr} + \tau_{fr}^{-1}}, \quad \tau_{fr} = t_{fr}\Omega_K.$$

Two limiting cases:

1. Small particles $a < \mathrm{cm},\ \tau_{fr} \ll 1$

They are well coupled to the gas and moving with almost gas velocity. But they don't experience pressure gradient and that's causes radial drift.

2. Rocks $a > \mathrm{m},\ \tau_{fr} \gg 1$

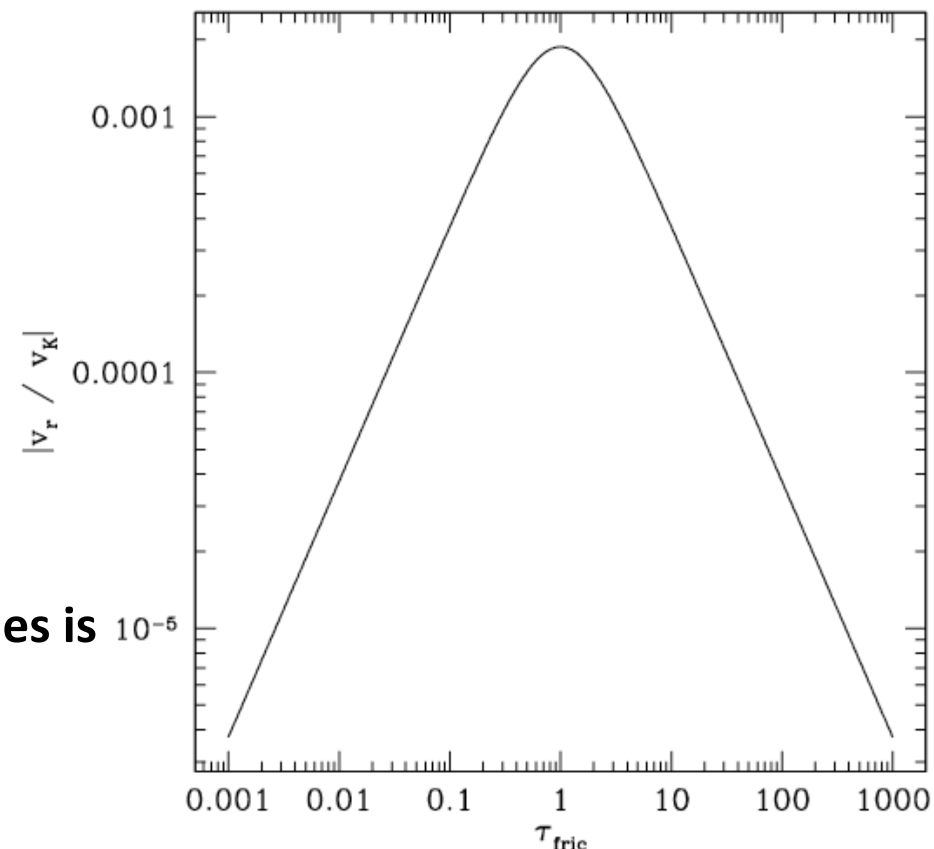
They are moving with almost Keplerian velocity and thus experience headwind from the gas, which saps their angular momentum.

Radial drift: conclusion

Maximum drift is obtained for grains of intermediate size

$$10 \text{ cm} < a < 1 \text{ m}, \ \tau_{fr} = 1$$

Assuming h/r = 0.05, at a distance 1 AU we get an estimate for the drifting time of 100 years.



Radial drift time for meter-scale bodies is much less than disk lifetime.

Planetesimal formation must be a rapid process!

(Armitage 2007)

Vertical settling: simple estimate

Consider vertically isothermal thin disk with surface mass density Σ and scale height h. The density profile is

$$\rho(z) = \frac{\Sigma}{h\sqrt{2\pi}} e^{-z^2/2h^2}$$

In the absence of turbulence vertical settling velocity is determined from the balance between vertical component of gravity and the drag force. In Epstein regime we have

$$m\Omega_K^2 z = \frac{4}{3}\pi a^2 \bar{v}\rho v \qquad\qquad v_{set} = \frac{\Omega_K^2}{\bar{v}}\frac{\rho_d}{\rho} a z$$

$$t_{set} = \frac{z}{v_{set}} = \frac{2}{\pi}\frac{\Sigma}{\Omega_K \rho_d a} e^{-z^2/2h^2}$$

For example, for micron sized particles at z=h at 1 AU the settling time scale is about 2×10^5 years, which is order of magnitude less than typical disk lifetime.

For sizes <0.1 micron these timescales are comparable.

Vertical settling: turbulent stirring

In reality, the disk is turbulent, and turbulence will stir up the settling particles. To make the simplest estimate of this effect, let's assume vertical diffusion coefficient to be equal to turbulent viscosity:

$$\nu_d = \nu = \alpha c_s h$$

To estimate the resulting thickness of the disk, let's equate the stirring time scale with the settling time scale:

$$t_{stir} \equiv \frac{z^2}{\nu_d} \approx t_{set} \qquad \frac{h^2}{z^2} e^{-z^2/h^2} = \frac{\pi \rho_d a}{2\alpha\Sigma}$$

For example, for the disk with $\alpha = 10^{-2},\ \Sigma = 10^3\ \mathrm{g/cm^2}$
and micron sized particles we have z≈3h. On the other hand, cm-sized particles will settle to z≈h even if the disk is fully turbulent.

Vertical settling: coagulation

Coagulation of particles can also significantly affect the vertical settling. Consider a particle moving through the dust background and accreting all encountered particles ("rain drop model").

The equations of motion are

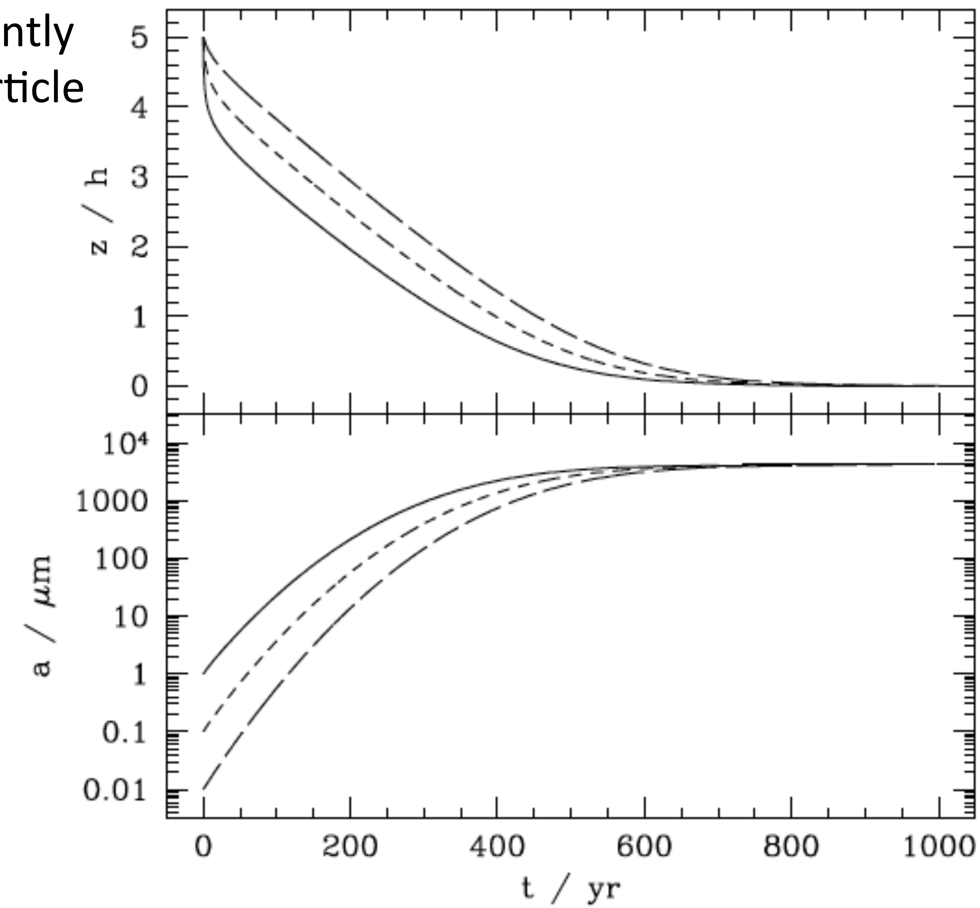
$$\frac{dm}{dt} = \pi a^2 |v_{set}| f \rho(z) = \frac{3}{4} \frac{\Omega^2 f}{\bar{v}} z m$$

$$\frac{dz}{dt} = -\frac{\rho_d}{\rho} \frac{a}{\bar{v}} \Omega^2 z$$

Here f is dust to gas ratio.

In the figure are results for the 0.01, 0.1 and 1 micron particles.

Coagulation can speed up the settling by two orders of magnitude!



(Armitage 2007)

Self-consistent modeling of coagulation (Dullemond, Dominik 2005)

To model the coagulation and settling processes self-consistently, we need to solve the coagulation equation and settling/mixing equation:

$$\frac{\partial f(m)}{\partial t}|_{coag} = \int_0^{m/2} f(m')f(m-m')\sigma(m',m-m')\Delta v(m',m-m')dm' -$$

$$-\int_0^{\infty} f(m')f(m)\sigma(m',m)\Delta v(m',m)dm',$$

$$\frac{\partial f(m)}{\partial t}|_{set} = -\frac{\partial(fv_{set})}{\partial z} + \frac{\partial}{\partial z}\left(D(z)\rho\frac{\partial(f/\rho)}{\partial z}\right).$$

Here f(r,z,m,t) is the distribution function of dusty particles, σ(m',m) is the collisional cross-section of particles m and m', Δv(m',m) is their average relative velocity, D is the turbulent diffusion coefficient.

Self-consistent modeling of coagulation (Dullemond, Dominik 2005)

Three processes leading to relative motion were taken into account: Brownian motion, differential settling and turbulence.

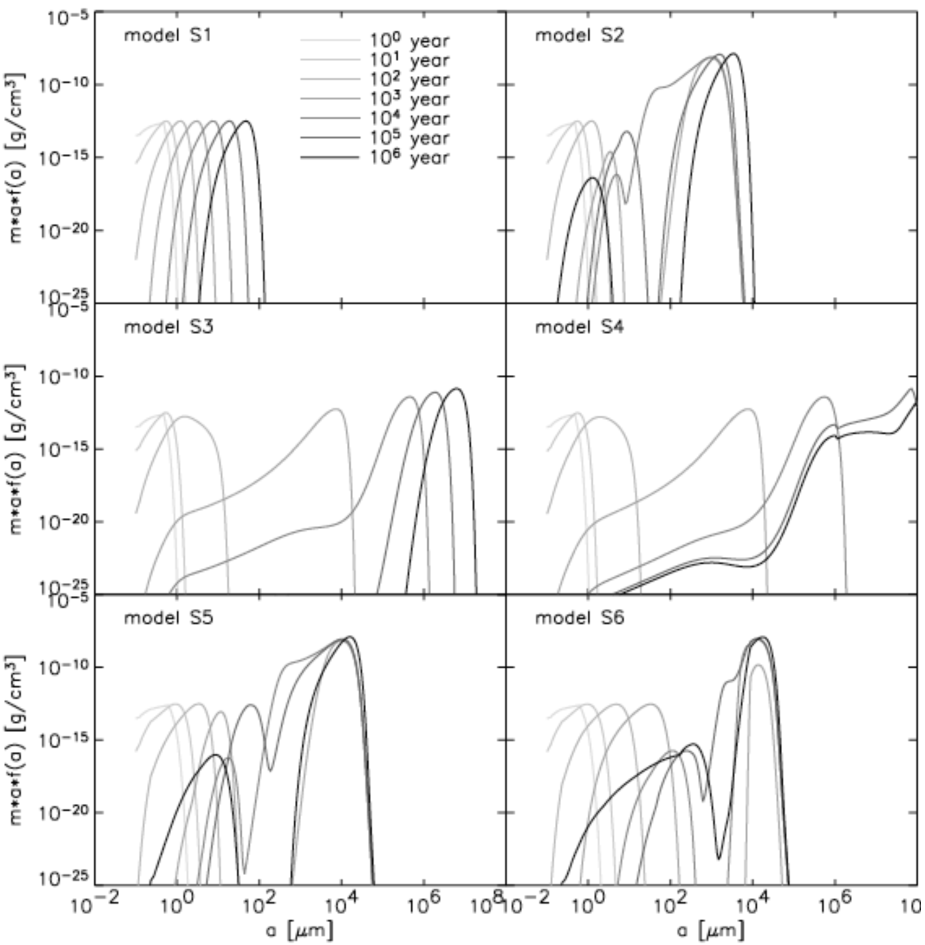
$$\Delta v_b(m_1, m_2) = \sqrt{\frac{8kT(m_1 + m_2)}{\pi m_1 m_2}},$$

$$\Delta v_s(m_1, m_2) = |v_{set}(m_1) - v_{set}(m_2)|.$$

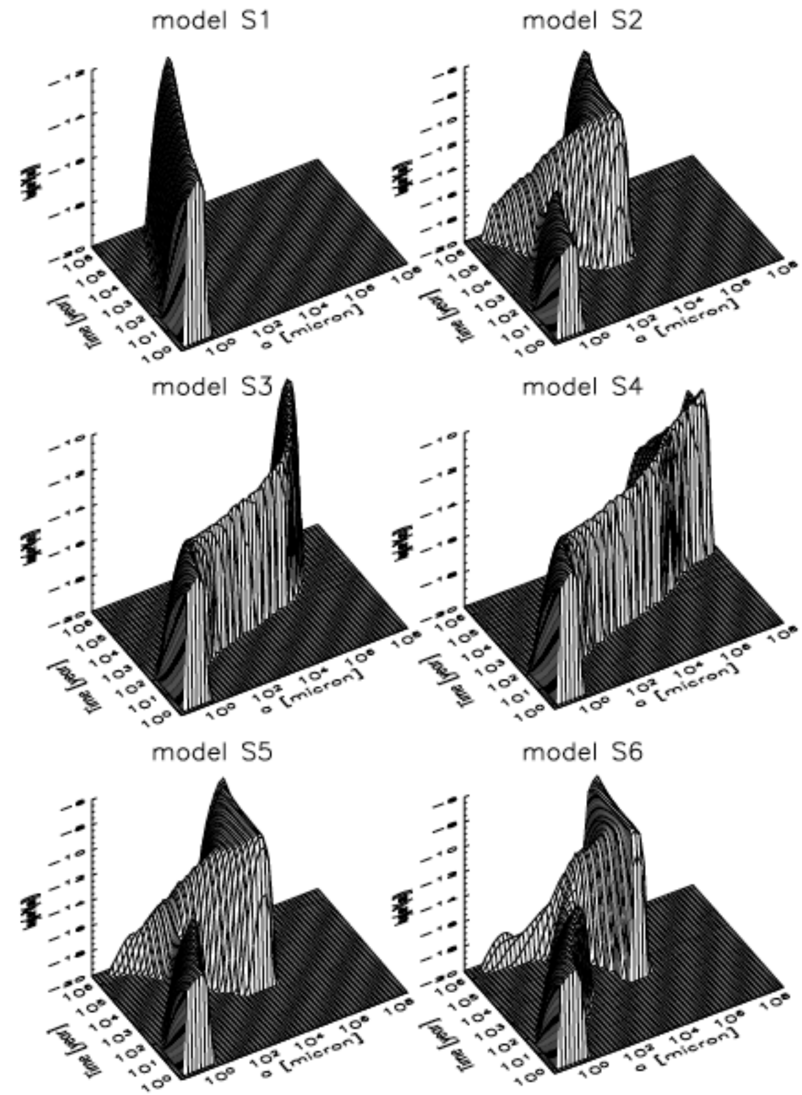
The expression for turbulent velocity is complicated and I don't show it here.

The coagulation and settling equations were solved simultaneously using operator splitting technique.

Self-consistent modeling of coagulation (Dullemond, Dominik 2005)



(Dullemond & Dominik 2005)



Self-consistent modeling of coagulation (Dullemond, Dominik 2005)

- Model S1: Brownian motion only. There is one peak in the distribution, which slowly moves toward larger sizes.
- Model S2: Brownian + dif. settling. Initial growth is due to Brownian motion, but at t = 500 years there is a “rain shower” of mm-sizes particles because of “rain drop effect”.
- Model S3: Brownian + settling + turbulent mixing. Rained down grains can be stirred up from the midplane and rain down again, growing in size even more.
- Model S4: Brownian + settling + mixing + coagulation by turbulence. Not very different from the previous case.
- Models S5 and S6: playing with different cross-sections.

Conclusion:

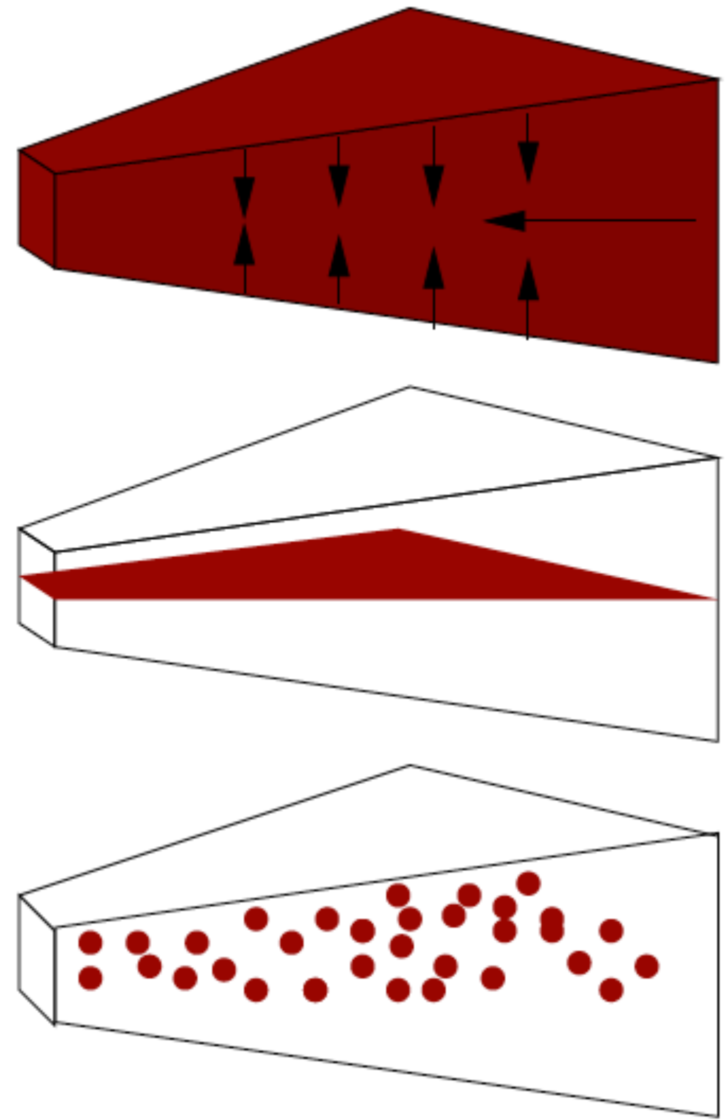
The coagulation in this model of protoplanetary disk proceeds too fast, the time scale is two to three magnitudes too short to be consistent with observations of T Tauri star disks. We need to include some processes in the model, which replenish small grains, for example fragmentation by collisions.

Goldreich-Ward mechanism

Possible resolution to the problem

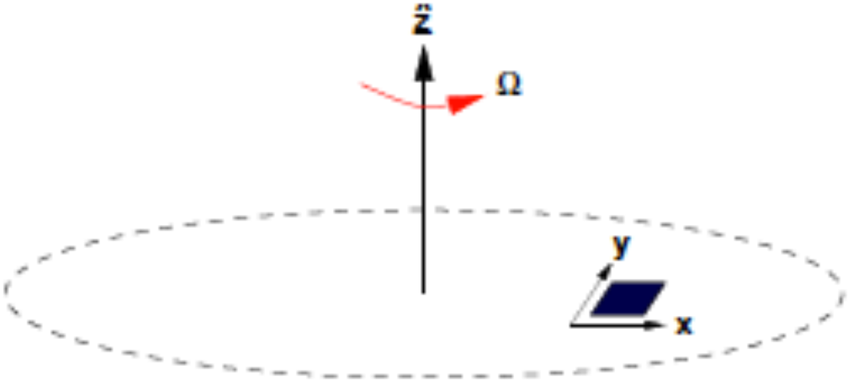
of planetesimal formation –
gravitational fragmentation of a dense particle sub-disk near the midplane of the gas disk (Goldreich, Ward 1973).

This mechanism can help grains successfully pass the dangerous zone of intermediate sizes (0.1-10 m). However, there are several problems with this idea in its original form, which we'll discuss later.



GW mechanism: classical analysis

For simplicity, let's consider rotating thin fluid sheet, with constant surface density and angular velocity.
Equations of motion (in corotating frame):



$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \vec{v}) = 0,$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla p}{\Sigma} - \nabla \Phi -$$

$$-2\vec{\Omega} \times \vec{v} + \Omega^2 (x\vec{e}_x + y\vec{e}_y),$$

$$\nabla^2 \Phi = 4\pi G \Sigma \delta(z).$$

Here p = p(Σ) is vertically integrated pressure, and we define sound speed as

$$c_s^2 = \frac{dp}{d\Sigma}$$

GW mechanism: classical analysis

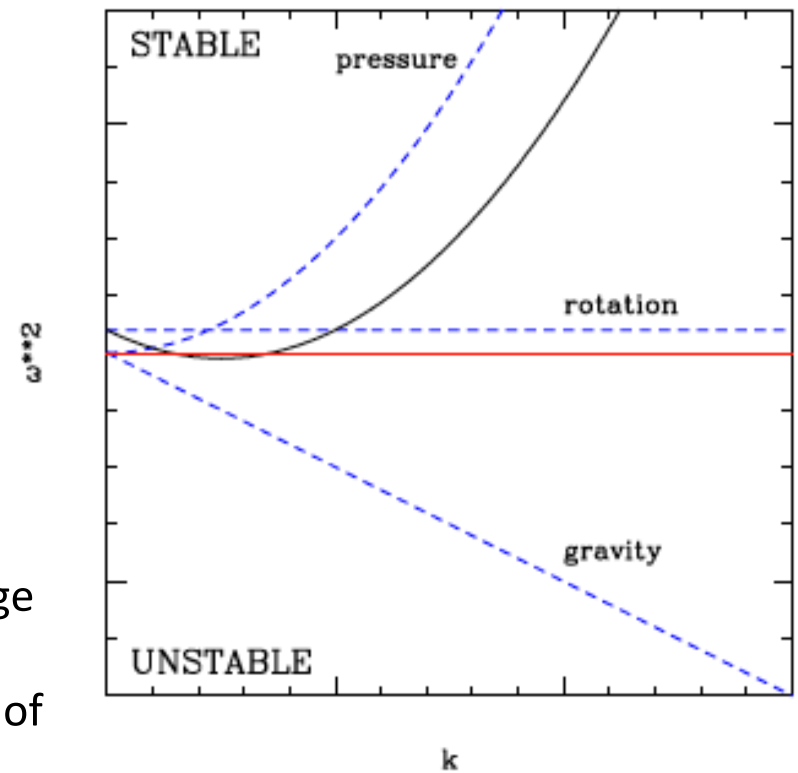
After introducing linear perturbations and decomposing them in Fourier modes, we obtain the dispersion relation:

$$\omega^2 = c_s^2 k^2 - 2\pi G \Sigma_0 |k| + 4\Omega^2$$

The disk is stable if

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} > Q_{crit} = 1/2$$

Differential rotation and global effects can only change critical value of Q. For the disk of solid particles, the expression contains the velocity dispersion σ instead of speed of sound.



(Armitage 2007)

GW mechanism: problems

Let's apply this stability criterion for the minimum mass Solar Nebula model with $\Sigma_{dust} \approx 10^{-2}\Sigma_{gas} \approx 10 \text{ g/cm}^2$ at 1 AU. We obtain for the dust velocity dispersion

$$\sigma \approx \frac{\pi G \Sigma_{dust}}{\Omega} \approx 10 \text{ cm/s}$$

This leads to the ratio of the scale heights for the dust and the gas:

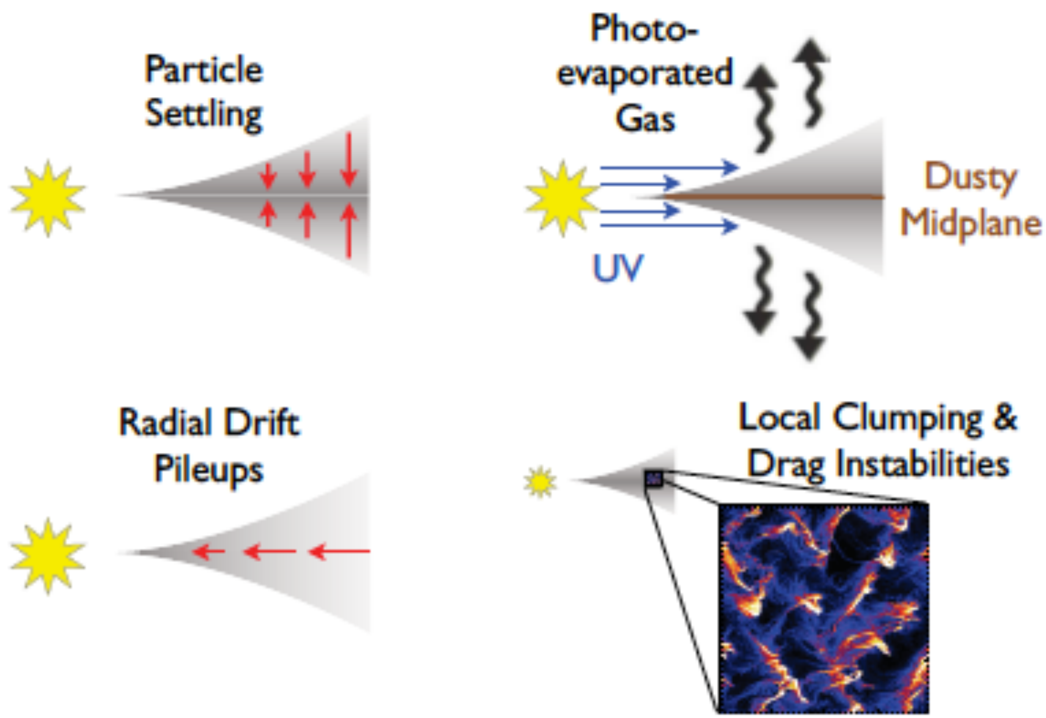
$$\frac{h_{dust}}{h_{gas}} = \frac{\sigma}{c_s} \sim 10^{-4}$$

The dusty sub-disk must be extremely thin!

But as we learned previously, turbulence will likely stir up the particles from the midplane.

Even if the gaseous disk is non-turbulent, the shear between solid sub-disk and the gas can give rise to **Kelvin-Helmholtz instability**.

Other ways to gravitational instability



(Chiang & Youdin 2010)

To initiate gravitational collapse, we need to provide high local concentration of dust. This can obtained by different ways.

Want to learn more about turbulent clumping and drag instabilities? Listen to Chris!

Summary

- Due to aerodynamic drag force, solid particles in protoplanetary disks are drifting inward. The maximum drift speed is for the particles of intermediate size (0.1 – 10 m), and corresponds to falling time of 100 years. This poses a problem for planetesimal formation.
- Small dusty particles in the disk interact with gas, collide and coagulate. But all these processes are not enough to explain the observed properties of disk around young stars. Some mechanism is needed to recreate small grains, e.g. fragmentation by collisions.
- One possible method to form planetesimals – gravitational collapse of dense solid sub-disk (Goldreich-Ward mechanism). But it is problematic to provide sufficiently thin and dense dusty sub-disk. However, we can also trigger gravitational collapse by local clumping of dust.

References

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